

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

# TRADING STRATEGIES INVOLVING OPTIONS

## Topic 43

### EXAM FOCUS

Traders and investors use option-based trading strategies to create an extraordinary spectrum of payoff profiles. This enables investors to take positions based on almost any possible expectation of the underlying stock over the life of the options. This topic describes the common option trading strategies and implementation. For the exam, know the general payoff graphs for each strategy discussed. In addition, know how to calculate the payoff for some of the more popular strategies including protective put, covered call, bull call spread, butterfly spread, and straddle.

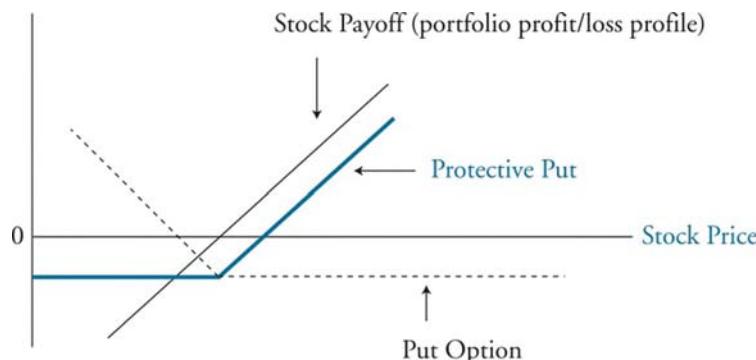
### COVERED CALLS AND PROTECTIVE PUTS

#### LO 43.1: Explain the motivation to initiate a covered call or a protective put strategy.

When an at-the-money long put position is combined with the underlying stock, we have created a **protective put strategy**. A protective put (also called *portfolio insurance* or a *hedged portfolio*) is constructed by holding a long position in the underlying security and buying a put option. You can use a protective put to limit the downside risk at the cost of the put premium,  $P_0$ . You will see by the diagram in Figure 1 that the investor will still be able to benefit from increases in the stock's price, but it will be lower by the amount paid for the put,  $P_0$ . Notice that the combined strategy looks very much like a call option. This should not be surprising since put-call parity requires that  $p + S_0$  be the same as  $c + Xe^{-rT}$ . Figure 1 illustrates this property.

**Figure 1: Protective Put Strategy**

Profit

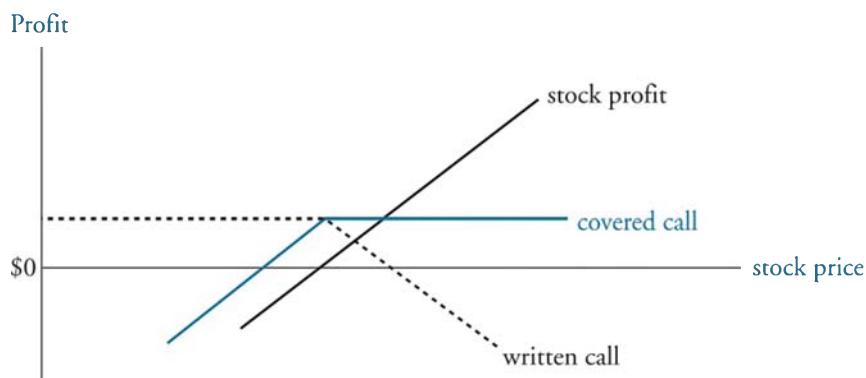


## Topic 43

## Cross Reference to GARP Assigned Reading – Hull, Chapter 12

Another common strategy is to sell a call option on a stock that is owned by the option writer. This is called a **covered call** position. By writing an out-of-the-money call option, the combined position caps the upside potential at the strike price. In return for giving up any potential gain beyond the strike price, the writer receives the option premium. This strategy is used to generate cash on a stock that is not expected to increase above the exercise price over the life of the option.

Figure 2: Profit Profile for a Covered Call



## SPREAD STRATEGIES

### LO 43.2: Describe the use and calculate the payoffs of various spread strategies.

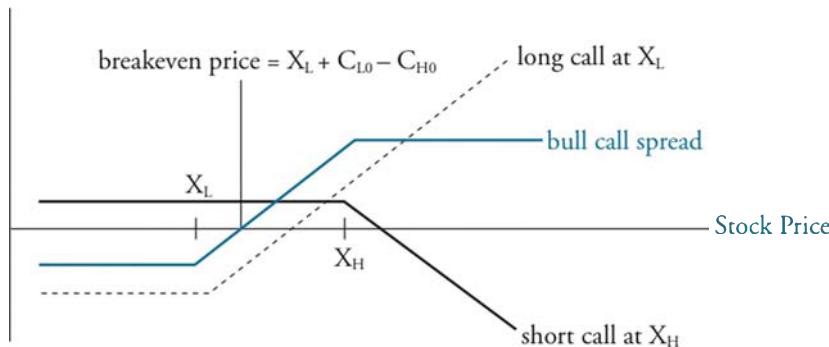
Several spread strategies exist. These strategies combine options positions to create a desired payoff profile. The differences between the options are either the strike prices and/or the time to expiration. We will discuss bull and bear spreads, butterfly spreads, calendar spreads, and diagonal spreads.

#### Bull and Bear Spreads

In a *bull call spread*, the buyer of the spread purchases a call option with a low exercise price,  $X_L$ , and subsidizes the purchase price of the call by selling a call with a higher exercise price,  $X_H$ . The buyer of a bull call spread expects the stock price to rise and the purchased call to finish in-the-money. However, the buyer does not believe that the price of the stock will rise above the exercise price for the out-of-the-money written call.

**Figure 3: Bull Call Spread**

Profit

**Example: Bull call spread**

An investor purchases a call for  $C_{L0} = \$3.00$  with a strike of  $X = \$40$  and sells a call for  $C_{H0} = \$1.00$  with a strike price of  $\$50$ . Compute the payoff of a bull call spread strategy when the price of the stock is at  $\$45$ .

