HUMAN MOTOR CONTROL

Emmanuel Guigon

Institut des Systèmes Intelligents et de Robotique
Sorbonne Université
CNRS / UMR 7222
Paris, France

emmanuel.guigon@sorbonne-universite.fr

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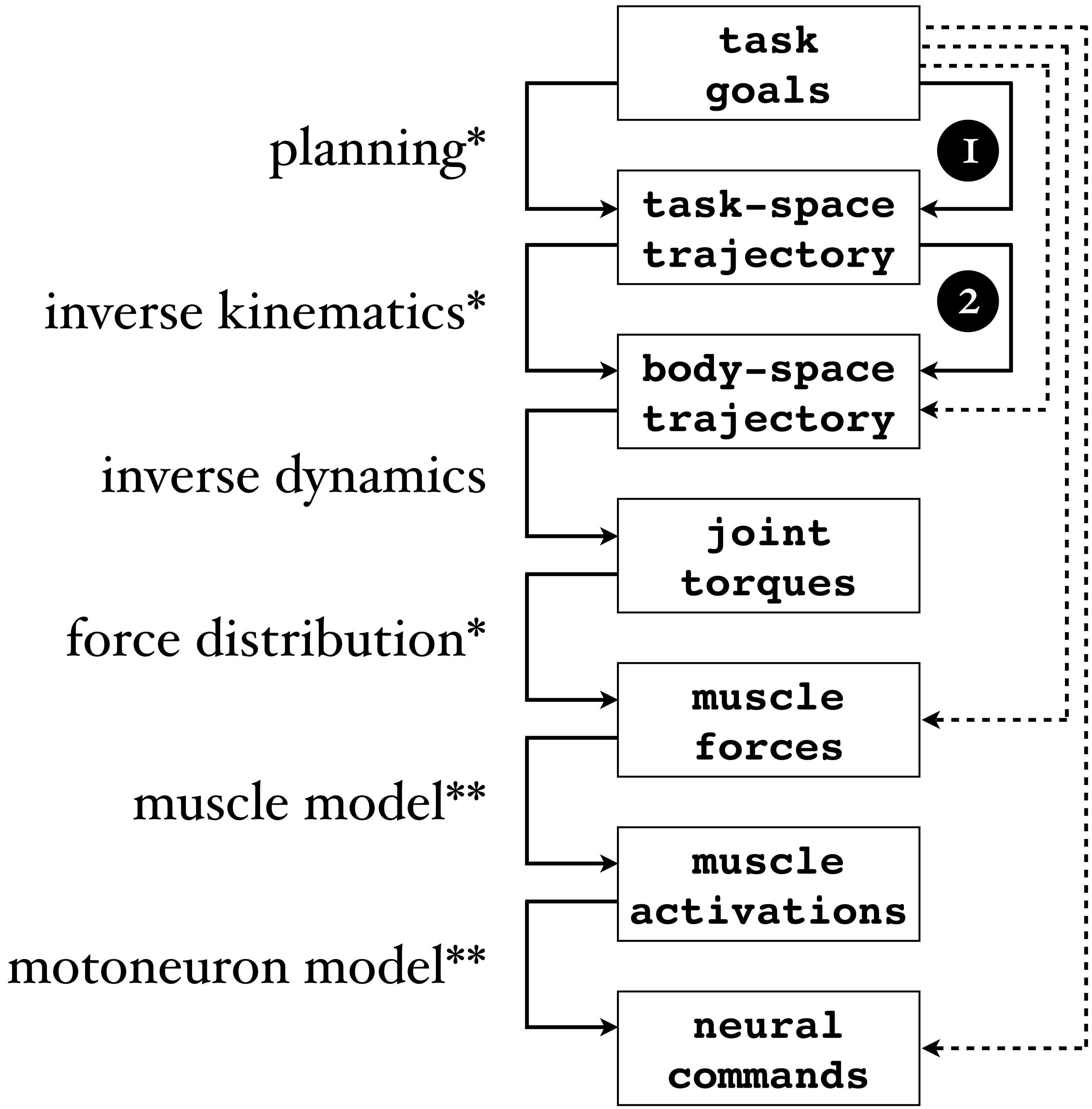
4. Models and theories

PRELIMINARY



1 e.g. minimum-jerk trajectory

2 e.g. pseudo-inverse of the Jacobian

* ill-posed problem, in general

** model-dependent

EVALUATION OF MODELS

- **Degrees-of-freedom problem [dof]**

coordination, redundancy

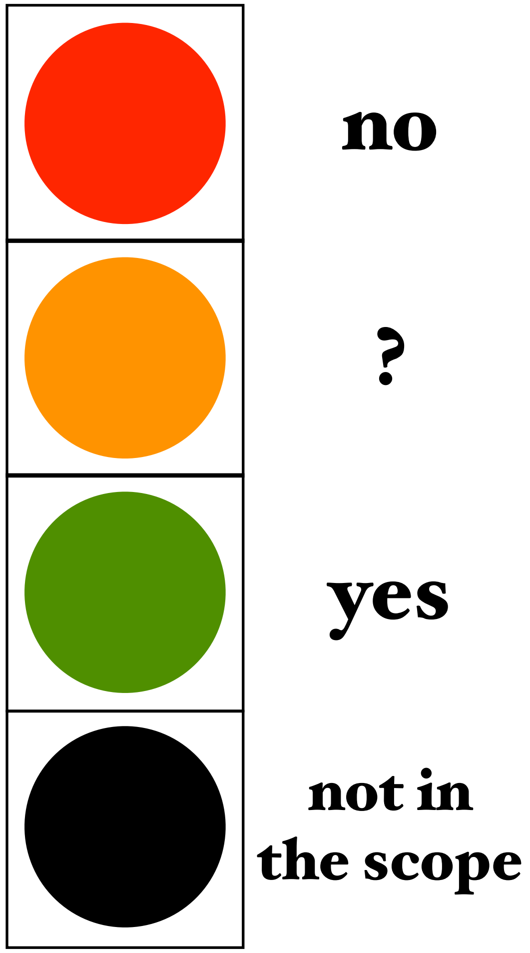
- **Kinematics [kin]**

— trajectories

- **Flexibility in space and time [flex]**

— perturbations

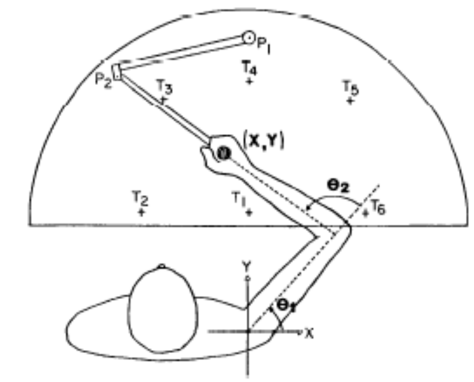
Does the model provide a solution to the problem?

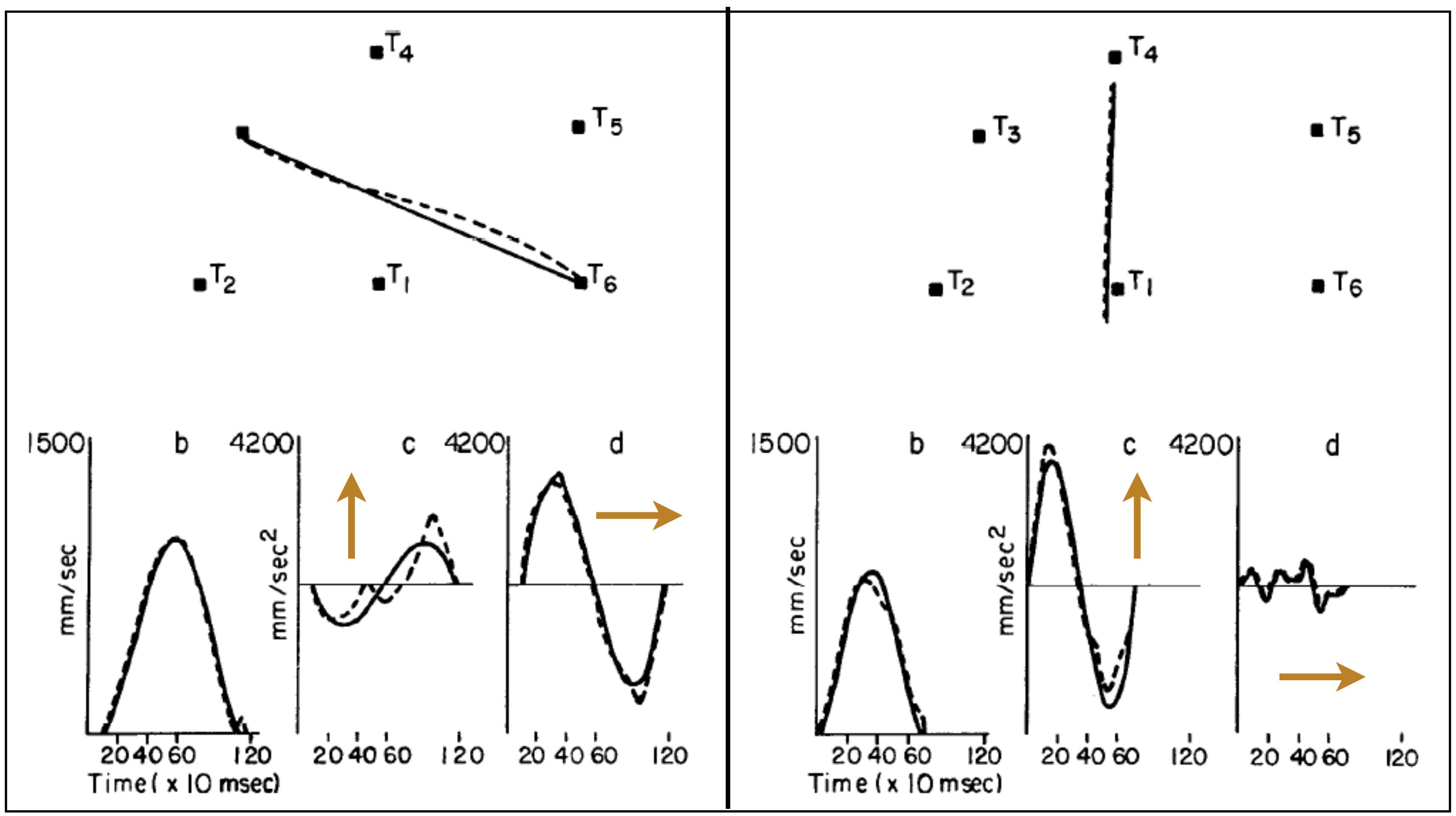


PLANNING: MINIMUM-JERK TRAJECTORY

$$C = \frac{1}{2}\int_0^{t_f}\left(\left(\frac{d^3x}{dt^3}\right)^2 + \left(\frac{d^3y}{dt^3}\right)^2\right)dt$$

$$x(t) = x_0 + (x_0 - x_f)(15\tau^4 - 6\tau^5 - 10\tau^3)$$
$$y(t) = y_0 + (y_0 - y_f)(15\tau^4 - 6\tau^5 - 10\tau^3)$$





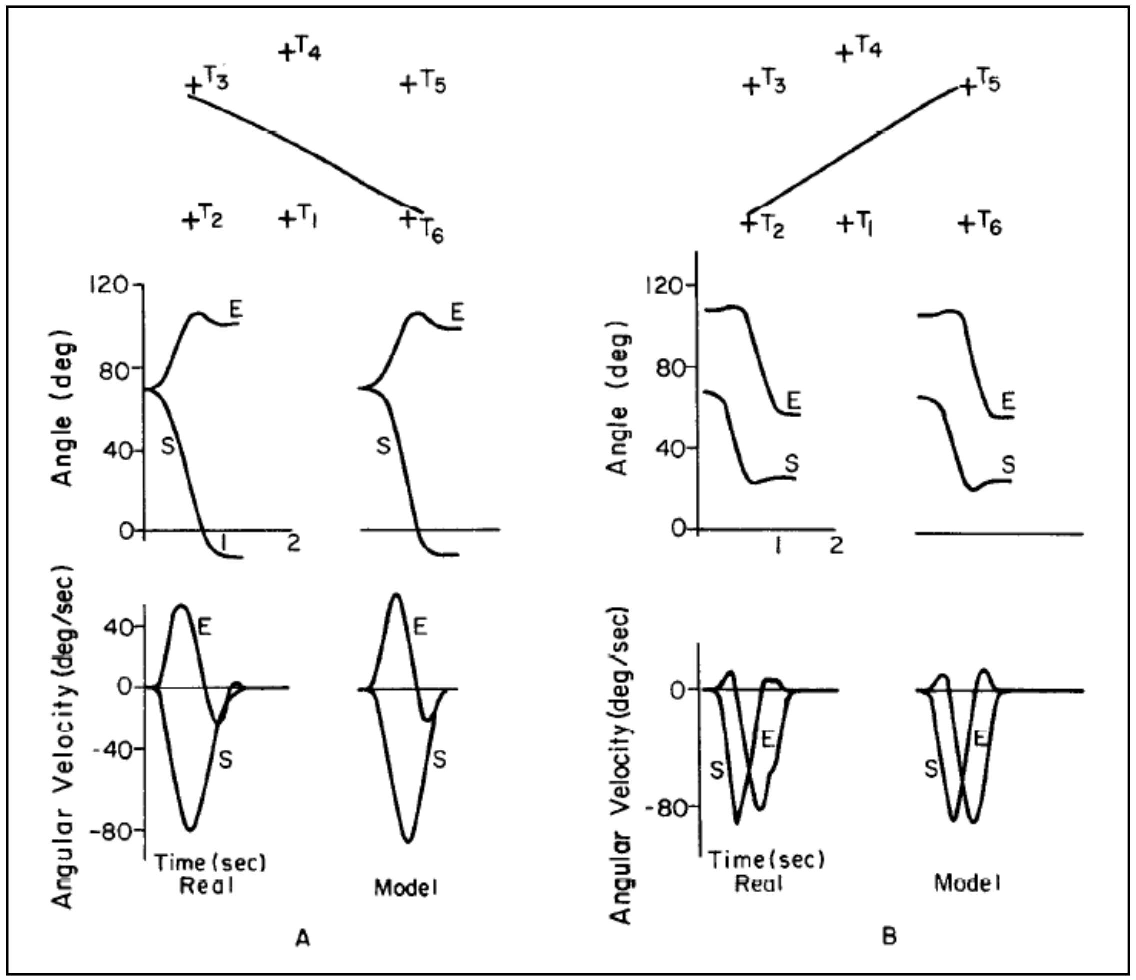
predicted

measured

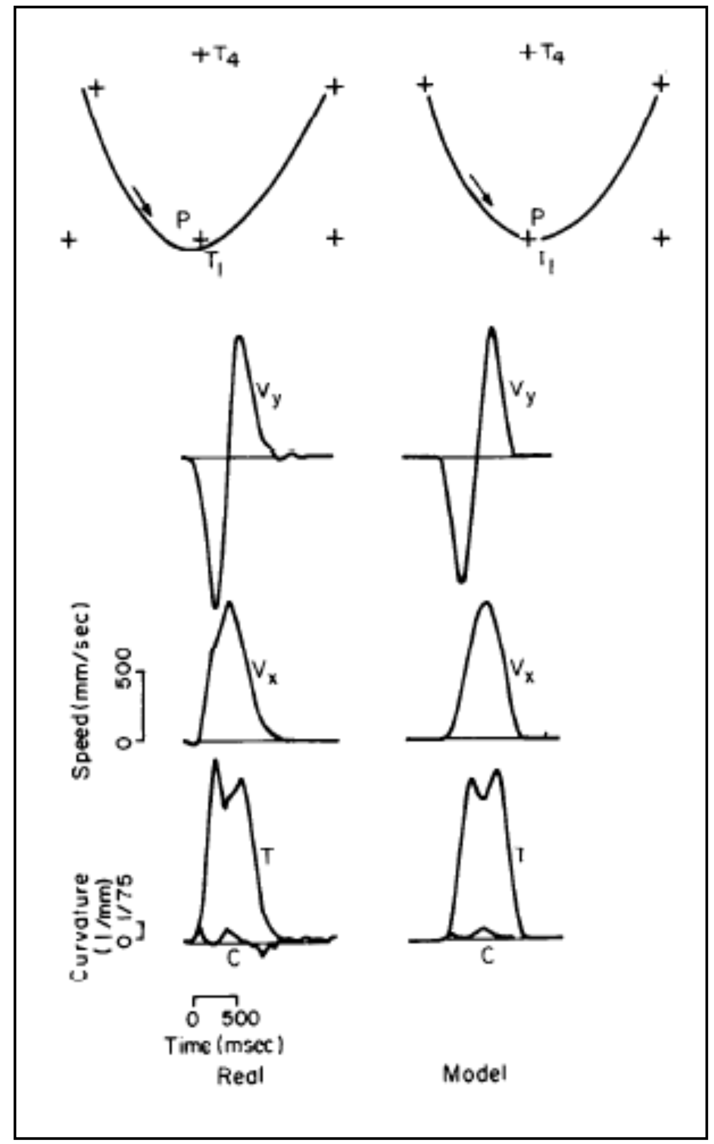
— Flash & Hogan, 1985, *J Neurosci* 5:1688

PLANNING: MINIMUM-JERK TRAJECTORY

measured and predicted
shoulder and elbow angles



curved movements
through via-points



dof

kin

flex

— Flash & Hogan, 1985, *J Neurosci* 5:1688

limitations — trajectories are exactly straight - velocity profiles are symmetric — time is given in advance — only in task space

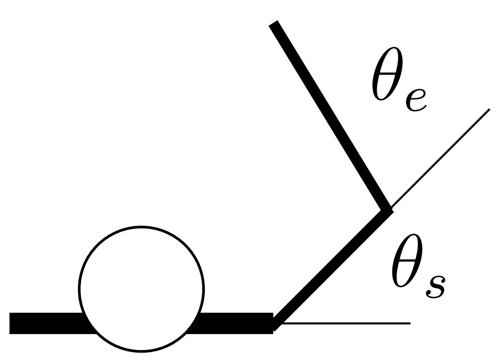
how to execute the plan?

task goals → **task-space trajectory**

INVERSE DYNAMICS

$$\tau_s = (I_s + I_e + m_e l_s l_e \cos\theta_e + \frac{m_s l_s^2 + m_e l_e^2}{4} + m_e l_s^2)\ddot{\theta}_s +$$
$$(I_e + \frac{m_e l_e^2}{4} + \frac{m_e l_s l_e}{2}\cos\theta_e)\ddot{\theta}_e -$$
$$\frac{m_e l_s l_e}{2}\dot{\theta}_e^2 \sin\theta_e - m_e l_s l_e \dot{\theta}_s \dot{\theta}_e \sin\theta_e$$
$$\tau_e = (I_e + \frac{m_e l_s l_e}{2}\cos\theta_e + \frac{m_e l_e^2}{4})\ddot{\theta}_s +$$
$$(I_e + \frac{m_e l_e^2}{4})\ddot{\theta}_e + \frac{m_e l_s l_e}{2}\dot{\theta}_s^2 \sin\theta_e$$

