ECE 201 – Spring 2014
Exam #3
April 17, 2014

Name (PRINT): Solutions ID#: ______________

I pledge on my honor that I have not given or received any unauthorized assistance on this examination.

Signature: ______________________________

Please identify your section number:

Prof. Peleato: 004   Prof. Peroulis: 003-006-007   Prof: Scott: 002

Instructions

1. DO NOT START UNTIL TOLD TO DO SO.

2. This is a multiple choice exam. If you mark six (6) or more questions with the same answer, e.g. all questions with answer (1), then you will receive a zero in this exam.

2. Write your name and ID# on the front page AND on your SCANTRON Sheet. IDENTIFY your section number.

3. This is a CLOSED BOOKS and CLOSED NOTES exam. No calculators are allowed.

4. Extra paper is available if needed.

5. Cheating will not be tolerated. Cheating in this exam will result in an F in this exam/course at the discretion of your instructor.

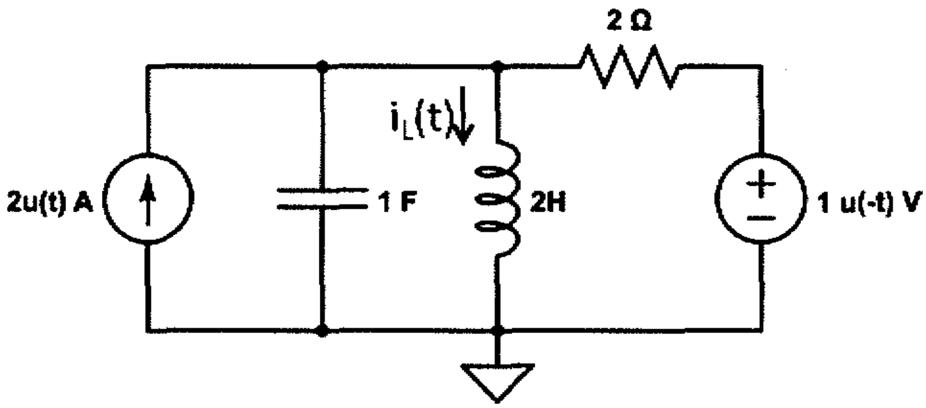
6. In the drawn circuits, the bottom node is the reference node unless otherwise specified.

7. To help you with time management we have calculated approximately how much time <u>on average</u> each question is likely to take you to solve. If you cannot solve a question, be sure to look at the other ones and come back to it if time permits.

Questions	Approximate time per question	Points per question
1 – 2 (ABET outcome iii)	2–3 minutes	11
3 – 5 (ABET outcome iii)	5–8 minutes	11
6 – 7 (ABET outcome i)	5–8 minutes	11
8-9 (ABET outcome iv)	8-12 minutes	11/12
TOTAL	**60 minutes**	**100**

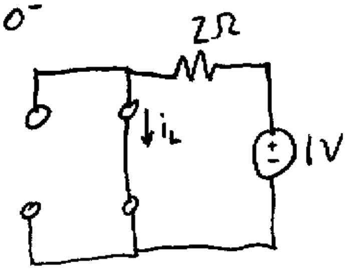
Question 1

For the current through the inductor $i_L(t)$, please find: the initial value ($i_L(0^+)$) the final value ($i_L(\infty)$), and the type of damping.



1) $i_L(0^+) = 0.5$ A, $i_L(\infty) = 1$A, underdamped
2) $i_L(0^+) = 0.5$ A, $i_L(\infty) = 1$A, overdamped
3) $i_L(0^+) = 0.5$ A, $i_L(\infty) = 2$A, underdamped
4) $i_L(0^+) = 0.5$ A, $i_L(\infty) = 2$A, overdamped
5) $i_L(0^+) = 1$ A, $i_L(\infty) = 1$A, underdamped
6) $i_L(0^+) = 1$ A, $i_L(\infty) = 1$A, overdamped
7) $i_L(0^+) = 1$ A, $i_L(\infty) = 2$A, underdamped
8) $i_L(0^+) = 1$ A, $i_L(\infty) = 2$A, overdamped
9) $i_L(0^+) = 1$ A, $i_L(\infty) = 2$A, critically-damped
10) None of the above

