Multiple linear regression

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Grading the professor

Many college courses conclude by giving students the opportunity to evaluate the course and the instructor anonymously. However, the use of these student evaluations as an indicator of course quality and teaching effectiveness is often criticized because these measures may reflect the influence of non-teaching related characteristics, such as the physical appearance of the instructor. The article titled, "Beauty in the classroom: instructors' pulchritude and putative pedagogical productivity" by Hamermesh and Parker found that instructors who are viewed to be better looking receive higher instructional ratings.

Here, you will analyze the data from this study in order to learn what goes into a positive professor evaluation.

Getting Started

Load packages

In this lab, you will explore and visualize the data using the **tidyverse** suite of packages. The data can be found in the companion package for OpenIntro resources, **openintro**.

Let's load the packages.

```
library(tidyverse)
library(openintro)
library(GGally)
```

This is the first time we're using the `GGally` package. You will be using the `ggpairs` function from this package later in the lab.

The data

The data were gathered from end of semester student evaluations for a large sample of professors from the University of Texas at Austin. In addition, six students rated the professors' physical appearance. The result is a data frame where each row contains a different course and columns represent variables about the courses and professors. It's called `evals`.

```
glimpse(evals)
```

```
## Rows: 463
## Columns: 23
## $ course_id     <int> 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 1~
## $ prof_id       <int> 1, 1, 1, 1, 2, 2, 2, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 5, 5,~
## $ score         <dbl> 4.7, 4.1, 3.9, 4.8, 4.6, 4.3, 2.8, 4.1, 3.4, 4.5, 3.8, 4~
## $ rank          <fct> tenure track, tenure track, tenure track, tenure track, ~
## $ ethnicity     <fct> minority, minority, minority, minority, not minority, no~
## $ gender        <fct> female, female, female, female, male, male, male, male, ~
## $ language      <fct> english, english, english, english, english, english, en~
```

```
## $ age           <int> 36, 36, 36, 36, 59, 59, 59, 51, 51, 40, 40, 40, 40, 40, ~
## $ cls_perc_eval <dbl> 55.81395, 68.80000, 60.80000, 62.60163, 85.00000, 87.500~
## $ cls_did_eval  <int> 24, 86, 76, 77, 17, 35, 39, 55, 111, 40, 24, 24, 17, 14,~
## $ cls_students  <int> 43, 125, 125, 123, 20, 40, 44, 55, 195, 46, 27, 25, 20, ~
## $ cls_level     <fct> upper, upper, upper, upper, upper, upper, upper, upper, ~
## $ cls_profs     <fct> single, single, single, single, multiple, multiple, mult~
## $ cls_credits   <fct> multi credit, multi credit, multi credit, multi credit, ~
## $ bty_f1lower   <int> 5, 5, 5, 5, 4, 4, 4, 5, 5, 2, 2, 2, 2, 2, 2, 2, 2, 7, 7,~
## $ bty_f1upper   <int> 7, 7, 7, 7, 4, 4, 4, 2, 2, 5, 5, 5, 5, 5, 5, 5, 5, 9, 9,~
## $ bty_f2upper   <int> 6, 6, 6, 6, 2, 2, 2, 5, 5, 4, 4, 4, 4, 4, 4, 4, 4, 9, 9,~
## $ bty_m1lower   <int> 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 7, 7,~
## $ bty_m1upper   <int> 4, 4, 4, 4, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 6, 6,~
## $ bty_m2upper   <int> 6, 6, 6, 6, 3, 3, 3, 3, 3, 2, 2, 2, 2, 2, 2, 2, 2, 6, 6,~
## $ bty_avg       <dbl> 5.000, 5.000, 5.000, 5.000, 3.000, 3.000, 3.000, 3.333, ~
## $ pic_outfit    <fct> not formal, not formal, not formal, not formal, not form~
## $ pic_color     <fct> color, color, color, color, color, color, color, color, ~
```

We have observations on 21 different variables, some categorical and some numerical. The meaning of each variable can be found by bringing up the help file:

```
?evals
```

Exploring the data

1. Is this an observational study or an experiment? The original research question posed in the paper is whether beauty leads directly to the differences in course evaluations. Given the study design, is it possible to answer this question as it is phrased? If not, rephrase the question.

WJ Response:

This is an observational study given that the experiment has no control or "treatment" groups. Given that observational studies use a sample of data to draw inferences about the general population, I think the research question could be rephrased as: does the attractiveness of a school professor have any influence on the ratings they recieve by students?

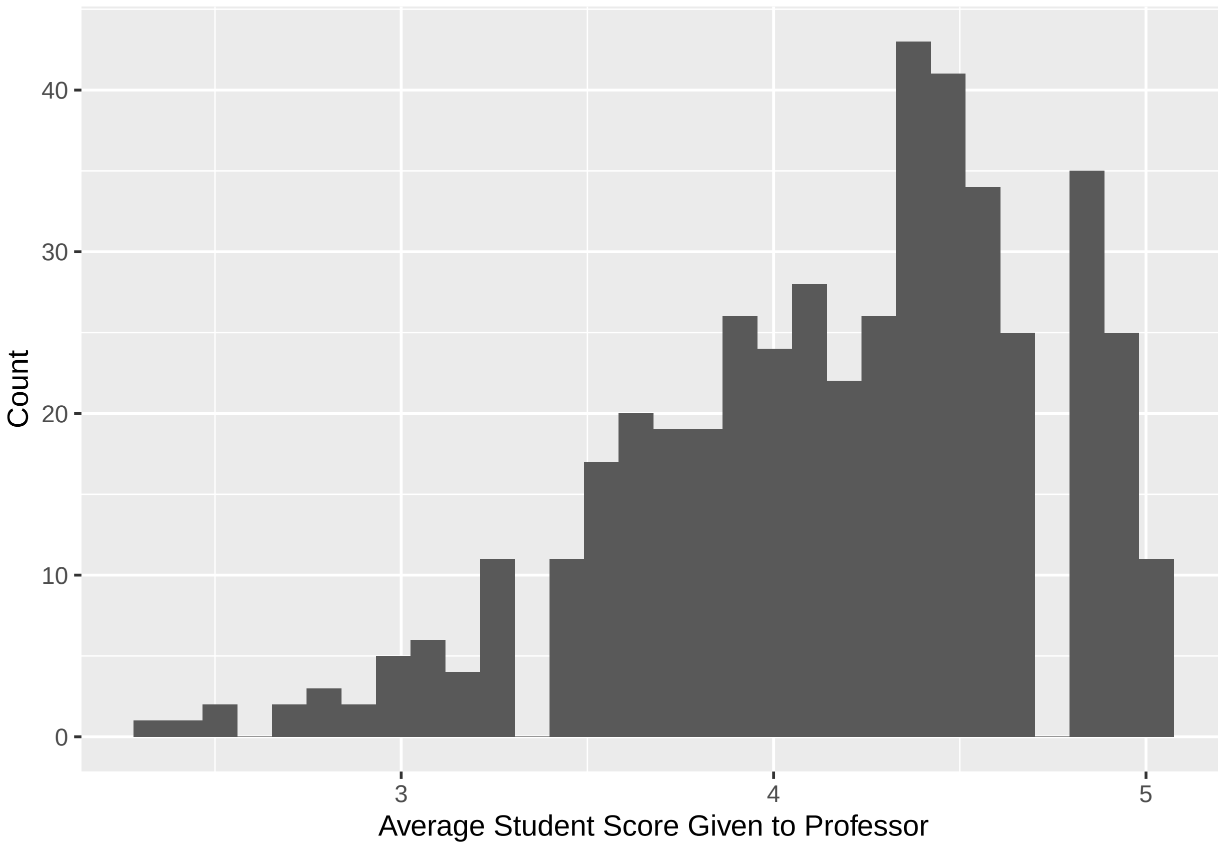
---

2. Describe the distribution of `score`. Is the distribution skewed? What does that tell you about how students rate courses? Is this what you expected to see? Why, or why not?

WJ Response:

The chunk below creates a histogram of the `score` variable:

```
ggplot(data = evals, aes(x = score)) +
  geom_histogram() +
    labs(
      x = "Average Student Score Given to Professor",
      y = "Count"
    )
```



The distribution is skewed to the right, indicating that the majority of students tend to rate professors on the upper end of the 0-5 scale.

