ECE 20100 –Fall 2014
Exam #2

October 22, 2014

Section (circle below)

Bermel – 002	Peleato-Inarrea – 004	Qi – 005
Gray – 006	Allen – 007	Lin - 010

Name Solutions PUID____________

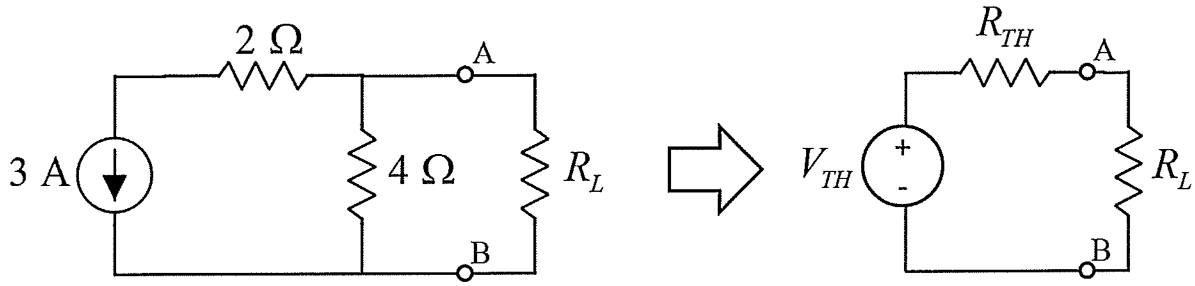
Instructions

1. DO NOT START UNTIL TOLD TO DO SO.
2. Write your name, section, professor, and student ID# on your **Scantron** sheet. We may check PUIDs.
3. This is a CLOSED BOOKS and CLOSED NOTES exam.
4. Calculators are NOT allowed (and not necessary).
5. If extra paper is needed, use the back of test pages.
6. Cheating will not be tolerated. Cheating in this exam will result in, at the minimum, an F grade for the course. In particular, **continuing to write after the exam time is up is regarded as cheating.**
7. If you cannot solve a question, be sure to look at the other ones, and come back to it if time permits.
8. Exam #2 provides evidence for satisfaction of this ECE 20100 Learning Objective:
   ii) An ability to analyze 1st order linear circuits with sources and/or passive elements.
   The minimum score needed to satisfy this objective will be posted on Blackboard after the exam has been graded. Remediation options will be posted in Blackboard if you fail to satisfy any of the course outcomes.
9. **The last page of the exam contains potentially useful formulas.**

Honor Pledge: I have neither given nor received unauthorized assistance on this exam

Signature:______________________________

1. For the Thevenin equivalent circuit seen by the load resistor R_L, what is the Thevenin equivalent voltage V_{TH}?

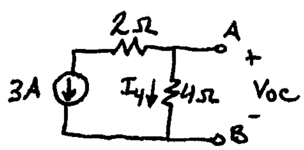


(1) $V_{TH} = 0$ V (2) $V_{TH} = -3$ V (3) $V_{TH} = -6$ V (4) $V_{TH} = -9$ V

(5) $V_{TH} = -12$ V (6) $V_{TH} = -15$ V (7) $V_{TH} = -18$ V (8) $V_{TH} = -\infty$ V

(9) None of the above

- V_{TH} is V_{oc}, the voltage across A & B when R_L is replaced by an open:



- $V_{TH} = V_{oc} = V_{AB}$ is the voltage across the 4Ω resistor:

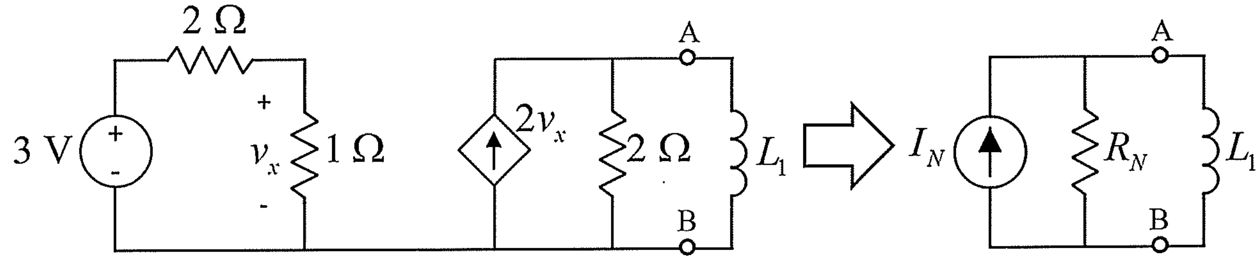
① $V_{AB} = I_4 (4\Omega)$ ← Ohm's law (w/ I_4 & V_{AB} in <u>passive</u> notation)