τ_s, τ_e shoulder, elbow torques
m_s, m_e segment masses
l_s, l_e segment lengths
I_s, I_e moments of inertia



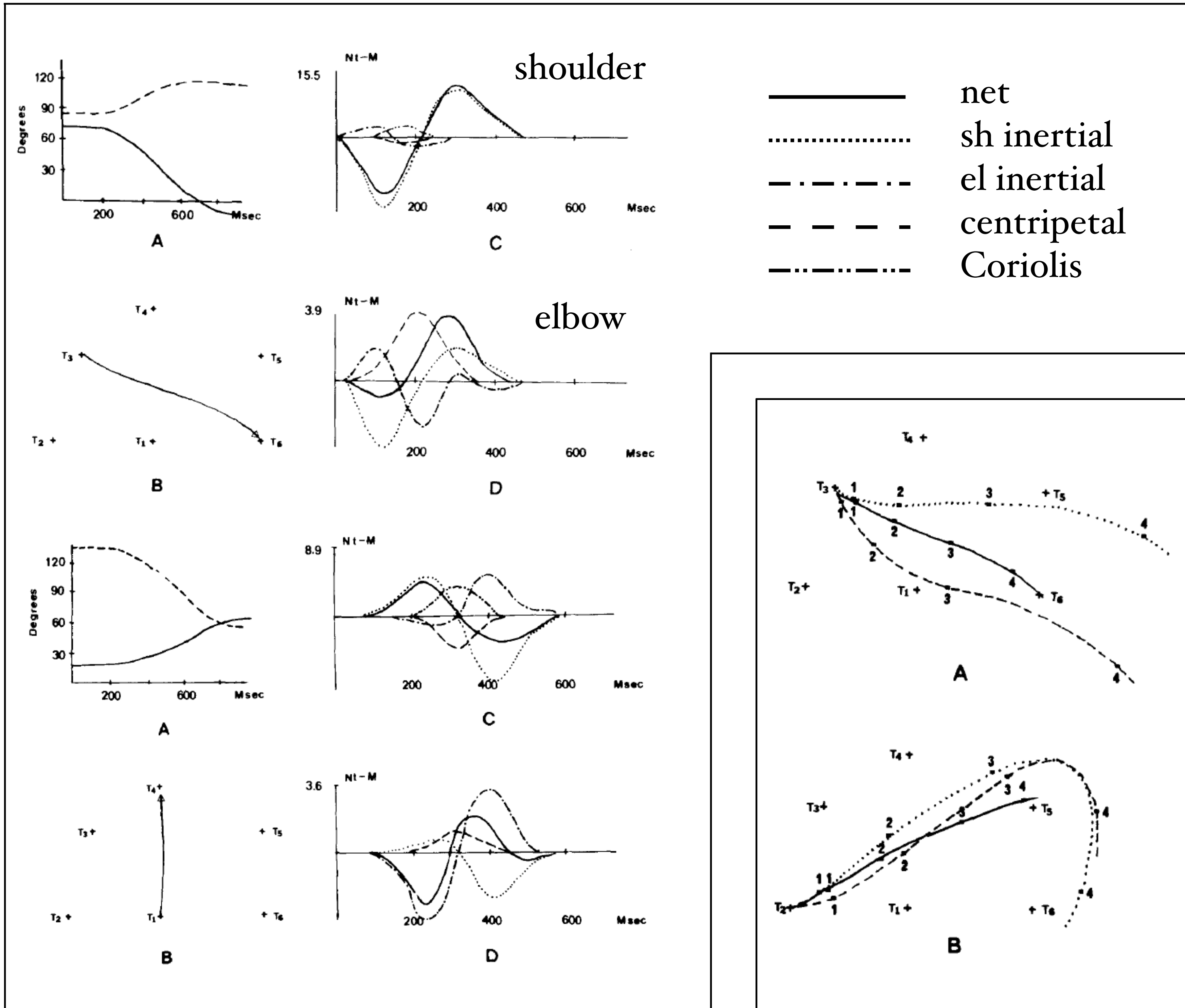
- Inertial torques: $\propto$ acceleration
  - Normal ($\tau_s = ... + [] \times \ddot{\theta}_s + ...$)
  - Interaction ($\tau_s = ... + [] \times \ddot{\theta}_e + ...$)
- Velocity-dependent torques
  - Coriolis ($\tau_s = ... + [] \times \dot{\theta}_s\dot{\theta}_e + ...$)
  - Centripetal ($\tau_s = ... + [] \times \dot{\theta}_s^2 + ...$)
- Gravity torques

INVERSE DYNAMICS

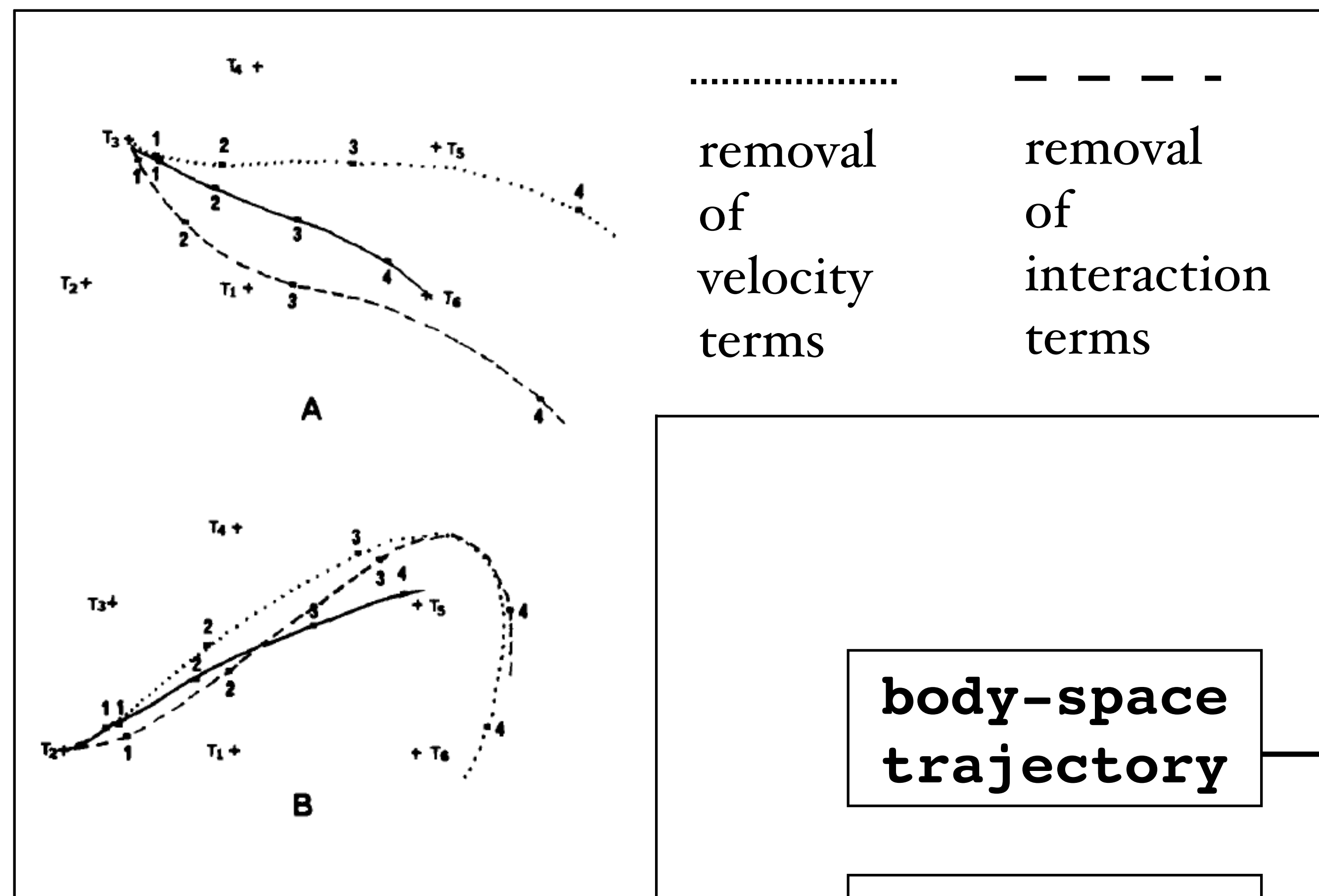
dof

kin

flex



— Hollerbach & Flash, 1982, *Biol Cybern* 44:67



limitations — only in body space

body-space trajectory

joint torques

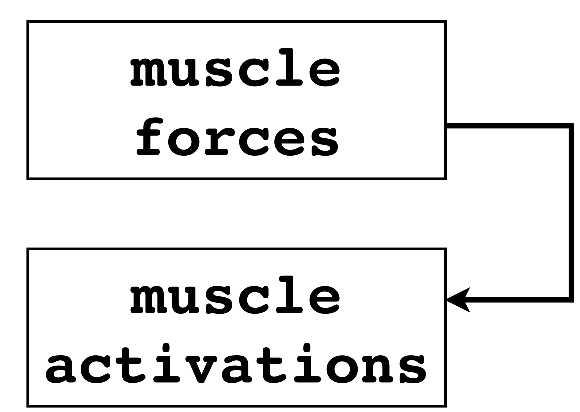
MUSCLE MODEL

- **3 types of model (by complexity order)**

— **input/output**: black box that reproduces the behavior of a muscle in specific conditions. In general, linear transfert function that translates nervous signals into force
— **lumped**: combination of linear mechanic elements that reproduces the viscoelastic properties of muscles. Sometimes nonlinear. Measurable parameters.
— **cross-bridge**: description of molecular aspects of muscular contraction. Parameters not directly measurable

- **How to choose?**

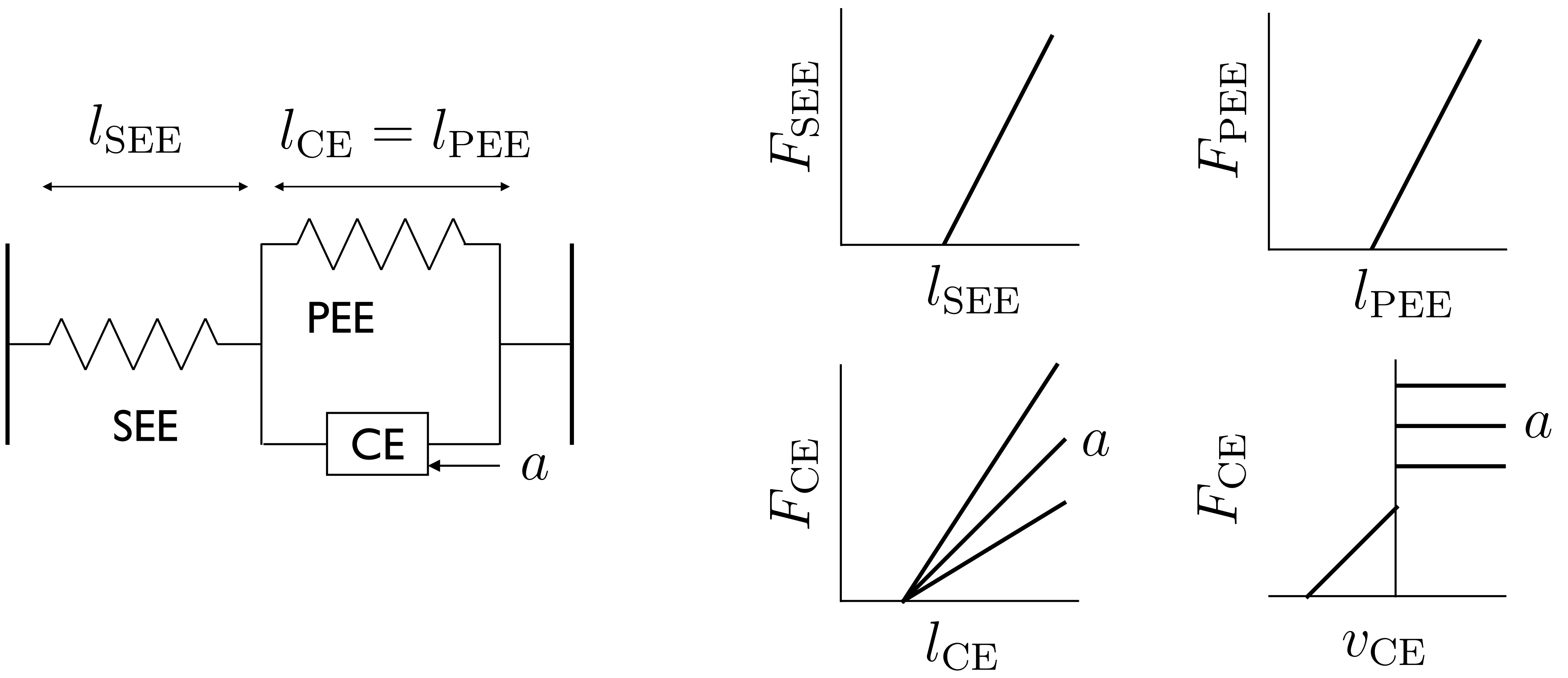
— a more complex model requires a larger number of parameters
— what is the expected influence of a complex model compare to a simpler one?



MUSCLE MODEL: LUMPED

The muscle is made of three elements

a contractile element (CE) which is a force generator; a parallel elastic element (PE) which represents the contribution of passive tissues; a serial elastic element (SEE) which represents the stiffness of tendon and cross-bridges acting in series with the CE

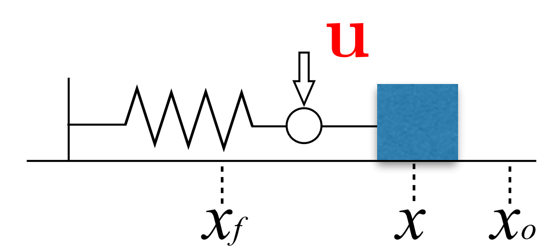


TWO DICHOTOMIES

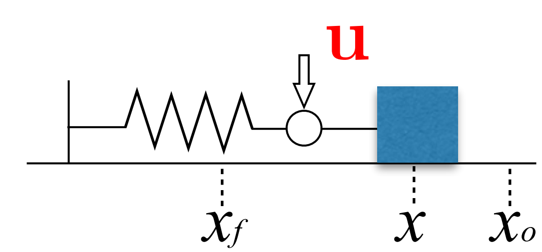
- **To model or not**
  — e.g. existence of an inverse model

- **To control or not**
  — constraints on the evolution of the system

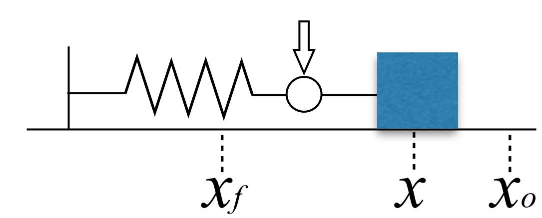
	Model	No
Control	inverse dyn. optimal c.	mass-spring classical FB c.
No	X	task dynamics



$$\mathbf{m}\ddot{x} + \mathbf{b}\dot{x} + \mathbf{k}(x - x_f) = \mathbf{u}$$
$$\mathbf{u} = \mathbf{u}(t, \mathbf{m}, \mathbf{b}, \mathbf{k})$$



$$\mathbf{m}\ddot{x} + \mathbf{b}\dot{x} + \mathbf{k}(x - x_f) = \mathbf{u}$$
$$\mathbf{u} = \mathbf{K}(x - x_f)$$



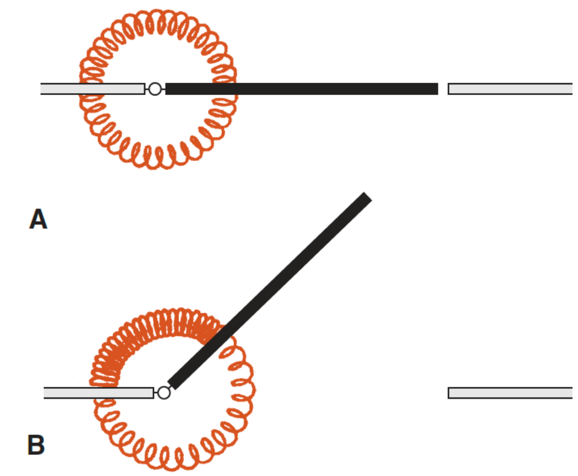
$$\mathbf{m}\ddot{x} + \mathbf{b}\dot{x} + \mathbf{k}(x - x_f) = 0$$
$$x = x(t, \mathbf{m}, \mathbf{b}, \mathbf{k})$$

MASS-SPRING MODELS

Endpoint location programming

— stiffness control

— rest-length control



cafe-door model

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details of trajectory are determined by
inertial and viscoelastic properties of
the limb and the muscles
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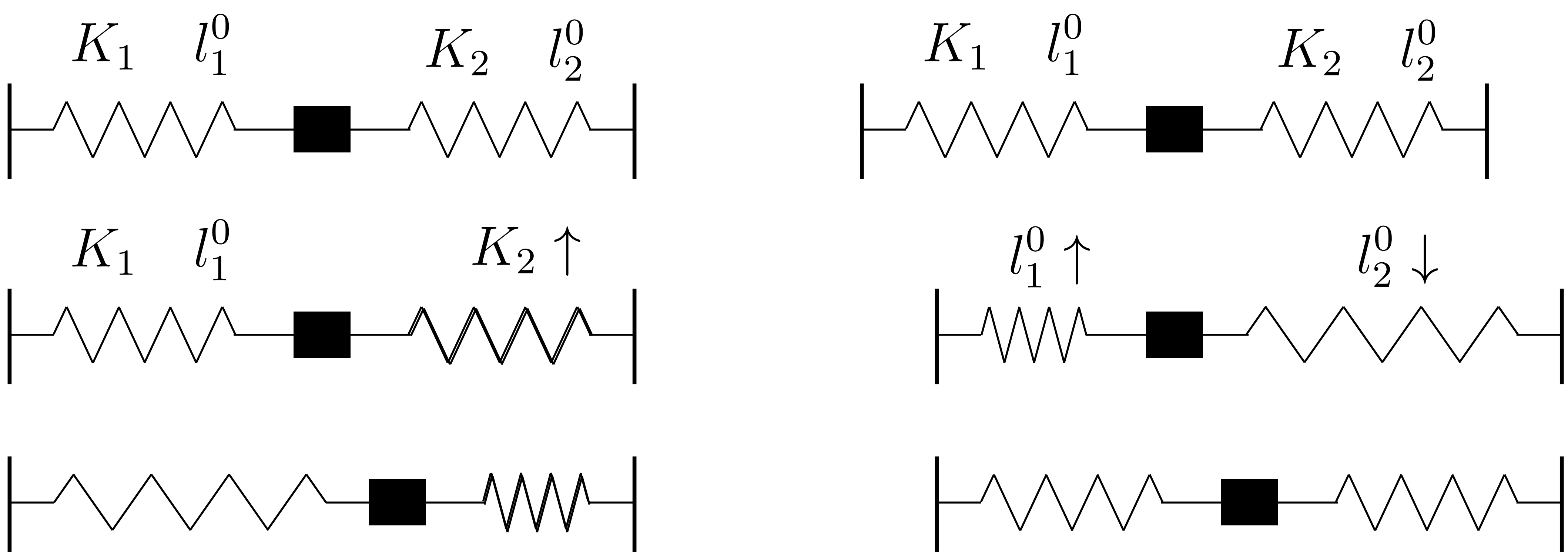