**Answer:**

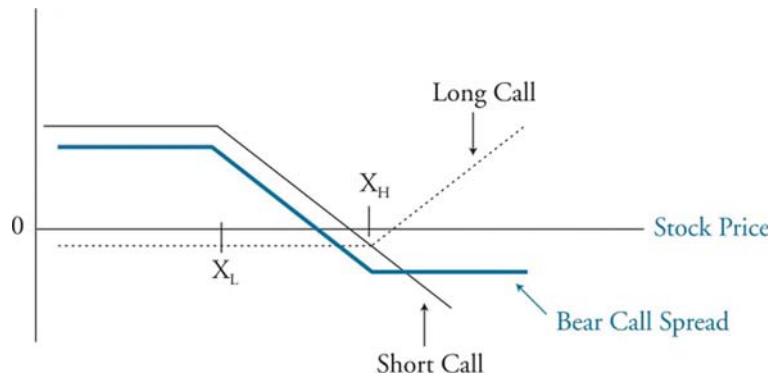
$$\text{profit} = \max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0}$$

$$\text{profit} = \max(0, 45 - 40) - \max(0, 45 - 50) - 3 + 1 = \$3.00$$

A *bear call spread* is the sale of a bull spread. That is, the bear spread trader will purchase the call with the higher exercise price and sell the call with the lower exercise price. This strategy is designed to profit from falling stock prices (i.e., a “bear” strategy). As stock prices fall, the investor keeps the premium from the written call, net of the long call’s cost. The purpose of the long call is to protect from sharp increases in stock prices. The payoff is the opposite (mirror image) of the bull call spread and is shown in Figure 4.

**Figure 4: Bear Call Spread**

Profit



Puts can also be used to replicate the payoffs for both a bull call spread and a bear call spread. In a *bear put spread* the investor buys a put with a higher exercise price and sells a put with a lower exercise price.

**Example: Bear put spread**

An investor sells a put for  $P_{L0} = \$3.00$  with a strike of  $X = \$20$  and purchases a put for  $P_{H0} = \$4.50$  with a strike price of \$40. Compute the payoff of a bear put spread strategy when the price of the stock is at \$35.

**Answer:**

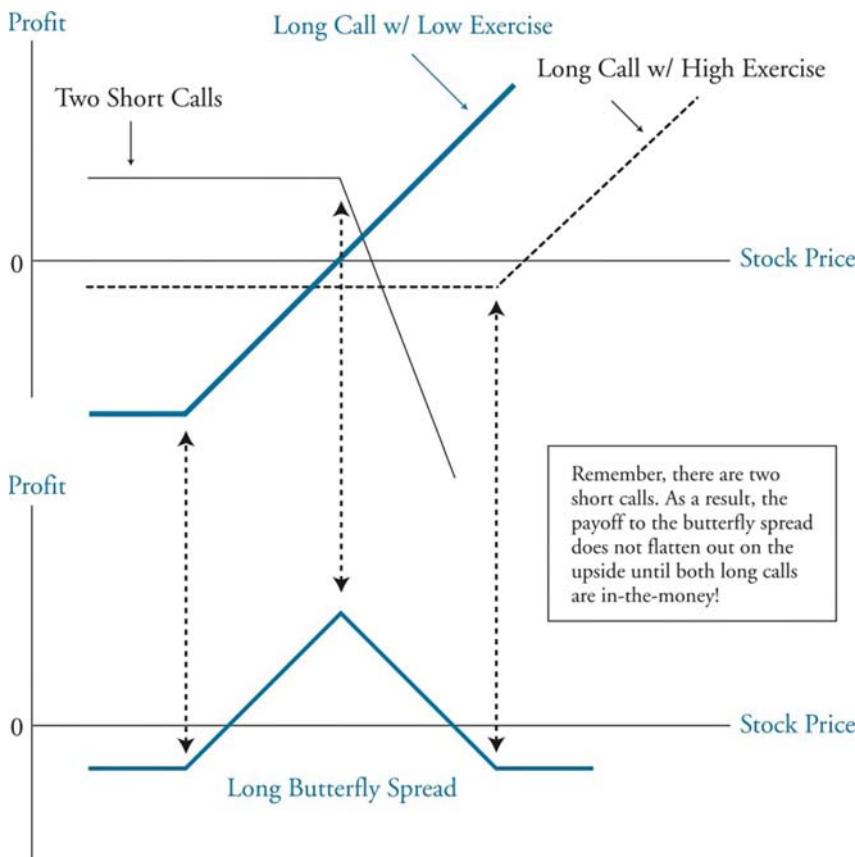
$$\text{profit} = \max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}$$

$$\text{profit} = \max(0, 40 - 35) - \max(0, 20 - 35) - 4.50 + 3 = \$3.50$$

**Butterfly Spreads**

A *butterfly spread* involves the purchase or sale of *three* different call options. Here, the investor buys one call with a low exercise price, buys another call with a high exercise price, and sells *two* calls with an exercise price in between. The buyer of a butterfly spread is essentially betting that the stock price will stay near the strike price of the written calls. However, the loss that the butterfly spread buyer sustains if the stock price strays from this level is limited. The two graphs in Figure 5 illustrate the construction and payoffs of a butterfly spread.

Figure 5: Butterfly Spread Construction and Behavior



### Example: Butterfly spread with calls

An investor makes the following transactions in calls on a stock:

- Buys one call defined by  $C_{L0} = \$7.00$  and  $X_L = \$55$ .
- Buys one call defined by  $C_{H0} = \$2.00$  and  $X_H = \$65$ .
- Sell two calls defined by  $C_{M0} = \$4.00$  and  $X_M = \$60$ .

Compute the payoff of a butterfly spread strategy with calls when the stock is at \$60.

**Answer:**

$$\text{profit} = \max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0}$$

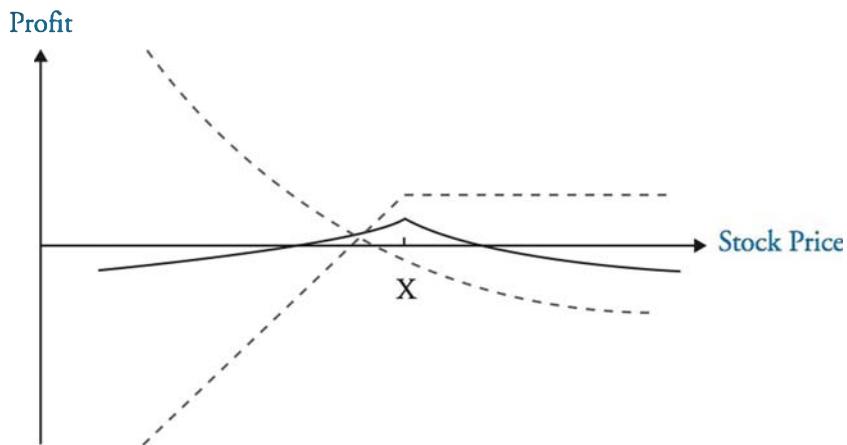
$$\text{profit} = \max(0, 60 - 55) - 2\max(0, 60 - 60) + \max(0, 60 - 65) - 7 + 2(4) - 2 = \$4.00$$

To create a butterfly spread with put options, the investor would buy a low and high strike put option and sell two puts with an intermediate strike price. Again, the combined position is constructed by summing the payoffs of the individual options at each stock price.