② $I_4 = -3$ A ← because 4Ω resistor is in series with 3A source

②→①: $V_{AB} = -3(4) = -12$ V

$\boxed{\therefore V_{TH} = -12\text{V}}$

2. For the Norton equivalent circuit seen by the inductor L_1, what is the Norton equivalent resistance R_N?

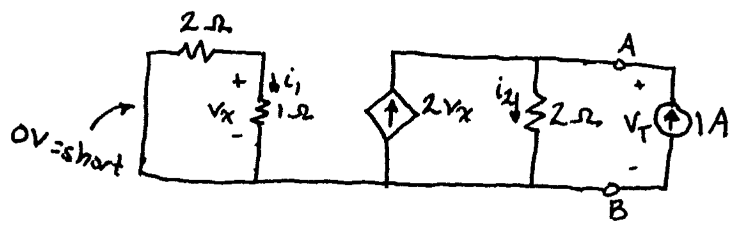


(1) $R_N = 1\ \Omega$ | $\boxed{(2)\ R_N = 2\ \Omega}$ | (3) $R_N = 3\ \Omega$ | (4) $R_N = 4\ \Omega$

(5) $R_N = 5\ \Omega$ | (6) $R_N = 6\ \Omega$ | (7) $R_N = 7\ \Omega$ | (8) $R_N = 0\ \Omega$

(9) $R_N = \infty\ \Omega$ | (10) None of the above

- To find R_N, zero all independent source and replace L_1 by a test source. $R_N = \frac{V_T}{I_T}$
- I choose a 1A source (arbitrarily) for my test source:



V_T is voltage across 2Ω resistor @ right (parallel w/ 1A source)

① $\therefore V_T = i_2\,(2)$ ← Ω's law

Due to KCL @ A:

② $i_2 = 2V_x + 1$

Because there is no source on the left side of the circuit,

③ $V_x = i_1\,(1)$ ← Ω's law

$= 0(1) = 0$

Plug ③ into ②:

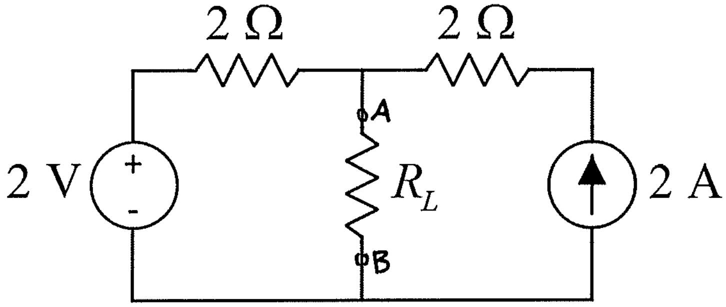
④ $i_2 = 0 + 1 = 1$

Plug ④ into ①:

$V_T = (1)(2) = 2$

$\boxed{\therefore R_N = \frac{V_T}{I_T} = \frac{2}{1} = 2\Omega}$

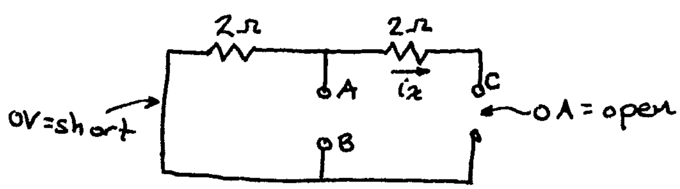
3. What value of resistor R_L will result in the maximum power being absorbed by R_L?



(1) 1 Ω $\boxed{\text{(2) } 2\ \Omega}$ (3) 3 Ω (4) 4 Ω (5) 8 Ω

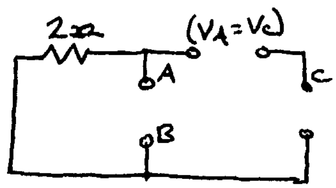
(6) 12 Ω (7) 16 Ω (8) 0 Ω (9) ∞ Ω (10) None of the above

- Max power absorbed by R_L when $R_L = R_{TH}$, where R_{TH} is the Thevenin equivalent resistance of the circuit to which R_L is connected.
- Thevenin equivalent resistance is found by zeroing out independent resistance and using resistor combination (since there are no dependent sources).



i_x is 0A since it is in series with an open.