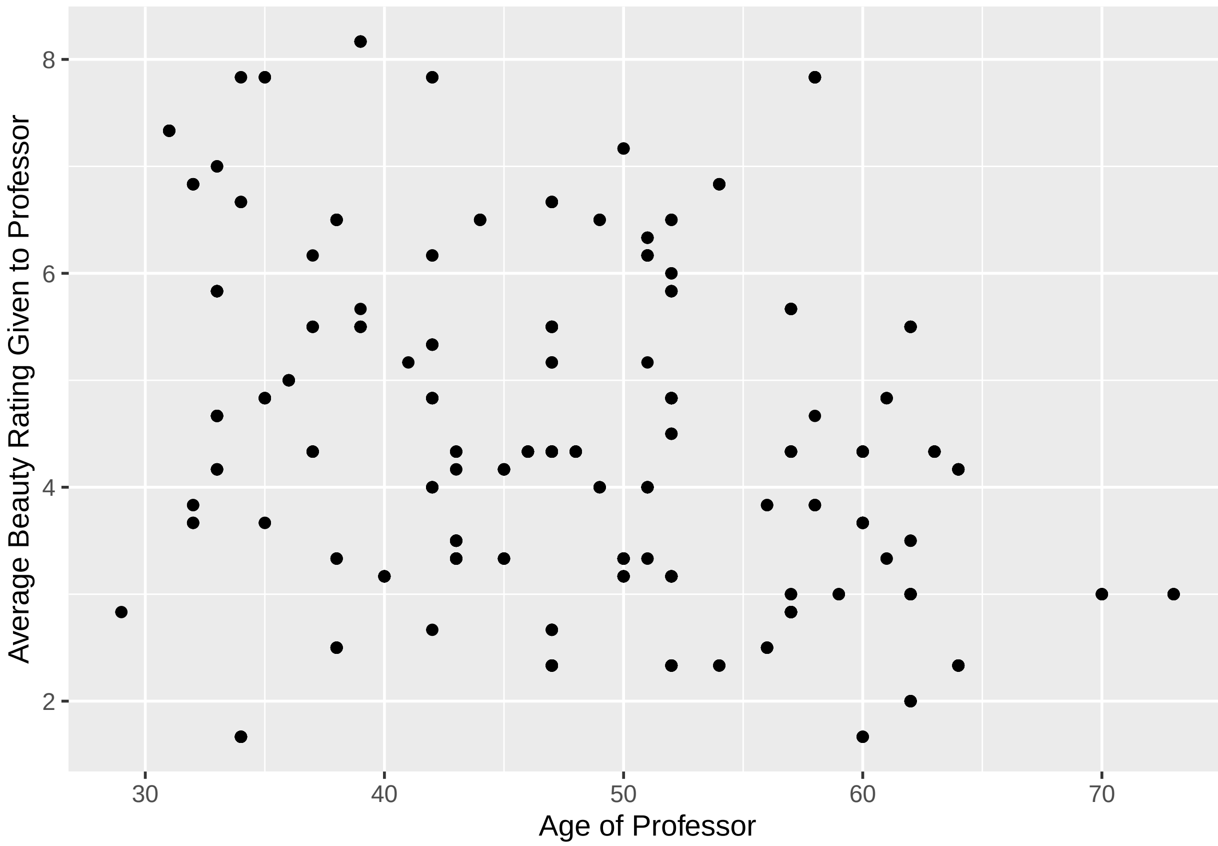
---

3. Excluding `score`, select two other variables and describe their relationship with each other using an appropriate visualization.

WJ Response:

The code chunk below shows the relationship between the age of the professor (`age`) and their average beauty rating (`bty_avg`) by means of a scatterplot:

```
ggplot(data = evals, aes(x = age, y= bty_avg)) +
  geom_point() +
    labs(
      x = 'Age of Professor',
      y = 'Average Beauty Rating Given to Professor'
    )
```



The plot above appears to show a slight negative correlation between the two variables. We can check for sure by looking at the correlation coefficient:

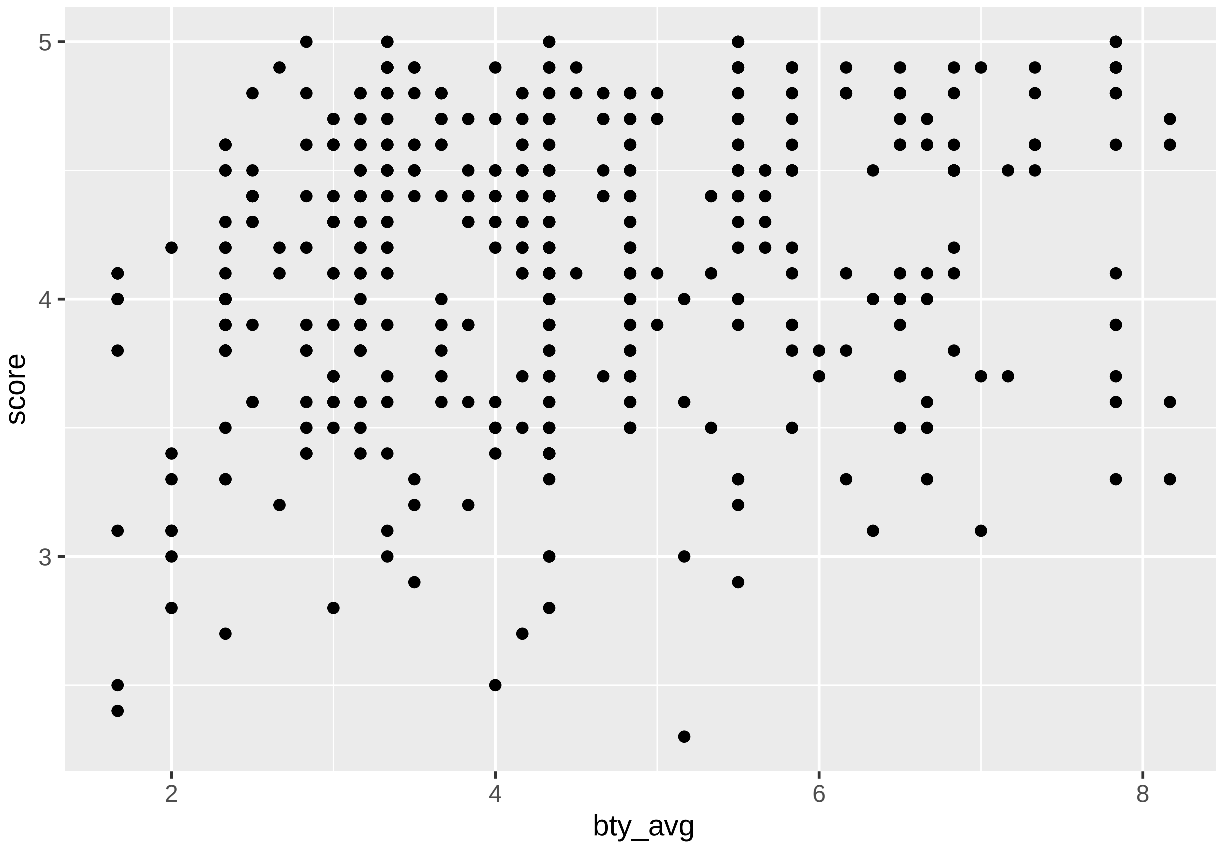
```
cor(evals$age, evals$bty_avg)
```

```
## [1] -0.3046034
```

Simple linear regression

The fundamental phenomenon suggested by the study is that better looking teachers are evaluated more favorably. Let's create a scatterplot to see if this appears to be the case:

```
ggplot(data = evals, aes(x = bty_avg, y = score)) +
  geom_point()
```



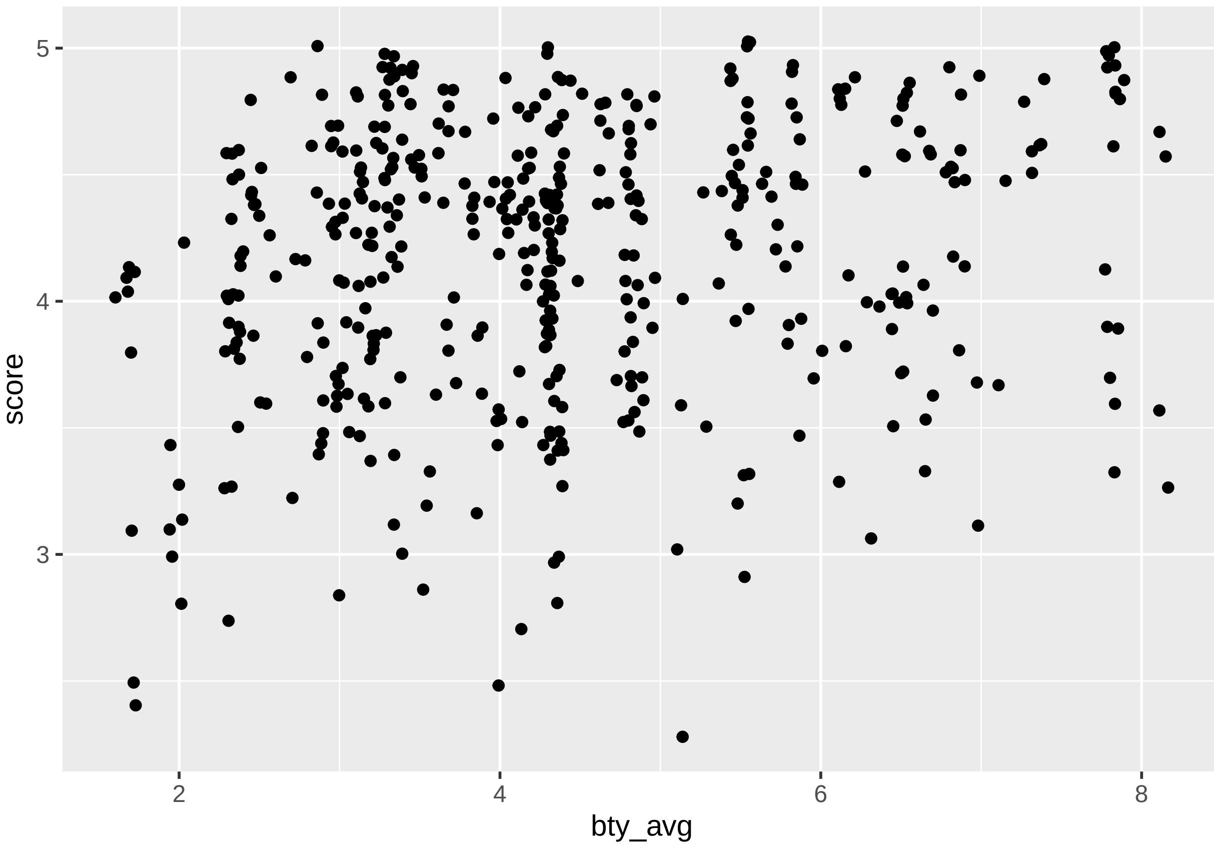
Before you draw conclusions about the trend, compare the number of observations in the data frame with the approximate number of points on the scatterplot. Is anything awry?