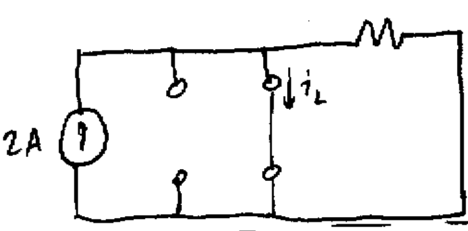
• $t = 0^-$



$i_L(0^-) = 0.5A$

$\boxed{i_L(0^+) = i_L(0^-) = 0.5A}$

• $t = \infty$



$\boxed{i_L(\infty) = 2A}$

• Damping

→ Parallel RLC → $s^2 + bs + c = 0$

$b = \frac{1}{RC}$

$c = \frac{1}{LC}$

$$\frac{-\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - \frac{4}{2}}}{2}$$

← roots are complex

→ $\boxed{\text{underdamped}}$

Question 2

Which of the following statements are true?

a) An undamped oscillator can be built with an inductor and a capacitor
b) An undamped oscillator can be built with just two inductors
c) An overdamped RLC circuit can oscillate (voltages and currents alternate between increasing and decreasing periodically)
d) An underdamped RLC circuit can oscillate (voltages and currents alternate between increasing and decreasing periodically)
e) The response of an RLC circuit with step-function sources does NOT depend on the capacitances C or the inductances L once all transient phenomena have died out.

1) all of them are true
2) Only (a), (b), (c), and (d) are true
3) Only (a), (b), (d), and (e) are true
4) Only (a), (c), and (e) are true
5) Only (a), (d), and (e) are true
6) Only (b) and (e) are true
7) Only (a) and (d) are true
8) Only (d) and (e) are true
9) Only (a) and (e) are true
10) none of the above

a) If there is no resistance in an LC circuit, roots are purely imaginary and the circuit will oscillate forever → $\boxed{\text{True}}$

b) two inductors can be combined and still make a first-order circuit. First order circuits do not oscillate → False

c) Overdamped RLC circuits have only real roots. Therefore, there can be no oscillation since it is the imaginary roots that cause this → False

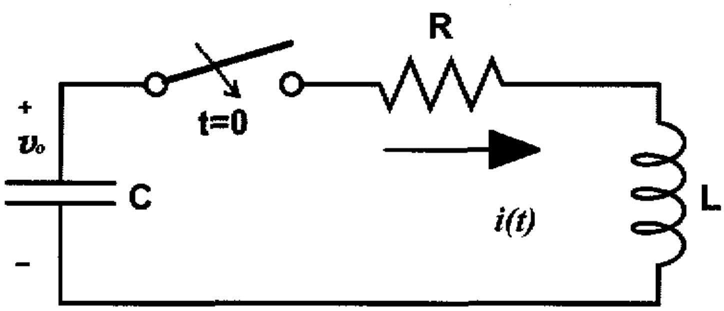
d) Underdamped circuits by definition have imaginary roots which result in some oscillation → $\boxed{\text{True}}$

e) When transients have died out (eg. @ $t=\infty$), capacitors have a fixed voltage and inductances have a fixed current regardless of their value (for example, they look like an open or short no matter their value). → $\boxed{\text{True}}$

$\therefore$ $\boxed{\text{A, D, E are true}}$

Question 3

In the circuit given below the capacitor is pre-charged at a voltage $v_c(0^-) = v_0$ while the inductor is not charged. The switch that is initially open is closed at *t=0*. Also L=C in terms of numerical values. Please find R (positive value) so the circuit is critically damped.



1) R = 1 Ω
2) R = 2 Ω
3) R = 3 Ω
4) R = 4 Ω
5) R = 5 Ω
6) R = 6 Ω
7) R = 7 Ω
8) R = 8 Ω
9) R = 0 Ω
10) None of the above

$v_c(0^+) = v_c(0^-) = v_0$

Series RLC → $s^2 + bs + c = 0$

$b = \frac{R}{L}$

$c = \frac{1}{LC} = \frac{1}{L^2}$

$$\frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{L^2}}}{2}$$

Critically-damped → $b^2 = 4c$ ⇒ $\frac{R^2}{L^2} = \frac{4}{L^2}$

$\therefore \boxed{R = 2}$

Question 4

For the circuit show in Question 3, please find the initial values $i(0^+)$ and $i'(0^+)$:

	$i(0^+)$ [A]	$i'(0^+)$ [A/s]		$i(0^+)$ [A]	$i'(0^+)$ [A/s]
1	0	0	2	v_0/R	0
3	v_0/R	v_0/L	4	$-v_0/R$	0
5	$-v_0/R$	v_0/L	6	0	v_0/L
7	0	$-v_0/L$	8	None of the above	

@ $t=0^-$, switch is open

$\Rightarrow i_L(0^-)=0$

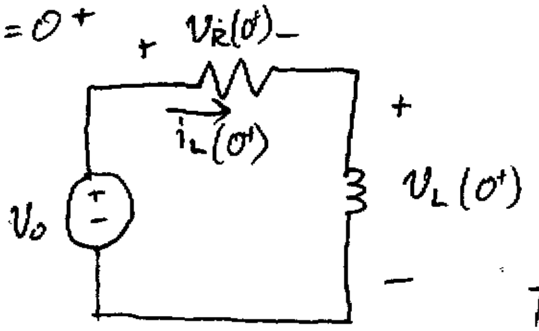
and $\boxed{i_L(0^+) = i_L(0^-) = 0}$

Can find $i_L'(0^+)$ from $V_L(0^+)$

$$V_L(0^+) = L\frac{\partial i_L(0^+)}{\partial t}$$

$$\star \quad \frac{V_L(0^+)}{L} = i_L'(0^+) = i'(0^+)$$

@ $t=0^+$



Since $i_L(0^+) = 0$ (from before),

$V_R(0^+) = i_L(0^+) \cdot R = 0$

So $V_L(0^+) = V_o$ from KVL

Therefore, (from $\star$) $\boxed{i'(0^+) = \frac{V_o}{L}}$

Question 5

For the circuit show in Question 3, please find the capacitor voltage $v_c(t), t \geq 0$:

1) $v_c(t) = v_0 \, e^{-\frac{R}{2L}t}$

2) $v_c(t) = v_0 \left[1 + \frac{R}{L}t\right] e^{-\frac{R}{L}t}$

3) $v_c(t) = v_0 \left[1 + \frac{R}{2L}t\right] e^{-\frac{R}{2L}t}$

4) $v_c(t) = v_0 \, e^{-\frac{R}{L}t}$

5$v_c(t) = v_0 \left[1 - \frac{R}{L}t\right] e^{-\frac{R}{2L}t}$

6) $v_c(t) = v_0 \left[1 - \frac{R}{2L}t\right] e^{-\frac{R}{L}t}$

7) $v_c(t) = v_0 \left[1 - \frac{R}{2L}t\right] e^{-\frac{R}{2L}t}$

8) None of the above

For this response, need:
1) Initial + final conditions $(v_c(0^+), v_c'(0^+), v_c(\infty))$
2) Type of damping (to get general form) and roots

1) $v_c(0^+) = V_0$ (from problem statement)

$v_c'(0^+) = \frac{i_c(0^+)}{C} = 0$ (from $i_c(t) = C\frac{dv_c(t)}{dt}$)

$v_c(\infty) = 0$ (no sources on, so resistor will eventually consume all energy)

2) Circuit is still critically-damped (from problem statement), $s = \frac{-b}{2} = \frac{-R}{2L}$

→ General form (from Table 9.2)