∴ The 2Ω resistor i_x is going through is an open (w/ $V_A = V_c$ due to Ω's law):

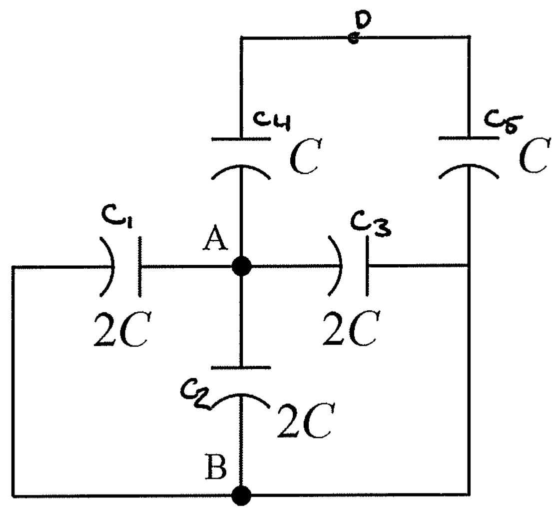


The only thing between A & B is a 2Ω resistor.

$$\boxed{\therefore\ R_L = R_{TH} = R_{eq} = 2\Omega}$$

For max power absorbed by R_L

4. Find the equivalent capacitance seen between nodes A and B, in terms of capacitance C.



(1) $0.5C$ (2) $2C$ (3) $2.5C$ (4) $4C$ (5) $4.5C$

(6) $6C$ $\boxed{(7)\ 6.5C}$ (8) $8C$ (9) $8.5C$ (10) None of the above

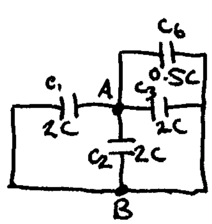
- $\frac{1}{C}$ combines like R
- C_4 and C_5 are connected to node D and nothing else is. Therefore, C_4 and C_5 are in series:

  R in series $\rightarrow$ $R_{eq} = R_4 + R_5$

  C in series $\rightarrow$ $\frac{1}{C_{eq}} = \frac{1}{C_4} + \frac{1}{C_5}$

  $\therefore C_6 = \left(\frac{1}{C_4} + \frac{1}{C_5}\right)^{-1} = 0.5C$

- So now,



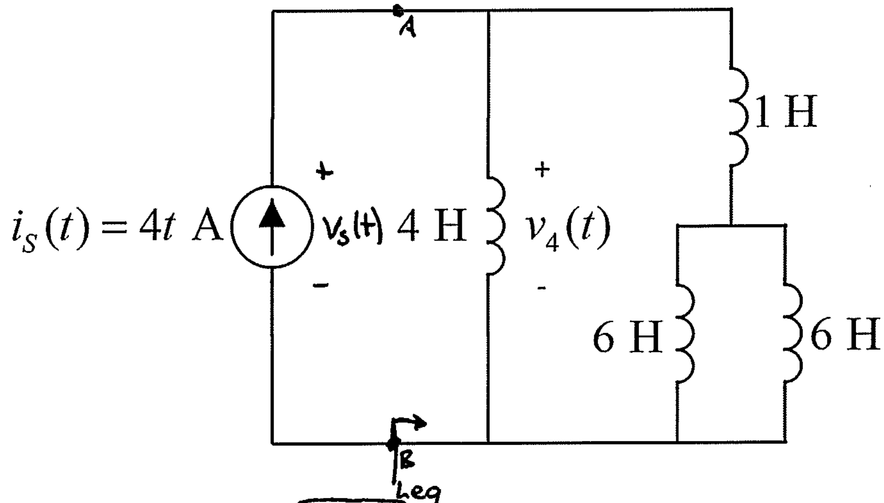
- C_1, C_2, C_3, and C_6 are all connected to both node A and node B. Therefore C_1, C_2, C_3, and C_6 are in parallel.

