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Pricing strategies for information products with network effects and complementary services in a duopolistic market

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In this paper, we model a fully covered duopoly market in which two firms offer a differentiated information product that exhibits positive network effects and a complementary premium service to consumers. For each firm, there are two marketing strategies: the freemium strategy and the bundling strategy. We find that, under the market equilibrium, a firms' decision whether to employ the freemium strategy or not depends largely on the quality of the information product compared to its rival. When the information product quality is similar and the products' intrinsic values are sufficiently large, both firms will be better off by adopting the freemium strategy, while the bundling strategy will prevail if the products' intrinsic values are sufficiently small. Additionally, when the magnitude of complementary effects or network effects exceeds a given threshold, both firms' profit can be enhanced by an increase in the degree of product complementarity or in the intensity of network effects. We also demonstrate that a firm can benefit from an increasing market size only if the intrinsic value of its information product is sufficiently large. Finally, we extend our model to the uncovered market and derive the equilibrium prices and profits.

Keywords: freemium products; premium service; bundling; product complementarity; network effects; duopolistic market

1. Introduction

During the past decade, the freemium business model, a pricing structure in which basic functionalities are given away for free but customers are charged for access to richer features, has become increasingly prevalent among software enterprises and internet start-ups. This business model, which is increasingly being taken up by vendors, can generate revenue by effectively building an audience base that potentially boosts the sales of premium services (Kumar 2014). As Gartner predicts, 94.5% of mobile applications will be downloaded globally at no cost in 2017, and the share of in-app purchases will grow from 11% of total revenue in 2012 to 48% in 2017 (Zhang et al. 2016). Additionally, other data show that as many as 736 out of 1000 (73.6%) top-grossing games in the Apple app store are free with in-app-purchases (Alha et al. 2016). Similar statistical results in the Google Play app store including apps for Android are also reported (Liu, Au, and Choi 2014).

The freemium strategy has achieved great popularity and success in the information product industry. For example, many famous computer games (e.g. League of Legends, Star Wars: The Old Republic, World of Warcraft and Dota 2), and many popular mobile games, such as Arena of Valour: 5v5 Arena Game and Temple, are all free to play but charge for complementary functionalities or additional services. Software developers also favour this business model when promoting their new products. Microsoft releases the Visual Studio and SQL server for free, but only with the basic performance. Consumers who want to use the more professional functionalities must pay for the subscription services. Adobe offers a free basic version (e.g. Acrobat Reader XI) but sells some premium functionalities that contain more capabilities in editing and writing. In the social networking software industry, the freemium strategy is also embraced with enthusiasm by many providers. LinkedIn, one of the earliest freemium companies, provides some premium subscriptions such as better e-mail functionality and deeper search capability to its free users. Skype users can communicate with each other for free but must pay for physical calls or the SkypeIN service. Additionally, some cloud storage companies, such as Amazon, Google and Microsoft, employ the freemium strategy to offer data storage services. A typical example is Dropbox, which attracts more than 200 million users by adopting this strategy. Each customer can access two gigabytes of data storage free of charge, but when the limited space runs out, he must spend $9.99 monthly (or, alternatively, $99 yearly) for another 100 G.

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Many scholars attribute the appeal of the freemium business model in the digital market to several specific features of information goods. Once developed, information goods have negligible costs in reproduction and distribution, which is significantly different from traditional industrial goods and makes tractable reproductions and unlimited offerings of the products possible (Jones and Mendelson 2011). In general, many information goods display positive network externalities, which refers to the effect that the utility a user obtains from a product is positively dependent on the number of other users (Shapiro and Varian 1999). The positive network externality allows a new venture to increase its product's value by attracting a large user base with the freemium strategy. Finally, information goods are highly differentiated. For example, each online game is different from most of the other games (or so the players think), and each movie is unique. Thus some information goods producers have huge monopolistic power so that they can adopt other alternative sales strategies, such as bundling and tying. In this paper, in addition to the freemium strategy, we also investigate the bundle strategy for two information goods enterprises in a duopoly market setting.

Despite the popularity and clear benefits of the freemium model, it is so inherently challenging to employ that some Web start-ups failed to benefit from it (Kumar 2014). However, when the complementarity between the free product and premium service is considered, the situation will improve since a strong functional fit between the free version and the paid associated service stimulates free customers to upgrade to the premium features and brings more subscription revenue (Wagner, Benlian, and Hess 2014). In fact, providers always encounter a dilemma in monetising freemium customers who are potentially interested in the premium offers by solely enhancing the quality of premium goods. Moreover, for many information products, the purpose of the premium service is not improving service quality but is strongly associated with the freemium products (Hamari, Hanner, and Koivisto 2017). Therefore, designers should invest more to strengthen the complementary effects between the free product and premium service so that freemium users are more incentivised to purchase the paid service. In addition, the sold bundle creates more value when the complementarity is exhibited. How much revenue do both selling tactics generate and which is more beneficial in this setting are additional interesting topics. This paper considers the complementary effect in freemium information products by comparing the freemium strategy with the bundling strategy and articulates the impact of complementary effects on market decisions and vendor's profit.

2. Literature review

The recent literature has included extensive studies on the freemium mechanism employed in the information product industry (Cheng and Liu 2012; Cheng and Tang 2010; Hamari, Hanner, and Koivisto 2017; Lee and Tan 2013; Lin and Sun 2011; Mäntymäki and Salo 2015; Niculescu and Wu 2014). Additionally, Wagner, Benlian, and Hess (2014) explore the impact that the limited functionalities of free accessed products have on the evaluation of premium services by examining 317 survey responses from freemium consumers. Their conclusions show that a strong coordination between the free product and premium service can increase the conversion rate of free users. By analysing 2791 player responses from 3 different game markets, Hamari (2015) studies the purchase behaviour of consumers in the virtual products industry and indicates that the enjoyment of a game attracts more players to join but weakens their purchase willingness and that the attitudes of users to virtual goods greatly enhance their willingness to buy.

Bundling, another popular business strategy in the information product market, has been widely investigated in the literature (Dewan and Freimer 2003; Ferrer, Mora, and Olivares 2010; Geng, Stinchcombe, and Whinston 2005; Hui et al. 2012; Jiang et al. 2015; Pérez et al. 2016; Wu et al. 2008). Additionally, Bhargava (2013) studies the mixed bundling strategy of two independent information products and derived the equilibrium solutions. Chao and Derdenger (2013) focus on the mixed bundling strategy in two-sided information markets, which exhibits installed base effects and obtain a unique price structure. The growing bundling literature also addresses the network effects that are displayed by most information goods (Etzion and Pang 2014; Jing 2007; Katz and Shapiro 1985; Prasad, Pang and Etzion 2012; Sundararajan 2004; Prasad, Venkatesh, and Mahajan 2010; Zhang et al. 2016). Pang and Etzion (2012) consider a monopolist firm that sells a product and offers an online service that exhibits positive network effects, and show that the monopolist often stops offering the bundle. Etzion and Pang (2014) develop a duopoly market in which each firm selling a differentiated product with non-negligible production cost can decide to provide a complementary service that displays positive network effects.

Our work contributes to the bundling literature on competition and complementarity because we examine a firm's bundling strategy when the information product and the premium service exhibit complementary effects in the duopoly setting. A great number of studies focus on the bundling strategy of complementary goods (Bakos and Brynjolfsson 1999; Halmenschlager and Mantovani 2017; Mukhopadhyay, Yue, and Zhu 2011; Yan et al. 2014; Yue, Mukhopadhyay, and Zhu 2006). Recently, Taleizadeh et al. (2017) examine the optimal problem for two complementary goods under three business strategies (i.e. separately selling, pure bundle and mixed bundle). There are also some papers investigating the complementary products' bundling strategy in a competitive context. Matutes and Regibeau (1992) initially analyse a duopoly market in which both firms sell two complementary components. Anderson and Leruth (1993) study the bundling of two complements in the duopoly setting and reveal that mixed bundling may be suboptimal. Afterwards, several researchers followed up this work with further studies in this field (i.e. Denicolo 2000; Farrell, Monroe, and Saloner 1998; Liao and Tauman 2002; Venkatesh and Kamakura 2003). Recently, Raghunathan and Sarkar (2016) give an analysis of the equilibrium bundling strategy in a symmetric duopolistic information market in which the products of both firms are complements or substitutes. The result shows that both firms choose bundling in equilibrium if there are at least a complement pair and a substitute pair among the provided information goods.

To our best knowledge, this paper contains the first model that considers complementarity between information products and premium services under the freemium strategy and the bundling strategy in a duopolistic market. Although several common features of this model may suggest an overlap with Zhang et al. (2016), which surveys a duopoly market where each firm either adopts a freemium strategy or follows a bundling strategy, our work has three significant differences. First, our paper considers the complementary effect between information goods and additive services, while Zhang et al. (2016), similar to most previous studies (e.g. Etzion and Pang 2014), assume that they are independent. Second, the sum value of an information product and a premium service is assumed to be equal for both firms by Zhang et al. (2016). In other words, both firms' bundled products consisting of an information product and a premium service are uniform in quality. However, in our model, bundled products are set to be differentiated in quality (vertical differentiation), which is a more typical assumption in the information product market. For instance, Microsoft employs a freemium strategy in selling its SQL Server while Oracle bundles its RDBMS, but most consumers believe that RDBMS performs better than SQL Server. Finally, we induce a marginal cost for offering the premium service which is neglected by Zhang et al. (2016). In general, as increasing numbers of subscribers of a premium service enter, a provider should invest more in upgrading the application server and network infrastructure to ensure an efficient service operation. Therefore, the expense of providing the premium service is not negligible, and increases with respect to the user base (Etzion and Pang 2014; Pang and Etzion 2012).