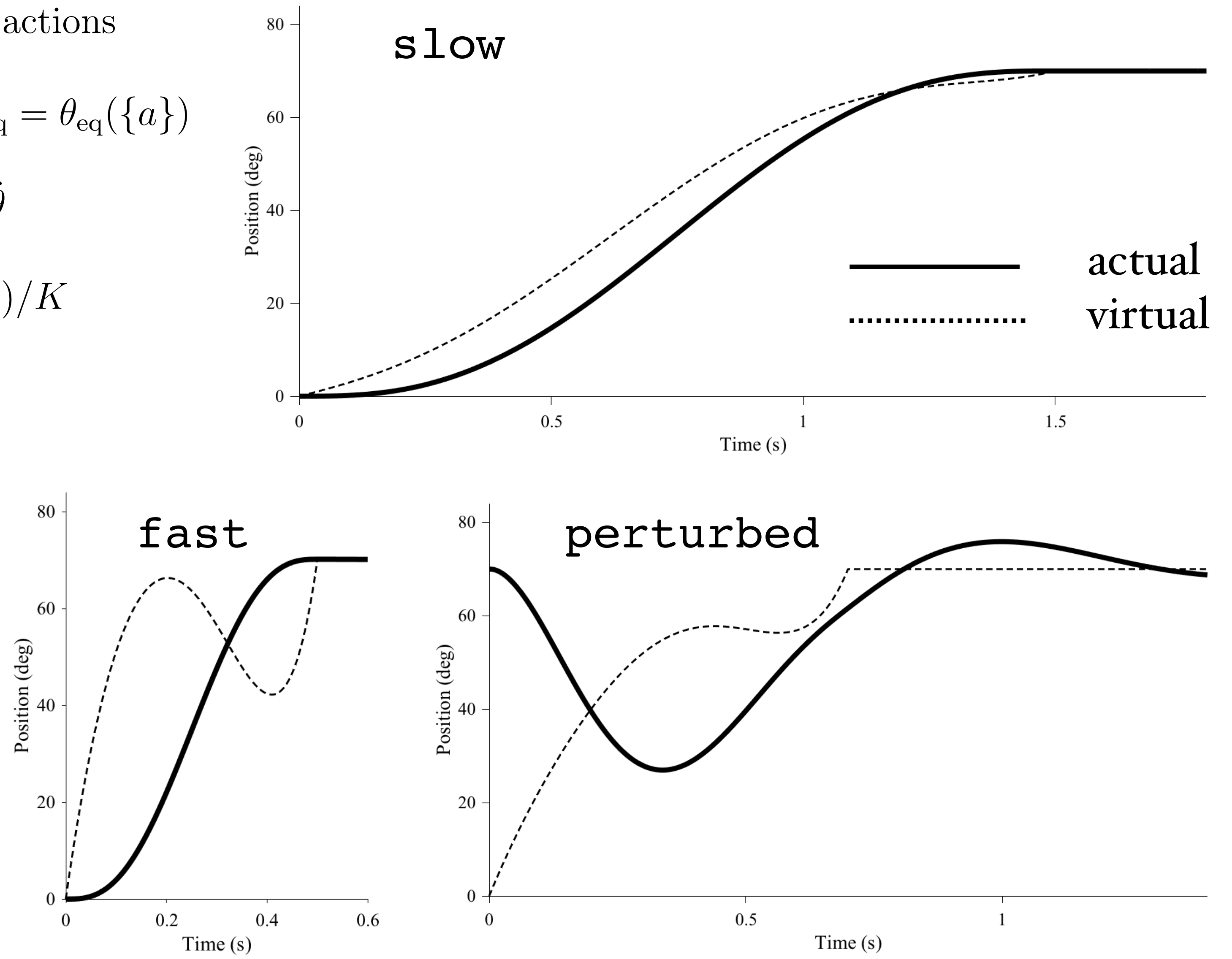
MASS-SPRING MODELS: SIMULATION

Equilibrium point model, single-joint

infer equilibrium from minimum-jerk desired trajectory

1. Dynamics $I\ddot{\theta} = T(\theta, \dot{\theta}, \{a\})$, $\{a\}$ set of muscle actions
2. At equilibrium, $\theta = \dot{\theta} = 0$, $T(\theta, \{a\}) = 0 \Rightarrow \theta_{\mathrm{eq}} = \theta_{\mathrm{eq}}(\{a\})$
3. Assumption: $T(\theta, \dot{\theta}, \{a\}) = T(\{a\}) - K\theta - B\dot{\theta}$
4. At equilibrium, $T(\{a\}) = K\theta$ and $\theta_{\mathrm{eq}} = T(\{a\})/K$
5. New dynamics $I\ddot{\theta} + B\dot{\theta} + K\theta = K\theta_{\mathrm{eq}}$
6. Calculate the minimum-jerk trajectory $\theta_{\mathrm{mj}}(t)$
7. Calculate the equilibrium trajectory
$\theta_{\mathrm{eq}} = (I\ddot{\theta}_{\mathrm{mj}} + B\dot{\theta}_{\mathrm{mj}} + K\theta_{\mathrm{mj}})/K$
8. Calculate the actual trajectory $\theta_{\mathrm{ac}}(t)$
$I\ddot{\theta}_{\mathrm{ac}} + B\dot{\theta}_{\mathrm{ac}} + K\theta_{\mathrm{ac}} = K\theta_{\mathrm{eq}}$

— Hogan, 1984, *J Neurosci* 4:2745



MASS-SPRING MODELS: SIMULATION

dof

kin

flex

Equilibrium point model, double-joint

— infer equilibrium from measured trajectory
— compare measured trajectory with trajectory derived from minimum-jerk

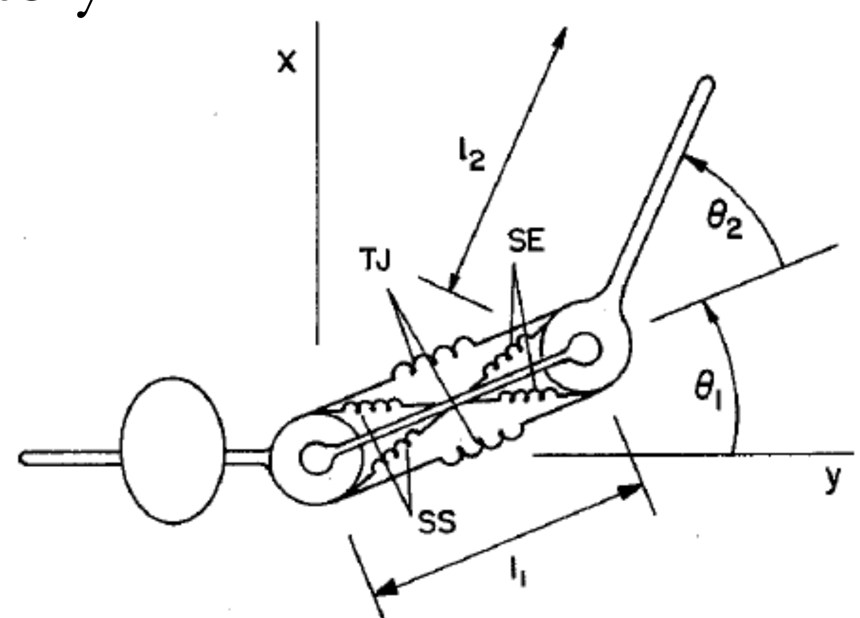
$$\mathbf{n} = \mathbf{I}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}}$$ dynamics

$$\mathbf{n}(t) = \mathbf{R}(\boldsymbol{\phi}(t) - \boldsymbol{\theta}(t)) - \mathbf{B}\dot{\boldsymbol{\theta}}(t)$$ control law

$$\boldsymbol{\phi}(t)$$ equilibrium angular trajectory

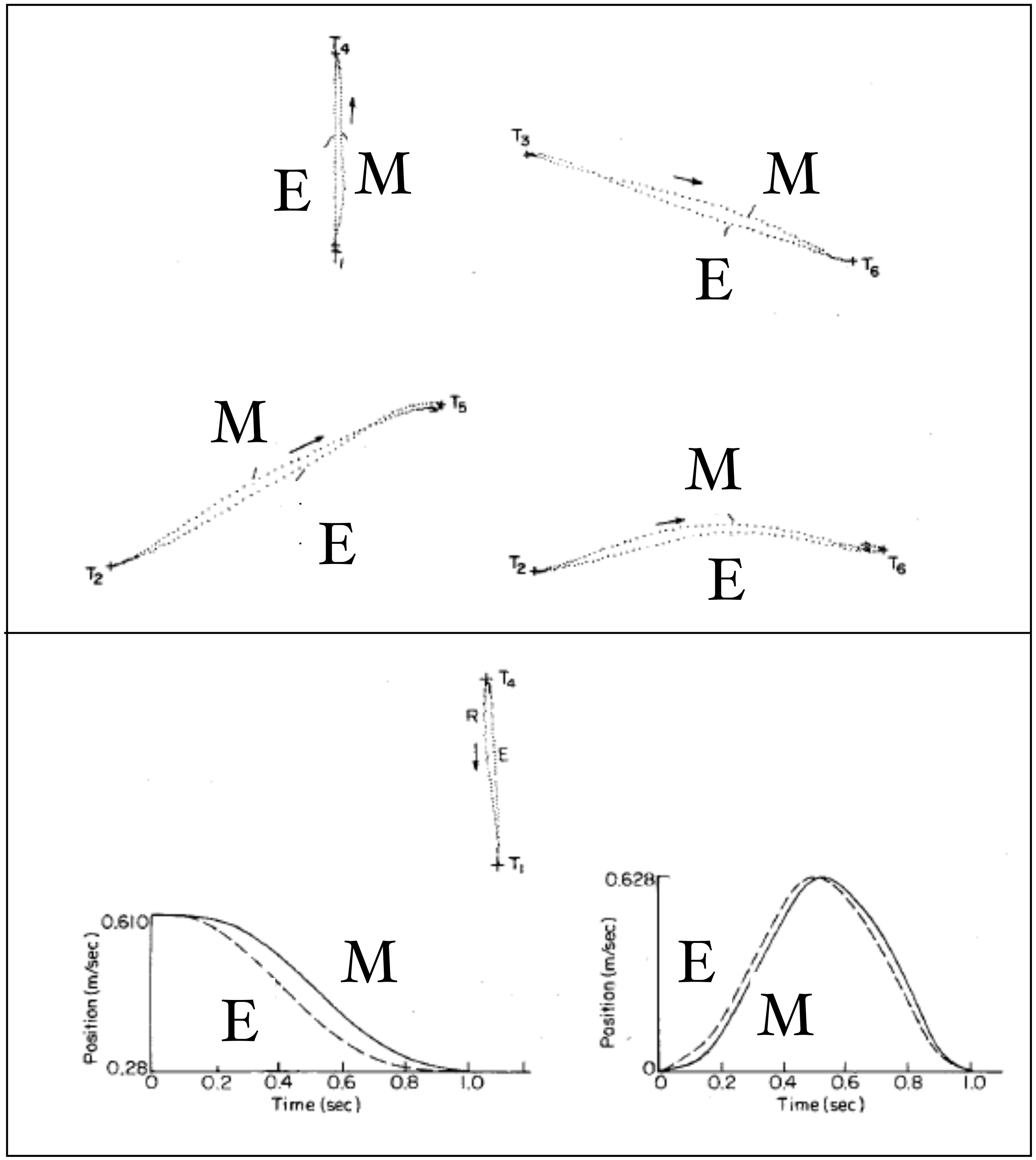
$$\mathbf{R} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

stiffness viscosity



— Flash, 1987, *Biol Cybern* 57:257

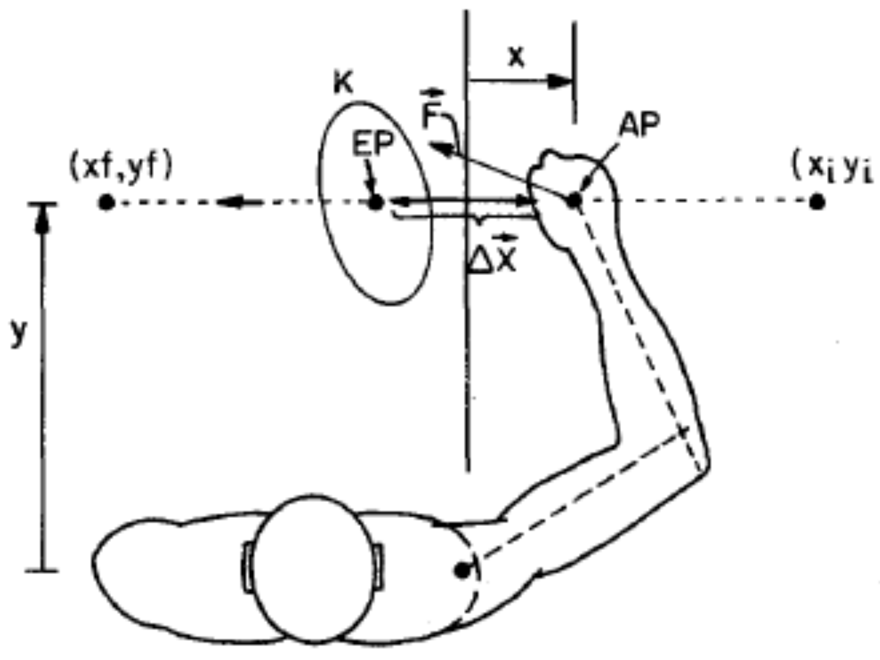
MASS-SPRING MODELS: SIMULATION



equilibrium trajectory (E) derived from measured trajectory (M)

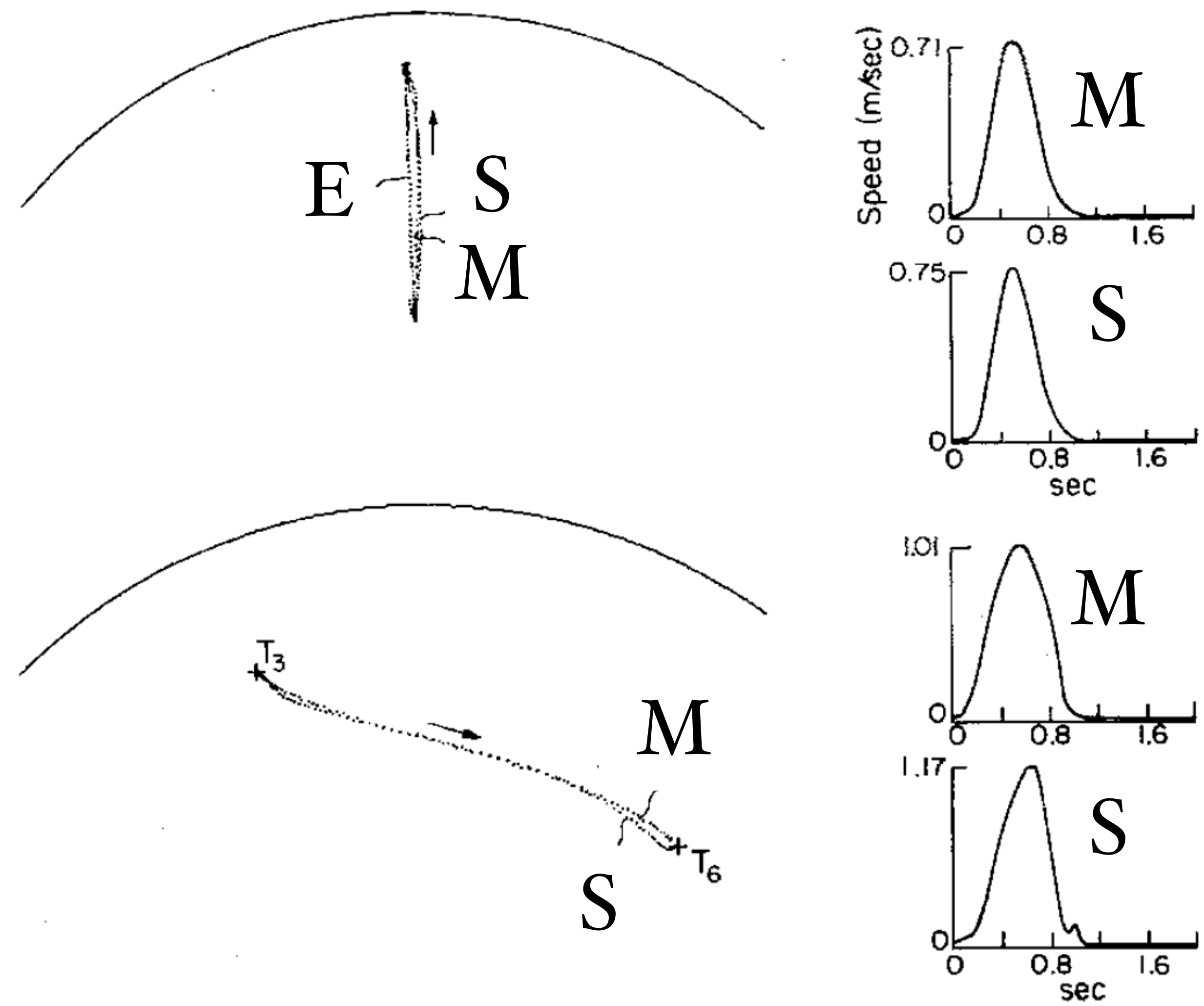
— Flash, 1987, *Biol Cybern* 57:257

MASS-SPRING MODELS: SIMULATION



limitations — Fast movements require a larger stiffness and viscosity. For 0.5–0.8 s movements, calculated trajectories are close to real trajectories. Below 0.5 s, differences are observed. The scaling strategy is not uniform. Some movements require a change in the shape and orientation of stiffness and viscosity ellipses.