  R in parallel $\rightarrow$ $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_6}$

  C in parallel $\rightarrow$ $C_{eq} = C_1 + C_2 + C_3 + C_6$

  $\boxed{\therefore C_{AB} = 2C + 2C + 2C + 0.5C = 6.5C}$

5. What is the voltage $v_4(t)$ across the 4 H inductor?



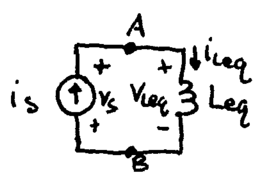
(1) 2 V (2) 4 V $\boxed{\text{(3) 8 V}}$ (4) 16 V (5) $2t$ V

(6) $4t$ V (7) $8t$ V (8) $16t$ V (9) 0 V (10) None of the above

- $V_4(t) = V_s(t)$ because they are parallel
- If I find $L_{eq} = L_{AB}$ then

$$V_s(t) = V_{L_{eq}}(t) = L_{eq} \frac{d\, i_{L_{eq}}}{dt} = L_{eq} \frac{d\, i_s}{dt} \quad ①$$

↑ because L_{eq} and I_s source are in series



- Finding L_{eq}: L combines like R

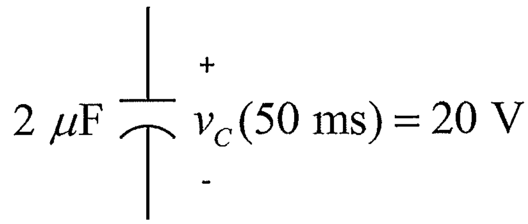
$$\therefore\ L_{eq} = 4 \,//\, (1 + 6//6) = 4 \,//\, (1+3) = 4 \,//\, 4 = 2\ H \quad ②$$

- Plug ② into ①:

$$V_4 = V_s = 2 \frac{d}{dt}(4t) = 2(4) = 8\ V$$

$$\boxed{\therefore\ V_4(t) = 8\,V}$$

6. A 2 μF capacitor is charged to a voltage of 20 V in 50 ms. Assuming the energy stored in the capacitor was initially 0 J, what is the energy stored in the capacitor at time $t = 50$ ms ?



(1) $W_C = 0$ J (2) $W_C = 20$ mJ (3) $W_C = 40$ mJ (4) $W_C = 80$ mJ

(5) $W_C = 100$ μJ (6) $W_C = 200$ μJ (7) $W_C = 300$ μJ $\boxed{(8)\ W_C = 400\ \mu\text{J}}$

(9) None of the above

- Total energy stored in a capacitor @ time t_1 is

$$① \; W_c(t_1) = \frac{1}{2} C v_c^2(t_1)$$

- In this problem,

$$t_1 = 50\text{ms} = 50 \times 10^{-3}\text{s}$$

$$v_c(t_1) = v_c(50\text{ms}) = 20\text{ V}$$

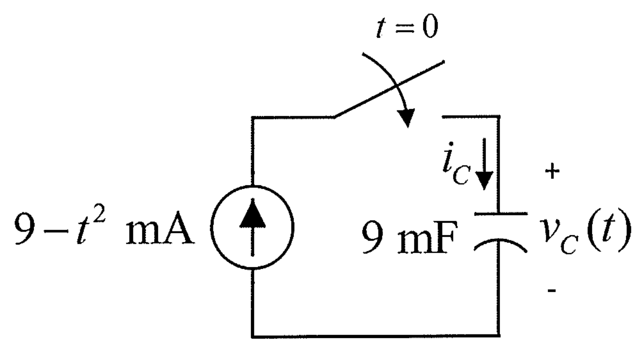
$$C = 2\mu\text{F} = 2 \times 10^{-6}\text{ F}$$

- Plugging those into ①:

$$W_c(50\text{ms}) = \frac{1}{2}(2 \times 10^{-6})(20)^2$$

$$\boxed{\therefore W_c(50\text{ms}) = 400 \times 10^{-6} = 400\mu\text{J}}$$

7. A 9 mF capacitor has an initial voltage of $v_C(0) = 5$ V. If the current through the capacitor is given by $i_C(t) = 9 - t^2$ mA for $t \geq 0$ s, what is $v_C(3)$?