$v_c(t) = (K_1 + K_2 t)e^{st} + X_F$

where $v_c(0^+) = K_1 + X_F$

$v_c'(0^+) = K_1 \cdot s_1 + K_2$

$X_F = 0$, so $K_1 = v_c(0^+) = V_0$

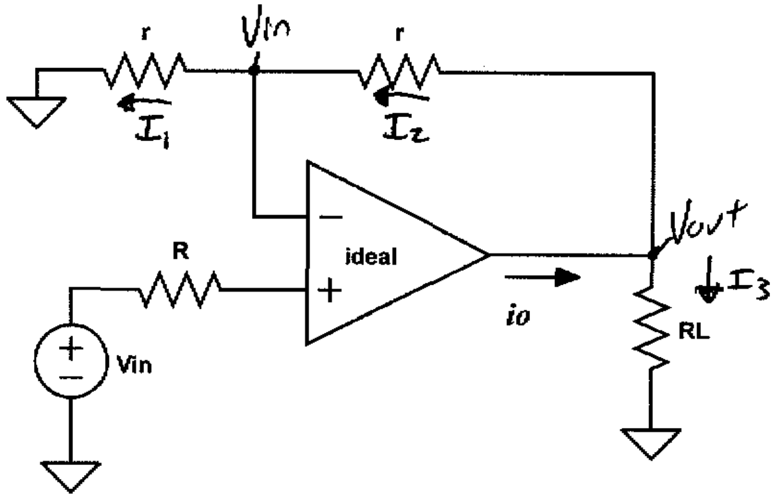
$v_c'(0^+) = 0 = V_0 \cdot s_1 + K_2$

$\Rightarrow K_2 = \frac{R}{2L} \cdot V_0$

$\therefore v_c(t) = \left(v_0 + v_0 \cdot \frac{R}{2L}t\right)e^{\frac{-R}{2L}t} \Rightarrow \boxed{v_c(t) = v_0\left(1 + \frac{R}{2L}t\right)e^{\frac{-R}{2L}t}}$

Question 6

In the circuit below, please find i_0. The opamp is ideal.



1) $i_0 = v_{in}\left(\frac{1}{r}+\frac{2}{RL}\right)$

2) $i_0 = v_{in}\left(\frac{1}{r}-\frac{2}{RL}\right)$

3) $i_0 = {}^{v_{in}}/_{r}$

4) $i_0 = {}^{v_{in}}/_{RL}$

5) $i_0 = v_{in}\left(\frac{1}{r}+\frac{1}{RL}\right)$

6) $i_0 = v_{in}\left(\frac{1}{r}-\frac{1}{RL}\right)$

7) $i_0 = v_{in}\left(\frac{2}{r}+\frac{2}{RL}\right)$

8) $i_0 = v_{in}\left(\frac{2}{r}-\frac{2}{RL}\right)$

9) None of the above

$i_+ = i_- = 0$, so no voltage drop across R

$\therefore V_+ = V_{in}$

also, $V_- = V_+$ (ideal op-Amp)

★ $i_0 = I_2 + I_3$

$I_2 = I_1 = \frac{V_{in}}{r} = \frac{V_{out} - V_{in}}{r} \Rightarrow V_{out} = 2V_{in}$

$i_0 = I_2 + I_3$

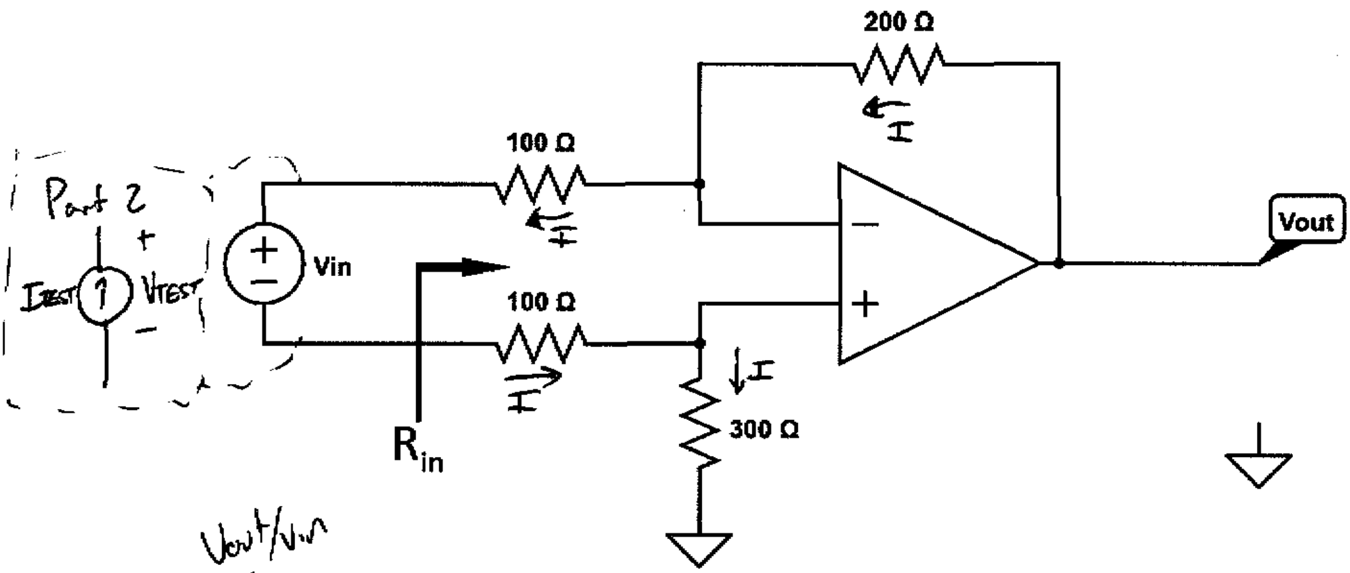
$= \frac{V_{in}}{r} + \frac{V_{out}}{R_L}$