— Flash, 1987, *Biol Cybern* 57:257

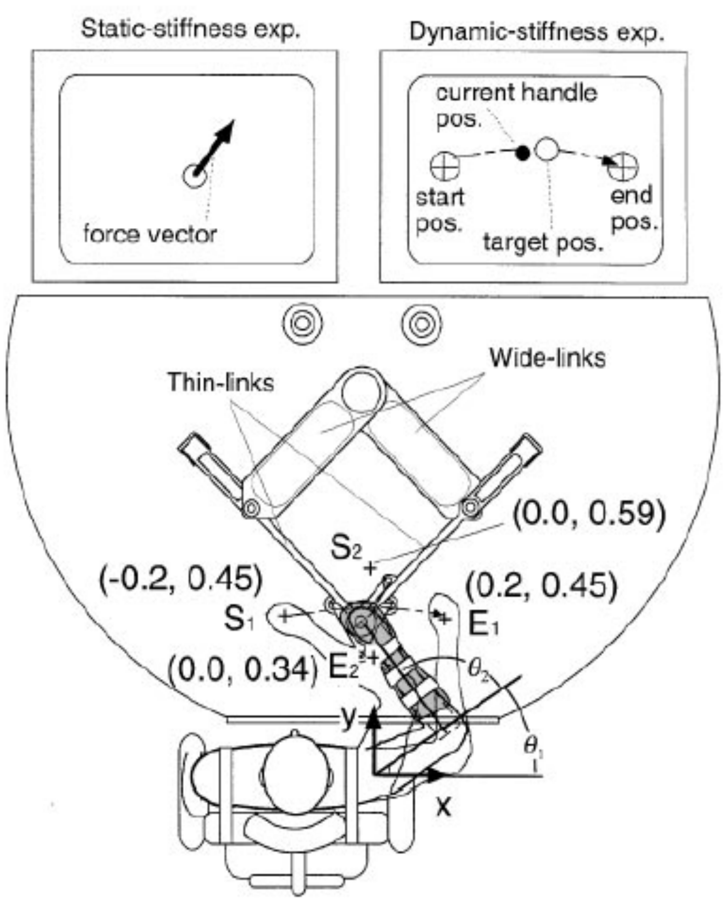


equilibrium (E)
measured (M)
simulated (S)

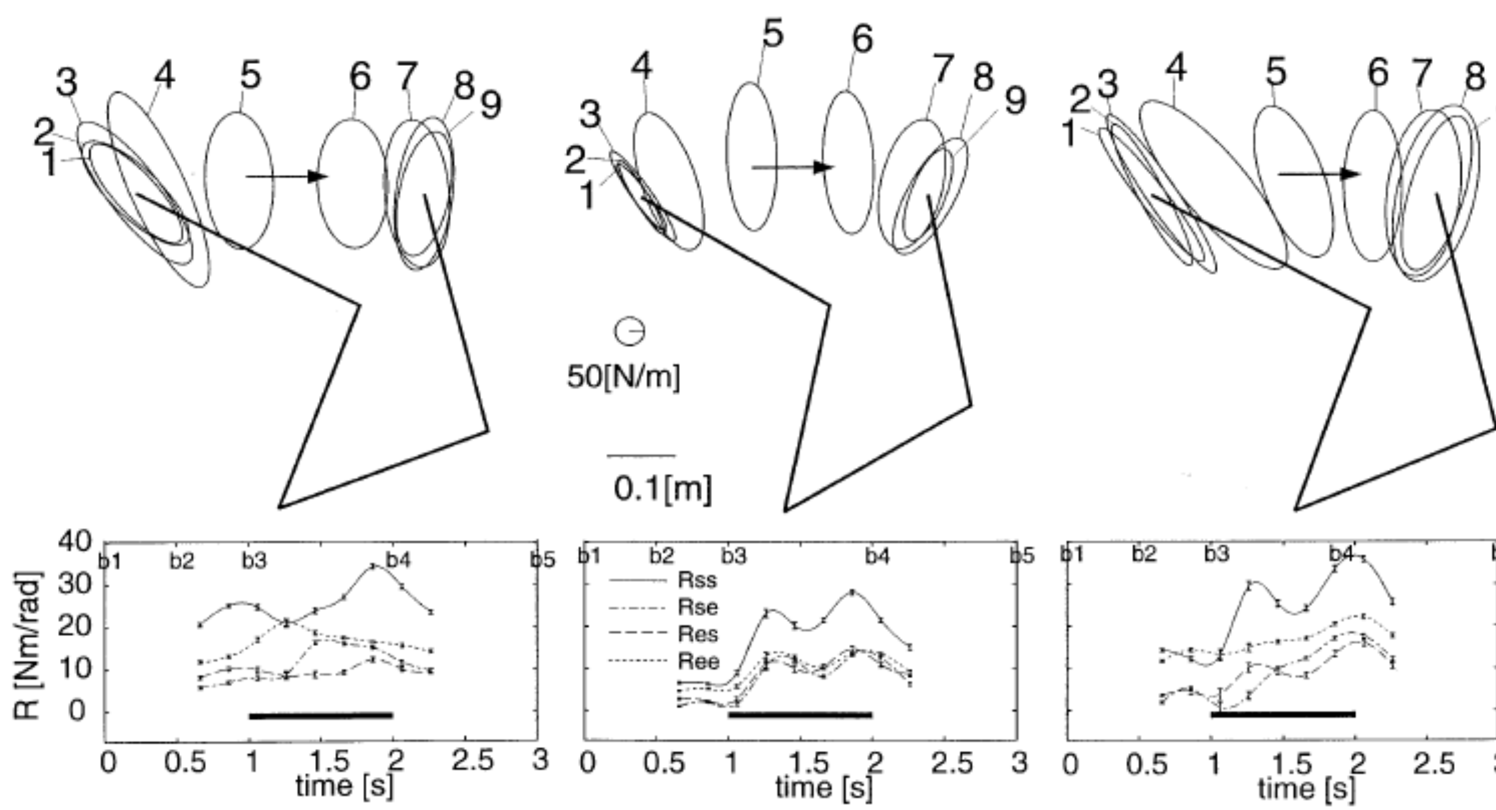
MASS-SPRING MODELS: ISSUES

Measuring stiffness *in vivo*

Is siffness large enough for the equilibrium point scenario?



3 subjects



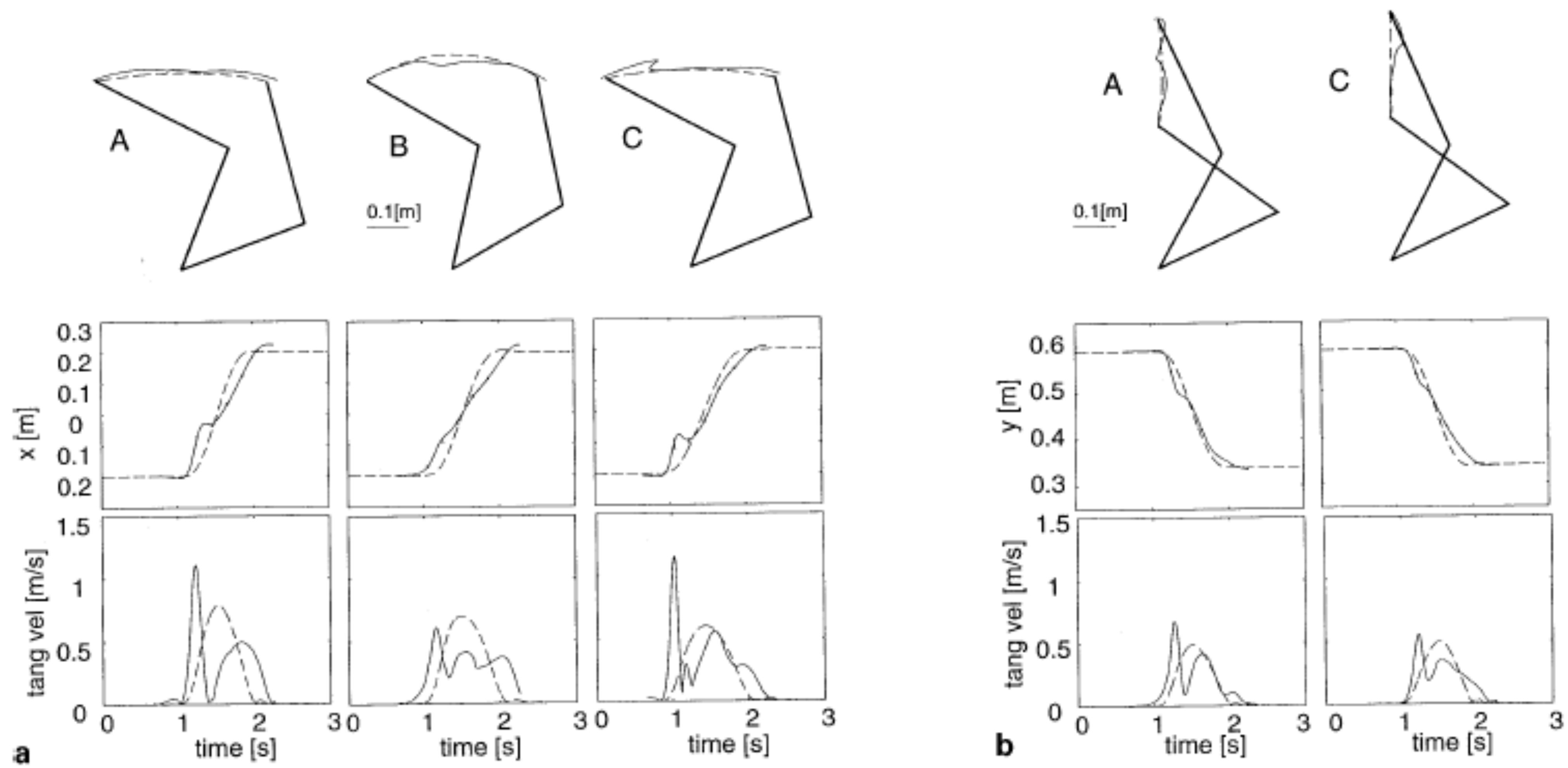
movement duration

— Gomi & Kawato, 1997, *Biol Cybern* 76:163

MASS-SPRING MODELS: ISSUES

Measuring stiffness *in vivo*

Is siffness large enough for the equilibrium point scenario? NO



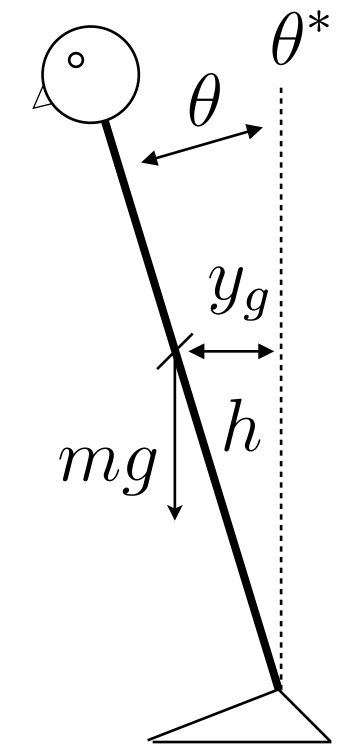
— Gomi & Kawato, 1997, *Biol Cybern* 76:163

—— equilibrium
– – – actual

CLASSICAL FEEDBACK CONTROL

Inverted pendulum — posture

maintain the pendulum to a reference position



dynamics

$$I\ddot{\theta}(t) = mgh\theta(t) + u(t) + \text{noise}$$

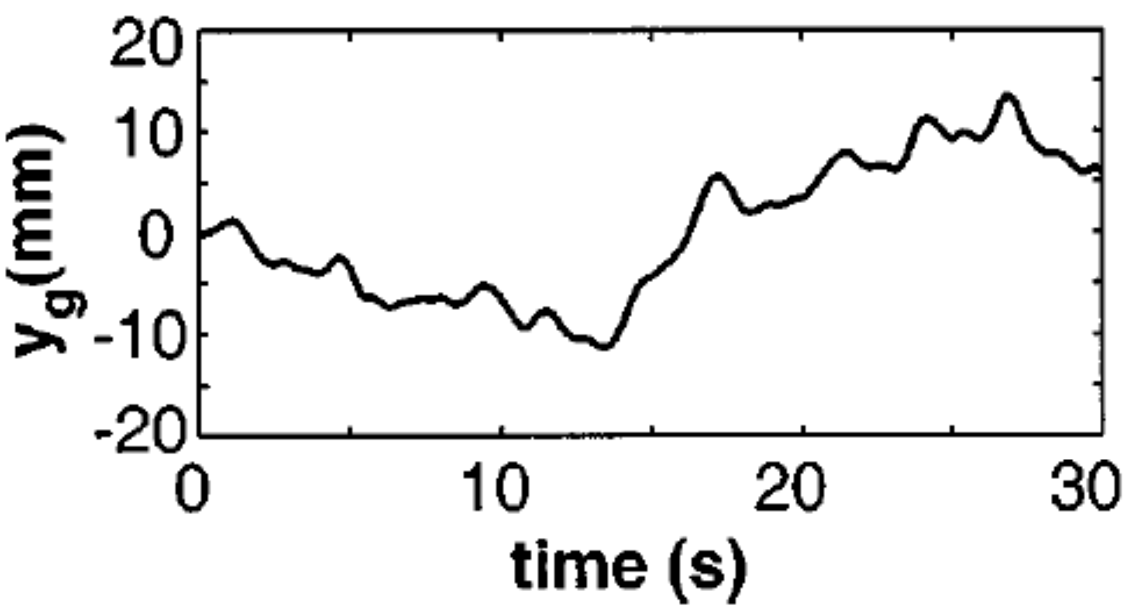
$$\theta^* = 0 \qquad \theta(t) \approx 0 \qquad K_P > mgh$$

control policy

$$u(t) = K_P(\theta^* - \theta(t)) - K_D\dot{\theta}(t) + K_I \int_{t_0}^{t} (\theta^*(\tau) - \theta(\tau))\, d\tau$$

- model of postural oscillations
- **reminder:** the controller has no knowledge of the system to be controlled (e.g. mass, height)

— Peterka, 2000, *Biol Cybern* 82:335



CLASSICAL FEEDBACK CONTROL

dof

kin

flex

Inverted pendulum — movement

displace the pendulum to a reference position

dynamics

$$I\ddot{\theta}(t) = mgh\sin\theta(t) + u(t)$$

$$\theta^* = 0$$

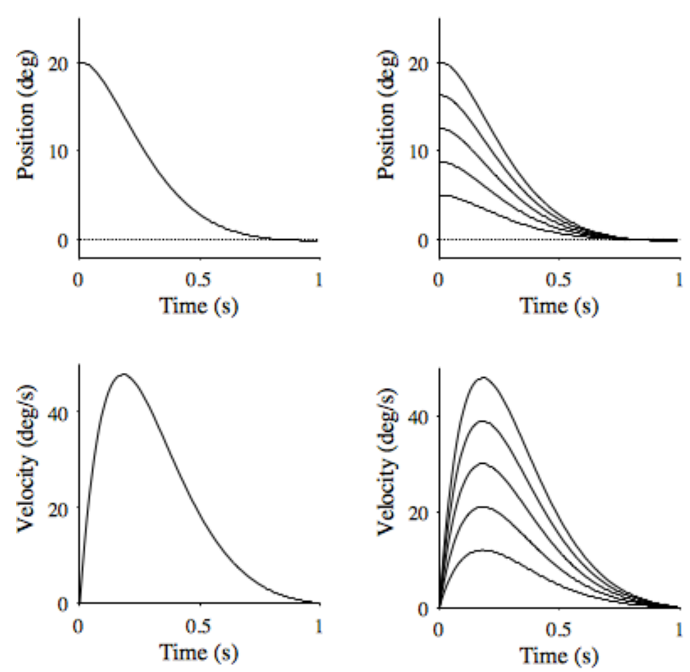
$$\theta(t) \not\approx 0$$

$$K_P > mgh$$

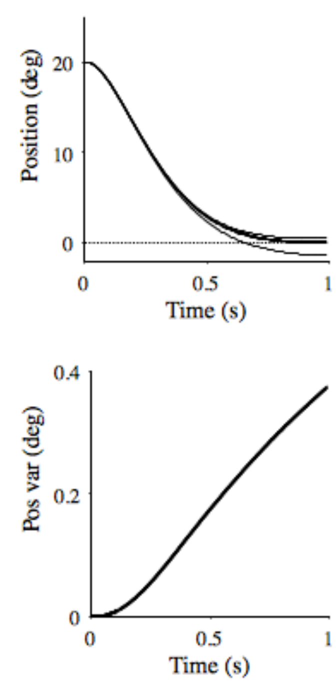
control policy

$$u(t) = K_P(\theta^* - \theta(t)) - K_D\dot{\theta}(t) + K_I\int_{t_0}^{t}(\theta^*(\tau) - \theta(\tau))\,d\tau$$

