Backpropagation - Steg för Steg

Nätverksarkitektur

```
[3, 2, 3] = 3 inputs → 2 hidden neurons → 3 output neurons

        Input Layer          Hidden Layer         Output Layer

           x₀ ──────────────┐
                            ├── Neuron 1 ──┬──── Neuron 3 (y₀)
                            │              │
           x₁ ──────────────┼── Neuron 2 ──┼──── Neuron 4 (y₁)
                            │              │
           x₂ ──────────────┘              └──── Neuron 5 (y₂)
```

Träningsexempel

	Värde
Input	[0.0000, 0.4492, 0.1630]
Target	[0, 1, 0]
Learning rate	0.2

STEG 1: Forward Pass

Lager 1 (Hidden)

Neuron 1

Input	Vikt	Input × Vikt
0.0000	0.1394	0.0000
0.4492	-0.4750	-0.2134
0.1630	-0.2250	-0.0367
	Summa:	**-0.2500**

	Värde
Bias	-0.2768
Total (Summa + Bias)	-0.5268
Output = sigmoid(-0.5268)	**0.3713**

Neuron 2

Input	Vikt	Input × Vikt
0.0000	0.2365	0.0000
0.4492	0.1767	0.0794
0.1630	0.3922	0.0639
	Summa:	**0.1433**

	Värde
Bias	-0.4131
Total (Summa + Bias)	-0.2697
Output = sigmoid(-0.2697)	**0.4330**

Lager 2 (Output)

Neuron 3

Input (från Hidden)	Vikt	Input × Vikt
0.3713	-0.0781	-0.0290
0.4330	-0.4702	-0.2036
	Summa:	**-0.2326**

	Värde
Bias	-0.2814
Total (Summa + Bias)	-0.5139
Output = sigmoid(-0.5139)	**0.3743**

Neuron 4

Input (från Hidden)	Vikt	Input × Vikt
0.3713	0.0054	0.0020
0.4330	-0.4735	-0.2050
	Summa:	**-0.2030**

	Värde
Bias	-0.3012
Total (Summa + Bias)	-0.5042

	Värde
Output = sigmoid(-0.5042)	**0.3766**

Neuron 5

Input (från Hidden)	Vikt	Input × Vikt
0.3713	0.1499	0.0556
0.4330	0.0449	0.0195
	Summa:	**0.0751**

	Värde
Bias	-0.2796
Total (Summa + Bias)	-0.2045
Output = sigmoid(-0.2045)	**0.4491**

STEG 2: Backward Pass (Felberäkning)

Output-lagret (Lager 2)

Formel: `Delta = (Target − Output) × Output × (1 − Output)`

Neuron 3

Steg	Formel	Värde
Target		0
Output		0.3743
Error	Target - Output	-0.3743
Sigmoid derivata	Output × (1 - Output)	0.2342
Delta	Error × Sigmoid derivata	**-0.0877**

Neuron 4

Steg	Formel	Värde
Target		1
Output		0.3766
Error	Target - Output	0.6234
Sigmoid derivata	Output × (1 - Output)	0.2348

Steg	Formel	Värde
Delta	Error × Sigmoid derivata	**0.1464**

Neuron 5

Steg	Formel	Värde
Target		0
Output		0.4491
Error	Target - Output	-0.4491
Sigmoid derivata	Output × (1 - Output)	0.2474
Delta	Error × Sigmoid derivata	**-0.1111**

Hidden-lagret (Lager 1)

Formel: `Delta = (Σ Delta_next × Vikt_till_next) × Output × (1 − Output)`

Neuron 1

Steg 1: Beräkna error contribution från output-lagret

Nästa Neuron	Delta	Vikt till Neuron 1	Bidrag
Neuron 3	-0.0877	-0.0781	0.0068
Neuron 4	0.1464	0.0054	0.0008
Neuron 5	-0.1111	0.1499	-0.0167
		Summa:	**-0.0090**

