The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Interest Rate Futures

Topic 39

Exam Focus

In this topic, we examine Treasury bonds (T-bonds) and Eurodollar futures contracts. These instruments are two of the most popular interest rate futures contracts that trade in the United States. Be able to define the cheapest-to-deliver bond for T-bonds and know how to use the convexity adjustment for Eurodollar futures. Duration-based hedging using interest rate futures is also discussed. Be familiar with the equation to calculate the number of contracts needed to conduct a duration-based hedge.

Day Count Conventions

LO 39.1: Identify the most commonly used day count conventions, describe the markets that each one is typically used in, and apply each to an interest calculation.

Day count conventions play a role when computing the interest that accrues on a fixed income security. When a bond is purchased, the buyer must pay any **accrued interest** earned through the settlement date.

$$\text{accrued interest} = \text{coupon} \times \frac{\text{\# of days from last coupon to the settlement date}}{\text{\# of days in coupon period}}$$

In the United States, there are three commonly used day count conventions.

1. U.S. Treasury bonds use **actual/actual.**
2. U.S. corporate and municipal bonds use **30/360**.
3. U.S. money market instruments (Treasury bills) use **actual/360**.

The following examples demonstrate the use of day count conventions when computing accrued interest.

Example: Day count conventions

Suppose there is a semiannual-pay bond with a $100 par value. Further assume that coupons are paid on March 1 and September 1 of each year. The annual coupon is 6%, and it is currently July 13. **Compute** the accrued interest of this bond as a T-bond and a U.S. corporate bond.

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Answer:

The T-bond uses actual/actual (in period), and the reference (March 1 to September 1) period has 184 days. There are 134 actual days from March 1 to July 13, so the accrued interest is:

$$\frac{134}{184} \times \$3 = \$2.1848$$

The corporate bond uses 30/360, so the reference period now has 180 days. Using this convention, there are 132 (= 30 × 4 + 12) days from March 1 to July 13, so the accrued interest is:

$$\frac{132}{180} \times \$3 = \$2.20$$

Quotations for T-Bonds

LO 39.3: Differentiate between the clean and dirty price for a US Treasury bond; calculate the accrued interest and dirty price on a US Treasury bond.

T-bond prices are quoted relative to a $100 par amount in dollars and 32nds. So a 95–05 is 95 5/32, or 95.15625. The quoted price of a T-bond is not the same as the cash price that is actually paid to the owner of the bond. In general:

cash price = quoted price + accrued interest

Clean and Dirty Prices

The cash price (a.k.a. **invoice price** or **dirty price**) is the price that the seller of the bond must be paid to give up ownership. It includes the present value of the bond (a.k.a. **quoted price** or **clean price**) plus the accrued interest. This relationship is shown in the equation above. Conversely, the clean price is the cash price less accrued interest:

quoted price = cash price – accrued interest

This relationship can also be expressed as:

clean price = dirty price – accrued interest

Example: Calculate the cash price of a bond

Assume the bond in the previous example is a T-bond currently quoted at 102–11. **Compute** the cash price.

Answer:

cash price = $102.34375 + $2.1848 = $104.52855

For a $100,000 par amount, this is $104,528.55.

Quotations for T-Bills

LO 39.2: Calculate the conversion of a discount rate to a price for a US Treasury bill.

T-bills and other money-market instruments use a discount rate basis and an actual/360 day count. A T-bill with a $100 face value with n days to maturity and a cash price of Y is quoted as:

$$\text{T-bill discount rate} = \frac{360}{n}(100 - Y)$$

This is referred to as the discount rate in annual terms. However, this discount rate is not the actual rate earned on the T-bill. The following example shows the calculation of the annualized yield on a T-bill, given its price.

Example: Calculating the cash price on a T-bill

Suppose you have a 180-day T-bill with a discount rate, or quoted price, of five (i.e., the annualized rate of interest earned is 5% of face value). If face value is $100, what is the true rate of interest and the cash price?