## Calendar Spreads

A *calendar spread* is created by transacting in two options that have the same strike price but different expirations. Figure 6 shows a calendar spread using put options. The strategy sells the short-dated option and buys the long-dated option. Notice that the payoff here is similar to the butterfly spread. The investor profits only if the stock remains in a narrow range, but losses are limited. In this case, the losses are not symmetrical as they are in the butterfly spread. A calendar spread based on calls is created in similar fashion.

**Figure 6: Calendar Spread (Using Two Put Options)**



Calendar spreads are categorized differently depending on the relationship between the strike price and the current stock price. The strategy is referred to as a **neutral calendar spread** if the strike price is close to the current stock price. A **bullish calendar spread** has a strike price above the current stock price, and a **bearish calendar spread** has a strike price below the current stock price.

A **reverse calendar spread** produces a payoff that is opposite of the graph shown in Figure 6. Instead of selling a short-dated option and buying a long-dated option, the investor of a reverse calendar spread will buy a short-dated option and sell a long-dated option. The investor will profit when the stock is well above or below the strike price and will suffer a loss if the stock is near the strike price.

## Diagonal Spreads

A *diagonal spread* is similar to a calendar spread except that instead of using options with the same strike price and different expirations, the options in a diagonal spread can have different strike prices in addition to different expirations.

## Box Spreads

A *box spread* is a combination of a bull call spread and a bear put spread on the same asset. This strategy will produce a constant payoff that is equal to the high exercise price ( $X_H$ ) minus the low exercise price ( $X_L$ ). Under a no arbitrage assumption, the present value of the payoff will equal the net premium paid (i.e., profit will equal zero).

When the profit from this strategy is different than zero, an investor can capitalize on the arbitrage opportunity by either buying or selling the box. If the profit is positive, the investor will create a long box spread by buying a call at  $X_L$ , selling a call at  $X_H$ , buying a put at  $X_H$ , and selling a put at  $X_L$ . If the profit is negative, the investor will create a short box spread by buying a call at  $X_H$ , selling a call at  $X_L$ , buying a put at  $X_L$ , and selling a put at  $X_H$ . Note that box spread arbitrage is only successful with European options.

## COMBINATION STRATEGIES

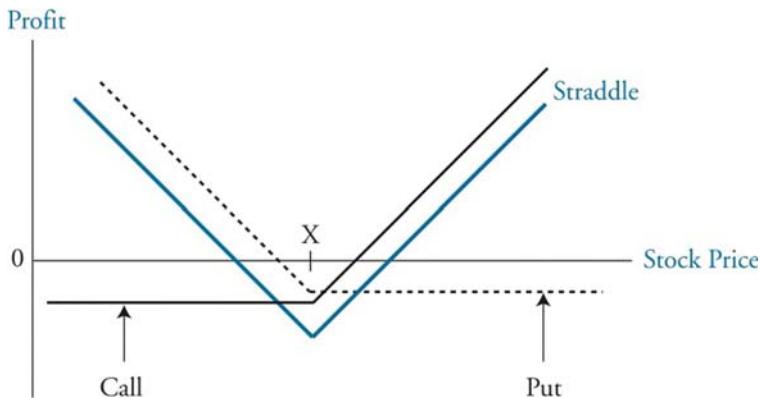
### LO 43.3: Describe the use and explain the payoff functions of combination strategies.

Combinations are option strategies involving both puts and calls. We will discuss straddles, strangles, strips, and straps.

#### Straddle

A long *straddle* (bottom straddle or straddle purchase) is created by purchasing a call and a put with the same strike price and expiration. Figure 7 illustrates the payoff for a long straddle position. Both options have the same exercise price and expiration. Note that this strategy is profitable when the stock price moves strongly in either direction. This strategy bets on volatility. A short straddle (top straddle or straddle write) sells both options and bets on little movement in the stock. A short straddle bets on the same thing as the butterfly spread or the calendar spread, except the losses are not limited. It is a bet that will profit more if correct but also lose more if it is incorrect. Straddles are symmetric around the strike price.

Figure 7: Long Straddle Profit/Loss



**Example: Straddle**

An investor purchases a call on a stock, with an exercise price of \$45 and a premium of \$3, and purchases a put option with the same maturity that has an exercise price of \$45 and a premium of \$2. Compute the payoff of a straddle strategy if the stock is at \$35.

**Answer:**

$$\text{profit} = \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$$

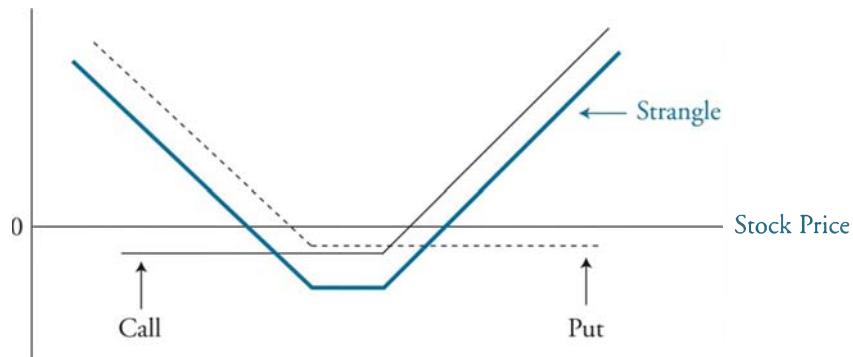
$$\text{profit} = \max(0, 35 - 45) + \max(0, 45 - 35) - 3 - 2 = \$5$$

**Strangle**

A *strangle* (or bottom vertical combination) is similar to a straddle except that the options purchased are slightly out-of-the-money, so it is cheaper to implement than the straddle. The payoff is similar to the straddle except for a flat section between the strike prices, as shown in Figure 8. Because it is cheaper, the stock will have to move more relative to the straddle before the strangle pays off. Strangles are also symmetric around the strikes.

**Figure 8: Long Strangle Profit/Loss**

Profit

**Example: Strangle**

An investor purchases a call on a stock, with an exercise price of \$50 and a premium of \$1.50, and purchases a put option with the same maturity that has an exercise price of \$45 and a premium of \$2. Compute the payoff of a strangle strategy if the stock is at \$40.