$= \frac{V_{in}}{r} + \frac{2V_{in}}{R_L}$

$\boxed{i_0 = V_{in}\left[\frac{1}{r} + \frac{2}{R_L}\right]}$

Question 7 Vout/Vin

Find ~~Vin/Vout~~ and the input resistance for the following circuit. The opamp is ideal.



Vout/Vin

1) Vin/Vout = -2, Rin = 100 Ω
2) Vin/Vout = 2, Rin = 100 Ω
3) Vin/Vout = -2, Rin = 200 Ω
4) Vin/Vout = 2, Rin = 200 Ω
5) Vin/Vout = -2.5, Rin = 100 Ω
6) Vin/Vout = 2.5, Rin = 100 Ω
7) Vin/Vout = -2.5, Rin = 200 Ω
8) Vin/Vout = 2.5, Rin = 200 Ω
9) Vin/Vout = -3, Rin = 100 Ω
10) none of the above

• $\frac{V_{out}}{V_{in}}$ since $i_+ = i_- = 0$, there is one big path for the current

KVL: $V_{out} = I(200 + 100 + 100 + 300) + V_{in}$

$V_{out} = 700I + V_{in}$

but $V_+ = V_-$, so $200I + V_{in} = 0$

$\Rightarrow I = \frac{-V_{in}}{200}$

$\rightarrow V_{out} = -\frac{V_{in}}{200} \cdot 700 + V_{in}$

$$\boxed{\frac{V_{out}}{V_{in}} = -3.5 + 1 = -2.5}$$

• $R_{in} \Rightarrow$ Assume I_{TEST} is on input + solve for $\frac{V_{TEST}}{I_{TEST}} \triangleq R_{in}$

$\Rightarrow$ Again, since $V_+ = V_-$,

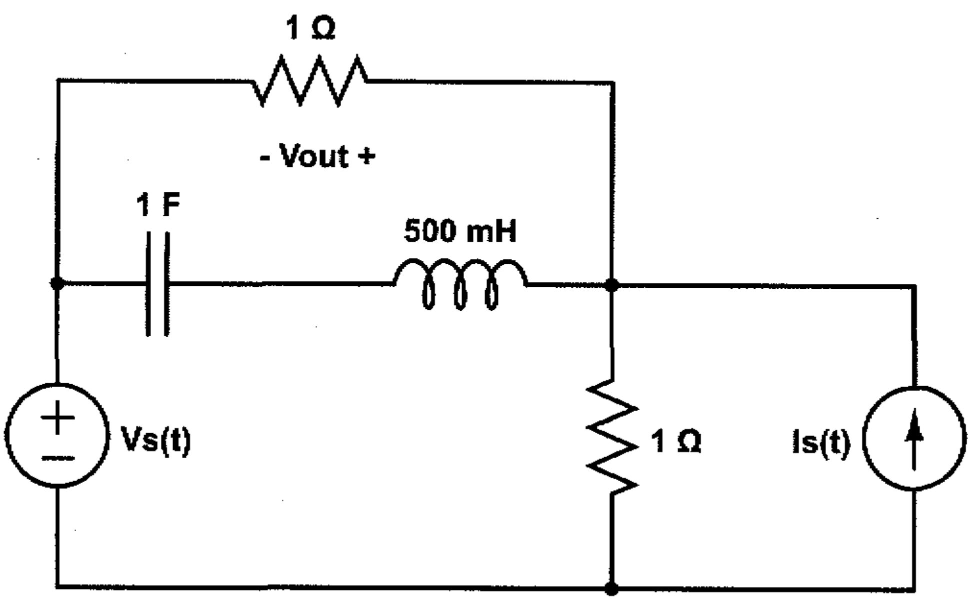
$V_{TEST} - 100I_{TEST} - 100I_{TEST} = 0$