4. Replot the scatterplot, but this time use `geom_jitter` as your layer. What was misleading about the initial scatterplot?

WJ Response:

Replotting the scatterplot:

```
ggplot(data = evals, aes(x = bty_avg, y = score)) +
  geom_jitter()
```



Yes. The `score` and `beauty_average` field have all of their values rounded to the nearest tenth, meaning that the first scatter plot likely had many overlapping points and giving it the appearance of not showing the full dataset. The `geom_jitter` function adds a small amount of random noise to the data so that it can be better visualized and prevent this "overplotting" feature.

---

5. Let's see if the apparent trend in the plot is something more than natural variation. Fit a linear model called `m_bty` to predict average professor score by average beauty rating. Write out the equation for the linear model and interpret the slope. Is average beauty score a statistically significant predictor? Does it appear to be a practically significant predictor?

Add the line of the bet fit model to your plot using the following:

WJ Response:

The code chunk below creates a linear model to relate `avg_bty` and `score`:

```
m_bty <- lm(score ~ bty_avg, data = evals)

b_1 <- unname(coefficients(m_bty)[2])
b_0 <- unname(coefficients(m_bty)[1])
eqn <- sprintf("$\\hat{y} = %.2f + (%.2f \\beta_1$)", b_0, b_1)

summary(m_bty)
```

```
##
## Call:
## lm(formula = score ~ bty_avg, data = evals)
```

```
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.9246 -0.3690  0.1420  0.3977  0.9309
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.88034    0.07614   50.96  < 2e-16 ***
## bty_avg      0.06664    0.01629    4.09 5.08e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5348 on 461 degrees of freedom
## Multiple R-squared:  0.03502,    Adjusted R-squared:  0.03293
## F-statistic: 16.73 on 1 and 461 DF,  p-value: 5.083e-05
```

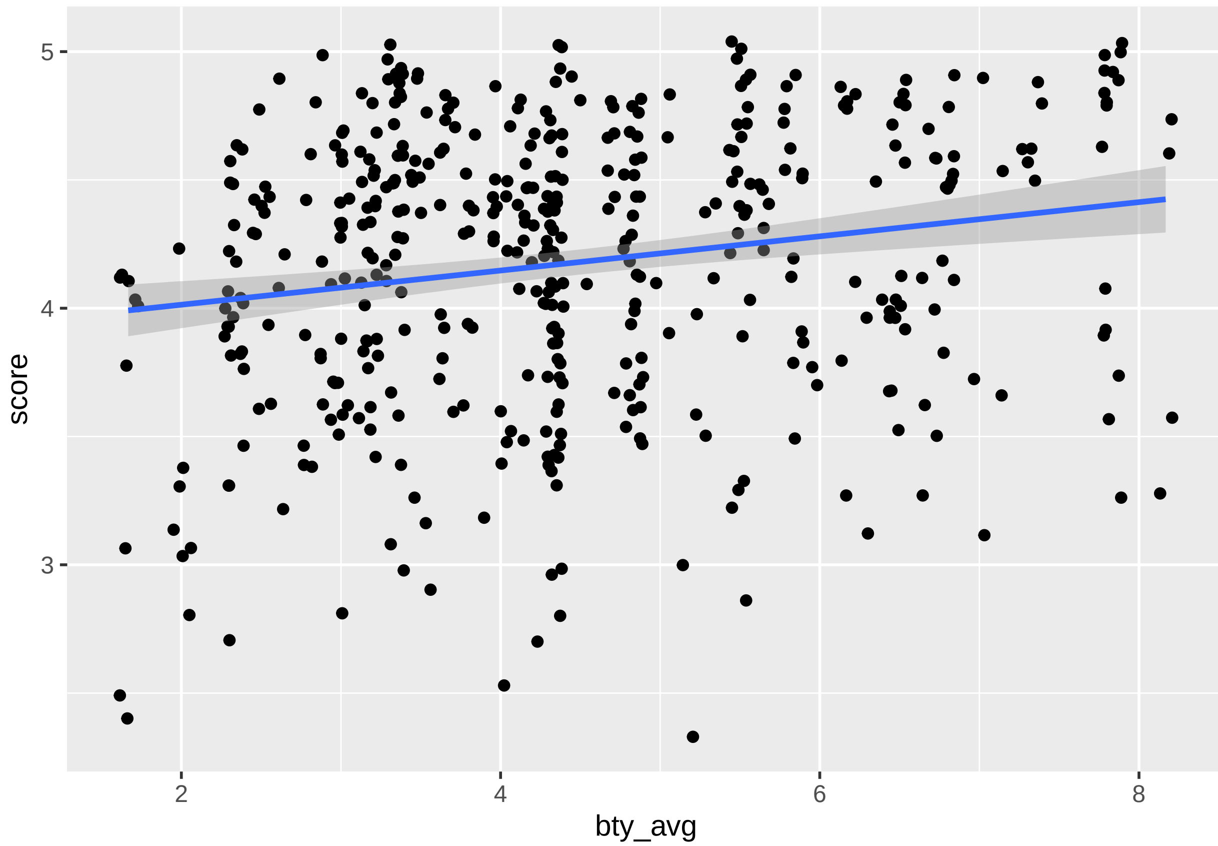
Based on the output of the above model summary, the equation relating `score` ($\hat{y}$) and `avg_bty` (β_1) is as follows:

$\hat{y} = 3.88 + (0.07\beta_1)$

The positive slope indicates that higher `bty_avg` scores tend to result in higher `score` values. This is further confirmed in model summary output, which indicates that the predictive effect of `bty_avg` on `score` is statistically significant.

The chunk below replots the scatterplot with this newly created model:

```
ggplot(data = evals, aes(x = bty_avg, y = score)) +
  geom_jitter() +
  geom_smooth(method = "lm")
```



---

6. Use residual plots to evaluate whether the conditions of least squares regression are reasonable. Provide plots and comments for each one (see the Simple Regression Lab for a reminder of how to make these).

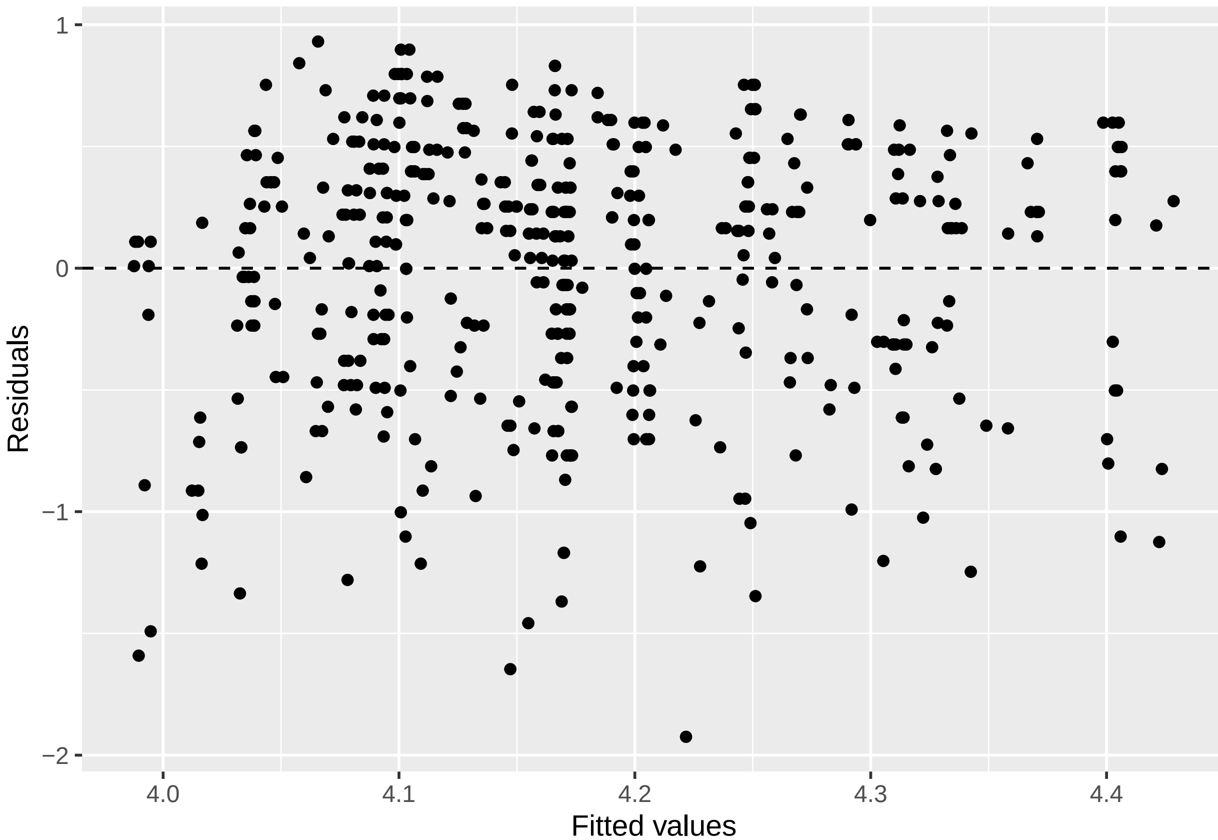
WJ Response:

The three conditions for least squares regression are linearity, nearly normal residuals, and constant variability. Each are evaluated below:

Linearity:

While we already checked the linearity between `score` and `avg_bty` by looking at a scatterplot, we can also check by plotting the residual values against the predicted values:

```
ggplot(data = m_bty, aes(x = .fitted, y = .resid)) +
  geom_jitter() +
  geom_hline(yintercept = 0, linetype = "dashed") +
  xlab("Fitted values") +
  ylab("Residuals")
```

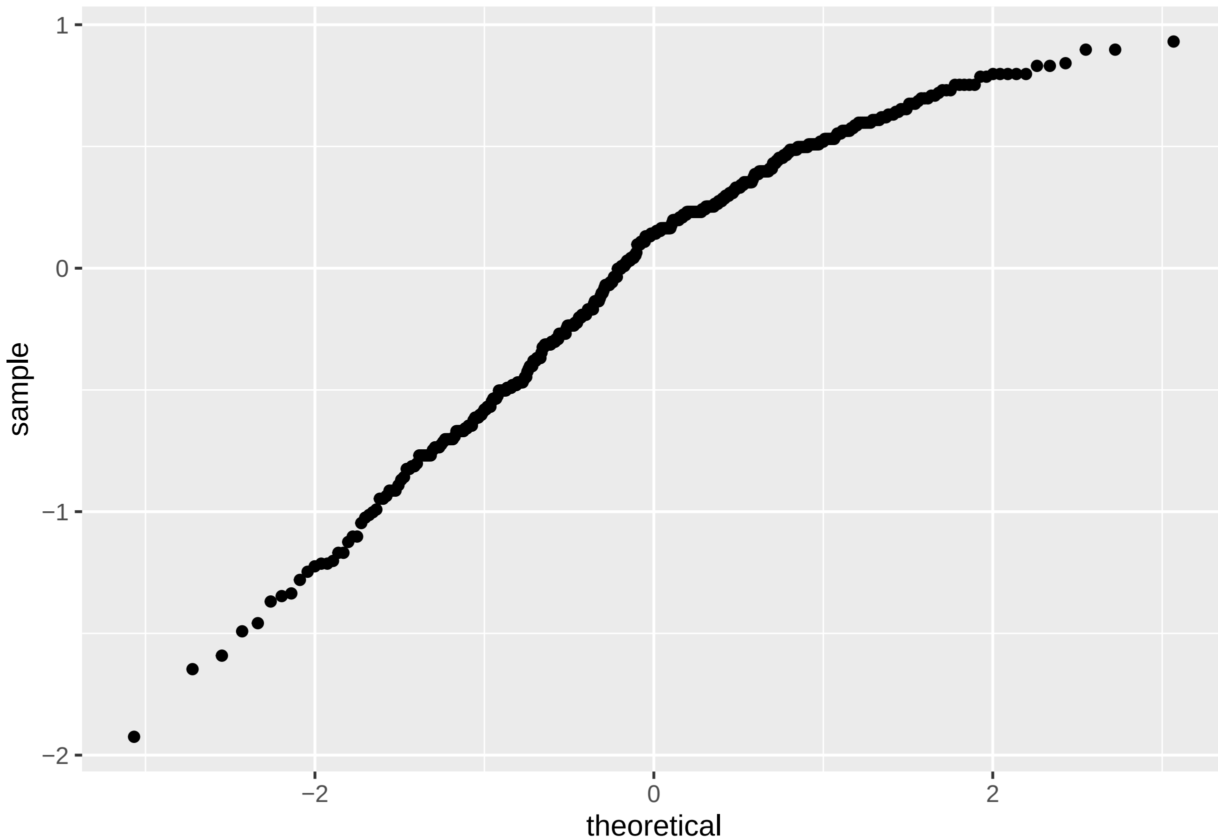


There appears to be no correlation in the residual plot, indicating that we do indeed have a linear relationship between our two variables.

Nearly normal residuals:

We can check this condition by making a normal probability plot of the residuals:

```
ggplot(data = m_bty, aes(sample = .resid)) +
  stat_qq()
```



While there is a slightly "curved over" structure apparent on the right side of the plot, the line is still quite linear. Overall, the slight deviation from a linearity does not seem enough to warrant the nearly normal residual condition as invalid.

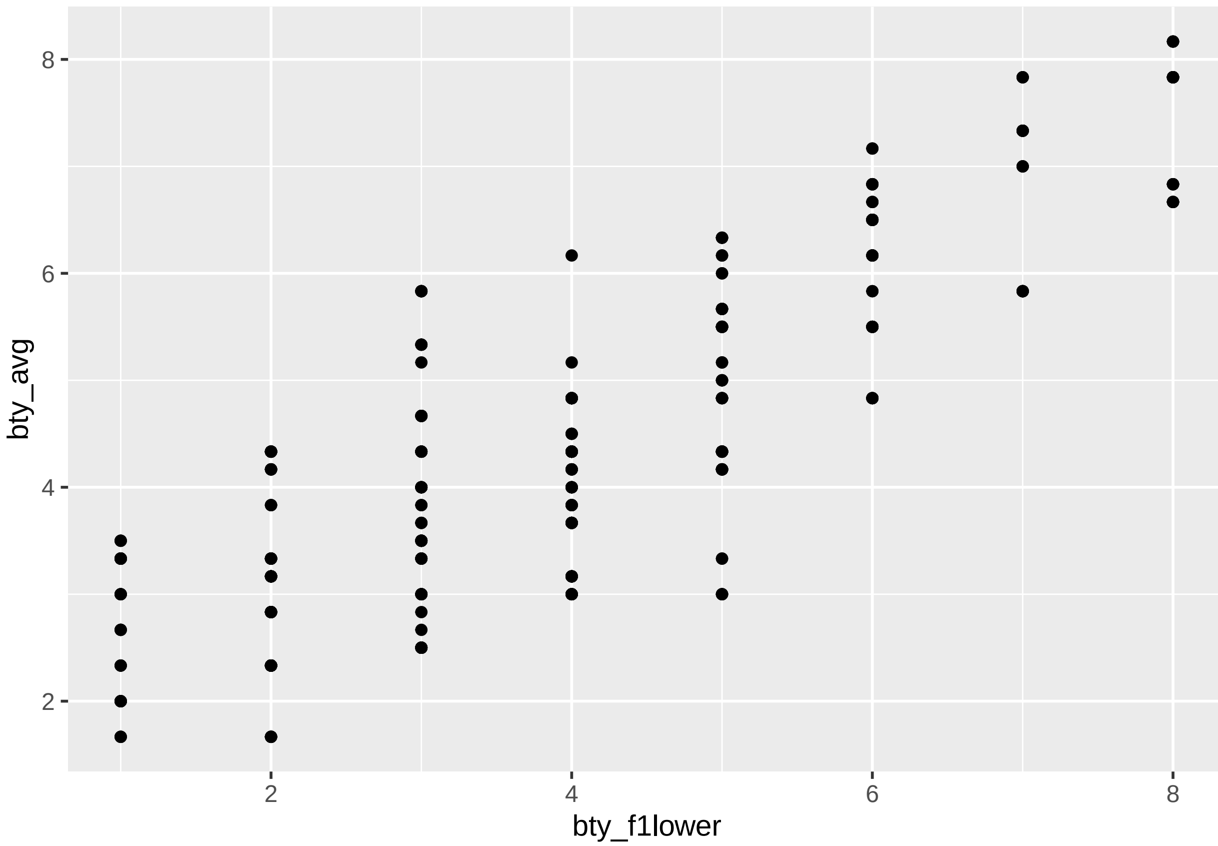
Constant variability:

Observing the residual plot created to determine linearity, we see that the spread of the residuals at each of the predicted values is relatively similar. This indicates that the constant variability condition is satisfied.

Multiple linear regression

The data set contains several variables on the beauty score of the professor: individual ratings from each of the six students who were asked to score the physical appearance of the professors and the average of these six scores. Let's take a look at the relationship between one of these scores and the average beauty score.

```
ggplot(data = evals, aes(x = bty_f1lower, y = bty_avg)) +
  geom_point()
```