Steg 2: Beräkna delta

Steg	Formel	Värde
Output		0.3713
Error contribution	Summa från ovan	-0.0090
Sigmoid derivata	Output × (1 - Output)	0.2334
Delta	Error contrib × Sigmoid derivata	**-0.0021**

Neuron 2

Steg 1: Beräkna error contribution från output-lagret

Nästa Neuron	Delta	Vikt till Neuron 2	Bidrag

Nästa Neuron	Delta	Vikt till Neuron 2	Bidrag
Neuron 3	-0.0877	-0.4702	0.0412
Neuron 4	0.1464	-0.4735	-0.0693
Neuron 5	-0.1111	0.0449	-0.0050
		Summa:	**-0.0331**

Steg 2: Beräkna delta

Steg	Formel	Värde
Output		0.4330
Error contribution	Summa från ovan	-0.0331
Sigmoid derivata	Output × (1 - Output)	0.2455
Delta	Error contrib × Sigmoid derivata	**-0.0081**

STEG 3: Viktuppdatering

Formel: `Ny vikt = Gammal vikt + Learning rate × Delta × Input`

Learning rate = 0.2

Lager 1 (Hidden)

Neuron 1 (δ = -0.0021)

Vikt	Gammal	Input	Ändring	Ny
w_0	0.1394	0.0000	0.2 × -0.0021 × 0.0000 = -0.0000	0.1394
w_1	-0.4750	0.4492	0.2 × -0.0021 × 0.4492 = -0.0002	-0.4752
w_2	-0.2250	0.1630	0.2 × -0.0021 × 0.1630 = -0.0001	-0.2250
bias	-0.2768	1	0.2 × -0.0021 = -0.0004	-0.2772

Neuron 2 (δ = -0.0081)

Vikt	Gammal	Input	Ändring	Ny
w_0	0.2365	0.0000	0.2 × -0.0081 × 0.0000 = -0.0000	0.2365
w_1	0.1767	0.4492	0.2 × -0.0081 × 0.4492 = -0.0007	0.1760
w_2	0.3922	0.1630	0.2 × -0.0081 × 0.1630 = -0.0003	0.3919
bias	-0.4131	1	0.2 × -0.0081 = -0.0016	-0.4147

Lager 2 (Output)

Neuron 3 (δ = -0.0877)

Vikt	Gammal	Input (från Hidden)	Ändring	Ny
w_0	-0.0781	0.3713	0.2 × -0.0877 × 0.3713 = -0.0065	-0.0846
w_1	-0.4702	0.4330	0.2 × -0.0877 × 0.4330 = -0.0076	-0.4778
bias	-0.2814	1	0.2 × -0.0877 = -0.0175	-0.2989

Neuron 4 (δ = 0.1464)

Vikt	Gammal	Input (från Hidden)	Ändring	Ny
w_0	0.0054	0.3713	0.2 × 0.1464 × 0.3713 = 0.0109	0.0162
w_1	-0.4735	0.4330	0.2 × 0.1464 × 0.4330 = 0.0127	-0.4608
bias	-0.3012	1	0.2 × 0.1464 = 0.0293	-0.2719

Neuron 5 (δ = -0.1111)

Vikt	Gammal	Input (från Hidden)	Ändring	Ny
w_0	0.1499	0.3713	0.2 × -0.1111 × 0.3713 = -0.0082	0.1416
w_1	0.0449	0.4330	0.2 × -0.1111 × 0.4330 = -0.0096	0.0353
bias	-0.2796	1	0.2 × -0.1111 = -0.0222	-0.3018

Sammanfattning

Neuron	Output (före)	Delta	Funktion
1	0.3713	-0.0021	Hidden
2	0.4330	-0.0081	Hidden
3	0.3743	-0.0877	Output (y_0)
4	0.3766	0.1464	Output (y_1)
5	0.4491	-0.1111	Output (y_2)