The rest of this paper is organised as follows. In Section 3, the equilibrium prices and profits for both firms are derived in detail. Section 4 provides an analysis of the market equilibrium and examines the impacts of product complementarity and network effects and the market size on both firms' profit. Section 5 extends our analysis to the uncovered market and obtains the equilibrium prices and profits for both firms.

3. The model

This paper considers a marketing model with two competing firms (Firm 1 and Firm 2) that offer differentiated information products. The information products are assumed to have no-cost in production. Each firm can provide a complementary premium service to consumers who use its information products, and has an option whether to do so. There are N potential customers who are heterogeneous in terms of their valuations for the information product and each consumer demands at most one unit of the product. We build our model based on the Hotelling location model (Hotelling 1929). Consumers' product preferences are assumed to be distributed on [0,1] uniformly, with Firm 1 located at 0 and Firm 2 located at 1. The location of each consumer corresponds to his ideal product. If a consumer uses a product that is different from the one he most desires, a misfit expense will be incurred. The misfit cost of a consumer increases with the increase of the distance between his ideal product and the seller. Let V_i represent the intrinsic value of the product from Firm i ($i = 1, 2$) and t denote the misfit cost per unit distance. Therefore, the utility of a consumer whose ideal product locates at x is $V_1 - tx$ when choosing Firm 1's product and $V_2 - t(1 - x)$ when consuming Firm 2's product.

We assume that the intrinsic value of the premium service provided by both firms (denoted by V) is equal and has a complementary effect on the information products.[1] A consumer gains an additional utility from receiving the premium service of a firm only when he consumes the information product offered by this firm. In other words, the service provided by Firm i will not benefit those consumers who use a product from Firm j. Following Venkatesh and Kamakura (2003), we formulate the intrinsic value of the bundle consisting of an information product and a premium service as $(1 + \theta)(V_i + V)$, where the parameter $\theta > 0$ records the degree of complementarity. Since a bundle contains an information product and a complementary premium service, the unit distance misfit cost of a bundle is denoted by $(1 + \theta)t$. In addition, two firms are assumed to have the same technological ability to provide a premium service and the marginal cost of providing the service is denoted by c.[2]

The information products provided by Firm i exhibit positive network effects. Similar to Etzion and Pang (2014), we assume that the network effects of a product only depend on the network size. In particular, a consumer obtains positive utility αN_k^i from consuming a product of Firm i, where α represents the intensity of network effects and N_k^i denotes the expected market size of the user base in a fulfilled expectation equilibrium. In our model, we employ the idea of

Fulfilled Expectations Equilibrium in which the realised demand equals the expected network size (Amir and Lazzati 2011; Katz and Shapiro 1985; Pang and Etzion 2012; Prasad, Venkatesh, and Mahajan 2010). This equilibrium concept has also become rather common in the context of IO (Industrial Organisation) static models studying network effects (Belleflamme and Peitz 2015).[3] The key notations are summarised in Table 1.

Each firm has two marketing strategies: (1) the freemium strategy, in which the firm offers a free information product and operates a paid premium service so that all customers who use the product for free can pay for the premium service if they wish, and (2) the bundling strategy, in which this firm can choose to provide a bundle consisting of an information product and a premium service. Two strategic options for both firms may result in four market configurations as follows. Case *BB*: Both firms sell a bundle containing an information product and the premium service; Case *FB*: Firm 1 provides a free information product and prices the premium service while Firm 2 sells a bundle; Case *BF*: Firm 1 sells a bundle while Firm 2 provides a free information product and prices the premium service; and Case *FF*: Both firms provide a free product and a paid service.

Let R_{pi} denotes the surplus a consumer gains when using only Firm i's information product and R_{bi} represents the consumer surplus of purchasing a bundle offered by Firm i, $i = 1, 2$. Then, we have the following consumer surplus functions:

$$R_{p1} = V_1 + \alpha N_k^1 - tx \tag{1}$$

$$R_{b1} = (V_1 + V)(1 + \theta) + \alpha N_k^1 - t(1 + \theta)x - P_k^1 \tag{2}$$

$$R_{p2} = V_2 + \alpha N_k^2 - t(1 - x) \tag{3}$$

$$R_{b2} = (V_2 + V)(1 + \theta) + \alpha N_k^2 - t(1 + \theta)(1 - x) - P_k^2 \tag{4}$$

3.1 *Case* BB*: both firms adopt the bundling strategy*

When both firms offer a bundle, the consumer surplus of using a bundle sold by Firm 1 and the surplus from consuming a Firm 2's bundle are given by (2) and (4), respectively. The consumer surplus functions, in this case, are shown in Figure 1.

Each consumer aims to maximise his own surplus. Therefore, the whole market is divided into two segments $(0, x_0)$ and $(x_0, 1)$. Consumers located in $(0, x_0)$ prefer to buy a bundle from Firm 1 rather than purchase a bundle from Firm 2 since consuming Firm 1's bundle brings a higher surplus. In contrast, consumers located in $(x_0, 1)$ prefer Firm 2's bundle to Firm 1's bundle. The marginal customer located at x_0 is indifferent about buying Firm 1's bundle and buying Firm 2's bundle.

Table 1. Key notations.

N	The market size
V_i	The intrinsic value of the information product of Firm i (i = 1,2)
V	The intrinsic value of the service
t	Unit misfit cost for using a given information product
θ	The degree of complementary effect
α	The intensity of network effects
c	The marginal cost of providing a premium service
N_k^i	The expected market size of the user base in a rational expectation equilibrium, where $k \in \{FF, FB, BF, FF\}$
P_k^i	The price charged by Firm i when the options regarding product provision are given by k, where $k \in \{FF, FB, BF, FF\}$
D_k^i	The demand for the premium service of Firm i when the options regarding product provision are k
π_k^i	The profit of Firm i when the options regarding product provision are k

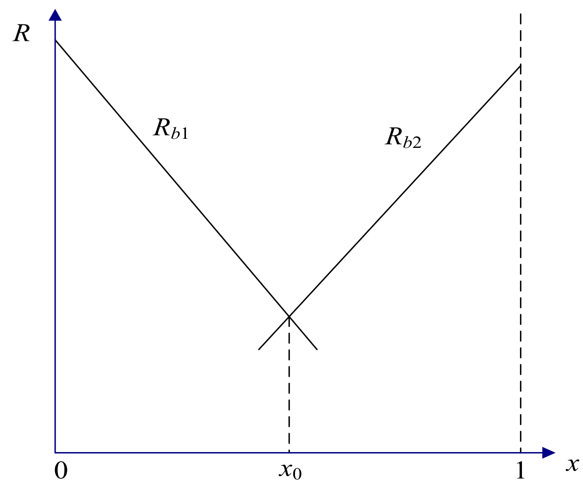

Figure 1. Market segmentation when both firms adopt the bundling strategy.

In the fulfilled expectation equilibrium, the demand for the information product of Firm i equals the expected market size N^i_{BB} ($i = 1,2$). Solving the indifference equation $R_{b1} = R_{b2}$ with $N^1_{BB} = x_0 N$ and $N^2_{BB} = (1 - x_0)N$ at equilibrium, we can express x_0 as a function of P^1_{BB} and P^2_{BB}:

$$x_0\left(P^1_{BB}, P^2_{BB}\right) = \frac{1}{2}\frac{(1+\theta)(V_1 - V_2 + t) + P^2_{BB} - P^1_{BB} - \alpha N}{t(1+\theta) - \alpha N}$$

The profit functions for both firms are given by:

$$\pi^1_{BB} = \left(P^1_{BB} - c\right)D^1_{BB} = \left(P^1_{BB} - c\right)x_0 N$$

$$\pi^2_{BB} = \left(P^2_{BB} - c\right)D^2_{BB} = \left(P^2_{BB} - c\right)(1 - x_0)N$$

$$s.t. \quad 0 < x_0 < 1$$

At equilibrium, each firm seeks an optimal price P^{i*}_{BB} to maximise its own profit. Substituting x_0 into the above profit functions, the equilibrium solutions, in this case, can be derived. We assume the second-order condition is positive (i.e. $t > \alpha N$), which guarantees that both products have positive demands. It is easy to obtain the optimal price for Firm i at equilibrium by solving the profit functions as follows:

$$P^{i*}_{BB} = \frac{\left(3t + V_i - V_j\right)(1+\theta) - 3\alpha N + 3c}{3}, \quad i,j \in \{1,2\} \text{ and } i \neq j$$

Accordingly, the market demand for the premium service of Firm i and its profit are:

$$D^{i*}_{BB} = \frac{\left(3t + V_i - V_j\right)(1+\theta) - 3\alpha N}{6(t(1+\theta) - \alpha N)}N$$

$$\pi^{i*}_{BB} = \frac{\left(\left(3t + V_i - V_j\right)(1+\theta) - 3\alpha N\right)^2}{18(t(1+\theta) - \alpha N)}N$$

To ensure both firms have a positive demand for the premium service (i.e. $D^{i*}_{BB} > 0$), which is equivalent to the condition of $0 < x_0 < 1$, the following assumption is necessary:

$$V_i - V_j < \frac{3(t(1+\theta) - \alpha N)}{1+\theta}$$

Finally, the condition for a fully covered market (i.e. the marginal customer has a positive surplus) under Case *BB* is

$$V > \frac{(3t + V_i - V_j)(1+\theta) - 3\alpha N + 2c}{2(1+\theta)} - V_i$$

3.2 *Case* FB *and* BF*: only one firm adopts the freemium strategy and the other chooses the bundling strategy*

Without loss of generality, we analyse the case where Firm 1 provides a free information product and prices the premium service while Firm 2 sells a bundle (i.e. Case *FB*). The result in Case *BF* can be obtained in the same way.