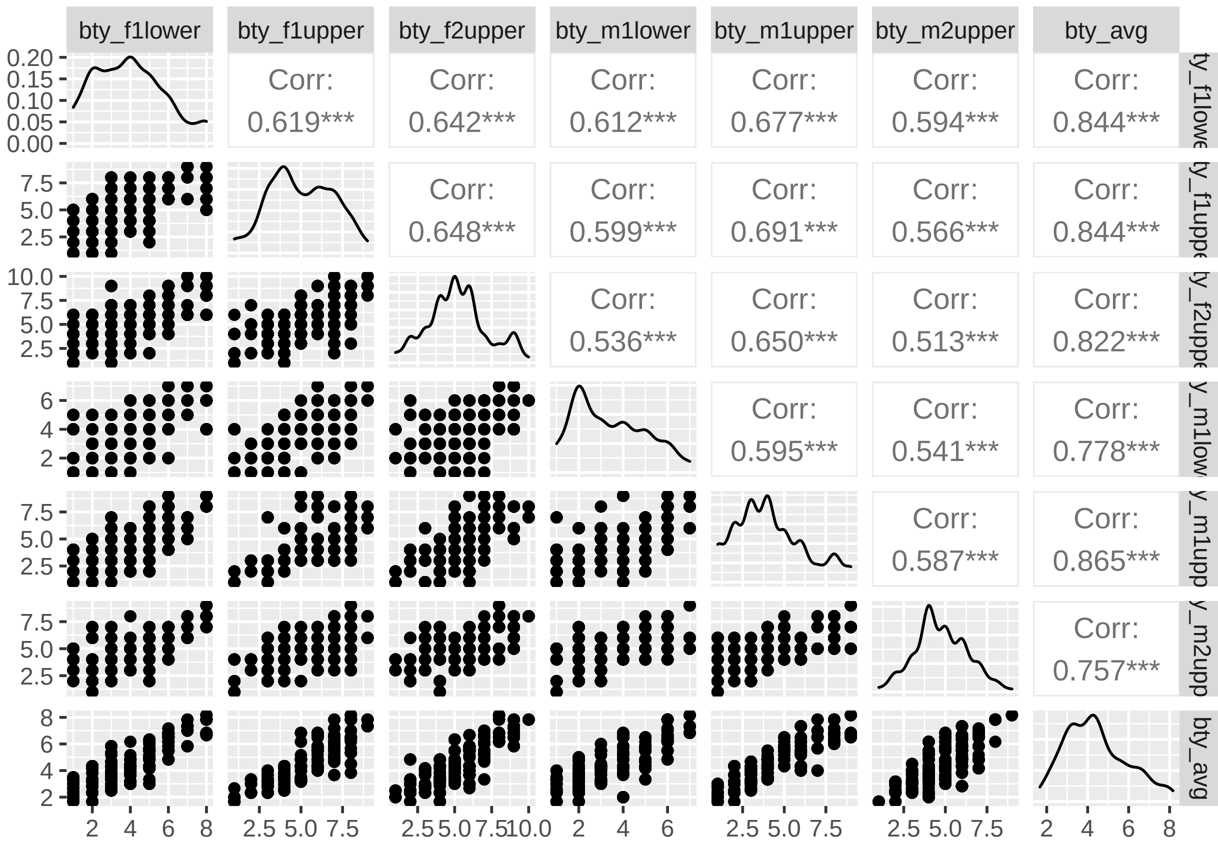



```
evals %>%
  summarise(cor(bty_avg, bty_f1lower))
```

```
## # A tibble: 1 x 1
##   `cor(bty_avg, bty_f1lower)`
##                         <dbl>
## 1                       0.844
```

As expected, the relationship is quite strong—after all, the average score is calculated using the individual scores. You can actually look at the relationships between all beauty variables (columns 13 through 19) using the following command:

```
evals %>%
  select(contains("bty")) %>%
  ggpairs()
```



These variables are collinear (correlated), and adding more than one of these variables to the model would not add much value to the model. In this application and with these highly-correlated predictors, it is reasonable to use the average beauty score as the single representative of these variables.

In order to see if beauty is still a significant predictor of professor score after you've accounted for the professor's gender, you can add the gender term into the model.

```r
m_bty_gen <- lm(score ~ bty_avg + gender, data = evals)
summary(m_bty_gen)
```

```
##
## Call:
## lm(formula = score ~ bty_avg + gender, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.8305 -0.3625  0.1055  0.4213  0.9314
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.74734    0.08466  44.266  < 2e-16 ***
## bty_avg      0.07416    0.01625   4.563 6.48e-06 ***
## gendermale   0.17239    0.05022   3.433 0.000652 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5287 on 460 degrees of freedom
## Multiple R-squared:  0.05912,    Adjusted R-squared:  0.05503
```

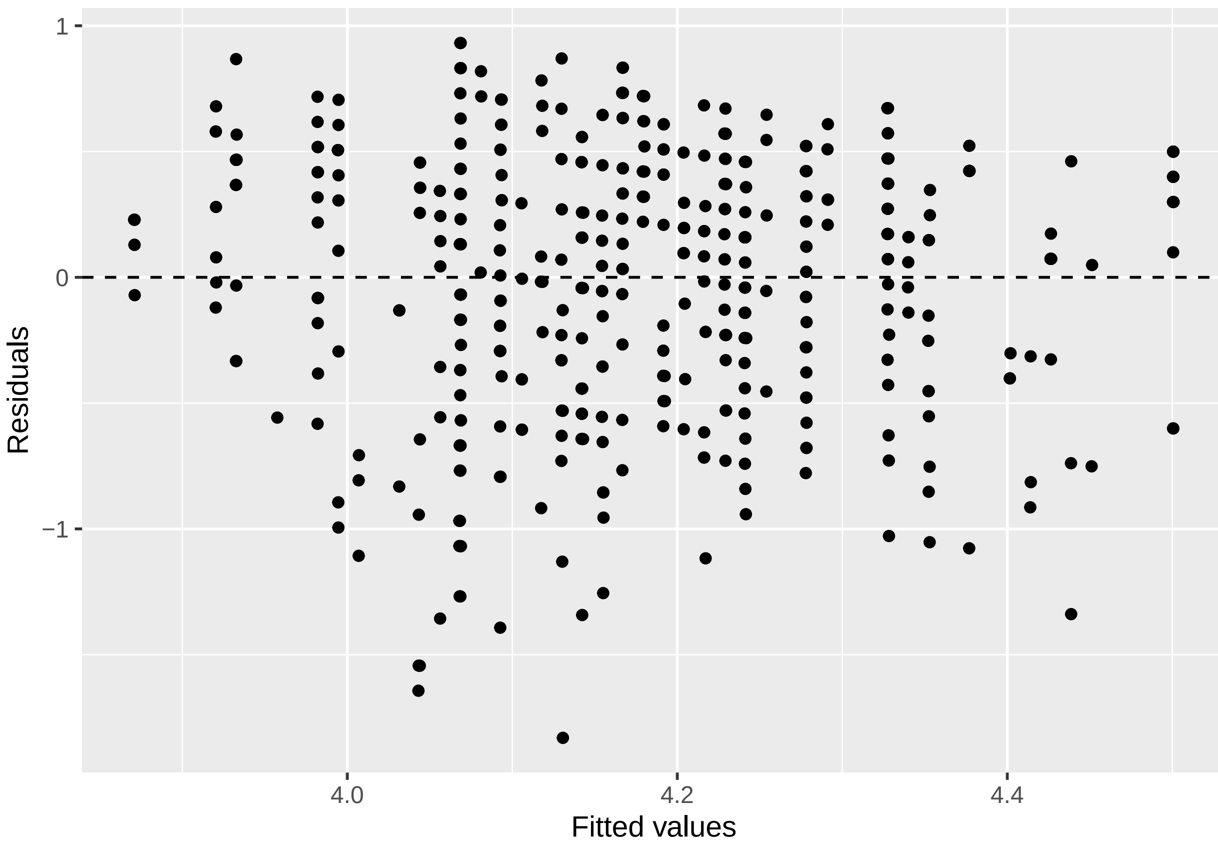
```
## F-statistic: 14.45 on 2 and 460 DF,  p-value: 8.177e-07
```

7. P-values and parameter estimates should only be trusted if the conditions for the regression are reasonable. Verify that the conditions for this model are reasonable using diagnostic plots.

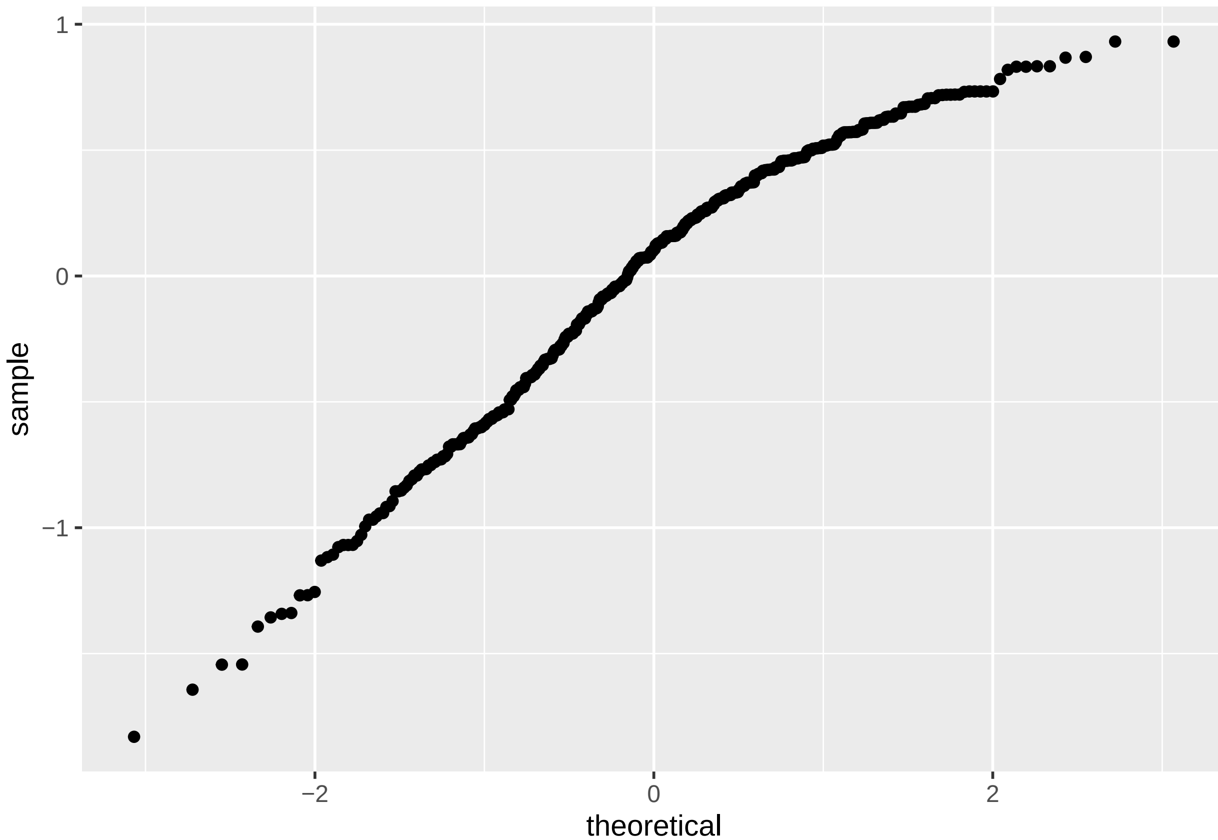
WJ Response:

The code chunk below creates two plots, one of the residuals vs. their predicted values, and a Q-Q plot of the residuals:

```
ggplot(data = m_bty_gen, aes(x = .fitted, y = .resid)) +
  geom_jitter() +
  geom_hline(yintercept = 0, linetype = "dashed") +
  xlab("Fitted values") +
  ylab("Residuals")
```




```
ggplot(data = m_bty_gen, aes(sample = .resid)) +
  stat_qq()
```



The lack of structure and even spread of the first plot indicates that we have satisfied our linearity and near-constant variability conditions, while the linear appearance of the residual Q-Q plot indicates that we have satisfied our nearly-normal residuals condition.