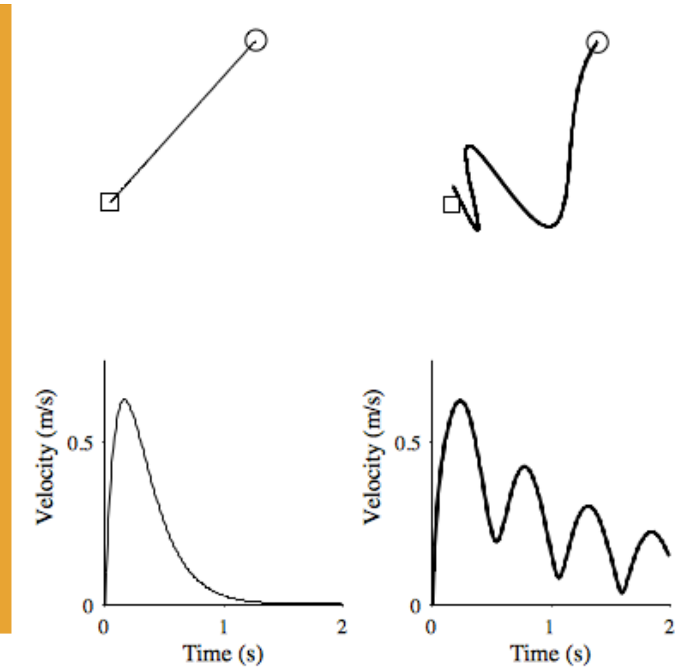
trajectory



variability



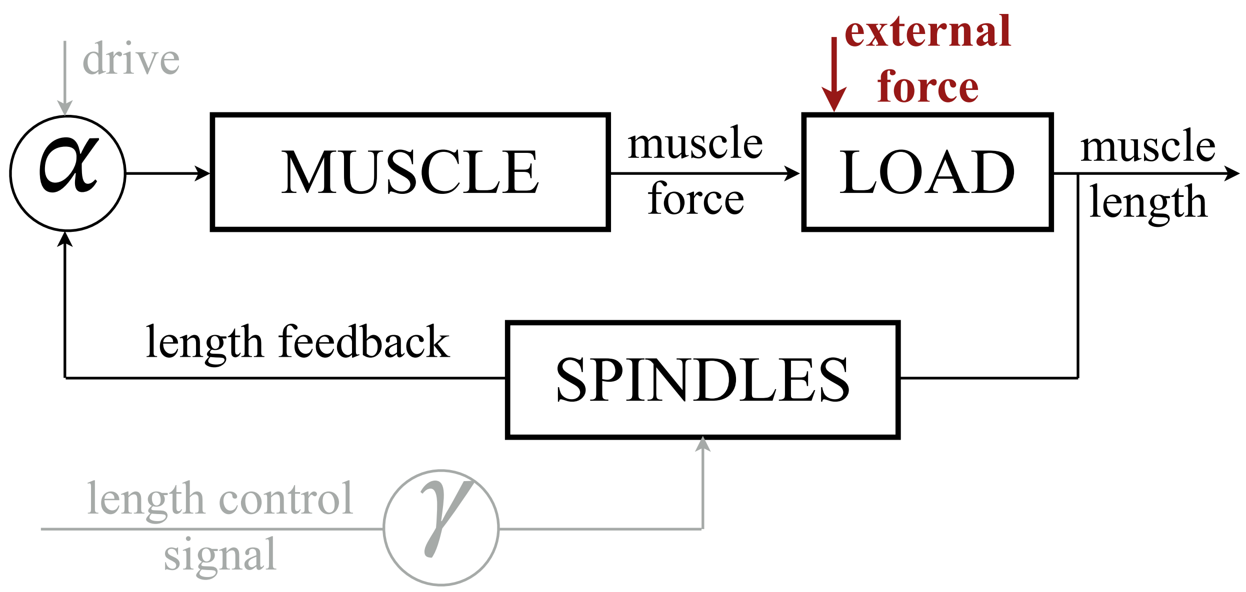
perturbation

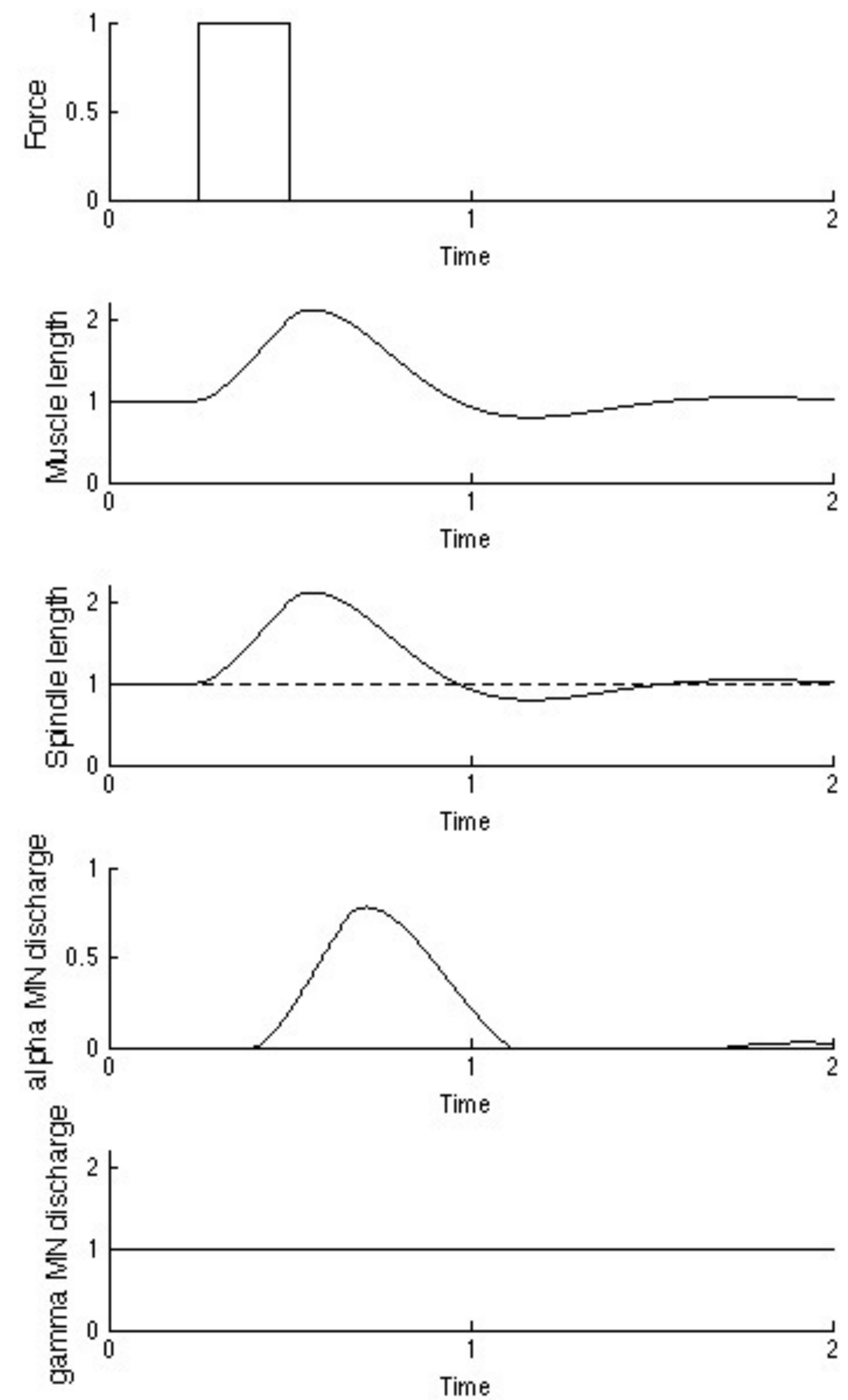


CLASSICAL FEEDBACK CONTROL

Neural implementation

stretch reflex



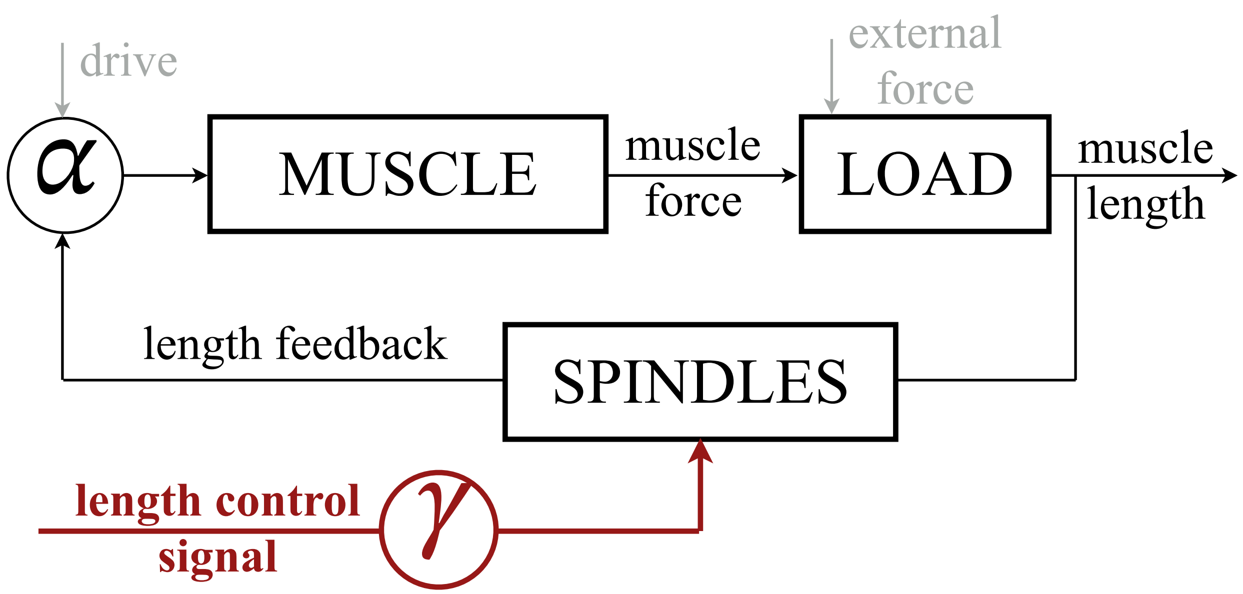


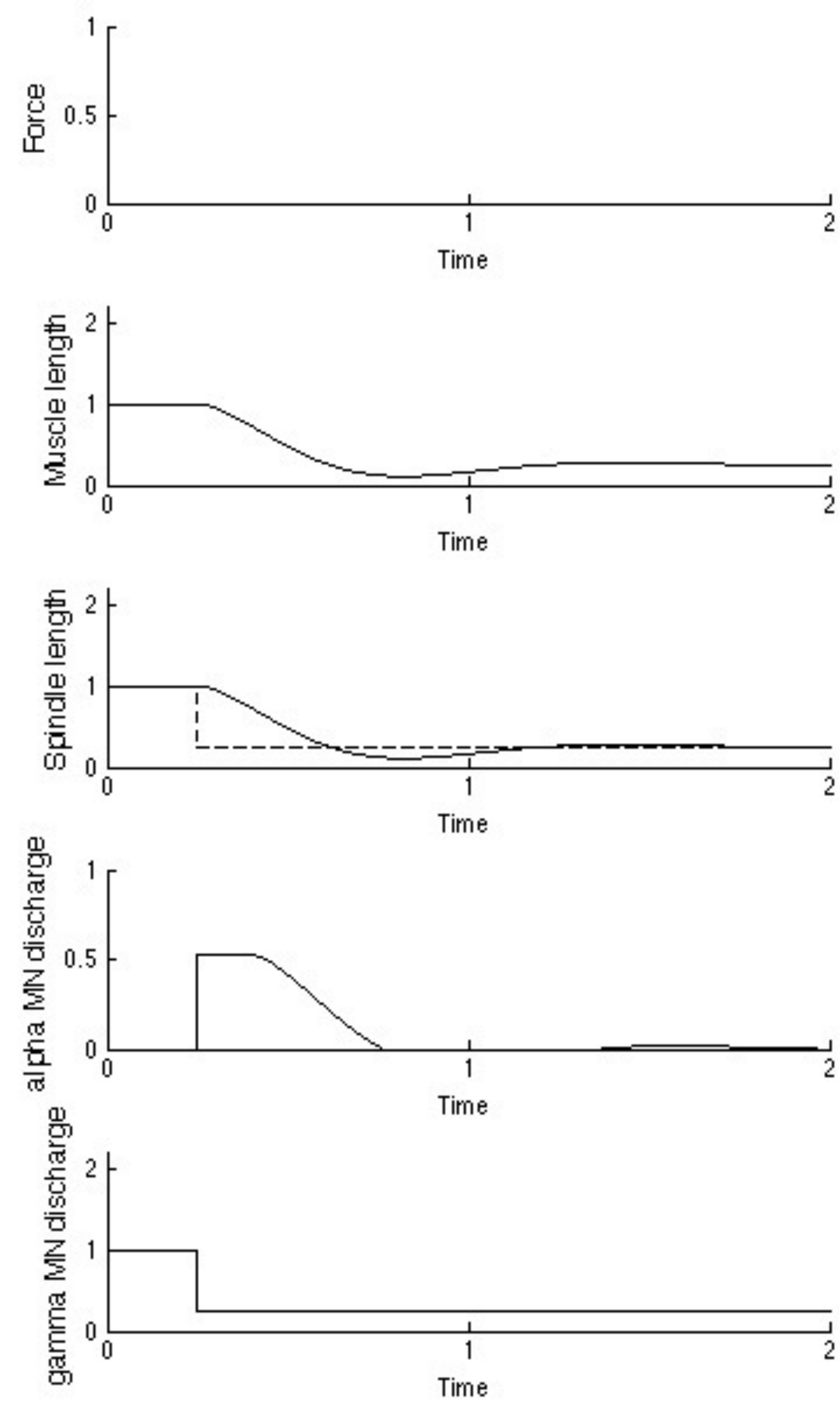
CLASSICAL FEEDBACK CONTROL

Neural implementation

stretch reflex = servo-control

Merton's model





CLASSICAL FEEDBACK CONTROL

Can servo-control be a general model of motor control?

No

— alpha/gamma coactivation

— Restricted to single-joint movements. Problem of intersegmental dynamics for multijoint movements. Rapid feedback is too slow to compensate for intersegmental dynamics

— Feedback can only have a modest effect on motor output

INVERSE DYNAMICS

Movement of a point mass

follow a desired trajectory

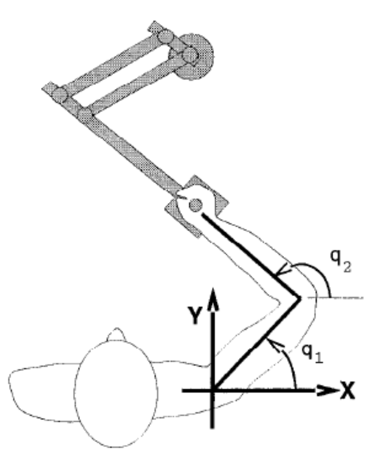
dynamics

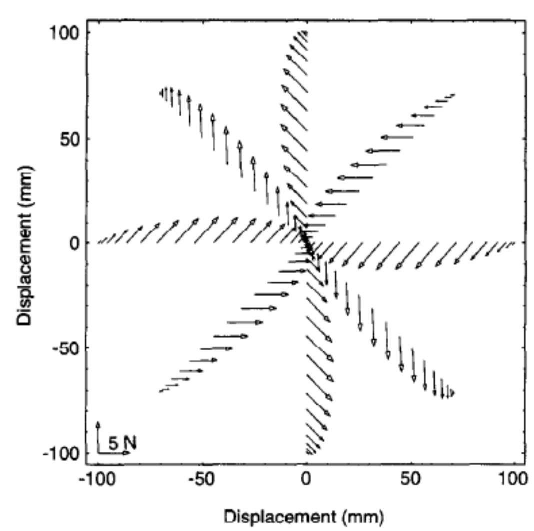
$$m\ddot{x}(t) = u(t)$$

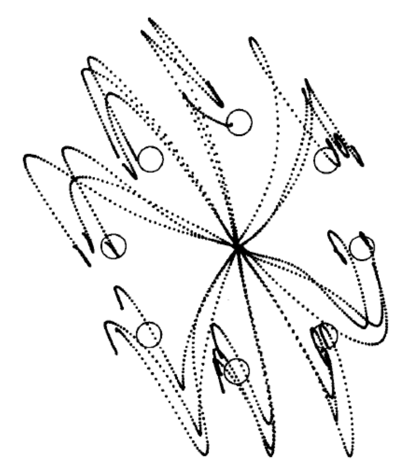
desired trajectory $x^*(t)$

control policy

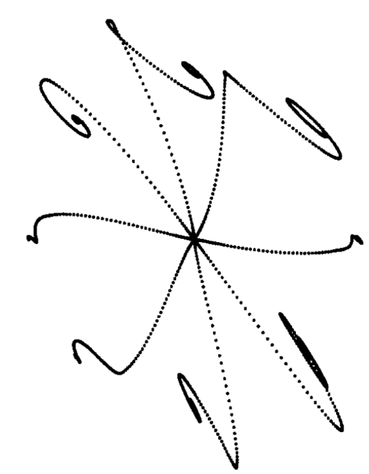
$$u(t) = \hat{m}\ddot{x}^*(t) + K_P(x^*(t) - x(t)) + K_D(\dot{x}^*(t) - \dot{x}(t))$$