(1) 1 V (2) 2 V (3) 3 V (4) 4 V (5) 5 V

(6) 6 V $\boxed{\text{(7) 7 V}}$ (8) 8 V (9) 9 V (10) None of these

- If the current through a capacitor is known, the voltage drop across it is:

$$① \quad v_c(t) = v_c(t_0) + \frac{1}{C}\int_{t_0}^{t} i_c(t')\,dt'$$ ← for v_c and i_c in passive notation

for $t \geq t_0$, if $v_c(t_0)$ is known (and, obviously, if the circuit doesn't change between t_0 and t)

- For our circuit,

$t_0 = 0$

$v_c(t_0) = v_c(0) = 5V$

$C = 9\,mF = 9 \times 10^{-3}$ F

$i_c(t') = 9 - t'^2$ mA $= (9 - t'^2) \times 10^{-3}$

- Plugging those into ①, for $t \geq 0$:

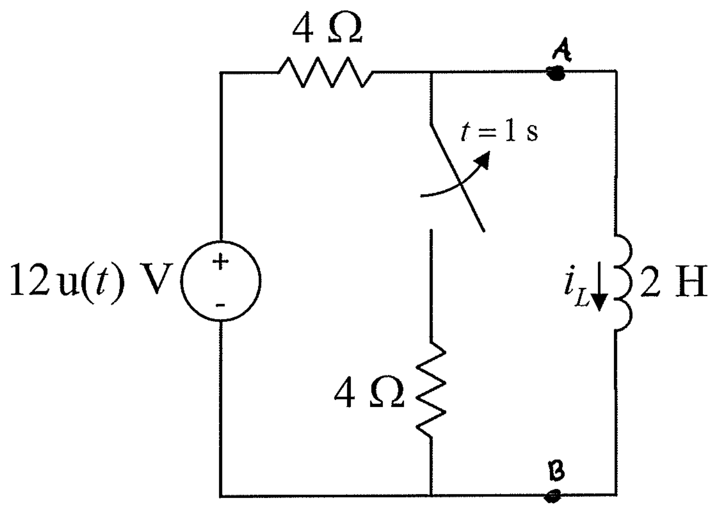
$$v_c(t) = 5 + \frac{1}{9 \times 10^{-3}} \int_0^t 10^{-3} \times (9 - t'^2)\,dt'$$

$$= 5 + \frac{1}{9}\left(9t' - \frac{1}{3}t'^3\right)\Big|_{t'=0}^{t}$$

$$= 5 + \frac{1}{9}\left(9t - \frac{t^3}{3}\right) = 5 + t - \frac{t^3}{27}$$

$$\boxed{\therefore v_c(3) = 5 + 3 - \frac{(3)^3}{27} = 7V}$$

Use this circuit for questions 8, 9, and 10. The switch is closed for a long time and then opened at $t = 1$ s. At $t = 0^-$, the inductor current is known to be $i_L(0^-) = 0$ A.

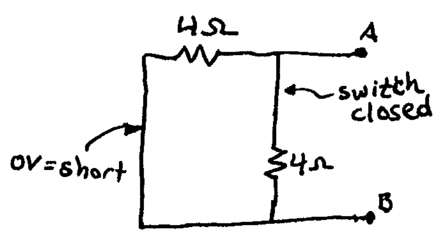


8. For $0 \le t < 1$ s, what is the time constant, τ?

(1) $\tau = 0$ s (2) $\tau = \frac{1}{4}$ s (3) $\tau = \frac{1}{2}$ s $\boxed{(4)\ \tau = 1\text{ s}}$ (5) $\tau = 2$ s

(6) $\tau = 4$s (7) $\tau = 8$s (8) $\tau = 16$s (9) $\tau = \infty$s (10) None of the above

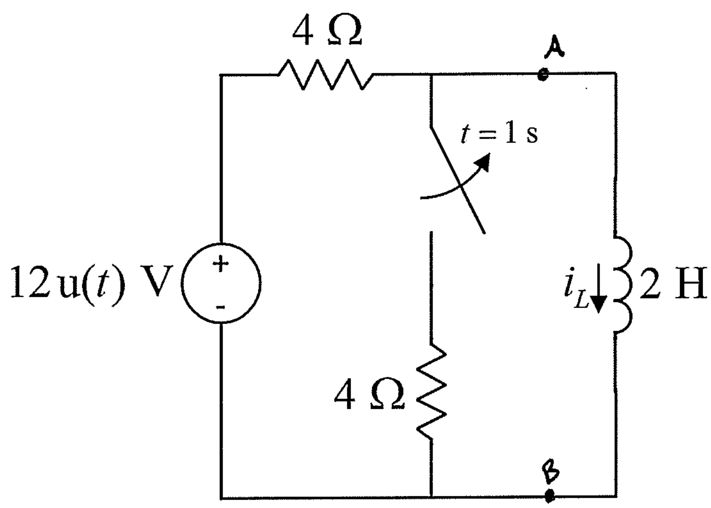
- τ for an inductor is $\frac{L}{R_{TH}}$ where R_{TH} is the Thevenin equivalent resistance of the circuit to which L is connected
- For $0 \le t \le 1$s, the circuit connected to A and B looks like (with the independent sources zeroed):