**Answer:**

$$\text{profit} = \max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$$

$$\text{profit} = \max(0, 40 - 50) + \max(0, 45 - 40) - 1.50 - 2 = \$1.50$$

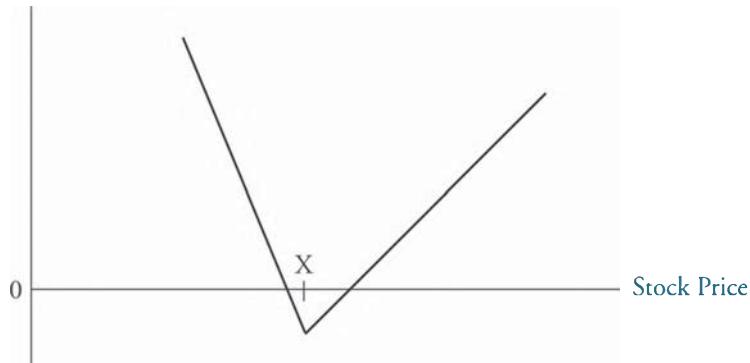
A short strangle (or a top vertical combination) is similar to the short straddle.

## Strips and Straps

A *strip* involves purchasing two puts and one call with the same strike price and expiration. Figure 9 illustrates a strip. Notice the asymmetry of the payoff. A strip is betting on volatility but is more bearish since it pays off more on the downside.

Figure 9: Strip Profit/Loss

Profit/Loss

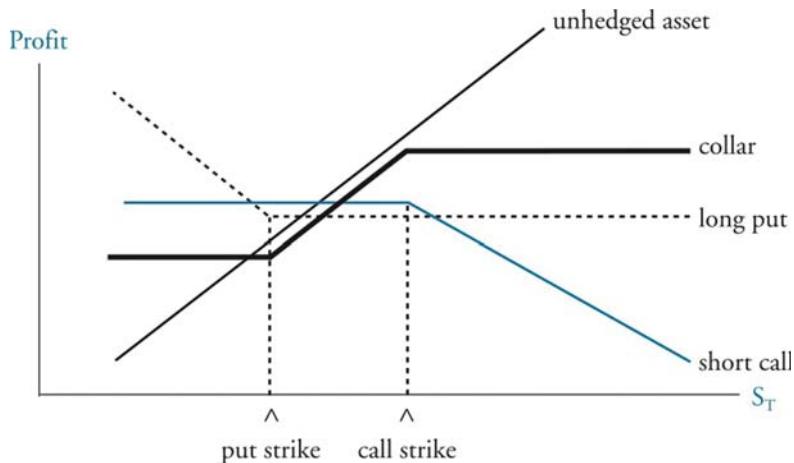


A *strap* involves purchasing two calls and one put with the same strike price and expiration. A strap is betting on volatility but is more bullish since it pays off more on the upside.

## Collar

A *collar* is the combination of a protective put and covered call. The usual goal is for the owner of the underlying asset to buy a protective put and then sell a call to pay for the put. If the premiums of the two are equal, it is called a **zero-cost collar**.

**Figure 10: Collar**



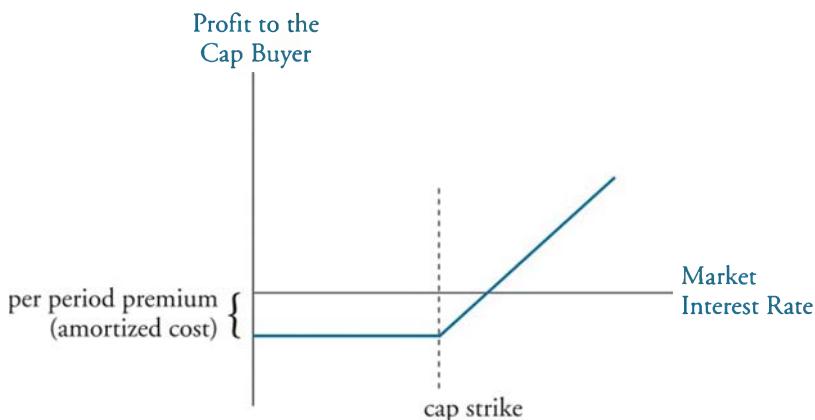
## INTEREST RATE CAPS AND FLOORS

An **interest rate cap** is an agreement in which one party agrees to pay the other at regular intervals over a certain period of time when the benchmark interest rate (e.g., LIBOR) exceeds the strike rate specified in the contract. This strike rate is called the **cap rate**. For example, the seller of a cap might agree to pay the buyer at the end of any quarter over the next two years if LIBOR is greater than a cap rate of 6%.

The buyer of a cap has a position similar to that of a buyer of a call on LIBOR, both of whom benefit when interest rates rise. Because an interest rate cap is a multi-period agreement, a cap is actually a portfolio of call options on LIBOR called **caplets**. For example, the 2-year cap discussed above is actually a portfolio of eight interest rate options with different maturity dates.

The cap buyer pays a premium to the seller and exercises the cap if the market rate of interest rises above the cap strike. The diagram in Figure 11 illustrates the profits of an interest rate cap at the end of one particular settlement period. It has the familiar shape of a long position in a call option.

Figure 11: Profit to a Long Cap

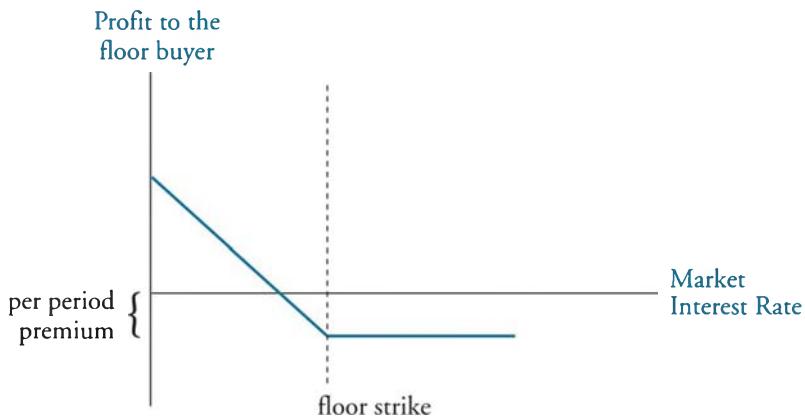


An **interest rate floor** is an agreement in which one party agrees to pay the other at regular intervals over a certain time period when the benchmark interest rate (e.g., LIBOR) falls below the strike rate specified in the contract. This strike rate is called the **floor rate**. For example, the seller of a floor might agree to pay the buyer at the end of any quarter over the next two years if LIBOR is less than a floor rate of 4%.