---

8. Is `bty_avg` still a significant predictor of `score`? Has the addition of `gender` to the model changed the parameter estimate for `bty_avg`?

WJ Response:

Yes, both `bty_avg` and `gender` are both significant predictors of `score`. The addition of `gender` as a variable has slightly increased the value of the `bty_avg` parameter estimate indicating that the new model attributes more of an effect of `bty_avg` on score.

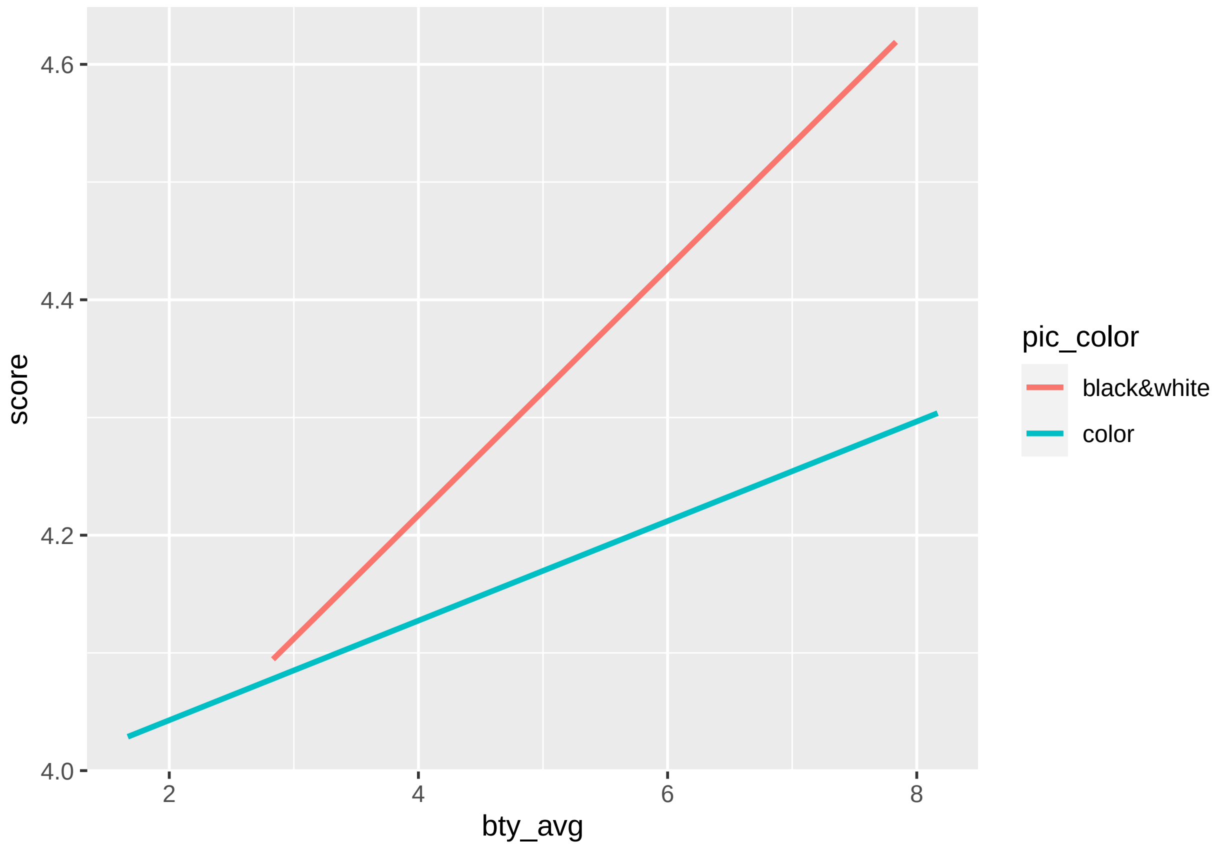
Note that the estimate for `gender` is now called `gendermale`. You'll see this name change whenever you introduce a categorical variable. The reason is that R recodes `gender` from having the values of `male` and `female` to being an indicator variable called `gendermale` that takes a value of 0 for female professors and a value of 1 for male professors. (Such variables are often referred to as "dummy" variables.)

As a result, for female professors, the parameter estimate is multiplied by zero, leaving the intercept and slope form familiar from simple regression.

$$
\begin{aligned}
\widehat{score} &= \hat{\beta}_0 + \hat{\beta}_1 \times bty\_avg + \hat{\beta}_2 \times (0) \\
&= \hat{\beta}_0 + \hat{\beta}_1 \times bty\_avg
\end{aligned}
$$

–

```r
ggplot(data = evals, aes(x = bty_avg, y = score, color = pic_color)) +
 geom_smooth(method = "lm", formula = y ~ x, se = FALSE)
```



9. What is the equation of the line corresponding to those with color pictures? (*Hint:* For those with color pictures, the parameter estimate is multiplied by 1.) For two professors who received the same beauty rating, which color picture tends to have the higher course evaluation score?

WJ Response:

```r
m_test <- lm(score ~ bty_avg + pic_color, data = evals)

b_0 <- unname(coefficients(m_test)[1])
b_1 <- unname(coefficients(m_test)[2])
b_2 <- unname(coefficients(m_test)[3])

eqn1 <- sprintf(
  "$\\hat{y} = %.2f + (%.2f \\beta_1) + (%.2f \\beta_2$)", b_0, b_1, b_2)
eqn2 <- sprintf("$\\hat{y} = %.2f + (%.2f \\beta_1$)", b_0, b_1)

summary(m_test)
```

```
##
## Call:
## lm(formula = score ~ bty_avg + pic_color, data = evals)
##
## Residuals:
```

```
##     Min      1Q  Median      3Q     Max
## -1.8892 -0.3690  0.1293  0.4023  0.9125
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)      4.06318    0.10908  37.249  < 2e-16 ***
## bty_avg          0.05548    0.01691   3.282  0.00111 **
## pic_colorcolor -0.16059    0.06892  -2.330  0.02022 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5323 on 460 degrees of freedom
## Multiple R-squared:  0.04628,    Adjusted R-squared:  0.04213
## F-statistic: 11.16 on 2 and 460 DF,  p-value: 1.848e-05
```

Based on the results above, if $\beta_1 =$ `bty_avg` and $\beta_2 =$ `pic_color`, the equation for professors with a color photo is:

$\hat{y} = 4.06 + (0.06\beta_1) + (-0.16\beta_2)$

and the equation for professors with a black and white photo is:

$\hat{y} = 4.06 + (0.06\beta_1)$

Thus, for professors with the same beauty rating, those with black and white photos on average were rated more highly.

The decision to call the indicator variable `gendermale` instead of `genderfemale` has no deeper meaning. R simply codes the category that comes first alphabetically as a 0. (You can change the reference level of a categorical variable, which is the level that is coded as a 0, using the`relevel()` function. Use `?relevel` to learn more.)

10. Create a new model called `m_bty_rank` with `gender` removed and `rank` added in. How does R appear to handle categorical variables that have more than two levels? Note that the rank variable has three levels: `teaching`, `tenure track`, `tenured`.

WJ Response:

```
m_bty_rank <- lm(score ~ bty_avg + rank, data = evals)
summary(m_bty_rank)
```

```
##
## Call:
## lm(formula = score ~ bty_avg + rank, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.8713 -0.3642  0.1489  0.4103  0.9525
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)        3.98155    0.09078  43.860  < 2e-16 ***
## bty_avg            0.06783    0.01655   4.098 4.92e-05 ***
## ranktenure track -0.16070    0.07395  -2.173   0.0303 *
## ranktenured      -0.12623    0.06266  -2.014   0.0445 *
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5328 on 459 degrees of freedom
## Multiple R-squared:  0.04652,    Adjusted R-squared:  0.04029
## F-statistic: 7.465 on 3 and 459 DF,  p-value: 6.88e-05
```

It appears that for a categorical variable with n categories, `lm` will create $n - 1$ binary dummy variables. In this case, it created two: `ranktenure track` and `ranktenured`. Having two binary variables actually provides information about all three categories, because if `ranktenuretrack` and `ranktenured` are both 0 we know that the only remaining value is `teaching`.

The interpretation of the coefficients in multiple regression is slightly different from that of simple regression. The estimate for `bty_avg` reflects how much higher a group of professors is expected to score if they have a beauty rating that is one point higher *while holding all other variables constant.* In this case, that translates into considering only professors of the same rank with `bty_avg` scores that are one point apart.

The search for the best model

We will start with a full model that predicts professor score based on rank, gender, ethnicity, language of the university where they got their degree, age, proportion of students that filled out evaluations, class size, course level, number of professors, number of credits, average beauty rating, outfit, and picture color.