As is shown in Figure 2, the whole market is divided into three segments $(0, x_{11})$, (x_{11}, x_{12}) and $(x_{12}, 1)$. Consumers located in $(0, x_{11})$ choose to use a free information product offered by Firm 1 and purchase its premium service. While consumers located in (x_{11}, x_{12}) only use Firm 1's free information product without buying the premium service. Consumers in this segment bring positive network effects, and therefore contribute to Firm 1's profit. In addition, consumers located in $(x_{12}, 1)$ prefer to purchase the bundle provided by Firm 2.

The demand for the information product of Firm i is N^i_{FB} $(i = 1,2)$ according to the fulfilled expectation equilibrium. Solving the equations $R_{b1} = R_{p1}$ and $R_{p1} = R_{b2}$ for x and accounting for $N^1_{FB} = x_{12}N$ and $N^2_{FB} = (1 - x_{12})N$, we can obtain:

$$x_{11} = \frac{(1+\theta)V + \theta V_1 - P^1_{FB}}{\theta t}, \quad x_{12} = \frac{V_1 + (1+\theta)(t - V - V_2) + P^2_{FB} - \alpha N}{(2+\theta)t - 2\alpha N}$$

At equilibrium, Firm 1 charges an optimal price for its premium service to maximise its profit, while Firm 2 selects an optimal price for its bundle to realise the maximum profit. The profit functions, in this case, are given by:

$$\pi^1_{FB} = (P^1_{FB} - c)D^1_{FB} = (P^1_{FB} - c)x_{11}N$$

$$\pi^2_{FB} = (P^2_{FB} - c)D^2_{FB} = (P^2_{FB} - c)(1 - x_{12})N$$

$$s.t. \quad 0 < x_{11} < x_{12} < 1$$

To ensure that the second-order condition for π^2_{FB} is positive, we need to assume that $(2 + \theta)t > 2\alpha N$. Solving the first-order conditions, the optimal prices and profits for both firms can be derived as follows:

$$P^{1*}_{FB} = \frac{\theta V_1 + (1+\theta)V + c}{2}, \quad P^{2*}_{FB} = \frac{(1+\theta)(V_2 + V) + t + c - V_1 - \alpha N}{2}$$

$$\pi^{1*}_{FB} = \frac{(\theta V_1 + (1+\theta)V - c)^2}{4\theta t}N, \quad \pi^{2*}_{FB} = \frac{((1+\theta)(V_2 + V) + t - c - V_1 - \alpha N)^2}{4((2+\theta)t - 2\alpha N)}N$$

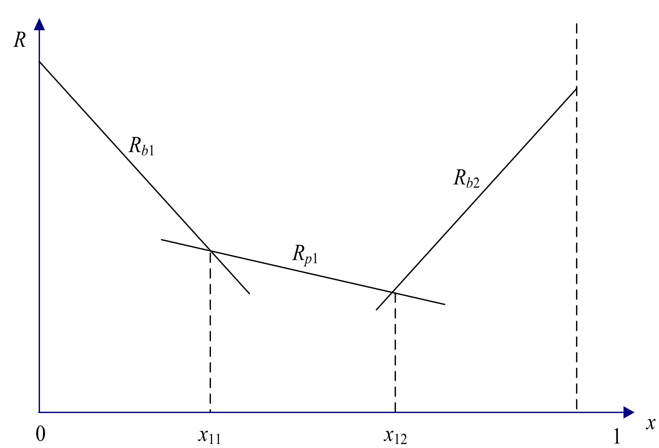

Figure 2. Market segmentation when only Firm 1 adopts the freemium strategy.

At the above prices, it is required that the freemium strategy of Firm 1 is effective and both firms have a positive demand for the premium service (i.e. $0 < N_{FB}^1 - D_{FB}^1 < N, 0$). These requirements are equivalent to the constraint on the indifference points (i.e. $0 < x_{11} < x_{12} < 1$). Thus, we require that:

$$\frac{V_1 + c + \alpha N - t}{1+\theta} - V < V_2 < \left(\frac{2\alpha N}{(1+\theta)t} - 1\right)V_1 + \frac{(3+2\theta)t - 3\alpha N}{1+\theta} + \frac{2(1+\theta)t - 2\alpha N}{\theta t}\left(\frac{c}{1+\theta} - V\right) \text{ and } V_1 > \frac{c-(1+\theta)V}{\theta}$$

The condition for a fully covered market (i.e. the marginal customer that is indifferent about the choice between Firm 1's free information product and Firm 2's bundle has a positive surplus) under Case *FB* is

$$V > 2t - V_2 + \frac{(3\alpha N - t - c)}{1+\theta} + \frac{(3\alpha N - (3+2\theta)t)}{(1+\theta)(t-\alpha N)}V_1$$

3.3 *Case* FF*: both firms adopt the freemium strategy*

If both firms provide a free information product and charge for the premium service separately, the whole market is divided into four segments (e.g. $(0, x_{11})$, (x_{11}, x_{12}), (x_{12}, x_{22}) and $(x_{22}, 1)$). Consumers located in $(0, x_{11})$ choose to use a free information product offered by Firm 1 and pay for its associated service. While consumers located in (x_{11}, x_{12}) only use Firm 1's free information product without buying the premium service. Consumers located in (x_{12}, x_{22}) prefer to only use the free information product offered by Firm 2. Whereas consumers located in $(x_{22}, 1)$ prefer to consume the free product and the paid service from Firm 2. Consumers in segments (x_{11}, x_{12}) and (x_{12}, x_{22}) only contribute to the network effects without increasing either firm's revenue. Figure 3 exhibits the market segmentation in Case *FF*.

Under the fulfilled expectation equilibrium requirement, the demand for the information product of Firm i is N_{FF}^i ($i = 1, 2$). By setting $R_{b1} = R_{p1}$, $R_{p1} = R_{p2}$ and $R_{p2} = R_{b2}$ and considering $N_{FF}^1 = x_{12}N$ with $N_{FF}^2 = (1 - x_{12})N$, we have

$$x_{11} = \frac{\theta V_1 + (1+\theta)V - P_{FF}^1}{\theta t}, \quad x_{12} = \frac{1}{2}\frac{t + V_1 - V_2 - \alpha N}{t - \alpha N}, \quad x_{22} = \frac{\theta t - \theta V_2 - (1+\theta)V + P_{FF}^2}{\theta t}.$$

According to consumers' above choices, either firm sets an optimal price $P_{FF}^{i*}(i = 1,2)$ for the premium service to maximise its own profit. The profit functions, in this case, are given by

$$\pi_{FF}^1 = \left(P_{FF}^1 - c\right)D_{FF}^1 = x_{11}N$$

$$\pi_{FF}^2 = \left(P_{FF}^2 - c\right)D_{FF}^2 = \left(P_{FF}^2 - c\right)(1 - x_{22})N$$

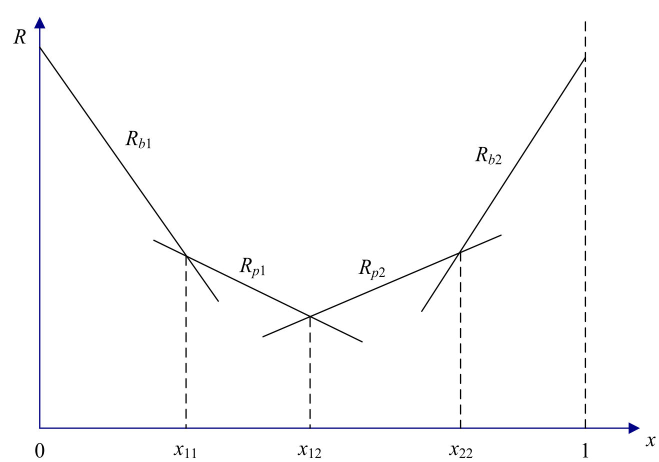

Figure 3. Market segmentation when both firms adopt the freemium strategy.

$$s.t. \quad 0 < x_{11} < x_{12} < x_{22} < 1$$

Differentiating the above profit functions with respect to prices, we obtain the following:

$$P_{FF}^{1*} = \frac{\theta V_1 + (1+\theta)V + c}{2}, \quad P_{FF}^{2*} = \frac{\theta V_2 + (1+\theta)V + c}{2},$$

$$D_{FF}^{1*} = \frac{\theta V_1 + (1+\theta)V - c}{2\theta t}N, \quad D_{FF}^{2*} = \frac{\theta V_2 + (1+\theta)V - c}{2\theta t}N.$$

The profits at the optimal prices are given by

$$\pi_{FF}^{1*} = \frac{(\theta V_1 + (1+\theta)V - c)^2}{4\theta t}N, \quad \pi_{FF}^{2*} = \frac{(\theta V_2 + (1+\theta)V - c)^2}{4\theta t}N.$$

The conditions for $D_{FF}^i > 0$ and $0 < N_{FF}^i - D_{FF}^i < N$ are given by

$$\frac{c - (1+\theta)V}{\theta} < V_i < \frac{\alpha N}{t}V_j + \frac{(t - \alpha N)(c - (1+\theta)V)}{\theta t} + t - \alpha N, \quad i,j \in \{1,2\} \text{ and } i \neq j$$

To ensure a fully covered market (i.e. the marginal customer that is indifferent about the choice between Firm 1's free information product and Firm 2's free information product has a positive surplus) under Case *FF*, the following condition is necessary

$$V_1 + V_2 > t - \alpha N.$$

4. Market equilibrium

The strategic game in our study has three stages. First, both firms simultaneously make their offering decisions: to adopt the freemium strategy or to choose the bundling strategy. Next, according to the decisions made in the first stage, both firms price their products simultaneously. Finally, consumers make their purchasing choices after acquiring the market information (i.e. product prices, firms' offering strategies and the expected network size).