The buyer of a floor benefits from an interest rate decrease and, therefore, has a position that is similar to that of a buyer of a put on LIBOR, who benefits when interest rates fall and the price of the instrument rises. Once again, because a floor is a multi-period agreement, a floor is actually a portfolio of put options on LIBOR called **floorlets**.

The floor buyer pays a premium and exercises the floor if the market rate of interest falls below the floor strike. The diagram in Figure 12 illustrates the profits of an interest rate floor at the end of one particular settlement period. It has the same shape as a long put option.

Figure 12: Profit to a Long Floor



Options are traded both on *interest rates* and on *prices* of fixed-income securities. So far we've talked about options on interest rates. The values of comparable options on rates and prices respond differently to changes in interest rates because of the inverse relationship

between bond yields and bond prices. Figure 13 outlines how each type of option responds to changes in yields and bond prices.

**Figure 13: Options on Rate vs. Options on Prices**

<i>Option</i>	<i>If Rates Increase and Bond Prices Decrease</i>	<i>If Rates Decrease and Bond Prices Increase</i>
Value of call on LIBOR	Increases	Decreases
Value of call on bond price	Decreases	Increases
Value of put on LIBOR	Decreases	Increases
Value of put on bond price	Increases	Decreases

We can also interpret caps and floors in terms of options on the prices of fixed-income securities:

- A long cap is equivalent to a portfolio of long put options on fixed-income security prices.
- A long floor is equivalent to a portfolio of long call options on fixed-income security prices.

An **interest rate collar** is a simultaneous position in a floor and a cap on the same benchmark rate over the same period with the same settlement dates. There are two types of collars:

- The first type of collar is to purchase a cap and sell a floor. For example, an investor with a LIBOR-based liability could purchase a cap on LIBOR at 8% and simultaneously sell a floor on LIBOR at 4% over the next year. The investor has now hedged the liability so that the borrowing costs will stay within the “collar” of 4% to 8%. If the cap and floor rates are set so that the premium paid from buying the cap is exactly offset by the premium received from selling the floor, the collar is called a “zero-cost” collar.
- The second type of collar is to purchase a floor and sell a cap. For example, an investor with a LIBOR-based asset could purchase a floor on LIBOR at 3% and simultaneously sell a cap at 7% over the next year. The investor has now hedged the asset so the returns will stay within the collar of 3% to 7%. The investor can create a zero-cost collar by choosing the cap and floor rates so that the premium paid on the floor offsets the premium received on the cap.

## KEY CONCEPTS

### LO 43.1

Stock options can be combined with their underlying stock to generate various payoff profiles. A protective put combines an at-the-money long put position with the underlying stock. A covered call involves selling a call option on a stock that is owned by the option writer.

### LO 43.2

Spread strategies combine options in the same option class to generate various payoff profiles.

The buyer of a bull call spread expects the stock price to rise and the purchased call to finish in-the-money. However, the buyer does not believe that the price of the stock will rise above the exercise price for the out-of-the-money written call.

The bear call spread trader will purchase the call with the higher exercise price and sell the call with the lower exercise price. This strategy is designed to profit from falling stock prices (i.e., a “bear” strategy). As stock prices fall, the investor keeps the premium from the written call, net of the long call’s cost.

A box spread is an extreme method of locking in value. The dollar return for a box spread is fixed. It is a combination of a bull call spread and a bear put spread.

A calendar spread is created by transacting in two options that have the same strike price but different expirations.

The buyer of a butterfly spread is essentially betting that the stock price will stay near the strike price of the written calls. However, the loss that the butterfly spread buyer sustains if the stock price strays from this level is not large.

In a diagonal spread, options can have different strike prices and different expirations.

Bull call spread payoff:

$$\text{profit} = \max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0}$$

Bear put spread payoff:

$$\text{profit} = \max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}$$

Butterfly spread payoff:

$$\text{profit} = \max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0}$$

**LO 43.3**

Combination strategies combine puts and calls to generate various payoff strategies.

A long straddle (bottom straddle or straddle purchase) is created by purchasing a call and a put with the same strike price and expiration. Note that this strategy only pays off when the stock moves in either direction.

A strangle (or bottom vertical combination) is similar to a straddle except that the option purchased is slightly out-of-the-money, so it is cheaper to implement than the straddle.

A strip is betting on volatility but is more bearish since it pays off more on the down side.

A strap is betting on volatility but is more bullish since it pays off more on the up side.

Straddle payoff:

$$\text{profit} = \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$$

Strangle payoff:

$$\text{profit} = \max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$$

## CONCEPT CHECKERS

1. An investor is very confident that a stock will change significantly over the next few months; however, the direction of the price change is unknown. Which strategies will most likely produce a profit if the stock price moves as expected?
  - I. Short butterfly spread.
  - II. Bearish calendar spread.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.
2. Which of the following will create a bear spread?
  - A. Buy a call with a strike price of  $X = 45$  and sell a call with a strike price of  $X = 50$ .
  - B. Buy a call with a strike price of  $X = 50$  and buy a put with a strike price of  $X = 55$ .
  - C. Buy a put with a strike price of  $X = 45$  and sell a put with a strike price of  $X = 50$ .
  - D. Buy a call with a strike price of  $X = 50$  and sell a call with a strike price of  $X = 45$ .
3. An investor believes that a stock will either increase or decrease greatly in value over the next few months, but believes a down move is more likely. Which of the following strategies will be the best for this investor?
  - A. A protective put.
  - B. An at-the-money strip.
  - C. An at-the-money strap.
  - D. A top vertical combination.
4. An investor constructs a long straddle by buying an April \$30 call for \$4 and buying an April \$30 put for \$3. If the price of the underlying shares is \$27 at expiration, what is the profit on the position?
  - A. -\$4.
  - B. -\$2.
  - C. \$2.
  - D. \$3.
5. Consider an option strategy where an investor buys one call option with an exercise price of \$55 for \$7, sells two call options with an exercise price of \$60 for \$4, and buys one call option with an exercise price of \$65 for \$2. If the stock price declines to \$25, what will be the profit or loss on the strategy?
  - A. -\$3.
  - B. -\$1.
  - C. \$1.
  - D. \$2.

