b) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & b \\ 1 & a & 1 \end{bmatrix}$

A^{-1} must exist

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & b \\ 0 & a & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & b \\ 0 & 0 & 1+ab \end{bmatrix}$

$ab \neq -1$

c) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & -2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & -1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & -1 & 0 \\ 0 & 0 & -1 & -1 & 1 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 2 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{array}\right]$

8a $A = \begin{bmatrix} 2/3 & 0 & 1/3 \\ 1/10 & 2/3 & 0 \\ 0 & 1/3 & 2/3 \end{bmatrix}$

not conservative b/c columns don't add to 1

b) $\det\left(\begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & \frac{1}{2}-\lambda & 1 \\ 0 & 1 & \frac{1}{2}-\lambda \end{bmatrix}\right) = 1-\lambda\left(\frac{1}{4}-\lambda+\lambda^2-1\right)$

$= 1-\lambda\left(\lambda^2-\lambda-\frac{3}{4}\right)$

$= (1-\lambda)\left(\lambda+\frac{1}{2}\right)\left(\lambda-\frac{3}{2}\right)$

$\lambda = 1$:

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 1 \\ 0 & 1 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\lambda = -\frac{1}{2}$:

$\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\lambda = \frac{3}{2}$

$\begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

c) i. $\operatorname{span}\left(\left\{\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}\right\}\right)$

is subspace of $\mathbb{R}^3$

ii $\operatorname{span}\left(\left\{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}\right\}\right)$

Not subspace of $\mathbb{R}^3$

5b. $\gamma = \begin{bmatrix} \frac{3}{5} \\ \frac{4}{5} \end{bmatrix}$

$a = \frac{3}{5}$

$b = \frac{4}{5}$

$|a|^2 = \frac{9}{25}$, $|b|^2 = \frac{16}{25}$

b is more likely

c. $\gamma^T S_z \gamma$

$= \frac{1}{5}\gamma^T \begin{bmatrix} \frac{h}{2} & 0 \\ 0 & -\frac{h}{2} \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$= \frac{1}{5}\gamma^T \begin{bmatrix} \frac{3h}{2} \\ -2h \end{bmatrix}$

$= \frac{1}{25}\begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} \frac{3h}{2} \\ -2h \end{bmatrix}$

$= \frac{1}{25}\left(\frac{9}{2}h - 8h\right)$

$= -\frac{7h}{50}$

d. $H\vec{\Psi}_1 = e\vec{\Psi}_1$

$H\vec{\Psi}_2 = e\vec{\Psi}_2$

$H\alpha\vec{\Psi}_1 = e\alpha\vec{\Psi}_1$

$H\beta\vec{\Psi}_2 = e\beta\vec{\Psi}_2$

$H(\alpha\vec{\Psi}_1 + \beta\vec{\Psi}_2) = e(\alpha\vec{\Psi}_1 + \beta\vec{\Psi}_2)$

6a) $\begin{bmatrix} \frac{3}{4} & 0 \\ \frac{3}{5} & \frac{3}{4} \end{bmatrix}$

b) $\begin{bmatrix} 6/7 & \frac{4}{5} \\ 0 & 1 \end{bmatrix}$

c) $C = BA$

$= \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{5} & 0 \\ \frac{7}{10} & \frac{3}{4} \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{15} + \frac{7}{30} & \frac{1}{4} \\ \frac{7}{10} & \frac{3}{4} \end{bmatrix}$

$= \begin{bmatrix} \frac{3}{10} & \frac{1}{4} \\ \frac{7}{10} & \frac{3}{4} \end{bmatrix}$

d) $\begin{bmatrix} \frac{1}{2} - \lambda & \frac{1}{4} \\ \frac{1}{2} & \frac{3}{4} - \lambda \end{bmatrix}$

$\frac{3}{8} - \frac{5}{4}\lambda + \lambda^2 - \frac{1}{8}$

$(\lambda - 1)(\lambda - \frac{1}{4})$

$\lambda = 1$:

$\begin{bmatrix} -\frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & -\frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\lambda = \frac{1}{4}$:

$\begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

steady state = $\lambda = 1$:

ratio carp to drag = 1:2

$\Rightarrow$ 300 carp

600 drag

7a) A^{-1} must exist:

$\begin{bmatrix} 1 & 0 & t \\ t & 1 & -1 \\ 1 & t & -1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & t \\ 0 & 1 & -t^2-1 \\ 0 & t & -t-1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & t \\ 0 & 1 & t^2+1 \\ 0 & 0 & t^3-1 \end{bmatrix}$

$t^3 - 1 \neq 0$,

$t \neq 1$

2. Worked alone throughout week.

3. a) $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -\ell & 0 & \ell & 0 \\ 0 & \ell & 0 & -\ell \\ k & k & -k & k \end{bmatrix}$

b) if A is invertible, B exists. If A is Linearly Indep, it is invertible:

$A = \begin{bmatrix} 10 & 10 & 10 & 10 \\ -2 & 2 & 2 & -2 \\ 2 & 2 & -2 & -2 \\ -0.1 & 0.1 & -0.1 & 0.1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 1 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

→ B exists.

c) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ -\ell & 0 & \ell & 0 \\ 0 & \ell & 0 & -\ell \\ -k & k & k & -k \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 1 & 1 & -1 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 1 & 1 & -1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ -1 & 1 & 1 & -1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 1 & 1 & -1 \end{bmatrix}$

Since it is linearly dep, B no longer exists and you can not determine the directions of P3 and P4

4. a) i. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ w \\ b \end{bmatrix} = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$

ii. basis of $nul(A) = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0.5 \end{bmatrix} \right\}$

iii. No, because the null space is non-trivial

b) If $UV = \vec{0}$, then

$U \begin{bmatrix} | & | & & | \\ v_1 & v_2 & \cdots & v_n \\ | & | & & | \end{bmatrix} = \vec{0}$

By def, col(V) are in nul(U) as $U\vec{x} = \vec{0}$ implies $\vec{x}$ is in the nullspace.

5a) $S_x - \lambda I = \begin{bmatrix} -\lambda & \frac{h}{2} \\ \frac{h}{2} & -\lambda \end{bmatrix}$

$\det(S_x - \lambda I) = \lambda^2 - \frac{h^2}{4} = 0.$

$\lambda = \pm \frac{h}{2}$

$\lambda = \frac{h}{2}$:

$\begin{bmatrix} -\frac{h}{2} & \frac{h}{2} \\ \frac{h}{2} & -\frac{h}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

vector.

$\lambda = -\frac{h}{2}$: $\begin{bmatrix} \frac{h}{2} & \frac{h}{2} \\ \frac{h}{2} & \frac{h}{2} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

vector

LE HW #5

1a. $A\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}a+b\\c+d\end{bmatrix}$

$= a+b\begin{bmatrix}1\\1\end{bmatrix}$

$= c+d\begin{bmatrix}1\\1\end{bmatrix}$

$\lambda_1 = a+b = c+d$

$\vec{v}_1 = \begin{bmatrix}1\\1\end{bmatrix}$

$A\begin{bmatrix}b\\-c\end{bmatrix} = (a-c)\begin{bmatrix}b\\-c\end{bmatrix}$

$= (d-b)\begin{bmatrix}b\\-c\end{bmatrix}$

$\lambda_2 = a-c = d-b$

$\vec{v}_2 = \begin{bmatrix}b\\-c\end{bmatrix}$

b. $\lambda_1 = 0.75+0.25 = 1$

$\vec{v}_1 = \begin{bmatrix}1\\1\end{bmatrix}$

$\lambda_2 = a-c = 0.5$

$\vec{v}_2 = \begin{bmatrix}1\\-1\end{bmatrix}$

c) if $\vec{s} \in \text{span}\{\begin{bmatrix}1\\1\end{bmatrix}\}$ then it is a steady state.

d) $s[0] \in \text{span}\{\vec{v}_2\}$

$s[1] = \alpha\lambda_2\vec{v}_2$

$s[n] = \alpha\lambda_2^n\vec{v}_2$

if $\lambda_2 = 0.5$, $\lambda_2^n \to 0$

$\Rightarrow s[n] \to \vec{0}$.

R and J will be neutral

e) $\lambda_1 = 1+1 = 2$

$\vec{v}_1 = \begin{bmatrix}1\\1\end{bmatrix}$

$\lambda_2 = 1-1 = 0$

$\vec{v}_2 = \begin{bmatrix}1\\-1\end{bmatrix}$

f) $s[0] \in \text{span}\{\vec{v}_2\}$

$\lambda_2 = 0 \Rightarrow s[1] = \vec{0}$.

$\Rightarrow s[n] = \vec{0}$

g) $s[0] \in \text{span}\{\vec{v}_1\}$

$s[n] = \alpha\lambda_1^n\vec{v}_1$

$= \alpha 2^n \vec{v}_1$

$n \to \infty \Rightarrow s[n] \Rightarrow \pm\infty$.

R and J will have "infinite hatred" or infinite love

Q3 Q1

h) $\lambda_1 = 1-2 = -1$

$\vec{v}_1 = \begin{bmatrix}1\\1\end{bmatrix}$

$\lambda_2 = 1+2 = 3$

$\vec{v}_2 = \begin{bmatrix}1\\-1\end{bmatrix}$

i) $s[0] \in \text{span}\{\vec{v}_2\}$

$s[n] = \alpha 3^n \vec{v}_2$

$= \pm\infty$

~~R and J will have~~

R will have infinite love for J and J will have ∞ hatred for R

OR vice versa

$R[0] > 0$ and $J[0] < 0$

$R[0] < 0$ and $J[0] > 0$

j) $s[0] \in \text{span}\{\vec{v}_1\}$

$s[n] = \alpha(-1)^n\vec{v}_1$

Same intensity, switch b/w love and hate.

They both feel the same way at each step.