4.1 *Market equilibrium analysis*

By solving this three-stage game in reverse, we can obtain the sub-perfect market equilibrium outcomes. In each subgame, we derive the optimal prices and the corresponding profits for both firms, which are summarised, respectively, in Tables 2 and 3. By solving the pay-off matrix in Table 3, we obtain the following proposition (See Appendix 1).

Proposition 1 (Market Equilibrium Conditions)

(1) *Both firms provide a free information product and a paid service separately. Case FF is an equilibrium iff* $V_1 > AV_2 + B$ *and* $V_2 > AV_1 + B$.
(2) *Both firms provide a bundle. Case BB is an equilibrium iff* $V_1 < CV_2 + D$ *and* $V_2 < CV_1 + D$.
(3) *Only Firm 1 provides a free information product and a paid service. Case FB is an equilibrium iff* $V_1 < AV_2 + B$ *and* $V_2 > CV_1 + D$.

Table 2. The equilibrium prices.

	Firm 2	
Firm 1	Freemium strategy	Bundling strategy
Freemium strategy	$P_{FF}^{1*} = \frac{\theta V_1+(1+\theta)V+c}{2}$ $P_{FF}^{2*} = \frac{\theta V_2+(1+\theta)V+c}{2}$	$P_{FB}^{1*} = \frac{\theta V_1+(1+\theta)V+c}{2}$ $P_{FB}^{2*} = \frac{(1+\theta)(V+V_2)+t+c-V_1-\alpha N}{2}$
Bundling strategy	$P_{BF}^{1*} = \frac{(1+\theta)(V+V_1)+t+c-V_2-\alpha N}{2}$ $P_{BF}^{2*} = \frac{\theta V_2+(1+\theta)V+c}{2}$	$P_{BB}^{1*} = \frac{(1+\theta)(3t+V_1-V_2)+3c-3\alpha N}{3}$ $P_{BB}^{2*} = \frac{(1+\theta)(3t+V_2-V_1)+3c-3\alpha N}{3}$

Table 3. The equilibrium profits.

Firm 1	Firm 2: Freemium strategy	Firm 2: Bundling strategy
Freemium strategy	$\pi_{FF}^{1*} = \frac{(\theta V_1+(1+\theta)V-c)^2 N}{4\theta t}$ $\pi_{FF}^{2*} = \frac{(\theta V_2+(1+\theta)V-c)^2 N}{4\theta t}$	$\pi_{FB}^{1*} = \frac{(\theta V_1+(1+\theta)V-c)^2 N}{4\theta t}$ $\pi_{FB}^{2*} = \frac{((1+\theta)(V+V_2)+t-c-V_1-\alpha N)^2 N}{4((2+\theta)t-2\alpha N)}$
Bundling strategy	$\pi_{BF}^{1*} = \frac{((1+\theta)(V+V_1)+t-c-V_2-\alpha N)^2 N}{4((2+\theta)t-2\alpha N)}$ $\pi_{BF}^{2*} = \frac{(\theta V_2+(1+\theta)V-c)^2 N}{4\theta t}$	$\pi_{BB}^{1*} = \frac{((3t+V_1-V_2)(1+\theta)-3\alpha N)^2 N}{18((1+\theta)t-\alpha N)}$ $\pi_{BB}^{2*} = \frac{((3t+V_2-V_1)(1+\theta)-3\alpha N)^2 N}{18((1+\theta)t-\alpha N)}$

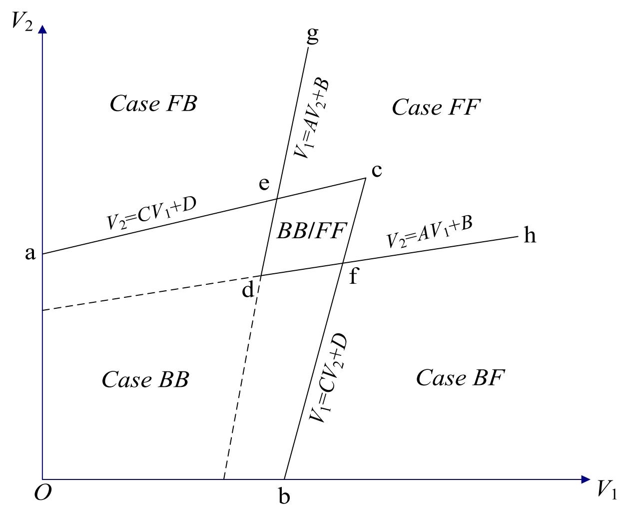

Figure 4. The market equilibrium in the V_1–V_2 space.

(4) *Only Firm 2 provides a free information product and a paid service. Case BF is an equilibrium iff* $V_1 > CV_2 + D$ *and* $V_2 < AV_1 + B$, where:

$$A = 1 + \theta\left(1 - \sqrt{\frac{(2+\theta)t-2\alpha N}{\theta t}}\right), \quad B = \left(\sqrt{\frac{(2+\theta)t-2\alpha N}{\theta t}} - 1\right)(c - (1+\theta)V) + t - \alpha N,$$

$$C = 1 - \frac{3\theta}{2(1+\theta)}\sqrt{\frac{2t(1+\theta)-2\alpha N}{\theta t}} \text{ and } D = \frac{3}{2}\sqrt{\frac{2t(1+\theta)-2\alpha N}{\theta t}}\left(\frac{c}{1+\theta} - V\right) + 3t - \frac{3\alpha N}{1+\theta}$$

Four possible market equilibria are illustrated in Figure 4. We can find that in the region O-a-c-b, the intrinsic values of the information product (i.e. V_1 and V_2) are so low that both firms choose to offer a bundle at equilibrium. In region g-e-d-f-h, V_1 and V_2 are sufficiently high and, therefore, both firms provide a free information product and a paid service separately. In region V_2-a-e-g, an equilibrium in which Firm 1 provides a free information product and a paid service while Firm 2 offers a bundle prevails. In region V_1-b-f-h, Firm 1 offers a bundle and Firm 2 provides a free information product and a paid service at equilibrium.

When making offering decisions, one firm must take the quality of the information product offered by its competitor into account. If there is a notable difference in terms of the information product's quality between the two firms, the firm with a higher product quality chooses to offer a bundle in equilibrium, while the competitor decides to adopt the freemium strategy. As depicted in region V_2-a-e-g of Figure 4, the information product quality of Firm 2 is much higher than that of Firm 1, so the optimal strategy for Firm 1 is to provide a bundle, and for Firm 2 is to offer a free information product and a paid service. However, in region V_1-b-f-h, Firm 1 has a more valuable information product than Firm 2, and the resulting equilibrium is that Firm 1 adopts the bundling strategy and Firm 2 chooses the freemium strategy. The freemium strategy increases a firm's market share by releasing a free product and further increases the network effect of the information product. Therefore, by offering a freemium strategy, a firm with a lower quality information product can attract more consumers to its product and enlarge the consumer base when facing a competitor with a higher quality information product. On the other hand, the information product with a higher quality is so attractive to consumers that they still prefer to buy the bundle even if no information products are offered for free. Thus, it is more profitable for the firm with the information product with a technological advantage to offer a bundle.

If the difference in the information product's quality is not significant, there are two possible situations: Both firms adopt the freemium strategy when the intrinsic values of their information product (i.e. the quality of information products) are large enough (in region g-e-d-f-h, labelled *FF*) and both firms offer a bundle when their information products are not sufficiently valuable to consumers (in region *O*-a-c-b, labelled *BB*). This result is in contrast to the previous literature, which suggested that a larger intrinsic value of an information product may result in the bundling strategy dominating the freemium strategy (Zhang et al. 2016). In this model, the intrinsic value of a bundle is formulated to be larger than the sum values of an information product and the associated service since the premium service has a complementary effect on the information product. When the information product quality is approximate and high enough (in region g-e-d-f-h, labelled *FF*) for both vendors, a free product released by a firm captures more marginal consumers from the competitor to purchase the premium service. Compared with purchasing an offered bundle, the marginal consumers gain more surplus value by consuming the information goods for free because of the strong positive network effects caused by the free product. In contrast, the freemium strategy will lead to the loss of the marginal consumer if both firms' information product quality is similar and sufficiently low (in region *O*-a-c-b, labelled *BB*). Under this case, the lost potential consumers prefer to only use the free information product rather than buy the premium service. Therefore, selling an information product and the premium service as a bundle is the best response for each firm regardless of the opposite firm's action.

For a group of parameter values that exist in region d-e-c-f of Figure 4, two feasible equilibria exist in which both firms decide to adopt the freemium strategy (i.e. Case *FF*) or choose the bundling strategy (i.e. Case *BB*) simultaneously. In this range of parameters, we show that the equilibrium in which both firms choose the bundling strategy is pay-off dominated.

Proposition 2 (Dominant Equilibrium Strategy)

If two equilibria coexist in which both firms sell the information product and the service as a bundle or both offer a free product and charge for the associated service, the former is a payoff dominated equilibrium strategy for both firms.

When the intrinsic value of an information product decreases to a relatively small level, releasing a free product will result in a purchaser loss because the free product is not sufficiently attractive to consumers. In other words, the freemium strategy will cannibalise potential sales of the paid services. If one firm chooses the freemium strategy, the other one will follow the freemium strategy at equilibrium, even if both adopting the bundling strategy would generate more benefits (in region d-e-c-f, labelled *BB*/*FF*). Both firms are caught in a prisoner's dilemma in this scenario and the condition is identified in the following proposition.