data



simulation

— Shadmehr & Mussa-Ivaldi, 1994, *J Neurosci* 14:3208

OPTIMAL CONTROL

dof

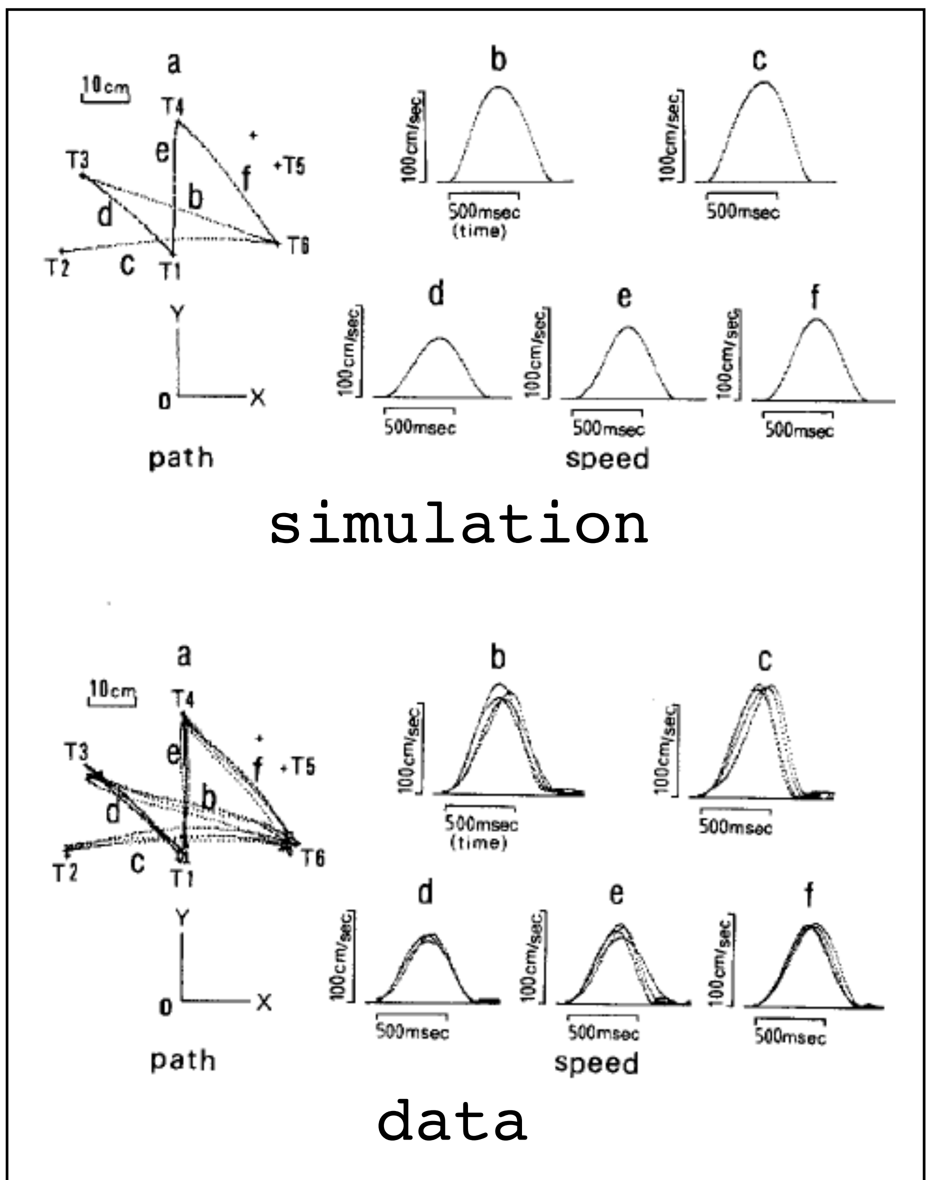
kin

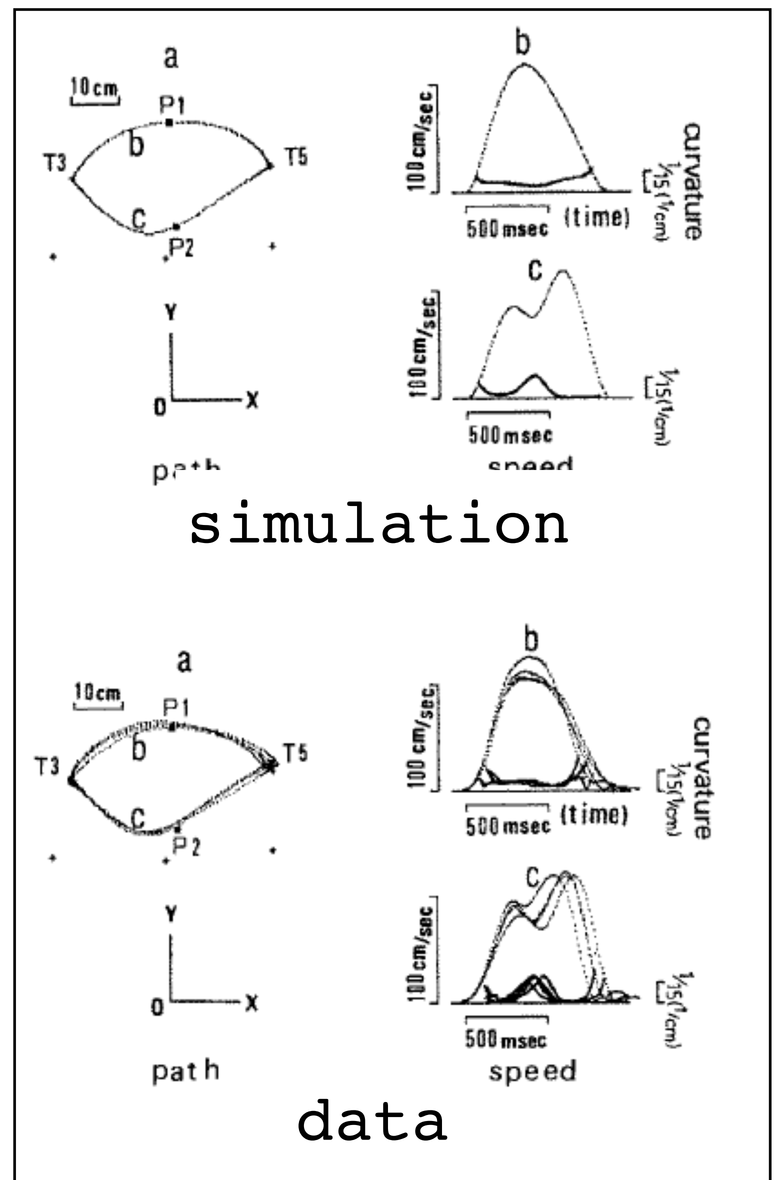
flex

Minimum torque change

to circumvent the limitations
of minimum jerk

$$C = \int_{t_0}^{t_f} \sum_i \left(\frac{d\tau_i}{dt} \right)^2 \, dt$$





— Uno et al., 1989,
Biol Cybern 61:89

OPTIMAL CONTROL

Minimum torque change

analytical solution

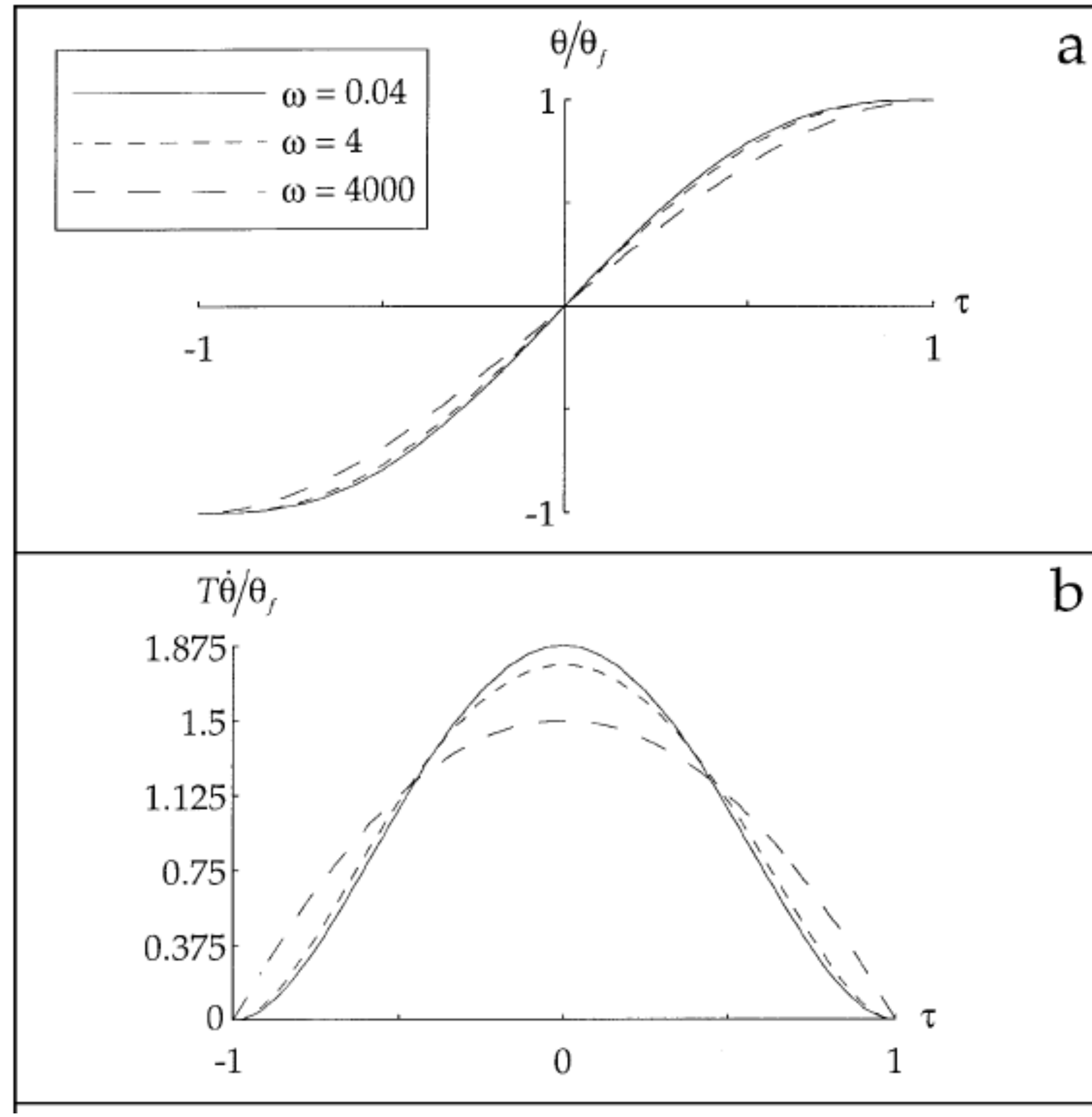
$$\tau = I\ddot{\theta} + B\dot{\theta} \qquad C = \int_{t_0}^{t_f} \dot{\tau}^2(t)\, dt$$

$$\frac{d^3}{dt^3}\frac{\partial \dot{\tau}^2}{\partial \dddot{\theta}} - \frac{d^2}{dt^2}\frac{\partial \dot{\tau}^2}{\partial \ddot{\theta}} = 0$$

$$\theta(t) = \theta_f Q(\omega)\left(\frac{\sinh \beta t}{\sinh \omega} - \frac{1}{6}\omega^2\left(\frac{t}{T}\right)^3 + \left(\frac{1}{2}\omega^2 - \omega \coth \omega\right)\frac{t}{T}\right)$$

$$t_0 = -T \quad tf = T \quad \beta = \frac{B}{I} \quad \omega = \beta(t_f - t_0)$$

— Engelbrecht & Fernandez, 1997, *Biol Cybern* 76:321



$\frac{\text{peak velocity}}{\text{mean velocity}}$	1.5-1.875 simulation	2.01-2.09 data

OPTIMAL CONTROL

dof

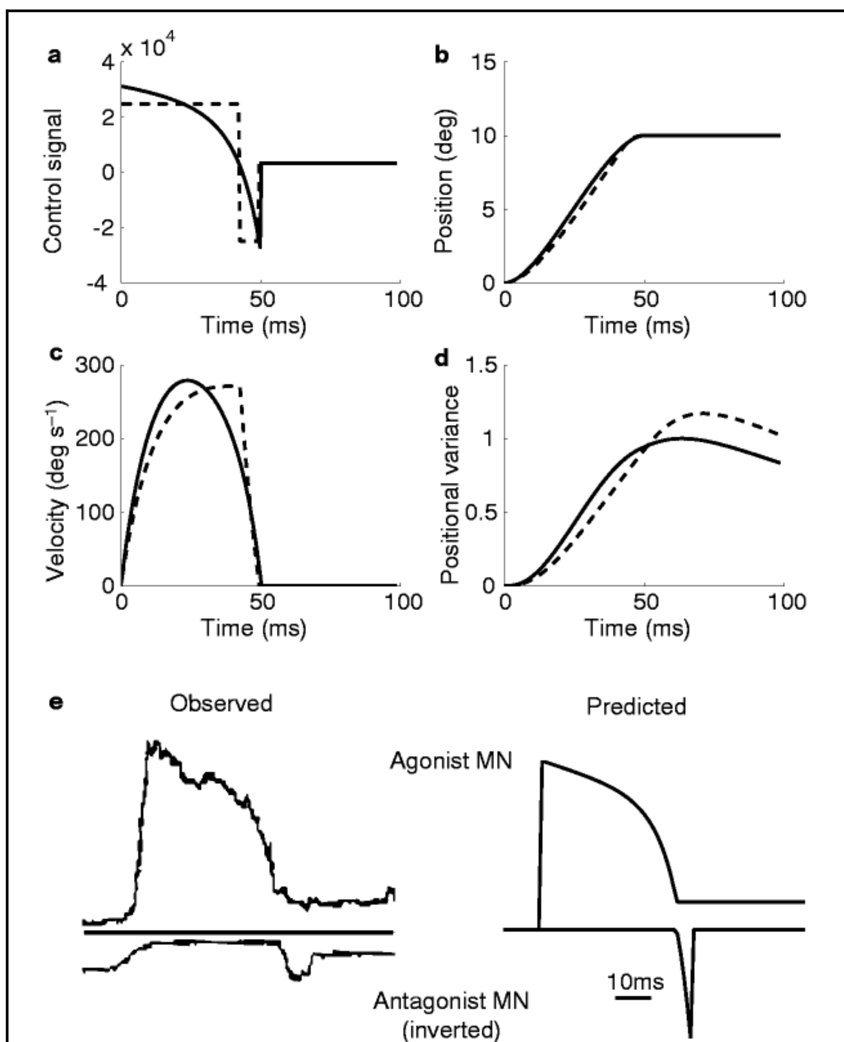
kin

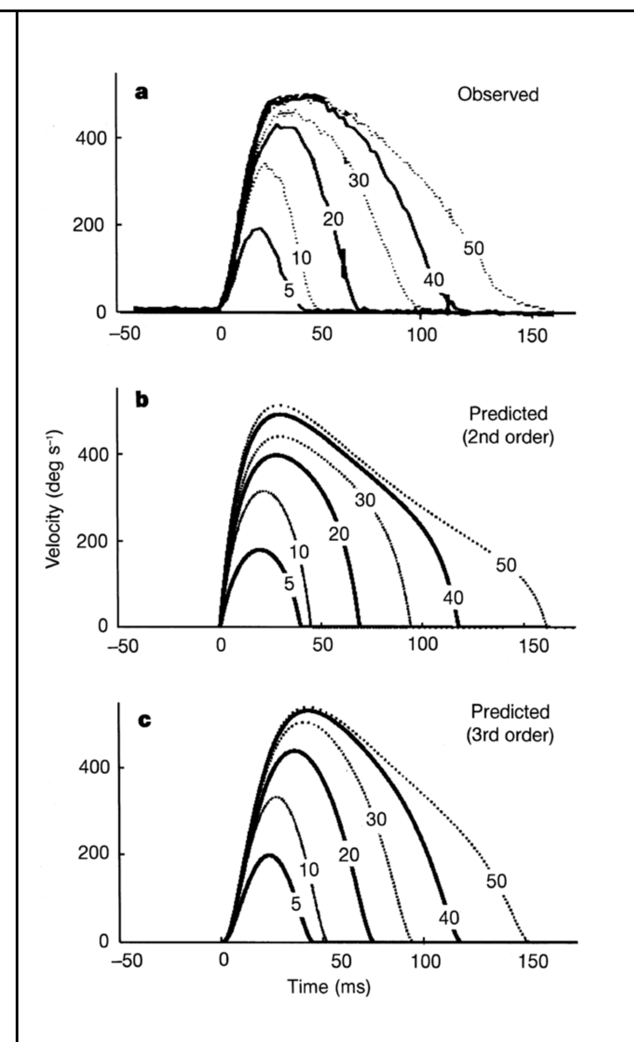
flex

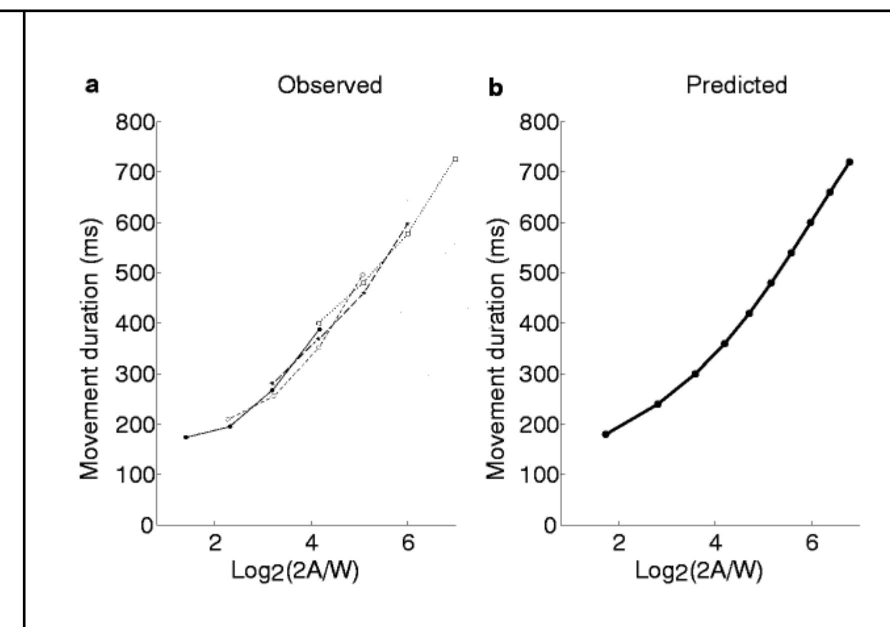
Minimum variance

minimize terminal variance in the presence of signal-dependent-noise = find the smallest command

Control signal | *Velocity profiles* | *Fitts' law*



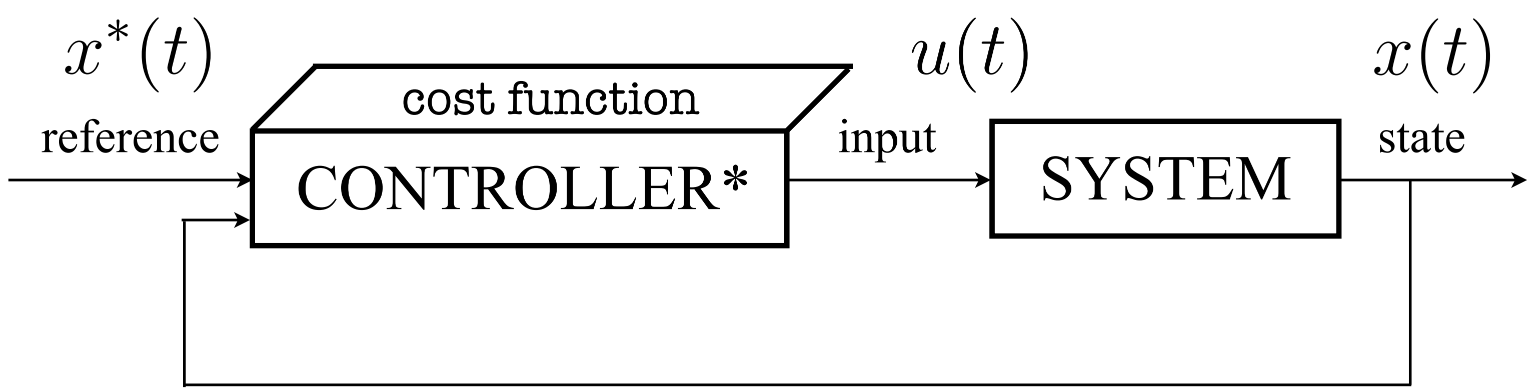




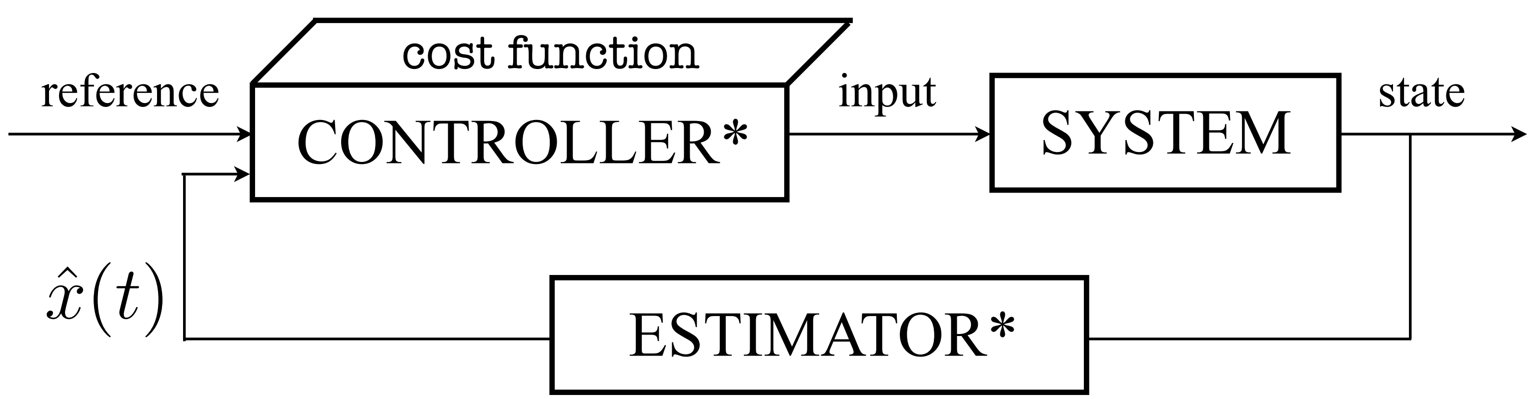
— Harris & Wolpert, 1998, *Nature* 394:780

