

## Lab3 Algorithm2 Clarification.

I provide some examples, Math part , and you will figure out how to implement program in  $O(n)$  complexity.

**1.**For example, you have sorted list, and need to find 2 elements that sum is, for example is equal 8.

|    |   |   |   |    |    |
|----|---|---|---|----|----|
| 1  | 5 | 7 | 8 | 12 | 13 |
| P1 |   |   |   |    | P2 |

, still

You add first and last elements :  $1+13 > 8$

So move pointer P2 to the left

Now

|    |   |   |   |    |    |
|----|---|---|---|----|----|
| 1  | 5 | 7 | 8 | 12 | 13 |
| P1 |   |   |   | P2 |    |

Now add  $1+12$  , still  $1+12 > 8$ , so again, move pointer P2 to the left

|    |   |   |    |    |    |
|----|---|---|----|----|----|
| 1  | 5 | 7 | 8  | 12 | 13 |
| P1 |   |   | P2 |    |    |

Now add  $1+8$  , still  $1+8 > 8$ , so again, move pointer P2 to the left

|    |   |    |   |    |    |
|----|---|----|---|----|----|
| 1  | 5 | 7  | 8 | 12 | 13 |
| P1 |   | P2 |   |    |    |

$1+7 == 8$  Yes! Bingo! We found our solution!

**2.** But, what if we do not have 7 in our list?

And the original list is:

|    |   |   |   |    |    |
|----|---|---|---|----|----|
| 1  | 5 | 6 | 8 | 12 | 13 |
| P1 |   |   |   |    | P2 |

We continue the same operations:

|    |   |   |   |    |    |
|----|---|---|---|----|----|
| 1  | 5 | 6 | 8 | 12 | 13 |
| P1 |   |   |   | P2 |    |

Now add  $1+12$ , still  $1+12 > 8$ , so again, move pointer P2 to the left

|    |   |   |    |    |    |
|----|---|---|----|----|----|
| 1  | 5 | 6 | 8  | 12 | 13 |
| P1 |   |   | P2 |    |    |

Now add  $1+8$ , still  $1+8 > 8$ , so again, move pointer P2 to the left

|    |   |    |   |    |    |
|----|---|----|---|----|----|
| 1  | 5 | 6  | 8 | 12 | 13 |
| P1 |   | P2 |   |    |    |

$1+6 < 8$ , so we move pointer P1 to the right

|   |    |    |   |    |    |
|---|----|----|---|----|----|
| 1 | 5  | 6  | 8 | 12 | 13 |
|   | P1 | P2 |   |    |    |

$5+6 > 8$ , move pointer P2 to the left

|   |        |   |   |    |    |
|---|--------|---|---|----|----|
| 1 | 5      | 6 | 8 | 12 | 13 |
|   | P1, P2 |   |   |    |    |

Now pointers P1 and P2 point to the same element: only one chance to get the solution is  $5+5$  is our number, but  $5+5$  is 10 and is not equal to 8, so we proved by construction, that there is no solutions with given list and given number 8.

You got the idea, how to build the algorithm? The key is, list is sorted, it is why we can move pointers, left, right.

Actually, since we have deal with the array, pointers, in our case, are actually indexes of the array.

**3.** Just for further clarification, let's consider one more array, and sum of two numbers is 13

Find all pairs, that their sum is 13

|    |   |   |   |   |    |    |
|----|---|---|---|---|----|----|
| 1  | 3 | 5 | 7 | 8 | 12 | 14 |
| P1 |   |   |   |   |    | P2 |

$1+14 > 13$ , move P2 to the left

|    |   |   |   |   |    |    |
|----|---|---|---|---|----|----|
| 1  | 3 | 5 | 7 | 8 | 12 | 14 |
| P1 |   |   |   |   | P2 |    |

$1+12 == 13$ , Bingo! Solution is found, but, what is we have more solutions?

If we move just one pointer, we violate equality, so we need to move both pointers, tha same time

|   |    |   |   |    |    |    |
|---|----|---|---|----|----|----|
| 1 | 3  | 5 | 7 | 8  | 12 | 14 |
|   | P1 |   |   | P2 |    |    |

$3+8 < 13$ , so we move P1 to the right

|   |   |    |   |    |    |    |
|---|---|----|---|----|----|----|
| 1 | 3 | 5  | 7 | 8  | 12 | 14 |
|   |   | P1 |   | P2 |    |    |

$5+8 = 13$ !, Bingo! We found one more solution!

We start like it!

Maybe we have even more solutions?

Move both pointers:

|   |   |   |       |   |    |    |
|---|---|---|-------|---|----|----|
| 1 | 3 | 5 | 7     | 8 | 12 | 14 |
|   |   |   | P1,P2 |   |    |    |

We almost end our investigation.

Now pointers point to the same number 7, only option, that it is our solution is  $7+7$  is our number, but  $7+7$  is 14, and it is not equal to 13, our number.

Even it is failure in our last case, on first glance, but it is actually, our success, because we proved that problem has only two solutions  $1+12$  and  $5+8$ , no more.

We proved it by construction.

Remind again, list is sorted, it is why we could move pointers (actually indexes of the array), to left, to right.. and all these calculations is  $O(n)$ .

Have the algorithm done!