Proposition 3 (Prisoner's Dilemma Conditions)

When both firms provide a free information product in equilibrium and $V_j < CV_i + D$ holds, Firm i would be better off if neither firm provides an information product for free, where:

$$C = 1 - \frac{3\theta}{2(1+\theta)}\sqrt{\frac{2t(1+\theta)-2\alpha N}{\theta t}}, D = \frac{3}{2}\sqrt{\frac{2t(1+\theta)-2\alpha N}{\theta t}}\left(\frac{c}{1+\theta} - V\right) + 3t - \frac{3\alpha N}{1+\theta},\ i,j \in \{1,2\} \text{ and } i \neq j$$

By comparing the pay-offs in Table 2, we can find that $\pi_{FF}^{1*} = \pi_{FB}^{1*}$ and $\pi_{FF}^{2*} = \pi_{BF}^{2*}$. Thus the result of Proposition 3 can be easily obtained. Proposition 3 suggests that when the quality of two firms' information products is approximate and sufficiently low (i.e. $V_j - CV_i < D$ with $C < 1$), both firms would be better off when neither releases a free information product compared to when both do.

4.2 *Complementary effects and profits*

In this model, we consider the complementary effects between the information product and the premium service. Here, we investigate what impact an increase in the degree of complementary effect has on a firm's profit. Proposition 4 reveals the conditions under which an increasing complementary effect influences a firm's profit positively.

Proposition 4 (Impact of Complementary Effects on Two Firms' Profits)

(1) *If both firms provide a free information product and a paid service separately at equilibrium, the profit of Firm i increases as the degree of complementary effects θ increases when:*

$$\theta > \frac{V - c}{V + V_i}$$

(2) *If both firms provide a bundle at equilibrium, the profit of Firm i increases as the degree of complementary effects* θ *increases when:*

$$\theta > \frac{3t(\alpha N - t) + \left(V_i - V_j\right)(2\alpha N - t)}{t\left(3t + V_i - V_j\right)}$$

(3) *If only Firm i offers a free information product and Firm j provides a bundle at equilibrium, the profit of Firm i increases as the degree of complementary effects* θ *increases when:*

$$\theta > \frac{V - c}{V + V_i}$$

while the profit of Firm j increases as the degree of complementary effects θ *increases when:*

$$\theta > \frac{t - c - V_i - \alpha N}{V + V_j} + \frac{4\alpha N - 3t}{t}$$

Proposition 4 suggests that there exists a fixed threshold with regard to the degree of complementary effects for a given equilibrium profit of a firm. If the degree of complementary effects is larger than this threshold, an increase in θ will cause a greater gain for this firm. However, when the degree of complementary effects is smaller than this threshold, an increasing θ brings less benefit. Intuitively, an exogenous increase in the degree of complementary effects results in the higher intrinsic value of a bundle and brings the firm more benefit. However, it also increases the misfit cost of consumers who consume an information product as well as its associated service and intensifies price competition between both firms. From Table 2, we can easily find that the equilibrium prices of both firms always increase with an increasing degree of complementary effects. When the degree of complementary effects is increasing below a given threshold for a firm, the negative influence of the increased misfit cost and the intensified competition in price dominates, and then this firm's demand will decrease. In contrast, if the degree of complementary effects is larger than the given threshold, a further increase enhances the firm's benefit.

We present a numerical example to investigate how the profit of Firm 1 varies with an increasing complementary effect. The system parameters are set as follows: $V_1 = V_2 = 3000$, $V = 6000$, $t = 8000$, $N = 10{,}000$, $\alpha = 0.7$, and $c = 7000$. The varying range of the degree of complementary effects in each case satisfies the requirements that both firms have positive demand and that the market is fully covered. For instance, it is only feasible for Case *BB* when the degree of complementary effects exists in the interval between 0 and 1/6. Case *FF* is feasible only if θ is varying in the interval of (1/9, 1), such as in Case *FB*/Case *BF* when $\theta \in (1/9, 1.4)$. From Figure 5, we can find that the profit of Firm 1 in each case always increases due to an increase in θ. This occurs because the thresholds given by Proposition 4 are all negative in this numerical example. When the degree of complementary effects is lower than 1/6 (i.e. $\theta \in (0, 1/6)$), Firm 1's profit under Case *BB* increases linearly in θ, and the *BB* strategy is always feasible and more beneficial. As θ exceeds 1/6 but is lower than 1(i.e. $\theta \in (1/6, 1)$), the conditions under Case *BB* are violated, and thus the *BF* strategy is the most beneficial. If θ increases further and is larger than 1(i.e. $\theta \in (1, 1.4)$), the conditions for Case *FF* are violated, and Case *FB* and Case *BF* are retained to be feasible. Furthermore, the profit of Firm 1 under Case *BF* is larger than that under Case *FB*. Finally, we can observe that if θ is in the region of (1/9, 1/6), the four market configurations (i.e. Case *BB*, Case *FB*, Case *BF* and Case *FF*) are all feasible. Firm 1 benefits more under Case *BB* and so does Firm 2 due to the symmetry in the parameter setting ($\pi_{BB}^{1*} > \pi_{FB}^{1*}$, $\pi_{BB}^{2*} = \pi_{BB}^{1*} > \pi_{FB}^{1*} = \pi_{BF}^{2*}$), and thus both firms are better off when they choose to offer a charged bundle simultaneously. In this scenario, the equilibrium status is the *BB* strategy.

4.3 *Network effects and profits*

Now we examine how the profit of a firm changes with the exogenous increasing intensity of network effects. Proposition 5 states the conditions under which one firm would benefit from strengthening network effects exhibited by the information product.

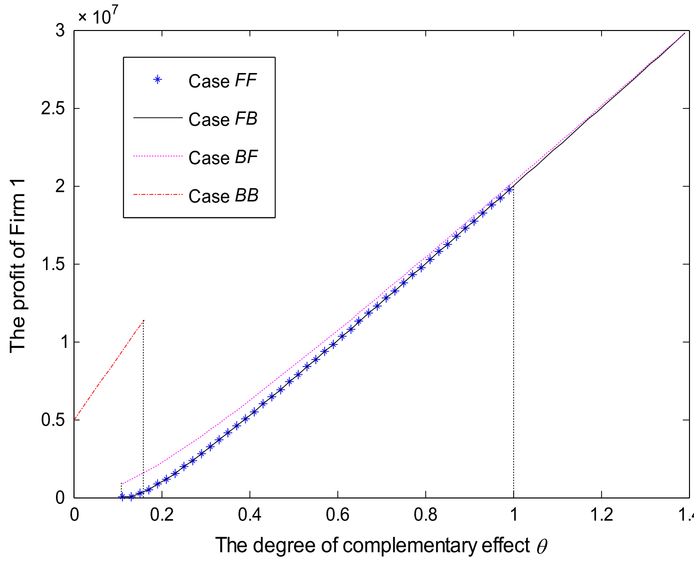

Figure 5. The profit of Firm 1 with respect to θ with four market configurations.

Proposition 5 (Impact of Network Effects on Two Firms' Profits)

(1) *If both firms provide a bundle at equilibrium, the profit of Firm i increases as the network intensity α increases when:*

$$\alpha > \frac{(1+\theta)\left(3t + V_j - V_i\right)}{3N}$$

(2) *If only Firm i offers a free information product and Firm j provides a bundle at equilibrium, the profit of Firm j increases as the network intensity α increases when:*

$$\alpha > \frac{(2+\theta)t - 2}{2N}$$

Proposition 5 shows that an increasing intensity of network effects only has an impact on the profit of a firm that chooses to sell a bundle, but the firm that adopts the freemium strategy does not receive benefits. There exists a certain threshold with respect to the network effects intensity for the equilibrium profit of a firm that offers a bundle. When the network effects intensity is increasing beyond the given threshold, this firm will obtain an increasing profit. While the network effects intensity grows below this threshold, the firm will have a decreased profit. Not only does an increasing intensity of network effects enhance the valuation of a product, but it also intensifies price competition in the market. The equilibrium outcomes in Table 2 suggest that when a firm adopts the bundling strategy, the equilibrium prices for its premium service are always decreasing in response to a growing intensity of network effects. Consequently, if the intensity of network effects is larger than the given threshold, the equilibrium demand for the service increases with respect to a further increase in α.

We apply the same numerical example performed in the previous Section to show how Firm 1's profits and the market equilibrium change with an exogenous increase in the intensity of network effects. All parameter values are the same except for the intensity of the network effects and the degree of the complementary effects. Specifically, we set $\theta = 0.5$ and $0 < \alpha < 1$. The network effects intensity varies in a special range in each case so that the conditions under which this case is feasible are satisfied. As shown in Figure 6, the profits of Firm 1 under Case *FF* and Case *FB* are independent of the varying intensity of the network effects. Whereas the profits under Case *BF* and Case *BB* for Firm 1 are both decreasing with an increasing α since the varying α is always smaller than the thresholds given in Proposition 5. When α falls in the region (0, 0.2), only Case *FB* and Case *BF* are feasible. The *BF* strategy is more beneficial than the *FB*

