32)

Approximate the indefinite Integral, using first four non-zero terms of the integrand's Taylor Series.

$\int_0^{\pi^2/4} \cos(\sqrt{x})\,dx$, we know TSE of $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots$ based off of theorem 8.8.2, the taylor series of $\cos(\sqrt{x})$ is then:

$$\cos(\sqrt{x}) = 1 - \frac{(\sqrt{x})^2}{2!} + \frac{(\sqrt{x})^4}{4!} - \frac{(\sqrt{x})^6}{6!} = 1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!}$$

Thus, to approximate the integral, we can solve the following:

$$\int_0^{\pi^2/4} 1 - \frac{x}{2} + \frac{x^2}{24} - \frac{x^3}{720}\,dx = x - \frac{x^2}{4} + \frac{x^3}{72} - \frac{x^4}{2,880}\Bigg]_0^{\pi^2/4} = \left(\frac{\pi^2}{4} - \frac{\left(\frac{\pi^2}{4}\right)^2}{4} + \frac{\left(\frac{\pi^2}{4}\right)^3}{72} - \frac{\left(\frac{\pi^2}{4}\right)^4}{2,880}\right) - 0$$

$$= \frac{\pi^2}{4} - \frac{\pi^4}{64} + \frac{\pi^6}{4,608} - \frac{\pi^8}{737,280} \approx \boxed{1.4114}$$

Plugging in $\int_0^{\pi^2/4} \cos(\sqrt{x})\,dx$ into online software gives the exact answer of $2-\pi$, meaning that the approximation is actually quite close ($2-\pi \approx 1.14$).

Theorem 8.8.2 (used above): $f(h(x)) = \sum_{n=0}^{\infty} a_n (h(x))^n$ for $|h(x)| < R$, iff $f(x) = \sum_{n=0}^{\infty} a_n x^n$ and $g(x) = \sum_{n=0}^{\infty} b_n x^n$ converge absolutely for $|x| < R$ s.t. R is the radius of convergence and $h(x)$ is continuous