$\therefore R_{AB} = R_{TH} = 4 \| 4 = 2\,\Omega$

$$\boxed{\therefore \tau = \frac{L}{R_{TH}} = \frac{2\text{ H}}{2\,\Omega} = 1\text{ s}}$$

Use this circuit for questions 8, 9, and 10. The switch is closed for a long time and then opened at $t = 1$ s. At $t = 0^-$, the inductor current is known to be $i_L(0^-) = 0$ A.

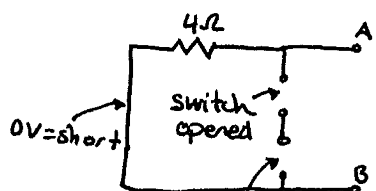


9. For $t \geq 1$ s, what is the time constant, τ?

(1) $\tau = 0$ s (2) $\tau = \frac{1}{4}$ s $\boxed{(3)\ \tau = \frac{1}{2}\text{ s}}$ (4) $\tau = 1$ s (5) $\tau = 2$ s

(6) $\tau = 4$s (7) $\tau = 8$s (8) $\tau = 16$s (9) $\tau = \infty$s (10) None of the above

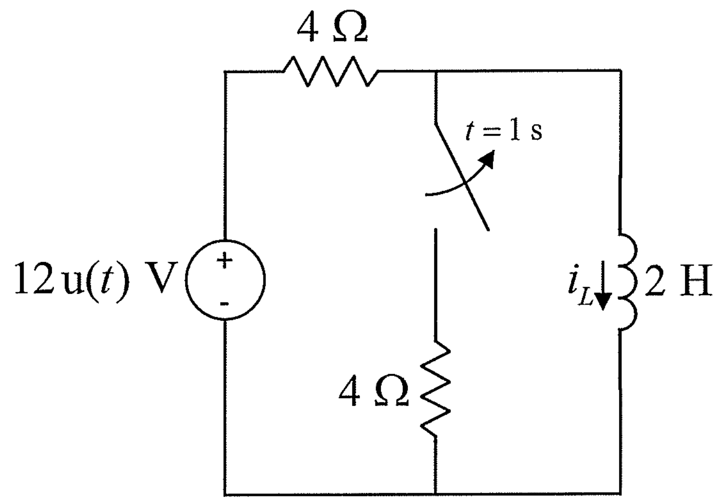
- Trying to find $\tau = \frac{L}{R_{TH}}$
- At $t \geq 1$s, with independent sources zeroed, circuit to left of A-B looks like:



$\therefore R_{TH} = R_{AB} = 4\Omega$

$$\boxed{\therefore\ \tau = \frac{L}{R_{TH}} = \frac{2H}{4\Omega} = \frac{1}{2}\text{ s}}$$

Use this circuit for questions 8, 9, and 10. The switch is closed for a long time and then opened at $t = 1$ s. At $t = 0^-$, the inductor current is known to be $i_L(0^-) = 0$ A.



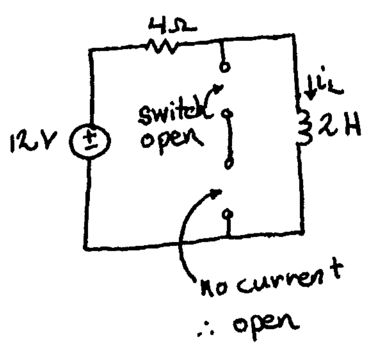
10. As $t \to \infty$, what is $i_L(\infty)$?