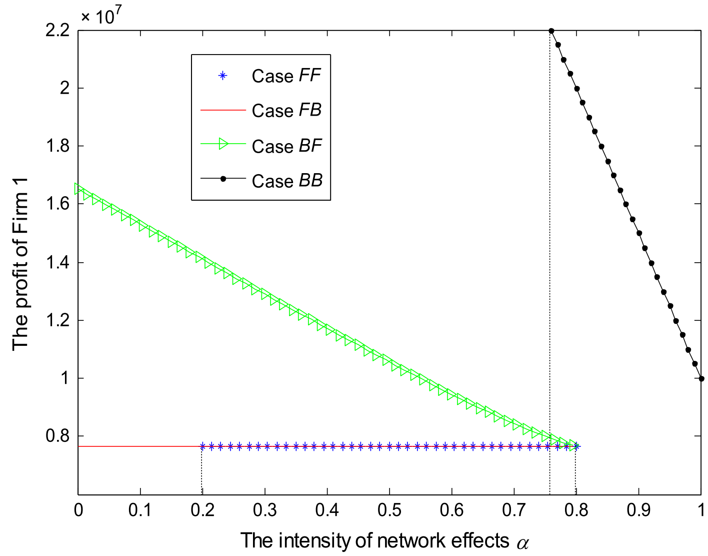

Figure 6. The profit of Firm 1 with respect to α with four market configurations.

strategy for Firm 1, whereas Firm 2 obtains more profit under Case *FB*. When α exceeds 0.2 but is lower than 23/30 (i.e. $\theta \in (0.2, 23/30)$), Case *FF* is feasible, and Firm 1 obtains the same profits when it offers a free information product regardless of its competitor's offering strategy. As α increases further and is not higher than 1, Case *BB* becomes feasible and is most beneficial, and Firm 1's profit decreases linearly with respect to α. If α is within the region (23/30, 0.8), all four cases are feasible and Case *BB* is the market equilibrium ($\pi_{BB}^{1*} > \pi_{FB}^{1*}$, $\pi_{BB}^{2*} = \pi_{BB}^{1*} > \pi_{FB}^{1*} = \pi_{BF}^{2*}$.).

4.4 *Market size and profits*

The following proposition demonstrates how the profit of a firm changes due to an exogenously increasing market size N.

Proposition 6 (Impact of Market Size on Two Firms' Profits)

(1) *If both firms provide a free information product and a paid service separately at equilibrium, the profit of each firm always increases as the market size N increases.*

(2) *If both firms provide a bundle at equilibrium, the profit of Firm i increases as the market size N increases when:*

$$V_i > V_j + \frac{9\alpha N}{1+\theta} - \frac{6\alpha^2 N^2 + 3(1+\theta)^2 t^2}{(1+\theta)^2 t}$$

(3) *If only Firm i offers a free information product and Firm j provides a bundle at equilibrium, the profit of Firm i always increases with respect to the market size N, while the profit of Firm j increases as the market size N increases when*

$$V_j > \frac{1}{1+\theta}\left(V_i + 3\alpha N - t - \frac{4\alpha^2 N^2}{(2+\theta)t}\right) - V$$

A firm that offers a free information product always benefits from increasing the market size. The equilibrium price is independent of N, and thus the equilibrium market share is increasing. However, when only one firm chooses to sell a bundle, its profit increases with respect to an increasing N only if the intrinsic value consumers gain from its information product is sufficiently large, while the competing firm can always benefit from an increase in the market size. If the quality of the information product from the firm that adopts the bundling strategy is not sufficiently high, the larger the market size, the more potential users who were intending to purchase the bundle from this firm are captured by its competitor's free product, which leads to a decrease in its own profit. When both firms decide to sell a bundle in equilibrium, the quality of the free information product offered by one firm has a direct effect on the competing firm's profit. In other words, if one firm has a sufficient advantage over its competitor in the quality of the information product, it can benefit from an exogenous increase in the market size.

To further investigate the impact of a varying market size on the profit of Firm 1, we perform another numerical example shown in Figure 7. We set $V_1 = V_2 = 3000$, $V = 6000$, $t = 8000$, $\alpha = 1$, $c = 7000$, $\theta = 1.5$ and $0 < N < 14{,}000$. When the market size is increasing in the region (0, 8000), Case *FB* and Case *BF* are both feasible and Firm 1's profits under two cases are increasing with respect to N. The *BF* strategy is always more beneficial than the *FB* strategy but Firm 1 obtains the same profits in both cases when $N = 8000$. As N increases further and does not exceed 14,000, the conditions for Case *FB* and Case *BF* are violated and Case *FF* becomes feasible. The profit of Firm 1 is increasing linearly with N in this range. When the market size is in the region (9667, 14,000), Case *BB* becomes feasible and the profit of Firm 1 decreases with an increasing N. Furthermore, both firms are better off adopting the *BB* strategy if the market size is varying in the range (9667, 13,490). Once the market size exceeds 13,490, the *FF* strategy becomes more beneficial than the *BB* strategy.

5. Extension: the market equilibrium in an uncovered market

A fully covered market is assumed to analyse the market equilibrium in the previous Sections. Here, the required conditions are relaxed to investigate both firms' pricing problem and the equilibrium strategies in which each firm acts as a local monopoly.

5.1 *Case* BB*: both local monopolies adopt the bundling strategy*

When both firms, who play as local monopolies decide to sell a bundle, the marginal customer that is indifferent about buying a bundle of Firm i ($i = 1, 2$) and making no purchase is located at:

$$x_1^u = \frac{(1+\theta)(V_1+V) - P_{BB}^{u1}}{t(1+\theta) - \alpha N}, x_2^u = \frac{(1+\theta)(t - V_2 - V) - \alpha N + P_{BB}^{u2}}{t(1+\theta) - \alpha N}.$$

The profit functions for both firms are given as follows:

$$\pi_{BB}^{u1} = x_1^u N\left(P_{BB}^{u1} - c\right), \pi_{BB}^{u2} = \left(1 - x_2^u\right)N\left(P_{BB}^{u2} - c\right).$$

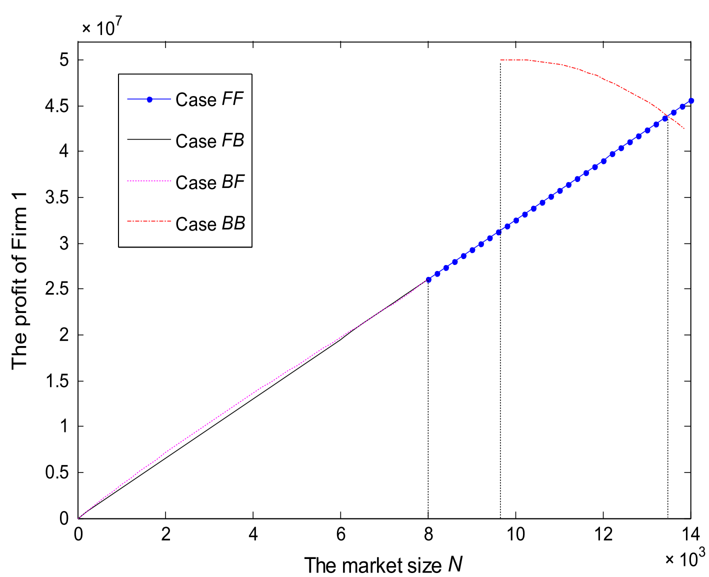

Figure 7. The profit of Firm 1 with respect to N with four market configurations.

Each firm sets an optimal price for its bundle (i.e. P_{BB}^{ui*}) to maximise its profit at equilibrium. To ensure that the second-order condition is positive, we need to assume that $(1 + \theta)t > \alpha N$. Differentiating the profit functions with respect to the price yields the equilibrium prices:

$$P_{BB}^{u1*} = \frac{(1+\theta)(V_1+V)+c}{2}, \; P_{BB}^{u2*} = \frac{(1+\theta)(V_2+V)+c}{2}.$$

Substituting the equilibrium prices into the above expressions, we obtain the marginal points and the equilibrium profits as follows:

$$x_1^u = \frac{(1+\theta)(V_1+V)-c}{2(t(1+\theta)-\alpha N)}, \; x_2^u = \frac{(1+\theta)(2t-V_2-V)+c-2\alpha N}{2(t(1+\theta)-\alpha N)},$$

$$\pi_{BB}^{u1*} = \frac{((1+\theta)(V_1+V)-c)^2 N}{4(t(1+\theta)-\alpha N)}, \; \pi_{BB}^{u2*} = \frac{((1+\theta)(V_2+V)-c)^2 N}{4(t(1+\theta)-\alpha N)}.$$

To guarantee that both firms act as local monopolies, the condition of $0 < x_1^u < x_2^u < 1$ must be satisfied, which is equivalent to:

$$V_i > \frac{c}{1+\theta} - V, \;\; V_1 + V_2 < 2t - 2V - \frac{2(\alpha N - c)}{1+\theta}, \; i = 1, 2.$$

Only if the above condition is met, will both firms who have positive demands not compete directly for the marginal customers.