## CONCEPT CHECKER ANSWERS

1. A A short butterfly spread will produce a modest profit if there is a large amount of volatility in the price of the stock. A bearish calendar spread is a play using options with different expiration dates.
2. D Spread strategies involve purchasing and selling an option of the same type. A bear spread with calls involves buying a call with a high strike price and selling a call with a low strike price. The investor profits if stock prices fall by keeping the premium from the written call, net of the premium from the purchased call. Note that a bear spread can also be constructed with put options by buying a put with a high strike price and selling a put with a low strike price. With a bear put spread, if the stock price declines and both puts are exercised, the investor receives the difference between the strike prices less the net premium paid.
3. B An at-the-money strip bets on volatility but is more bearish since it pays off more on the downside.
4. A The sum of the premiums paid for the position is \$7. With the underlying stock at \$27, the put will be worth \$3, while the call option will be worthless. The value of the position is  $(-\$7 + \$3) = -\$4$ .
5. B The strategy described is a butterfly spread where the investor buys a call with a low exercise price, buys another call with a high exercise price, and sell two calls with a price in between. In this case, if the option moves to \$25, none of the call options will be in the money, so the profit is equal to the net premium paid, which is  $-\$7 + (2 \times \$4) - \$2 = -\$1$ .

---

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

# EXOTIC OPTIONS

## Topic 44

### EXAM FOCUS

In this topic, we define and discuss the important characteristics of a variety of exotic options. The difference between exotic options and more traditional exchange-traded instruments is also highlighted. Be familiar with the payoff structures for the various exotic options discussed.

---

### EVALUATING EXOTIC OPTIONS

**LO 44.1: Define and contrast exotic derivatives and plain vanilla derivatives.**

**LO 44.2: Describe some of the factors that drive the development of exotic products.**

---

Plain vanilla derivatives include listed futures contracts and commonly used forwards and other over-the-counter (OTC) derivatives that are traded in fairly liquid markets. Exotic derivatives are customized to fit a specific firm need for hedging that cannot be met by plain vanilla derivatives. With plain vanilla derivatives, there is little uncertainty about the cost, the current market value, when they will pay, how much they will pay, and the cost of exiting the position. With exotic derivatives, some or all of these may be in question.

Exotic derivatives are developed for several reasons. The main purpose is to provide a unique hedge for a firm's underlying assets. Other reasons include addressing tax and regulatory concerns as well as speculating on the expected future direction of market prices.

Four questions that should be considered when evaluating exotic derivative strategies are:

- Will the strategy pay in the right circumstances to provide an effective hedge? Problems with understanding the payoff of the exotic derivative and credit risk of the derivative strategy can lead to a difference between the payoff the user expects and the actual payoff received.
- What is the cost of the exotic derivative hedging strategy?
- Is a pricing model needed, and does the user have the appropriate pricing model to estimate dealer cost and monitor the value of non-traded derivatives over time?
- How is a derivative position reversed? Note that the costs of exiting a position or strategy may involve penalties and large bid-ask spreads or require a pricing model to evaluate alternatives.

## USING PACKAGES TO FORMULATE A ZERO-COST PRODUCT

### LO 44.3: Explain how any derivative can be converted into a zero-cost product.

A package is defined as some combination of standard European options, forwards, cash, and the underlying asset. Bull, bear, and calendar spreads, as well as straddles and strangles, are examples of packages. Packages usually consist of selling one instrument with certain characteristics and buying another with somewhat different characteristics. Because packages often consist of a long position and a short position, they can be constructed so that the initial cost to the investor is zero.

For example, consider a zero-cost short collar. A short collar combines a long standard put option with an exercise price  $X_L$  and a short standard call option with exercise price  $X_H$  (where  $X_L < X_H$ ). If the premium the investor pays for the put option is exactly offset by the premium the investor receives for the short call position, the investor's net cost for implementing the short collar strategy is zero. In any case where the investor's cash outflows from long positions are offset by cash inflows from short positions, the investor can use a package to create a zero-cost product.

## TRANSFORMING STANDARD AMERICAN OPTIONS INTO NONSTANDARD AMERICAN OPTIONS

### LO 44.4: Describe how standard American options can be transformed into nonstandard American options.

Recall that standard exchange-traded American options can be exercised at any time prior to expiration. If some of the available expiration periods are restricted, or changes are made to other standard features, standard options become what we refer to as **nonstandard options**. Nonstandard options are common in the over-the-counter (OTC) market.

There are three common features that transform standard American options into nonstandard options:

- The most common transformation can be made to restrict early exercise to certain dates (e.g., a three month call option may only be exercised on the last day of each month.) This type of transformation results in a **Bermudan option**.
- Early exercise can be limited to a certain portion of the life of the option (e.g., there is a “lock out” period that does not allow a 6-month call option to be exercised in the first three months of the call’s life).
- The option’s strike price may change (e.g., the strike price of a 3-year call option with a strike price of 40 at initiation may rise to 44 in year 2 and 48 in year 3).

## EXOTIC OPTION PAYOFF STRUCTURES

**LO 44.5:** Identify and describe the characteristics and pay-off structure of the following exotic options: gap, forward start, compound, chooser, barrier, binary, lookback, shout, Asian, exchange, rainbow, and basket options.

### Gap Options

A gap option has two strike prices,  $X_1$  and  $X_2$ . ( $X_2$  is sometimes referred to as the trigger price.) If these two strike prices are equal, the gap option payoff will be the same as an ordinary option. If the two strike prices differ and the payoff for a gap option is non-zero, there will be a gap in the payoff graph that is either increased or decreased by the difference between the strike prices. Gap options can be valued with a slight modification to the Black-Scholes-Merton option pricing model, which will be discussed in Book 4.

For a *gap call option*, if  $X_2$  is greater than  $X_1$ , and the stock price at maturity,  $S_T$ , is greater than the trigger price,  $X_2$ , then the payoff for the call option will be equal to  $S_T - X_1$ . If the stock price is less than or equal to  $X_2$ , the payoff will be zero. Note that a negative payoff can occur if the stock price is greater than  $X_2$  and  $X_2$  is less than  $X_1$ . In this case, the payoff will be reduced by  $X_2 - X_1$ .

For a *gap put option*, if  $X_2$  is less than  $X_1$ , and the stock price at maturity,  $S_T$ , is less than the trigger price,  $X_2$ , then the payoff for the put option will be equal to  $X_1 - S_T$ . If the stock price is greater than or equal to  $X_2$ , the payoff will be zero. A negative payoff can occur if the stock price is less than  $X_2$  and  $X_2$  is greater than  $X_1$ . Like with a gap call option, if this is the case, the payoff will be reduced by  $X_2 - X_1$ .