(1) $i_L(\infty) = 1$ A (2) $i_L(\infty) = 2$ A $\boxed{(3)\ i_L(\infty) = 3\text{ A}}$ (4) $i_L(\infty) = 4$ A

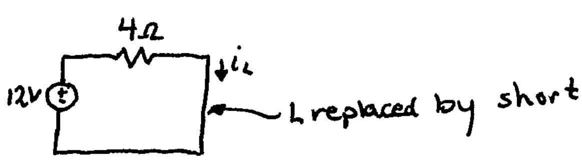
(5) $i_L(\infty) = 5$ A (6) $i_L(\infty) = 6$ A (7) $i_L(\infty) = 8$ A (8) $i_L(\infty) = 12$ A

(9) $i_L(\infty) = 0$ A (10) None of the above

• In the time interval of $t \to \infty$, the circuit looks like:



• $i_L(\infty)$ is i_L if the inductor is replaced by a short (in the circuit of this time interval):

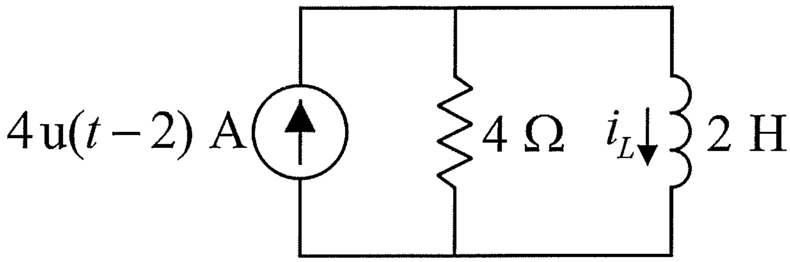


• i_L is same as current through the 4Ω resistor:

$i_L = \frac{12}{4}$ ← Ω's law

∴ $\boxed{i_L(\infty) = 3A}$ ← for $t \to \infty$

11. The current through an inductor was kept at $i_L(t) = 8$ A for all time until $t = 0^-$. The inductor was then connected to the circuit as shown below at $t = 0$. Find the current through the inductor, $i_L(t)$, for $t \geq 2$ s.



(1) $i_L(t) = 1 - e^{-2t} - 8e^{-2t-4}$ A

(2) $i_L(t) = 1 - e^{-2t} + 8e^{-2t-4}$ A

(3) $i_L(t) = 1 - 8e^{-2t} - e^{-2t+4}$ A

(4) $i_L(t) = 1 + 8e^{-2t} - e^{-2t+4}$ A

(5) $i_L(t) = 4 - 4e^{-2t} - 8e^{-2t-4}$ A

(6) $i_L(t) = 4 - 4e^{-2t} + 8e^{-2t-4}$ A

(7) $i_L(t) = 4 - 8e^{-2t} - 4e^{-2t+4}$ A

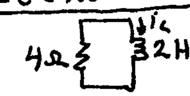
(8) $\boxed{i_L(t) = 4 + 8e^{-2t} - 4e^{-2t+4} \text{ A}}$

(9) None of the above

- During a time interval when the circuit does not change (beginning @ $t = t_0$),

$$i_L(t) = i_L(\infty) + (i_L(t_0^+) - i_L(\infty))\, e^{-\frac{t-t_0}{\tau}}$$

- For $0 \leq t \leq 2$s: Circuit same for $t = 0$ to $t = 2$s



← (current source zero ∴ open)

$t_0 = 0$

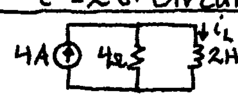
$i_L(\infty) = i_{L,short} = 0$ ← no sources

$i_L(0^+) = i_L(0^-) = 8A$ ← current continuity in inductors

$\tau = \frac{L}{R_{TH}} = \frac{2}{4} = \frac{1}{2}$ s

$\therefore\ i_L(t) = 0 + (8 - 0)e^{-2t} \overset{①}{=} 8e^{-2t}$ (A), $0 \leq t \leq 2$ s

- For $t \geq 2$ s: Circuit same for all time after $t = 2$s



← (current source on)

$t_0 = 2$

$i_L(\infty) = i_{L,short} = 4A$ ← replace L by short

$i_L(2^+) = i_L(2^-) \xrightarrow[\text{①}]{\text{from}} i_L(2^+) = 8e^{-2(2)} = 8e^{-4}$