$V_{TEST} = 200I_{TEST}$

$$\boxed{\frac{V_{TEST}}{I_{TEST}} = 200\Omega} = R_{in}$$

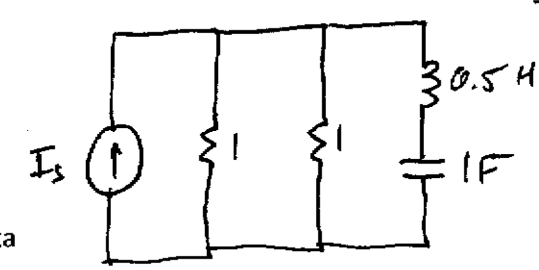
Question 8

In the circuit below, $Vs(t) = \cos(t)$ and $Is(t) = \sqrt{2}\cos(2t - 45^0)$. Find the output voltage Vout(t) due to Is(t) ONLY. You might need the following identities:
$\cos(45^0) = 1/\sqrt{2}$, $\sin(45^0) = 1/\sqrt{2}$

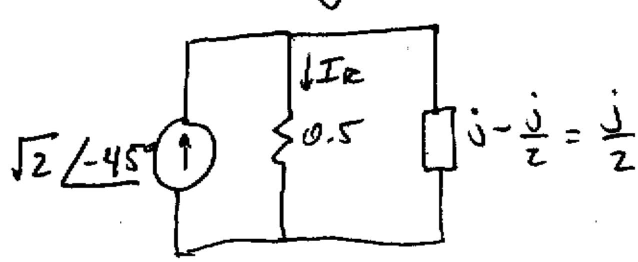


1) Vout(t) = 0.5cos(2t)
2) Vout(t) = 0.5cos (t)
3) Vout(t) = 0.5cos(2t-45°)
4) Vout(t) = sin(2t)
5) Vout(t) = cos(2t)
6) Vout(t) = $\sqrt{2}/2$ cos(2t)
7) Vout(t) = $\sqrt{2}/2$ sin(2t-45°)
8) Vout(t) = $\sqrt{2}/2$
9) It cannot be found with the given data
10) None of the above

Finding only from I_s so V_s is off
⇒ short-circuit



⇓



From current division

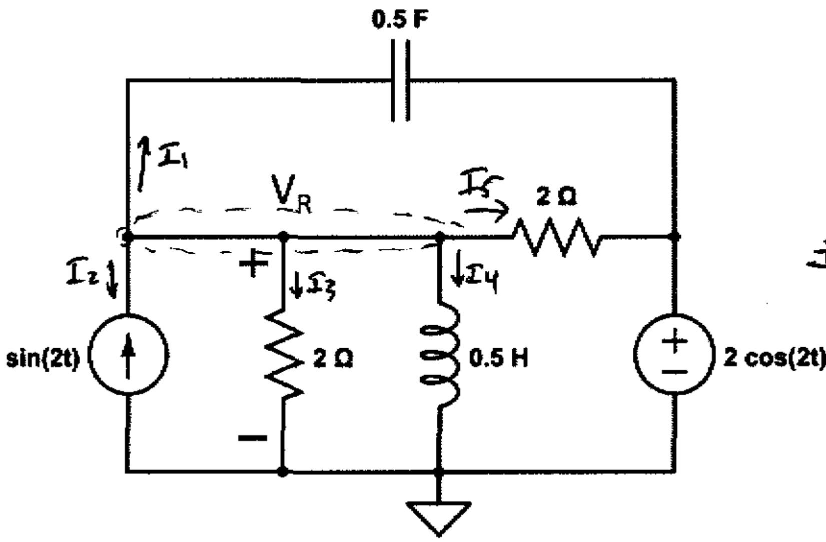
$$I_R = \sqrt{2}\angle{-45} \cdot \frac{\frac{1}{0.5}}{\frac{1}{0.5} + \frac{2}{j}} = \sqrt{2}\angle{-45} \cdot \frac{2}{2+j2} = \sqrt{2}\angle{-45} \cdot \frac{2}{2\sqrt{2}\angle 45} = 1\angle 0^\circ A$$