11. Which variable would you expect to have the highest p-value in this model? Why? *Hint:* Think about which variable would you expect to not have any association with the professor score.

WJ Response:

I would expect ethnicity to have the highest p-value. In other words, I expect it to be the predictor variable that has the least effect on a professor's overall score.

Let's run the model…

```
m_full <- lm(score ~ rank + gender + ethnicity + language + age + cls_perc_eval
             + cls_students + cls_level + cls_profs + cls_credits + bty_avg
             + pic_outfit + pic_color, data = evals)
summary(m_full)
```

```
##
## Call:
## lm(formula = score ~ rank + gender + ethnicity + language + age +
##     cls_perc_eval + cls_students + cls_level + cls_profs + cls_credits +
##     bty_avg + pic_outfit + pic_color, data = evals)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.77397 -0.32432  0.09067  0.35183  0.95036
##
## Coefficients:
##                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)           4.0952141  0.2905277  14.096  < 2e-16 ***
## ranktenure track     -0.1475932  0.0820671  -1.798  0.07278 .
## ranktenured          -0.0973378  0.0663296  -1.467  0.14295
```

```
## gendermale             0.2109481  0.0518230   4.071 5.54e-05 ***
## ethnicitynot minority  0.1234929  0.0786273   1.571  0.11698
## languagenon-english   -0.2298112  0.1113754  -2.063  0.03965 *
## age                   -0.0090072  0.0031359  -2.872  0.00427 **
## cls_perc_eval          0.0053272  0.0015393   3.461  0.00059 ***
## cls_students           0.0004546  0.0003774   1.205  0.22896
## cls_levelupper         0.0605140  0.0575617   1.051  0.29369
## cls_profssingle       -0.0146619  0.0519885  -0.282  0.77806
## cls_creditsone credit  0.5020432  0.1159388   4.330 1.84e-05 ***
## bty_avg                0.0400333  0.0175064   2.287  0.02267 *
## pic_outfitnot formal  -0.1126817  0.0738800  -1.525  0.12792
## pic_colorcolor        -0.2172630  0.0715021  -3.039  0.00252 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.498 on 448 degrees of freedom
## Multiple R-squared:  0.1871, Adjusted R-squared:  0.1617
## F-statistic: 7.366 on 14 and 448 DF,  p-value: 6.552e-14
```

12. Check your suspicions from the previous exercise. Include the model output in your response.

WJ Response:

Based on the model output, ethnicity was not a statistically significant predictor of professor score. However, there were other variables with higher p values.

---

13. Interpret the coefficient associated with the ethnicity variable.

WJ Response:

The `ethnicity` field was transformed to the `ethnicity_not_minority` field, which is 1 if the professor is not a minority and 0 if the professor is. Given the positive value of this coefficient, the data suggests that non-minority professors were given higher scores. However, as stated previously, `ethnicity` was not categorized as being a statistically significant predictor.

---

14. Drop the variable with the highest p-value and re-fit the model. Did the coefficients and significance of the other explanatory variables change? (One of the things that makes multiple regression interesting is that coefficient estimates depend on the other variables that are included in the model.) If not, what does this say about whether or not the dropped variable was collinear with the other explanatory variables?

WJ Response:

Refitting the model, but dropping the `cls_profs` column (number of professors teaching the class), since it had the highest p-value:

```r
m_full <- lm(score ~ rank + gender + ethnicity + language + age + cls_perc_eval
             + cls_students + cls_level + cls_credits + bty_avg
             + pic_outfit + pic_color, data = evals)
summary(m_full)
```

```
##
## Call:
## lm(formula = score ~ rank + gender + ethnicity + language + age +
##     cls_perc_eval + cls_students + cls_level + cls_credits +
##     bty_avg + pic_outfit + pic_color, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.7836 -0.3257  0.0859  0.3513  0.9551
##
## Coefficients:
##                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)           4.0872523  0.2888562  14.150  < 2e-16 ***
## ranktenure track     -0.1476746  0.0819824  -1.801 0.072327 .
## ranktenured          -0.0973829  0.0662614  -1.470 0.142349
## gendermale            0.2101231  0.0516873   4.065 5.66e-05 ***
## ethnicitynot minority  0.1274458  0.0772887   1.649 0.099856 .
## languagenon-english  -0.2282894  0.1111305  -2.054 0.040530 *
## age                  -0.0089992  0.0031326  -2.873 0.004262 **
## cls_perc_eval         0.0052888  0.0015317   3.453 0.000607 ***
## cls_students          0.0004687  0.0003737   1.254 0.210384
## cls_levelupper        0.0606374  0.0575010   1.055 0.292200
## cls_creditsone credit  0.5061196  0.1149163   4.404 1.33e-05 ***
## bty_avg               0.0398629  0.0174780   2.281 0.023032 *
## pic_outfitnot formal -0.1083227  0.0721711  -1.501 0.134080
## pic_colorcolor       -0.2190527  0.0711469  -3.079 0.002205 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4974 on 449 degrees of freedom
## Multiple R-squared:  0.187,  Adjusted R-squared:  0.1634
## F-statistic: 7.943 on 13 and 449 DF,  p-value: 2.336e-14
```

When removing the `cls_profs` variable, both a number of p values decreasesed, and the adjusted R^2 value actually increased. This indicates that the `cls_profs` column was likely collinear with a number of other explanatory columns, and provided no additional information to the model (in fact, it actually made it worse).

---

15. Using backward-selection and p-value as the selection criterion, determine the best model. You do not need to show all steps in your answer, just the output for the final model. Also, write out the linear model for predicting score based on the final model you settle on.

WJ Response:

The model below uses the p-value to select only those explanatory variables that have a statistically significant effect on `score`.

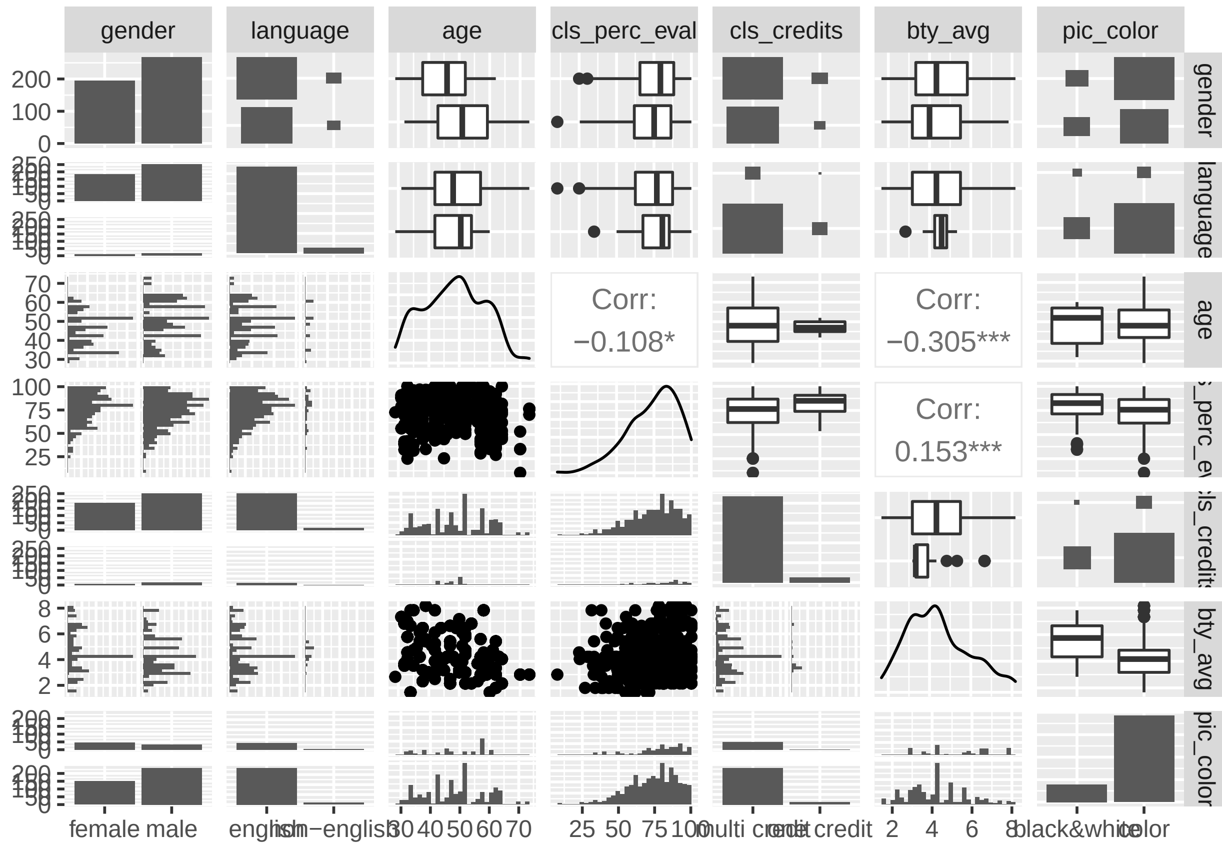
```r
m_final <- lm(score ~ gender + language + age + cls_perc_eval +
              cls_credits + bty_avg + pic_color, data = evals)
summary(m_final)
```

```
##
## Call:
## lm(formula = score ~ gender + language + age + cls_perc_eval +
##     cls_credits + bty_avg + pic_color, data = evals)
```

```
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.81919 -0.32035  0.09272  0.38526  0.88213
##
## Coefficients:
##                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)            3.967255   0.215824  18.382  < 2e-16 ***
## gendermale             0.221457   0.049937   4.435 1.16e-05 ***
## languagenon-english   -0.281933   0.098341  -2.867  0.00434 **
## age                   -0.005877   0.002622  -2.241  0.02551 *
## cls_perc_eval          0.004295   0.001432   2.999  0.00286 **
## cls_creditsone credit  0.444392   0.100910   4.404 1.33e-05 ***
## bty_avg                0.048679   0.016974   2.868  0.00432 **
## pic_colorcolor        -0.216556   0.066625  -3.250  0.00124 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5014 on 455 degrees of freedom
## Multiple R-squared:  0.1631, Adjusted R-squared:  0.1502
## F-statistic: 12.67 on 7 and 455 DF,  p-value: 6.996e-15
```