$\tau = \frac{L}{R_{TH}} = \frac{2}{4} = \frac{1}{2}$ s

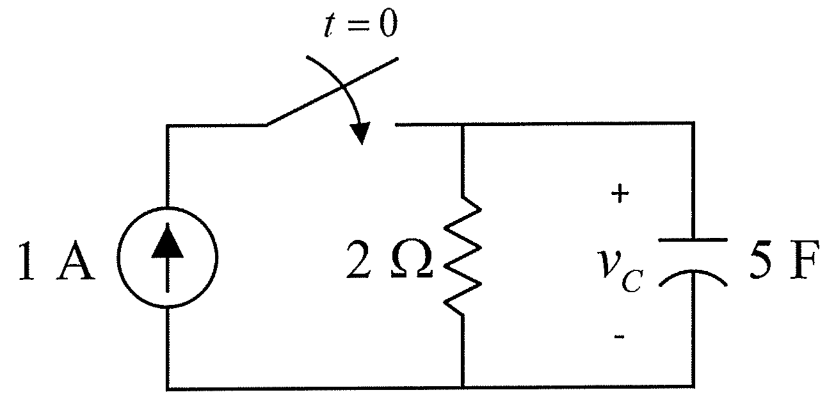
$\underbrace{R_{TH}}_{\text{zero out 4A source}}$

$\therefore\ i_L(t) = 4 + (8e^{-4} - 4)e^{-\frac{t-2}{1/2}}$

$= 4 + (8e^{-4} - 4)e^{-2t+4}$

$\boxed{i_L(t) = 4 + 8e^{-2t} - 4e^{-2t+4}\ A,\quad t \geq 2s}$

12. A capacitor is charged to $v_C(0^-) = 4$ V. At $t = 0$, the switch in the circuit below is closed. At what time t_1 in seconds will the capacitor voltage be $v_C(t_1) = 3$ V?



(1) $\frac{1}{2}\ln\frac{5}{2}$ (2) $\frac{1}{2}\ln 10$ (3) $2\ln\frac{5}{2}$ (4) $2\ln 10$

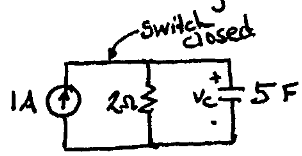
(5) $\frac{5}{2}\ln\frac{1}{2}$ (6) $\frac{5}{2}\ln 2$ (7) $10\ln\frac{1}{2}$ $\boxed{\text{(8) } 10\ln 2}$

(9) None of the above

• During a time interval when the circuit does not change (beginning @ $t = t_0$):

$$v_c(t) = v_c(\infty) + (v_c(t_0^+) - v_c(\infty))e^{-\frac{t-t_0}{\tau}}$$

• For $t \geq 0$, the circuit stays the same:



$t_0 = 0$

$V_c(\infty) = V_{c,open} = \overset{\Omega\text{'s law}}{2(1)} = 2V$ ← replace C by open

$V_c(0^+) = V_c(0^-) = 4V$ ← voltage continuity across capacitors

$\tau = R_{TH}C = 2(5) = 10$ s

zero-out 1A source

$\therefore v_c(t) = 2 + (4-2)e^{-t/10}$

$\overset{①}{=} 2 + 2e^{-t/10}$ V

• Want to find t_1 such that $v_c(t_1) = 3V$. Use ①:

$v_c(t_1) = 2 + 2e^{-t_1/10} = 3$

$e^{-t_1/10} = \frac{1}{2}$ ⟶ $-\frac{t_1}{10} = \ln\frac{1}{2}$

$t_1 = -10\ln\frac{1}{2}$ $\xrightarrow{-\ln x = \ln\frac{1}{x}}$ $\boxed{t_1 = 10\ln 2}$

Potentially Useful Formulas

$$x(t) = x(\infty) + \left[x(t_0^+) - x(\infty) \right] e^{-(t-t_0)/\tau} \text{ , where } \tau = R_{TH}C \text{ or } \tau = \frac{L}{R_{TH}}$$

$$v_L(t) = L\frac{di_L(t)}{dt}$$

$$i_L(t) = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t} v_L(t')dt'$$

$$W_L(t_0, t_1) = \frac{L}{2}\left[\left(i_L(t_1)\right)^2 - \left(i_L(t_0)\right)^2 \right]$$

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

$$v_C(t) = v_C(t_0) + \frac{1}{C}\int_{t_0}^{t} i_C(t')dt'$$

$$W_C(t_0, t_1) = \frac{C}{2}\left[\left(v_C(t_1)\right)^2 - \left(v_C(t_0)\right)^2 \right]$$

$$-\ln x = \ln\frac{1}{x}$$