5.2 *Case* FB *or* BF*: only one local monopoly adopts the freemium strategy and the other chooses the bundling strategy*

We focus on the case in which Firm 1 offers an information product for free with charging the associated service and Firm 2 sells a bundle (i.e. Case *FB*). The marginal customer that is indifferent about either only using the information product or consuming the information product and purchasing the associated service from Firm 1 is located at:

$$x_{11}^u = \frac{(1+\theta)V + \theta V_1 - P_{FB}^{u1}}{\theta t},$$

while the marginal customer who is indifferent about either using the information product from Firm 1 or not using it is located at:

$$x_1^u = \frac{V_1}{t - \alpha N}.$$

The indifferent marginal customer considering whether to buy a bundle from Firm 2 or not is located at:

$$x_2^u = \frac{(1+\theta)(t-V_2-V) - \alpha N + P_{FB}^{u2}}{t(1+\theta)-\alpha N}$$

The profit functions for both firms are:

$$\pi_{FB}^{u1} = x_{11}^u N\left(P_{FB}^{u1} - c\right), \pi_{FB}^{u2} = \left(1 - x_2^u\right) N\left(P_{FB}^{u2} - c\right).$$

Differentiating the profit functions with respect to the price yields the following equilibrium prices, which also needs the condition of $(1 + \theta)t > \alpha N$ to ensure the positive second-order condition:

$$P_{FB}^{u1*} = \frac{(1+\theta)V + \theta V_1 + c}{2}, \; P_{FB}^{u2*} = \frac{(1+\theta)(V_2+V)+c}{2}.$$

Accordingly, the values of marginal points and the equilibrium profits are:

$$x_1^u = \frac{(1+\theta)V + \theta V_1 - c}{2\theta t}, \; x_2^u = \frac{(1+\theta)(2t-V_2-V)+c-2\alpha N}{2(t(1+\theta)-\alpha N)},$$

$$\pi_{FB}^{u1*} = \frac{((1+\theta)V + \theta V_1 - c)^2 N}{4\theta t}, \; \pi_{FB}^{u2*} = \frac{((1+\theta)(V_2+V)-c)^2 N}{4(t(1+\theta)-\alpha N)}.$$

Both local monopolies have positive demands only if $0 < x_{11}^u < x_1^u < x_2^u < 1$, which is equivalent to:

$$t > \alpha N,$$

$$V_2 > \frac{c}{1+\theta} - V,$$

$$\frac{(1+\theta)(2t - V_2 - V) + c - 2\alpha N}{2(t(1+\theta) - \alpha N)} > V_1 > \max\{\frac{c - (1+\theta)V}{\theta}, \frac{(t-\alpha N)((1+\theta)V - c)}{\theta(t+\alpha N)}\}.$$

5.3 *Case* FF*: both local monopolies adopt the freemium strategy*

When two local monopolies choose to offer a free product and charge for the service, the whole market is divided into five segments. The marginal customer that is indifferent to either only using the information product or consuming the information product and purchasing the associated service from Firm 1 is located at:

$$x_{11}^u = \frac{(1+\theta)V + \theta V_1 - P_{FF}^{u1}}{\theta t},$$

while the indifferent marginal customer deciding whether to only use the information product or consume the information product with purchasing the associated service from Firm 2 is located at:

$$x_{22}^u = \frac{\theta(t - V_2 - V) - V + P_{FF}^{u2}}{\theta t}.$$

The indifferent marginal customer considering whether to use an information product from Firm 1 or not is located at:

$$x_1^u = \frac{V_1}{t - \alpha N},$$

while the marginal customer that is indifferent regarding whether to use an information product from Firm 2 or not is located at:

$$x_2^u = \frac{t - V_2 - \alpha N}{t - \alpha N}.$$

The profit functions for both firms are given by:

$$\pi_{FF}^{u1} = x_{11}^u N\left(P_{FF}^{u1} - c\right), \pi_{FF}^{u2} = \left(1 - x_{22}^u\right)N\left(P_{FF}^{u2} - c\right).$$

Maximising the profit function with respect to the price for each firm yields:

$$P_{FF}^{u1*} = \frac{(1+\theta)V + \theta V_1 + c}{2}, \quad P_{FF}^{u2*} = \frac{(1+\theta)V + \theta V_2 + c}{2}.$$

Then, the values of marginal points and the equilibrium profits are:

$$x_{11}^u = \frac{(1+\theta)V + \theta V_1 - c}{2\theta t}, \; x_{22}^u = \frac{\theta(2t - V_2 - V) - V + c}{2\theta t},$$

$$\pi_{FF}^{u1*} = \frac{((1+\theta)V + \theta V_1 - c)^2 N}{4\theta t}, \pi_{FF}^{u2*} = \frac{((1+\theta)V + \theta V_2 - c)^2 N}{4\theta t}.$$

Finally, this case is feasible only when $0 < x_{11}^u < x_1^u < x_2^u < x_{22}^u < 1$ is met, which is equivalent to:

$$V_1 + V_2 < t - \alpha N,$$

$$V_i > \max\{\frac{c - (1+\theta)V}{\theta}, \frac{(t-\alpha N)((1+\theta)V - c)}{\theta(t+\alpha N)}\}, \quad i = 1, 2.$$

We summarise the equilibrium prices and the equilibrium profits for both firms in the uncovered market in Tables 4 and 5, respectively.

Table 4. The equilibrium prices in an uncovered market.

Firm 1	Firm 2	
	Freemium strategy	Bundling strategy
Freemium strategy	$P_{FF}^{u1^*} = \frac{\theta V_1+(1+\theta)V+c}{2}$	$P_{FB}^{u1^*} = \frac{\theta V_1+(1+\theta)V+c}{2}$
	$P_{FF}^{u2^*} = \frac{\theta V_2+(1+\theta)V+c}{2}$	$P_{FB}^{u2^*} = \frac{(1+\theta)(V+V_2)+c}{2}$
Bundling strategy	$P_{BF}^{u1^*} = \frac{(1+\theta)(V+V_1)+c}{2}$	$P_{BB}^{u1^*} = \frac{(1+\theta)(V+V_1)+c}{2}$
	$P_{BF}^{u2^*} = \frac{\theta V_2+(1+\theta)V+c}{2}$	$P_{BB}^{u2^*} = \frac{(1+\theta)(V+V_2)+c}{2}$

Table 5. The equilibrium profits in an uncovered market.

Firm 1	Firm 2	
	Freemium strategy	Bundling strategy
Freemium strategy	$\pi_{FF}^{u1^*} = \frac{(\theta V_1+(1+\theta)V-c)^2 N}{4\theta t}$	$\pi_{FB}^{u1^*} = \frac{(\theta V_1+(1+\theta)V-c)^2 N}{4\theta t}$
	$\pi_{FF}^{u2^*} = \frac{(\theta V_2+(1+\theta)V-c)^2 N}{4\theta t}$	$\pi_{FB}^{u2^*} = \frac{((1+\theta)(V+V_2)-c)^2 N}{4((1+\theta)t-\alpha N)}$
Bundling strategy	$\pi_{BF}^{u1^*} = \frac{((1+\theta)(V+V_1)-c)^2 N}{4((1+\theta)t-\alpha N)}$	$\pi_{BB}^{u1^*} = \frac{((1+\theta)(V+V_1)-c)^2 N}{4((1+\theta)t-\alpha N)}$
	$\pi_{BF}^{u2^*} = \frac{(\theta V_2+(1+\theta)V-c)^2 N}{4\theta t}$	$\pi_{BB}^{u2^*} = \frac{((1+\theta)(V+V_2)-c)^2 N}{4((1+\theta)t-\alpha N)}$

6. Conclusions

In this paper, we attempt to examine a duopolistic information product market in which two firms offer a differentiated information product exhibiting positive network effects and a complementary premium service to consumers. Each firm has two marketing options: a freemium strategy or a bundling strategy. Four possible market configurations follow when two firms are making offering decisions (i.e. Case *FF*, Case *FB*, Case *BF*, and Case *BB*). Both vendors simultaneously set optimal prices to maximise their own profits according to the observed offering decisions.

By applying a game theoretical method, we derive the equilibrium prices and profits and analyse the market equilibrium. The conditions under which each marketing strategy prevails are identified. We show that a firm's decision whether to employ the freemium strategy or adopt the bundling strategy depends largely on the quality of the information product from its rival. A high product quality firm should adopt the bundling strategy. Moreover, we find that in contrast to the prior literature, when information products are similar in quality, both firms will be better off by adopting the freemium strategy if the products' intrinsic values are sufficiently large while they prefer the bundling strategy if the products' intrinsic values are sufficiently small. However, two firms adopting consistent strategies with a similar information product quality may be caught in a prisoner's dilemma. We also have identified the conditions under which a prisoner's dilemma occurs. In addition, we reveal that a firm' profit can be enhanced by an increase in the degree of the complementary effect or in the intensity of network effects, only when complementary effect or network effects are sufficiently strong. We also demonstrate that a firm can benefit from an increasing market size only if the intrinsic value of its information product is large enough. Several numerical examples are presented to illustrate how Firm 1's profit and the market equilibrium change with several underlying exogenous factors (i.e. product complementarity, network effects, the market size). Finally, we briefly investigate the uncovered market and obtain the equilibrium profits for both firms.

Our findings can provide firms with several important managerial suggestions on their marketing strategies in a competitive information product market. First, when a firm is planning the bundling strategy or freemium strategy, it needs to compare its information product quality or intrinsic value with its rival ones. If the firm's information product quality has an advantage over its rival, it should adopt the bundling strategy to maximise the profit. Conversely, it should choose the freemium strategy to build its consumer base and thus enhance the benefit. Second, when the firm has a same level or almost same level quality of information product with its competitor, it would be better to follow the rival's adopted strategy according to their intrinsic value. Otherwise both firms' profit would be worse off. Last, the effect of the degree of product complementarity or the intensity of network effects on the firm's profit depends on the magnitude of complementary effects or network effects. For example, if a firm's degree of product complementarity or intensity of network effects is more than some threshold, it can choose any one strategy. In all, these management insights expand the marketing knowledge of information products under different scenarios.

To further expand this study, future research can examine the substitution effects between information goods and premium services. Some online start-ups may encounter the mismatch between a core information product and premium services when offering new information goods. A functional unfit between the core product and the premium service may reduce the total value, and thus, they can be regarded as substitutes. In this situation, the strategies that a firm will adopt and the resulting market equilibrium changes can be investigated.

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Notes

1. This paper analyses cases where the premium service is uniform in quality for both firms. An alternative formulation is to add vertical differentiation by assuming that the service has different intrinsic values (e.g. V_{s1} and V_{s2}) for both firms. This model can be analysed in a similar way, but the analysis becomes unnecessarily complicated and the main insights remain unchanged. We also acknowledge that a firm may provide a premium service that is horizontally differentiated; however, it is beyond the scope of this paper. Special thanks to an anonymous reviewer for suggesting this statement.
2. We make this assumption by considering that the same quality provision has the same marginal cost. A similar type of assumption can be found in Gabszewicz and Thisse (1979) and Etzion and Pang (2014).
3. We thank an anonymous reviewer for bringing this interpretation to our attention.