In addition, the following ‘`ggpairs` plot confirms that none of our explanatory variables seem to have overwhelming high levels of collinearity:

```
evals %>%
  select(gender, language, age, cls_perc_eval,
         cls_credits, bty_avg, pic_color) %>%
  ggpairs()
```



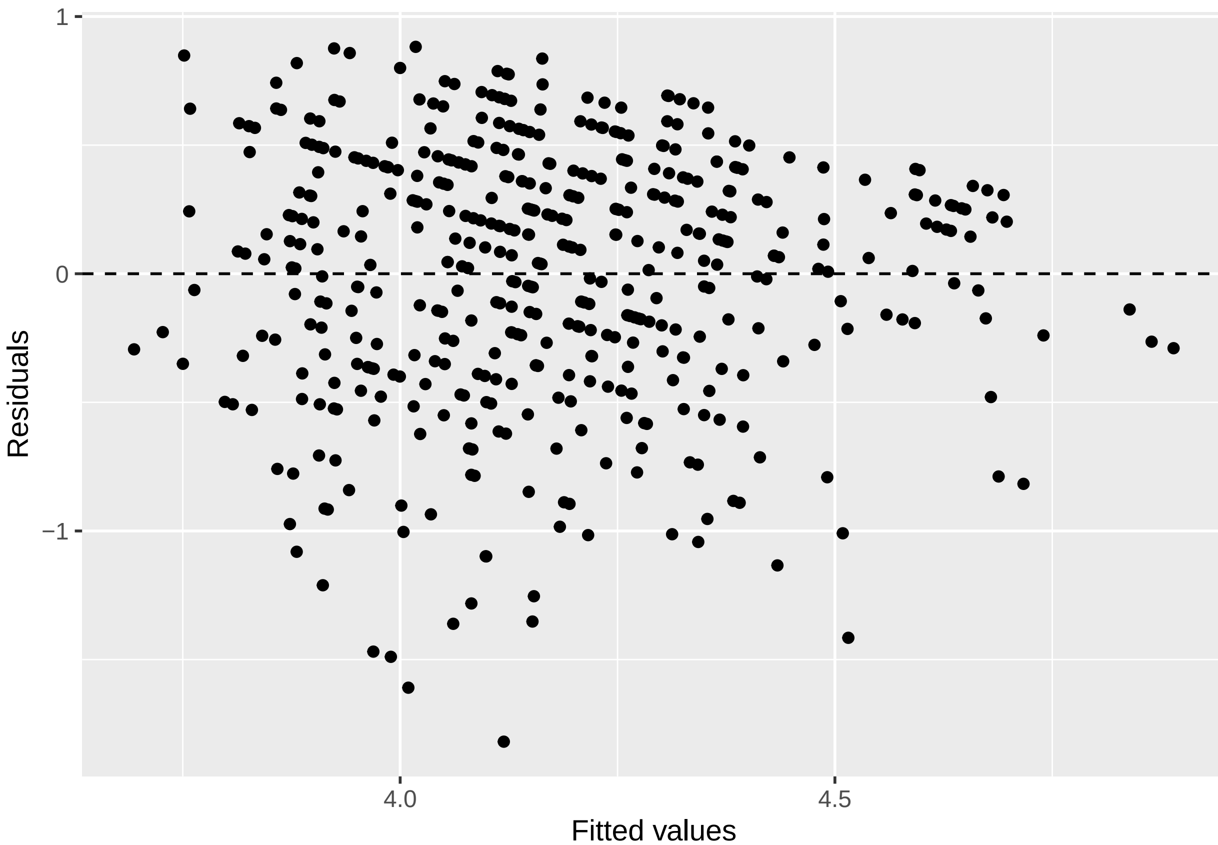
---

16. Verify that the conditions for this model are reasonable using diagnostic plots.

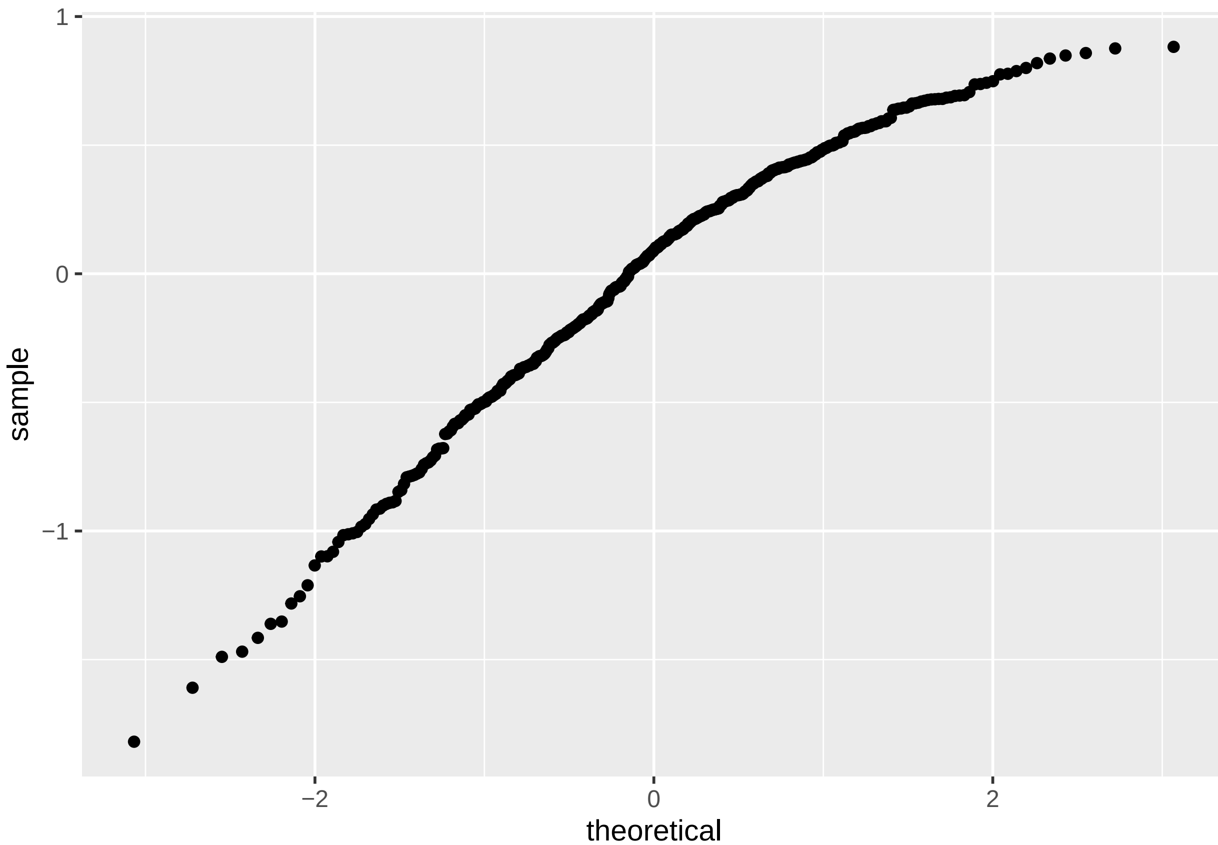
WJ Response:

The code chunk below creates two plots, one of the residuals vs. their predicted values, and a Q-Q plot of the residuals:

```
ggplot(data = m_final, aes(x = .fitted, y = .resid)) +
  geom_jitter() +
  geom_hline(yintercept = 0, linetype = "dashed") +
  xlab("Fitted values") +
  ylab("Residuals")
```




```
ggplot(data = m_final, aes(sample = .resid)) +
  stat_qq()
```



The lack of structure and even spread of the first plot indicates that we have satisfied our linearity and near-constant variability conditions, while the linear appearance of the residual Q-Q plot indicates that we have satisfied our nearly-normal residuals condition.

---

17. The original paper describes how these data were gathered by taking a sample of professors from the University of Texas at Austin and including all courses that they have taught. Considering that each row represents a course, could this new information have an impact on any of the conditions of linear regression?

WJ Response:

Yes, this could pose an issue in which scores are replicated even though they come from the same professor. The same professor is likely to get evaluated similarly for all courses that they teach, meaning that the dataset can be skewed by these repeated scores for the same professor. A better approach might have been to average all the professor class scores before we created the model.

---

18. Based on your final model, describe the characteristics of a professor and course at University of Texas at Austin that would be associated with a high evaluation score.

WJ Response:

Based off the results of the model, a professor at the University of Texas at Austin who is most likely to have a high evaluation score would be one who:

1. is male
2. got their degree in an English speaking country
3. is young
4. teaches single credit courses
5. has a black and white profile picture
6. is attractive

---

19. Would you be comfortable generalizing your conclusions to apply to professors generally (at any university)? Why or why not?

WJ Response:

I would not. The data represents those scores from students and professors from only a single university. I would be much more inclined to apply the model more generally if it was based off random samplings of data from other universities across the country.