OPTIMAL FEEDBACK CONTROL

Recalculate optimal control at each time step



$$u(t) = \pi(x(t), x^*(t))$$



$$u(t) = \pi(\hat{x}(t), x^*(t))$$

OPTIMAL FEEDBACK CONTROL

Linear Quadratic Regulator (LQR)

algorithm (discrete case)

$$x_{k+1} = Ax_k + Bu_k$$

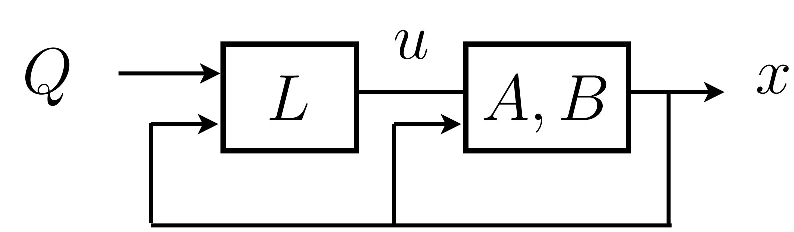
controlled object
discrete time linear system

$$J = \sum_{k=0}^{N} (x_k^T Q x_k + u_k^T R u_k)$$

performance index

$$u_k = -L_k x_k$$

feedback control law



$$L_k = (R + B^T P_k B)^{-1} B^T P_k A$$

$$P_{k-1} = Q + A^T (P_k - P_k B (R + B^T P_k B)^{-1} B^T P_k) A$$

$$P_N = Q$$

solution

- requires backward evaluation
- time is fixed in advance
- nonstationarity:
the control law depends on time

OPTIMAL FEEDBACK CONTROL

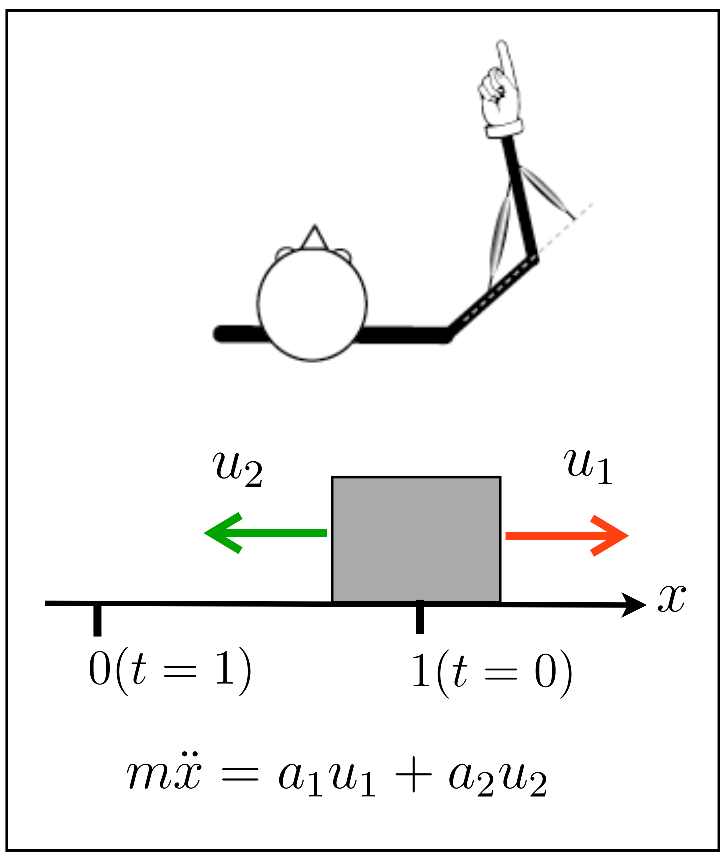
dof

kin

flex

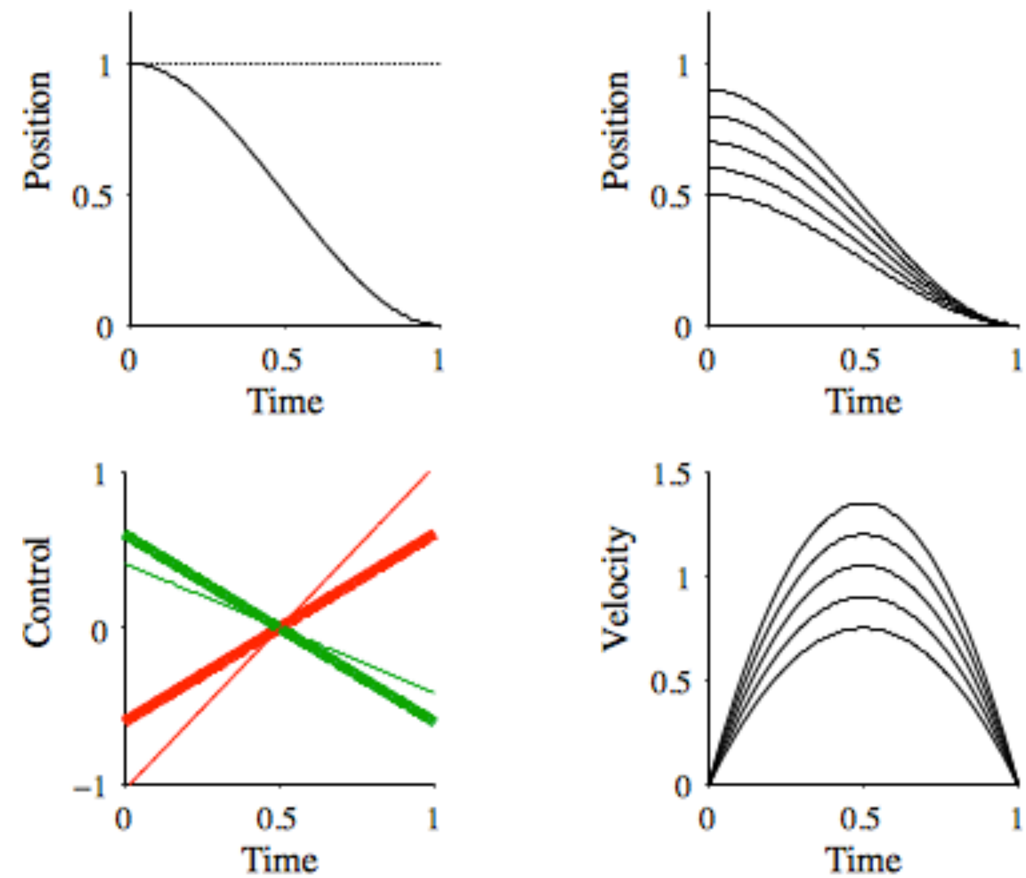
Linear Quadratic Regulator (LQR)

simulation (continuous case)



$$a_1 = +5$$
$$a_2 = -5$$

$$a_1 = +5$$
$$a_2 = -2$$



OPTIMAL FEEDBACK CONTROL

Linear Quadratic Gaussian (LQG)

continuous/discrete case

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x}(t) + B\mathbf{u}(t) + \overset{noise}{\mathbf{w}(t)}$$

controlled object — continuous time linear system

$$x_{k+1} = Ax_k + Bu_k$$

controlled object
discrete time linear system

$$J = E\left\{\int_{t_0}^{t_f} \left[\mathbf{x}^T(t)Q\mathbf{x}(t) + \mathbf{u}^T(t)R\mathbf{u}(t)\right]\ dt\right\}$$

performance index

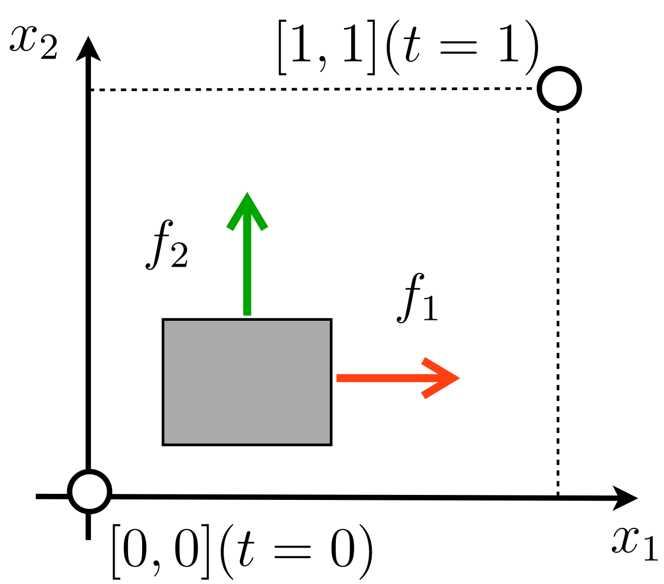
$$J = \sum_{k=0}^{N}(x_k^T Q x_k + u_k^T R u_k)$$

performance index

OPTIMAL FEEDBACK CONTROL

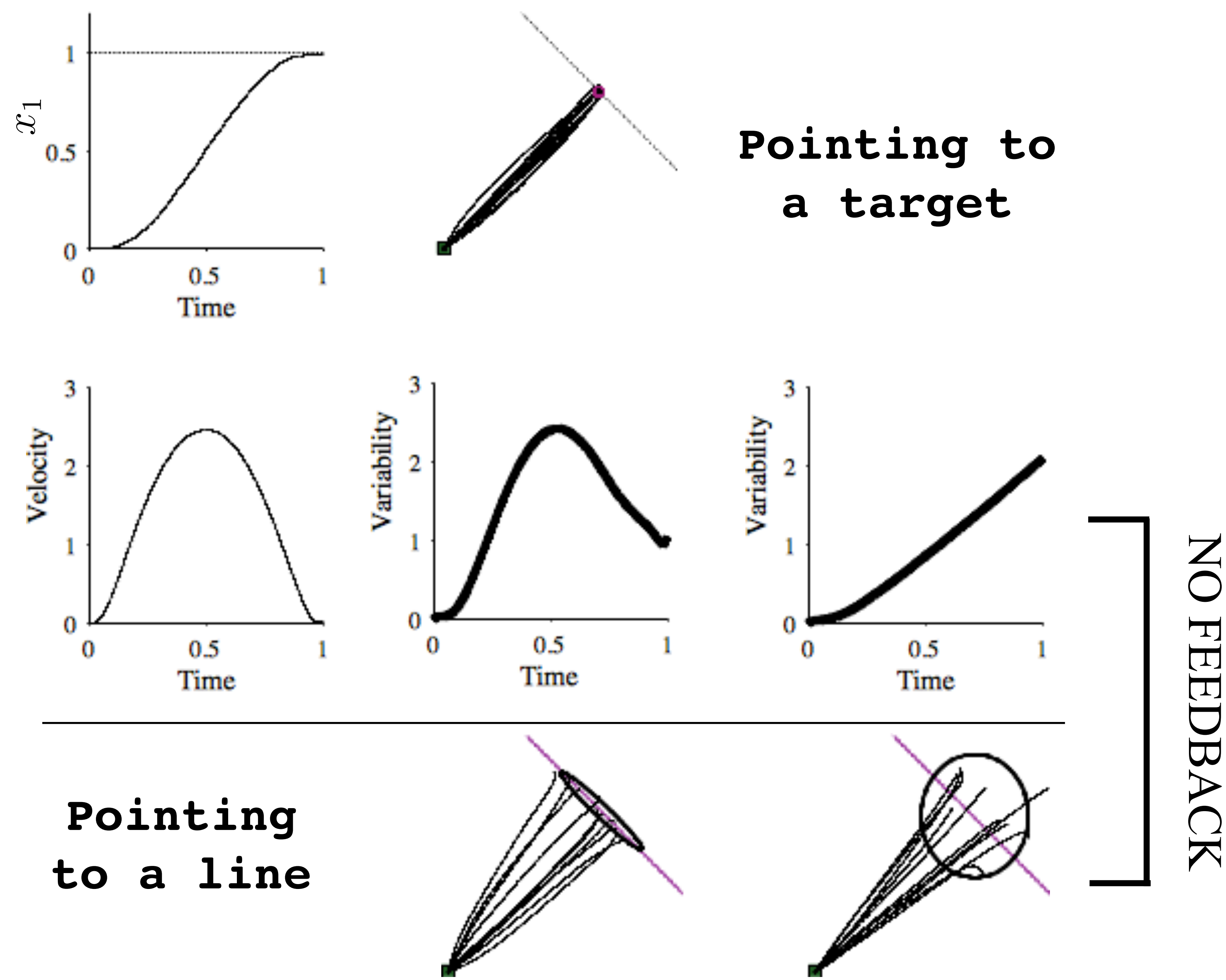
Linear Quadratic Gaussian (LQG)

simulation (continuous case)



$$\begin{cases} m_1\ddot{x}_1 = f_1 \\ m_2\ddot{x}_2 = f_2 \end{cases}$$

$$\begin{cases} f_1 = \texttt{lowpass}(u_1) \\ f_2 = \texttt{lowpass}(u_2) \end{cases}$$

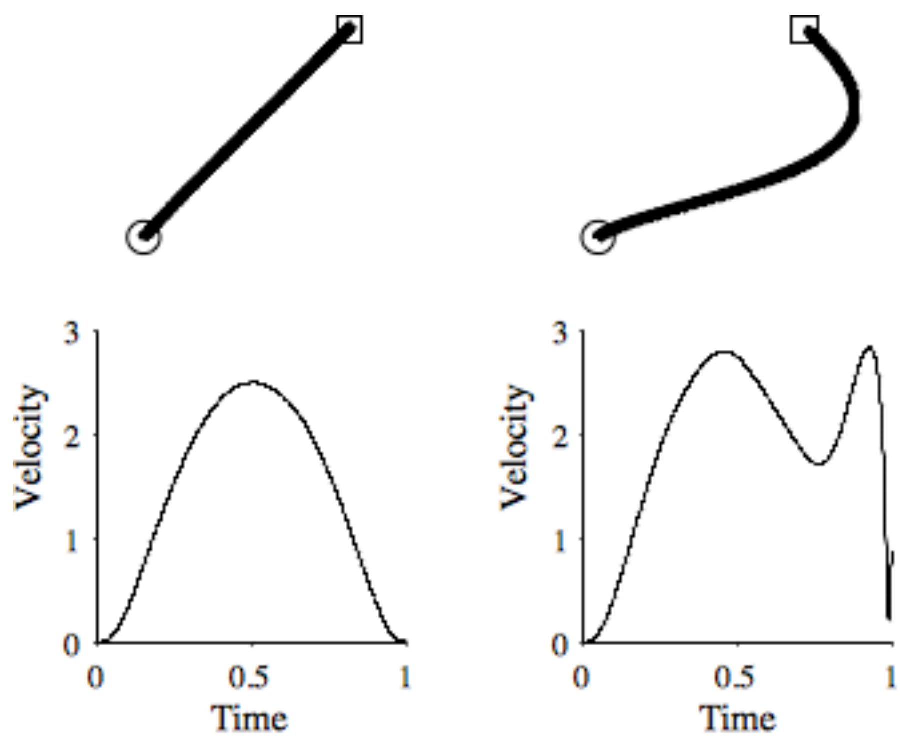


OPTIMAL FEEDBACK CONTROL

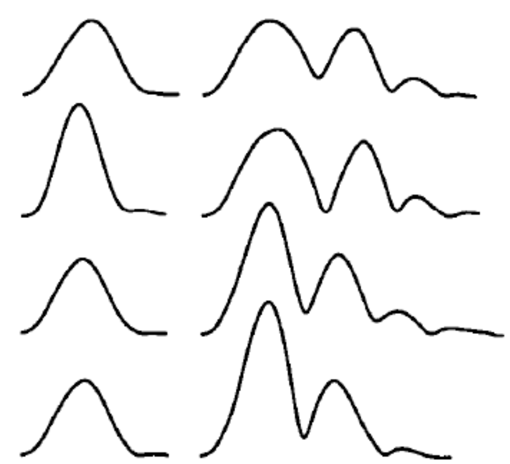
dof
kin
flex

Linear Quadratic Gaussian (LQG)

simulation (continuous case)



Perturbation (force field)