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Appendix 1

Proof of Proposition 1

In equilibrium, neither firm has the incentive to deviate, or it will bear profit loss. Therefore, we have:

(a) Case *FF* is an equilibrium when $\pi_{FF}^{1*} > \pi_{BF}^{1*}$, and $\pi_{FF}^{2*} > \pi_{FB}^{2*}$.
(b) Case *BB* is an equilibrium when $\pi_{BB}^{1*} > \pi_{FB}^{1*}$, and $\pi_{BB}^{2*} > \pi_{BF}^{2*}$.
(c) Case *FB* is an equilibrium when $\pi_{FB}^{1*} > \pi_{BB}^{1*}$, and $\pi_{FB}^{2*} > \pi_{FF}^{2*}$.
(d) Case *BF* is an equilibrium when $\pi_{BF}^{1*} > \pi_{FF}^{1*}$, and $\pi_{BF}^{2*} > \pi_{BB}^{2*}$.

By solving a group of inequalities in (a), the condition in Proposition 1-(1) can be obtained; by solving a group of inequalities in (b), the condition in Proposition 1-(2) can be obtained. Conditions in (3) and (4) in Proposition 1 can be derived similarly.

Proof of Proposition 4

(a) When both firms release a free information product and charge for the associated service separately at equilibrium, we differentiate the equilibrium profit function with respect to θ:

$$\frac{\partial}{\partial\theta}\pi_{FF}^{i*} = \frac{\partial}{\partial\theta}\frac{(\theta V_i + (1+\theta)V - c)^2 N}{4\theta t} = \frac{(\theta V_i + (1+\theta)V - c)(\theta V_i + (\theta-1)V + c)}{4\theta^2 t}$$

Noting that θV_i+$(1+\theta)$ $V - c$ is positive (i.e. $P_{FF}^{i*} - c > 0$), the above derivative is positive when $\theta V_i + (\theta - 1)V + c$ is positive, which is equivalent to the condition presented in Proposition 4-(1).

(a) By differentiating the equilibrium profit function, when both firms sell an information product and the premium service as a bundle at equilibrium, with respect to θ, we have:

$$\begin{aligned}\frac{\partial}{\partial\theta}\pi_{BB}^{i*} &= \frac{\partial}{\partial\theta}\frac{\left(\left(3t+V_i-V_j\right)(1+\theta)-3\alpha N\right)^2 N}{18((1+\theta)t-\alpha N)} \\ &= \frac{\left(\left(3t+V_i-V_j\right)(1+\theta)-3\alpha N\right)\left(t(1+\theta)\left(3t+V_i-V_j\right)-\alpha N\left(3t+2V_i-2V_j\right)\right)}{18((1+\theta)t-\alpha N)^2}N\end{aligned}$$

Because $(3t + V_i - V_j)(1 + \theta) - 3\alpha N$ is positive (i.e. $P_{BB}^{i*} - c > 0$), the derivative is ensured to be positive if $t(1 + \theta)$ $(3t + V_i - V_j) - \alpha N(3t + 2V_i - 2V_j)$ is positive, which is equivalent to the condition in Proposition 4-(2) after some calculations.

Suppose that Firm 1 releases a free information product and Firm 2 offers a bundle at equilibrium, it is easily determined that Firm 1 obtains the same profit as it does under Case *FF* (i.e. $\pi_{FF}^{1*} = \pi_{FB}^{1*}$). Thus, it has the same condition in both cases. The derivative of the profit for Firm 2 is:

$$\begin{aligned}\frac{\partial}{\partial\theta}\pi_{FB}^{2*} &= \frac{\partial}{\partial\theta}\frac{((1+\theta)(V+V_2)+t-c-V_1-\alpha N)^2 N}{4((2+\theta)t-2\alpha N)} \\ &= \frac{((1+\theta)(V+V_2)+t-c-V_1-\alpha N)((V+V_2)((3+\theta)t-4\alpha N)-t(t-c-V_1-\alpha N))}{4((2+\theta)t-2\alpha N)^2}N\end{aligned}$$

Given that $(1 + \theta)(V + V_2) + t - c - V_1 - \alpha N$ is positive (i.e. $P_{FB}^{2*} - c > 0$), the above is positive if $(V + V_2)$ $(t(3 + \theta) - 4\alpha N) - t$ $(t - c - V_1 - \alpha N)$ is positive.

The condition when only Firm 2 offers a free information product and Firm 1 provides a bundle at equilibrium can be derived similarly. The conditions under both cases are summarised in Proposition 4-(3).

Proof of Proposition 5

(a) When both firms choose to offer a bundle at equilibrium, the derivative of the profit function of Firm i is given by:

$$\begin{aligned}\frac{\partial}{\partial\alpha}\pi_{BB}^{i*} &= \frac{\partial}{\partial\alpha}\frac{\left(\left(3t+V_i-V_j\right)(1+\theta)-3\alpha N\right)^2 N}{18((1+\theta)t-\alpha N)} \\ &= \frac{\left(\left(3t+V_i-V_j\right)(1+\theta)-3\alpha N\right)\left(3\alpha N-(1+\theta)\left(3t+V_j-V_i\right)\right)}{18((1+\theta)t-\alpha N)^2}N^2\end{aligned}$$

Since $(3t + V_i - V_j)(1 + \theta) - 3\alpha N$ is positive, the above is positive when $3\alpha N - (1 + \theta)(3t + V_j - V_i) > 0$, which is equivalent to the condition in Proposition 5-(1).

(a) If only Firm 1 releases a free information product and Firm 2 offers a bundle at equilibrium, the derivative of the profit function of Firm j is given by:

$$\begin{aligned}\frac{\partial}{\partial\theta}\pi_{FB}^{2*} &= \frac{\partial}{\partial\theta}\frac{((1+\theta)(V+V_2)+t-c-V_1-\alpha N)^2 N}{4((2+\theta)t-2\alpha N)} \\ &= \frac{((1+\theta)(V+V_2)+t-c-V_1-\alpha N)(2+2\alpha N-(2+\theta)t)}{2((2+\theta)t-2\alpha N)^2}N^2\end{aligned}$$

Provided that $(1+\theta)(V+V_2)+t-c-V_1-\alpha N$ is positive, the above is positive if and only if $2+2\alpha N-(2+\theta)t>0$, which is equivalent to the condition in Proposition 5-(2). When Firm 1 offers a bundle and Firm 2 releases a free information product at equilibrium, we can derive the same condition stated in Proposition 5-(2).

Proof of Proposition 6

(a) If both firms offer a free information product and charge for the associated service separately at equilibrium, the derivative of the profit function of Firm i is:

$$\frac{\partial\pi_{FF}^{i*}}{\partial N} = \frac{\partial}{\partial N}\frac{(\theta V_i+(1+\theta)V-c)^2 N}{4\theta t} = \frac{(\theta V_i+(1+\theta)V-c)^2}{4\theta t}$$

The above derivative is a positive constant when the market size N varies. Thus the profit of each firm always increases in an increasing market size.

(a) If both firms offer a bundle at equilibrium, the derivative of the profit function of Firm i is:

$$\begin{aligned}\frac{\partial\pi_{BB}^{i*}}{\partial N} &= \frac{\partial}{\partial N}\frac{\left(\left(3t+V_i-V_j\right)(1+\theta)-3\alpha N\right)^2 N}{18((1+\theta)t-\alpha N)} \\ &= \frac{\left(\left(3t+V_i-V_j\right)(1+\theta)-3\alpha N\right)\left(t(1+\theta)\left(\left(3t+V_i-V_j\right)(1+\theta)-3\alpha N\right)-6\alpha N((1+\theta)t-\alpha N)\right)}{18((1+\theta)t-\alpha N)^2}\end{aligned}$$

Noting that $(1+\theta)(3t+V_i-V_j)-3\alpha N$ is positive, the above derivative is positive if $t(1+\theta)((1+\theta)\ (3t+V_i-V_j)-3\alpha N)-6\alpha N((1+\theta)\ t-\alpha N)>0$, which is equivalent to the condition in Proposition 6-(2).

If only Firm 1 releases a free information product and Firm 2 offers a bundle at equilibrium, the derivative of the profit function of Firm 1 is:

$$\frac{\partial\pi_{FB}^{1*}}{\partial N} = \frac{\partial}{\partial N}\frac{(\theta V_1+(1+\theta)V-c)^2 N}{4\theta t} = \frac{(\theta V_1+(1+\theta)V-c)^2}{4\theta t},$$

which is always positive with respect to N. The derivative of the profit function of Firm 2 is:

$$\begin{aligned}\frac{\partial\pi_{FB}^{2*}}{\partial N} &= \frac{\partial}{\partial N}\frac{((1+\theta)(V+V_2)+t-c-V_1-\alpha N)^2 N}{4((2+\theta)t-2\alpha N)} \\ &= \frac{((1+\theta)(V+V_2)+t-c-V_1-\alpha N)(t(2+\theta)((1+\theta)(V+V_2)+t-c-V_1-\alpha N)-2\alpha N((2+\theta)t-2\alpha N))}{4((2+\theta)t-2\alpha N)^2}\end{aligned}$$

We can check that the first term of the numerator of the above derivative is positive. Thus the fraction is positive on the condition that the second term of the numerator is positive. The condition of Proposition 6-(3) under Case *FB* is obtained after some calculation. The result in Case *BF* can be derived similarly, and we conclude the general condition in both cases in Proposition 6-(3).