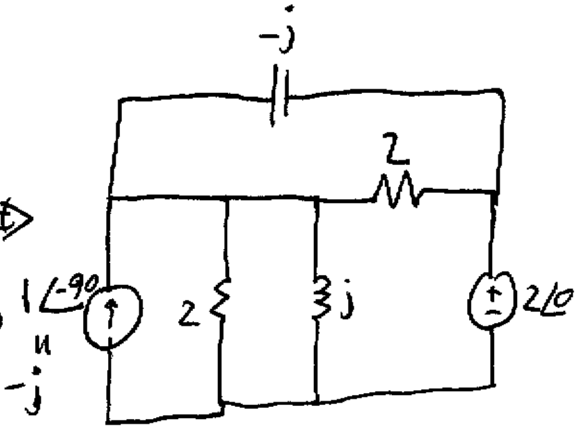
$$\therefore \tilde{V}_{out} = 0.5\Omega \cdot 1\angle 0^\circ = 0.5\angle 0^\circ$$

$\boxed{v(t) = 0.5\cos(2t)\ V}$

Question 9

What is the voltage across the left resistor ($V_R(t)$) in the circuit below? Hint: use nodal analysis





1) cos (2t) V
2) cos (2t - pi/4) V
3) cos (2t + pi/4) V
4) cos (2t - pi/2) V
5) cos (2t + pi/2) V
6) √2 cos (2t - pi/4) V
7) √2 cos (2t + pi/4) V
8) √2 cos (2t - pi/2) V
9) √2 cos (2t + pi/2) V
10) None of the above

★ Nodal Analysis: $I_1 + I_2 + I_3 + I_4 + I_5 = 0$

$$I_1 = \frac{\tilde{V}_R - 2}{-j}$$

$$I_2 = +j$$

$$I_3 = \frac{\tilde{V}_R}{2}$$

$$I_4 = \frac{\tilde{V}_R}{j}$$

$$I_5 = \frac{\tilde{V}_R - 2}{2}$$

★ $\therefore\ j\tilde{V}_R - 2j + j + \frac{\tilde{V}_R}{2} - j\tilde{V}_R + \frac{\tilde{V}_R}{2} - 1 = 0$

$$\tilde{V}_R\left(j + \tfrac{1}{2} - j + \tfrac{1}{2}\right) = 2j - j + 1$$

$$\tilde{V}_R = 1 + j = \sqrt{2}\angle 45^\circ$$

$$V_r(t) = \sqrt{2}\cos(2t + 45^\circ)\ \text{V}$$

$$\boxed{V_r(t) = \sqrt{2}\cos(2t + \pi/4)\ \text{V}}$$

TABLE 9.2 General Solutions for Constant-Source Second-Order Networks

General solution of the driven differential equation

$$\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = F$$

having characteristic equation $s^2 + bs + c = (s - s_1)(s - s_2) = 0$, with roots

$$s_1, s_2 = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$$

Case 1. Real and distinct roots; $b^2 - 4c > 0$:

$$x(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} + X_F$$

where $X_F = F/c$, and

$$x(0^+) = K_1 + K_2 + X_F$$
$$x'(0^+) = s_1 K_1 + s_2 K_2$$

Case 2. The roots $s_1 = -\sigma + j\omega_d$ and $s_2 = -\sigma - j\omega_d$ of the characteristic equation are distinct but complex; $b^2 - 4c < 0$:

$$x(t) = e^{-\sigma t}[A\cos(\omega_d t) + B\sin(\omega_d t)] + X_F$$

where again $X_F = F/c$, and

$$x(0^+) = A + X_F$$
$$x'(0^+) = -\sigma A + \omega_d B$$

Case 3. The roots are real and equal; $s_1 = s_2$ and $b^2 - 4c = 0$:

$$x(t) = (K_1 + K_2 t)e^{s_1 t} + X_F$$

where again $X_F = F/c$, and

$$x(0^+) = K_1 + X_F$$
$$x'(0^+) = s_1 K_1 + K_2$$