

Given the matrix  $B = \begin{pmatrix} 4-x & -4 & -4 \\ 2 & -2-x & -4 \\ 3 & -3 & -4-x \end{pmatrix}$ , find all values of  $x$  such that  $\det(B) = 0$

$$\det(B) = (4-x) \cdot \det \begin{pmatrix} -2-x & -4 \\ -3 & -4-x \end{pmatrix} + 4 \cdot \det \begin{pmatrix} 2 & -4 \\ 3 & -4-x \end{pmatrix} - 4 \cdot \det \begin{pmatrix} 2 & -2-x \\ 3 & -3 \end{pmatrix}$$

$$0 = (4-x) \left( (-2-x)(-4-x) - 12 \right) + 4 \left( 2(-4-x) + 12 \right) - 4 \left( -6 - 3(-2-x) \right)$$

$$0 = (4-x) (x^2 - 6x - 4) + 4(-2x + 4) - 12x$$

$$0 = (4-x)(x^2 - 6x - 4) - 8x + 16 - 12x$$

$$0 = (4-x)(x^2 - 6x - 4) - 20x + 16$$

→ Thus,  $\det(B) = 0$  when  $x = -4$  or  $x = -2$

$$0 = -x^3 - 2x^2 + 28x - 16 - 20x + 16$$

$$0 = -x^3 - 2x^2 + 8x$$

$$0 = x^3 + 2x^2 - 8$$

$$0 = (x+4)(x-2)$$

$$x = -4, 2$$