### Forward Start Options

Forward start options are options that begin their existence at some time in the future. For example, today an investor may purchase a 3-month call option that will not come into existence until six months from today. Employee incentive plans commonly incorporate forward start options in which at-the-money options will be created after some period of employment has passed. Note that when the underlying asset is a nondividend paying stock, the value of a forward start option will be identical to the value of a European at-the-money option with the same time to expiration as the forward start option.

### Compound Options

Compound options are options on options. There are four key types of compound options:

- A *call on a call* gives the investor the right to buy a call option at a set price for a set period of time.
- A *call on a put* gives the investor the right to buy a put option at a set price for a set period of time.

## Topic 44

## Cross Reference to GARP Assigned Reading – Hull, Chapter 26

- A *put on a call* gives the investor the right to sell a call option at a set price for a set period of time.
- A *put on a put* gives the investor the right to sell a put option at a set price for a set period of time.

Compound options have two levels of the underlying that determine their value—the value of the underlying option, which in turn is determined by the value of the underlying asset.

Compound options consist of two strike prices and two exercise dates. The first strike price and exercise date are used by the holder to evaluate whether to exercise the first option to receive the second option, where the second option is an option on the underlying asset, or just let the compound option expire. For example, a call on a call would be exercised if the price of the call on the underlying for the second call option were greater than the strike price of the initial option. The strike price and exercise date on the second call, however, are related to the value of the underlying asset.

### Chooser Options

This interesting option allows the owner, after a certain period of time has elapsed, to choose whether the option is a call or a put. The option with the greater value after the requisite time has elapsed will determine whether the owner will choose the option to be a put or a call.

### Barrier Options

Barrier options are options whose payoffs (and existence) depend on whether the underlying's asset price reaches a certain barrier level over the life of the option. These options are usually less expensive than standard options, and essentially come in either *knock-out* or *knock-in* flavors. Specific types of barrier options are:

- *Down-and-out call (put)*. A standard call (put) option that ceases to exist if the underlying asset price hits the barrier level, which is set below the current stock value.
- *Down-and-in call (put)*. A standard call (put) option that only comes into existence if the underlying asset price hits the barrier level, which is set below the current stock value.
- *Up-and-out call (put)*. A standard call (put) option that ceases to exist if the underlying asset price hits a barrier level, which is set above the current stock value.
- *Up-and-in call (put)*. A standard call (put) option that only comes into existence if the underlying asset price hits the above-current stock-price barrier level.

Barrier options have characteristics that can be very different from those of standard options. For example, vega, the sensitivity of an option's price to changes in volatility, is always positive for a standard option but may be negative for a barrier option. Increased volatility on a down-and-out option and an up-and-out option does not increase value because the closer the underlying gets to the barrier price, the greater the chance the option will expire.

Note that the value of a down-and-out call combined with the value of a down-and-in call is equal to the value of a standard call option. In other words, by knowing the value of two of these three options you can calculate the value of the remaining option (e.g., down-and-out call = standard call – down-and-in call). Similarly, the value of a standard put option is equal to the value of an up-and-out put plus the value of an up-and-in put.

## Binary Options

Binary options generate discontinuous payoff profiles because they pay only one price at expiration if the asset value is above the strike price. The term binary means that the option payoff has one of two states: the option pays a set dollar amount at expiration if the option is above the strike price, or the option pays nothing if the price is below the strike price. Hence, a payoff discontinuity results from the fact that the payoff is only one value—it does not increase continuously with the price of the underlying asset as in the case of a traditional option.

In the case of a **cash-or-nothing call**, a fixed amount,  $Q$ , is paid if the asset ends up above the strike price. Since the Black-Scholes-Merton formula denotes  $N(d_2)$  as the probability of the asset price being above the strike price, the value of a cash-or-nothing call is equal to  $Qe^{-rT}N(d_2)$ .

An **asset-or-nothing call** pays the value of the stock when the contract is initiated if the stock price ends up above the strike price at expiration. The corresponding value for this option is  $S_0e^{-qT}N(d_1)$ , where  $q$  is the continuous dividend yield.

## Lookback Options

Lookback options are options whose payoffs depend on the maximum or minimum price of the underlying asset during the life of the option. A **floating lookback call** pays the difference between the expiration price and the minimum price of the stock over the horizon of the option. This essentially allows the owner to purchase the security at its lowest price over the option's life. On the other hand, a **floating lookback put** pays the difference between the expiration and maximum price of the stock over the time period of the option. This translates into allowing the owner of the option to sell the security at its highest price over the life of the option.

Lookback options can also be fixed when an exercise price is specified. A **fixed lookback call** has a payoff function that is identical to a European call option. However, for this exotic option, the final stock price (or expiration price) in the European call option payoff is replaced by the maximum price during the option's life. Similarly, a **fixed lookback put** has a payoff like a European put option but replaces the final stock price with the minimum price during the option's life.

## Shout Options

A shout option allows the owner to pick a date when he “shouts” to the option seller, which then translates into an intrinsic value of the option at the time of the shout. At option expiration, the owner receives the maximum of the shout intrinsic value or the option expiration intrinsic value. In other words, for a shout call option, even if the price of the stock falls after the shout, the investor has locked in the difference between the price of the stock and the shout price. If the stock continues to rise, the shout option will have a payoff consistent with a standard call option. Note that most shout options allow for one “shout” during the option's life.

## Asian Options

Asian options have payoff profiles based on the average price of the security over the life of the option. *Average price* calls and puts pay off the difference between the average stock price and the strike price. Note that the average price will be much less volatile than the actual price. This means that the price for an Asian average price option will be lower than the price of a comparable standard option. *Average strike* calls and average strike puts pay off the difference between the stock expiration price and average price, which essentially represents the strike price in a typical intrinsic value calculation. If the average price or strike price for an Asian option is based on a geometric average, then using an option pricing model is not a problem because a geometric average is lognormal. However, most Asian options base their average calculations on arithmetic averages, which complicates the pricing process. In this case, a lognormal distribution of prices is assumed, which provides an adequate approximation.

## Exchange Options

A common use of an option to exchange one asset for another, often called an exchange option, is to exchange one currency with another. For example, consider a U.S. investor who holds an option to purchase euros with yen at a specified exchange rate. In this particular case, the option will be exercised if euros are more valuable to the U.S. investor than yen. Other applications, such as tender offers to exchange one stock for another, also arise in certain situations.

## Basket Options

Basket options are simply options to purchase or sell baskets of securities. These baskets may be defined specifically for the individual investor and may be composed of specific stocks, indices, or currencies. Any exotic options that involve several different assets are more generally referred to as **rainbow options**.