— Shadmehr & Mussa-Ivaldi, 1994, *J Neurosci* 14:3208

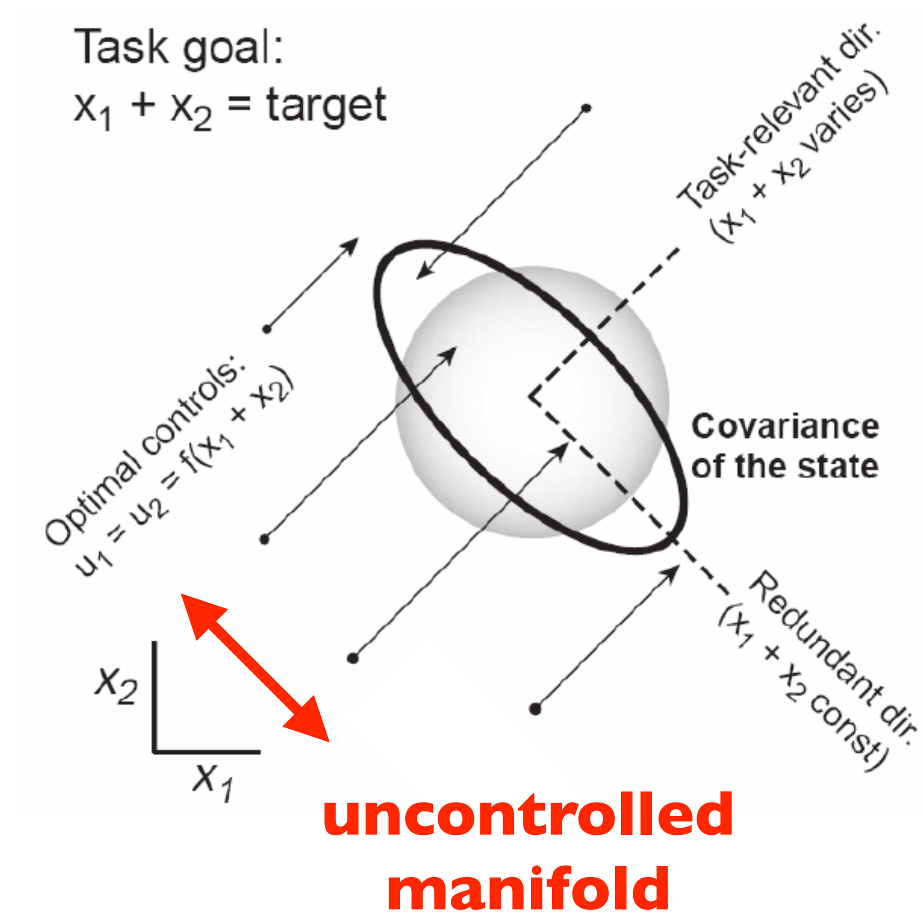
OPTIMAL FEEDBACK CONTROL

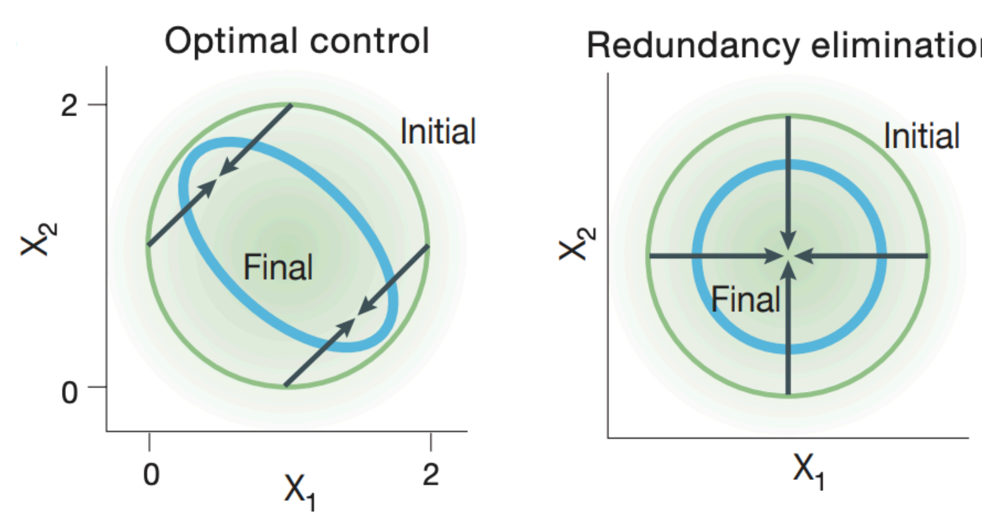
- **Paradox**

— ability to reach a goal in a fiable and repetitive way vs. variability of each trial

- **Minimum intervention**

— fluctuations on individual dof are larger than on the parameters to be controlled (i.e. specified by the task). Variability is constrained to a redundant subspace rather than being suppressed in a nonspecific manner





— Todorov & Jordan, 2002, *Nat Neurosci* 5:1226

— Scott, 2004, *Nat Rev Neurosci* 5:532

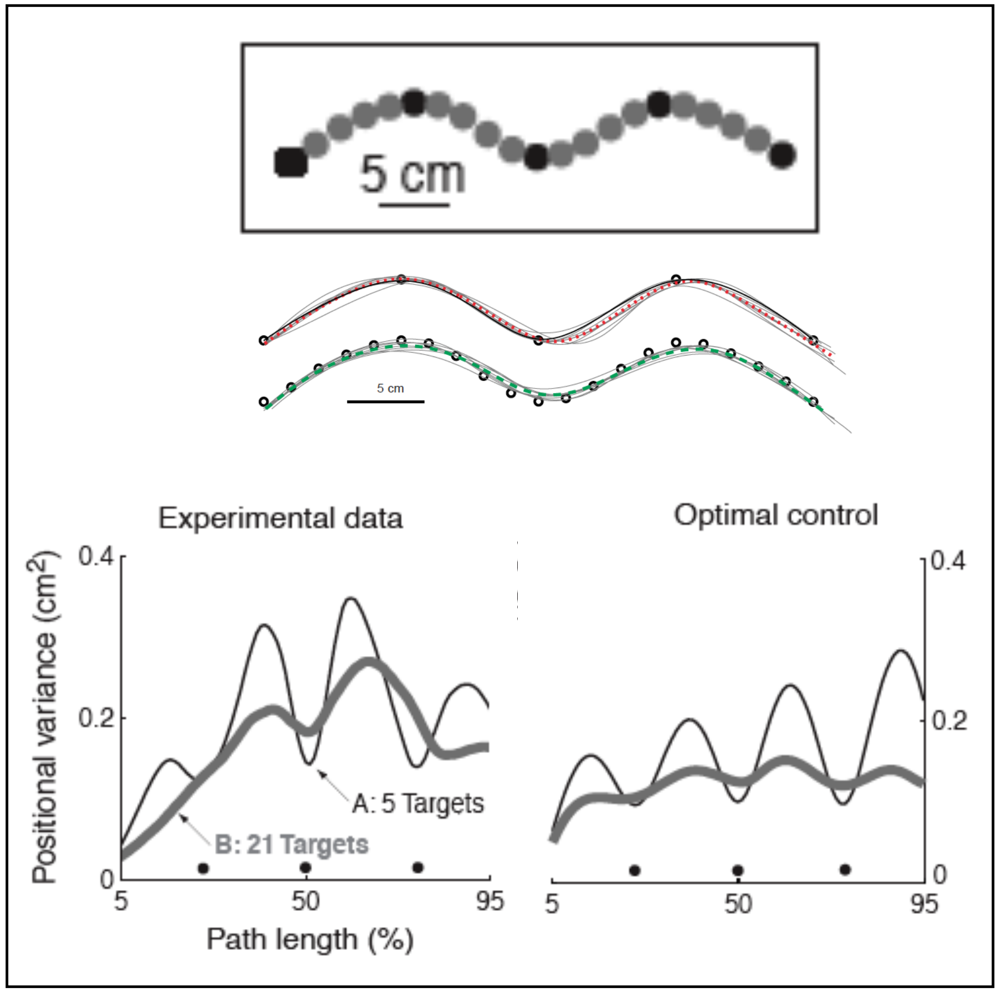
OPTIMAL FEEDBACK CONTROL

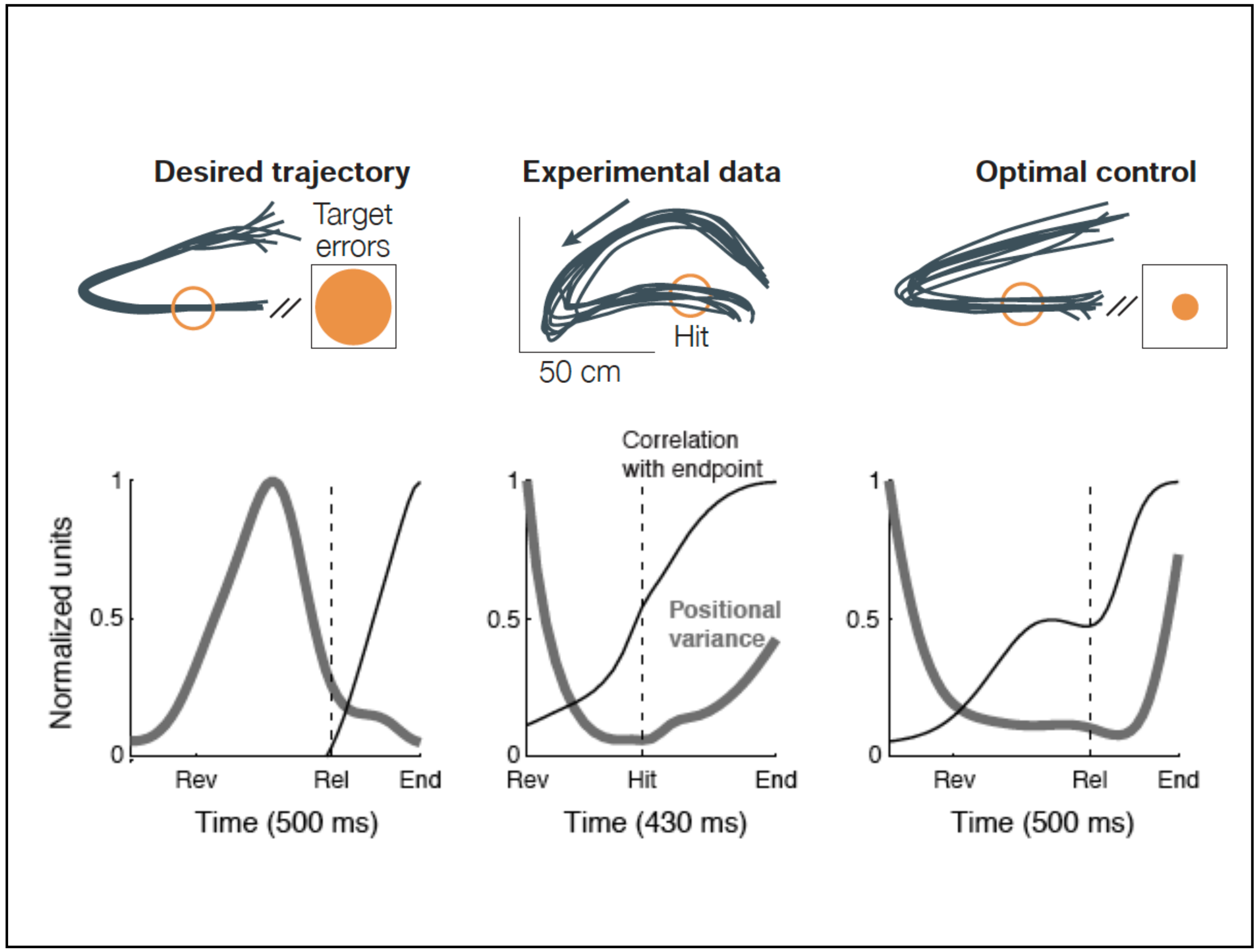
dof

kin

flex

LQG, signal-dependent noise





—Todorov & Jordan, 2002, *Nat Neurosci* 5:1226

OPTIMAL CONTROL: ISSUES

- **Energy, effort, force, force change, duration, …**

arbitrary nature, no underlying principle

- **How can the nervous system measure a cost?**

— e.g. no sensor for acceleration or jerk

- **Is it possible to prove that an observed movement is optimal for a given cost function?**

— difficult («data are accurately depicted by the model»)
— depends on the nature of the data

- **Too complex calculus?**

«good-enough»

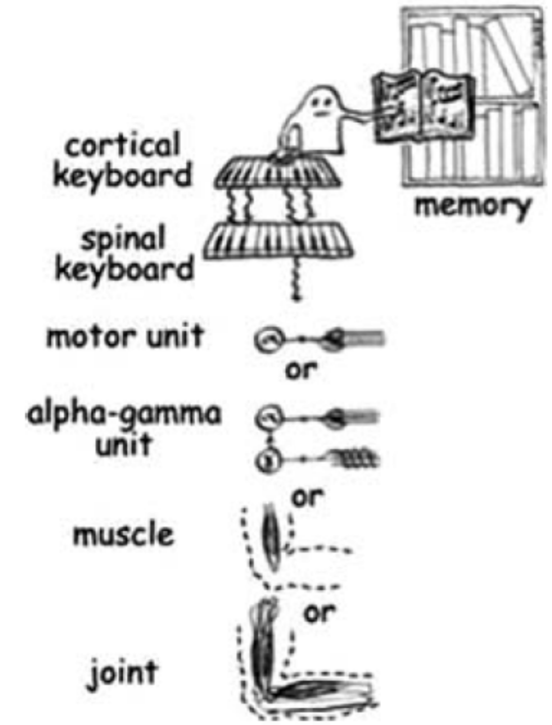
— Loeb, 2012, *Biol Cybern* 106:757

CONTROL: ISSUES

Control theory approach

— the nervous system performs computations
— two parts in the body: a controller (nervous system?) and a controlled object
— actions are represented and stored in the nervous system

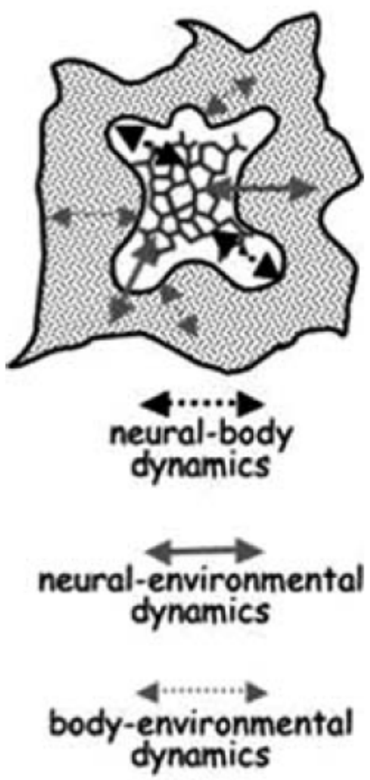
Information processing — Cognitive — Motor programs



Physical approach

— the nervous system does not perform computations
— actions are not *represented* in the brain by way of a plan or a program but emerge naturally (or self-organize) out of the physical properties of the body, the environment and the brain

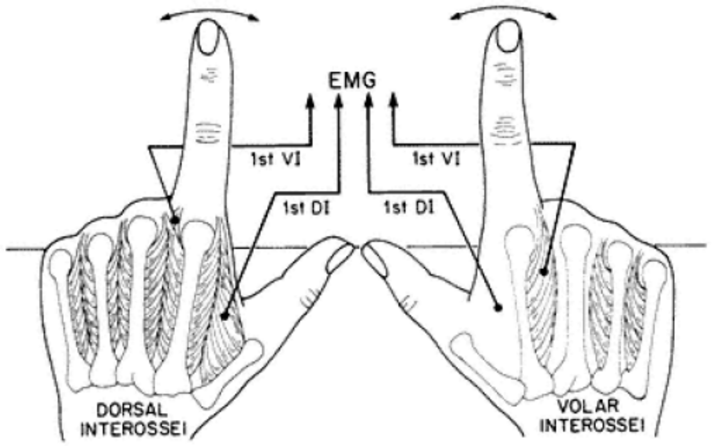
Action systems — Dynamical systems — Task dynamics

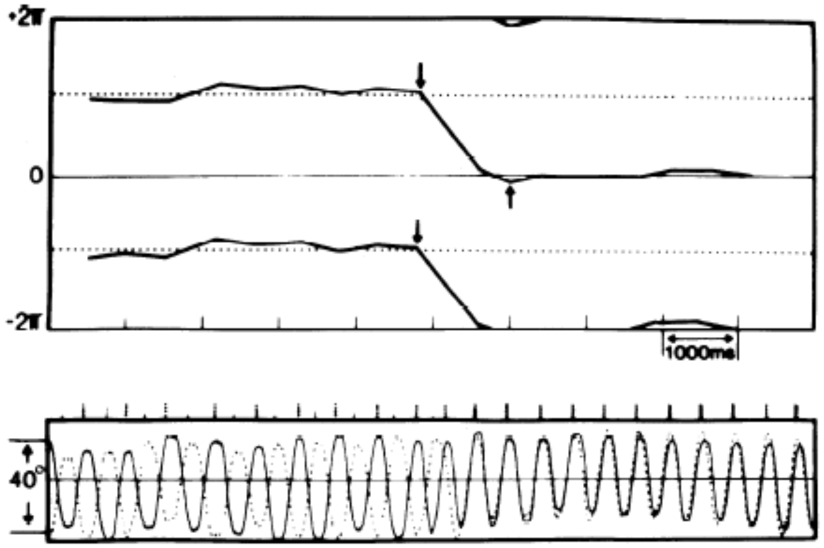


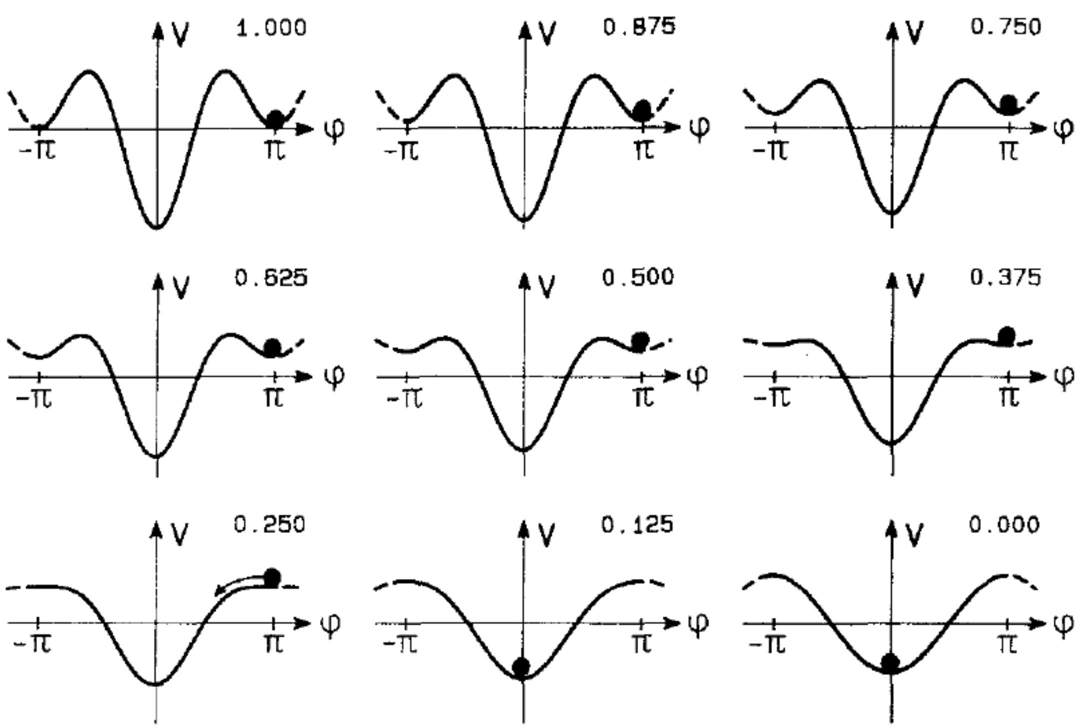
TASK DYNAMICS

Bimanual coordination

— start in opposition phase
— increasing frequency (1-5 Hz)







$$\dot{\phi} = -\frac{dV}{dt}$$

$$V = -a\cos\phi - b\cos 2\phi$$

phenomenological model

— Haken et al., 1985, *Biol Cybern* 51:347