## Volatility and Variance Swaps

---

### LO 44.6: Describe and contrast volatility and variance swaps.

---

A **volatility swap** involves the exchange of volatility based on a notional principal. One side of the swap pays based on a pre-specified fixed volatility while the other side pays based on realized volatility. Unlike the exotic options we have discussed thus far, volatility swaps are a bet on volatility alone as opposed to a bet on volatility and the price of the underlying asset.

Much like a volatility swap, a **variance swap** involves exchanging a pre-specified fixed variance rate for a realized variance rate. The variance rate being exchanged is simply the square of the volatility rate. However, unlike volatility swaps, variance swaps are easier to price and hedge since they can be replicated using a collection of call and put options.

## ISSUES IN HEDGING EXOTIC OPTIONS

**LO 44.7: Explain the basic premise of static option replication and how it can be applied to hedging exotic options.**

The typical dynamic option-hedging situation uses option Greeks to measure sensitivity of the option value to changes in underlying asset characteristics (i.e., creating a delta-neutral portfolio). Hedging is simpler with some exotic options than it is with plain vanilla options. Asian options, for instance, depend on the average price of the underlying. Through time, the uncertainty of the average value gets smaller. Hence, the option begins to become less sensitive to changes in the value of the security because the payoff can be estimated more accurately.

Hedging positions in barrier and other exotic options are not so straightforward. This type of hedging requires the replication of a portfolio that is exactly opposite to the option position. When the replication portfolio requires frequent adjustments to the holdings in the underlying assets, the hedging procedure is referred to as dynamic options replication. Dynamic options replication requires frequent trading, which makes it costly to implement.

As an alternative, a static options replication approach may be used to hedge positions in exotic options. In this case, a short portfolio of actively traded options that approximates the option position to be hedged is constructed. This short replication options portfolio is created once, which drastically reduces the transaction costs associated with dynamic rebalancing.

## KEY CONCEPTS

---

### LO 44.1

Plain vanilla derivatives include listed futures contracts and commonly used forwards and other OTC derivatives that are traded in fairly liquid markets. Exotic derivatives are customized to fit a specific firm need.

---

### LO 44.2

The main purpose for the development of exotic derivatives is to provide a unique hedge for a firm's underlying assets. Additional reasons include addressing tax and regulatory concerns as well as speculating on the expected future direction of market prices.

---

### LO 44.3

Packages are portfolios of European options, forwards, cash, and the underlying asset. Given that packages often consist of a long position and a short position, they can be constructed so that the initial cost to the investor is zero.

---

### LO 44.4

Restricting exercise dates and changing strike prices can transform standard options into nonstandard options.

---

### LO 44.5

A gap option has two strike prices. If the two strike prices differ and the payoff is non-zero, there will be a gap in the payoff graph that is either increased or decreased by the difference between the strike prices.

Forward start options are options that commence in the future.

A compound option is defined as an option on another option.

Chooser options allow the owner to choose whether the option is a call or a put, after option initiation.

Barrier options are options whose payoffs (and existence) depend on whether the underlying's asset price reaches a certain barrier level over the life of the option.

Binary options either pay nothing (if price is below strike price) or a fixed amount at expiration.

Lookback options depend on the maximum or minimum value of the underlying asset during the life of the option.

Shout options allow the owner to receive either the intrinsic value of the option at the shout date or at expiration, whichever is greater.

Asian options have payoff profiles that depend on the average underlying asset price over the life of the option.

An exchange option is an option to exchange one asset for another.

Basket options allow the owner to buy or sell portfolios of assets. Exotic options that involve several different assets are more generally referred to as rainbow options.

---

#### LO 44.6

A volatility swap involves the exchange of volatility based on a notional principal. A variance swap involves exchanging a pre-specified fixed variance rate for a realized variance rate.

---

#### LO 44.7

Exotic options can be hedged in either a dynamic or static context, depending on the characteristics of the option.

## CONCEPT CHECKERS

1. A down-and-in call option is an option that comes into existence only when the underlying asset price:
  - A. rises to a set barrier level.
  - B. falls to a set barrier level.
  - C. falls to a set average barrier level.
  - D. rises to a set average barrier level.
  
2. A cash-or-nothing put option has a payout profile equivalent to zero or:
  - A. the underlying asset price if the value of the asset ends below the strike price.
  - B. the underlying asset price if the value of the asset ends above the strike price.
  - C. a set amount if the value of the asset ends below the strike price.
  - D. a set amount if the value of the asset ends above the strike price.
  
3. An Asian option can be hedged dynamically because the:
  - A. average value of the underlying asset price decreases uncertainty the closer the option gets to expiration.
  - B. average value of the underlying asset price increases uncertainty the closer the option gets to expiration.
  - C. maximum value of the underlying asset price decreases uncertainty the closer the option gets to expiration.
  - D. minimum value of the underlying asset price increases uncertainty the closer the option gets to expiration.
  
4. Which of the following options is most likely to have a negative vega?
  - A. A chooser option close to expiration.
  - B. A forward start put option before the start date.
  - C. An Asian put option close to the beginning of the option's life.
  - D. An up and out put when the stock price is close to the barrier.
  
5. Under which of the following circumstances would the value of an up and out call option be zero?
  - A. The strike price is above the barrier price.
  - B. The stock price is below the barrier price.
  - C. The stock price is above the strike price.
  - D. The stock price is below the strike price.

## CONCEPT CHECKER ANSWERS

1. B Down-and-in call options are standard options that come into existence only if the asset price falls to a set barrier price level, which is set below the current stock price.
2. C Cash-or-nothing put options pay only a set amount if the stock price ends below the strike price. These options differ from standard put options because the payment is a set amount that does not continuously increase with the decrease in stock price.
3. A Dynamic hedging can be used to hedge Asian options because uncertainty in the expiration value is decreased the closer one gets to expiration. This occurs because the intrinsic value becomes “set” due to the averaging effect over the life of the option.
4. D Vega is the sensitivity of the price of an option to changes in volatility of the underlying stock. For most options, vega is always positive—as volatility of the underlying stock increases, the price of the option also increases. An exception would be a knockout barrier option when the stock price is close to the barrier. Higher volatility means the barrier is more likely to be reached and the option will cease to exist.
5. A With an up and out call, if the stock price rises beyond the barrier price, the option ceases to exist. It therefore follows that if the strike price is above the barrier price, the option will never come into the money because the option will cease to exist before the option will ever come into the money.