Answer:

Interest is equal to $2.5 (= $100 × 0.05 × 180 / 360) for a 180-day period. The true rate of interest for the period is therefore 2.564% [= 2.5 / (100 – 2.5)].

Cash price: 5 = (360 / 180) × (100 – Y); Y = $97.5.

Treasury Bond Futures

LO 39.4: Explain and calculate a US Treasury bond futures contract conversion factor.

LO 39.5: Calculate the cost of delivering a bond into a Treasury bond futures contract.

LO 39.6: Describe the impact of the level and shape of the yield curve on the cheapest-to-deliver Treasury bond decision.

In a T-bond futures contract, any government bond with more than 15 years to maturity on the first of the delivery month (and not callable within 15 years) is deliverable on the contract. This produces a large supply of potential bonds that are deliverable on the contract and reduces the likelihood of market manipulation. Since the deliverable bonds have very different market values, the Chicago Board of Trade (CBOT) has created **conversion factors**. The conversion factor defines the price received by the short position of the contract (i.e., the short position is delivering the contract to the long). Specifically, the cash received by the short position is computed as follows:

cash received = (QFP × CF) + AI

where:
QFP = quoted futures price (most recent settlement price)
CF = conversion factor for the bond delivered
AI = accrued interest since the last coupon date on the bond delivered

Conversion factors are supplied by the CBOT on a daily basis. Conversion factors are calculated as: (discounted price of a bond – accrued interest) / face value. For example, if the present value of a bond is \$142, accrued interest is \$2, and face value is \$100, the conversation factor would be: (142 – 2) / 100 = 1.4.

Cheapest-to-Deliver Bond

The conversion factor system is not perfect and often results in one bond that is the cheapest (or most profitable) to deliver. The procedure to determine which bond is the cheapest-to-deliver (CTD) is as follows:

cash received by the short = (QFP × CF) + AI

cost to purchase bond = (quoted bond price + AI)

The CTD bond minimizes the following: quoted bond price – (QFP × CF). This expression calculates the cost of delivering the bond.

Example: The cheapest-to-deliver bond

Assume an investor with a short position is about to deliver a bond and has four bonds to choose from which are listed in the following table. The last settlement price is $95.75 (this is the quoted futures price). **Determine** which bond is the cheapest-to-deliver.

Bond	*Quoted Bond Price*	*Conversion Factor*
1	99	1.01
2	125	1.24
3	103	1.06
4	115	1.14

Answer:

Cost of delivery:

Bond 1: 99 – (95.75 × 1.01) = $2.29

Bond 2: 125 – (95.75 × 1.24) = $6.27

Bond 3: 103 – (95.75 × 1.06) = $1.51

Bond 4: 115 – (95.75 × 1.14) = $5.85

Bond 3 is the cheapest-to-deliver with a cost of delivery of $1.51.

Finding the cheapest-to-deliver bond does not require any arcane procedures but could involve searching among a large number of bonds. The following guidelines give an indication of what type of bonds tend to be the cheapest-to-deliver under different circumstances:

- When yields > 6%, CTD bonds tend to be low-coupon, long-maturity bonds.
- When yields < 6%, CTD bonds tend to be high-coupon, short-maturity bonds.
- When the yield curve is upward sloping, CTD bonds tend to have longer maturities.
- When the yield curve is downward sloping, CTD bonds tend to have shorter maturities.

Treasury Bond Futures Price

LO 39.7: Calculate the theoretical futures price for a Treasury bond futures contract.

Recall the cost-of-carry relationship, where the underlying asset pays a known cash flow, as was presented in the previous topic. The futures price is calculated in the following fashion:

$$F_0 = (S_0 - I)e^{rT}$$

where:
I = present value of cash flow

We can use this equation to calculate the theoretical futures price when accounting for the CTD bond's accrued interest and its conversion factor.

Example: Theoretical futures price

Suppose that the CTD bond for a Treasury bond futures contract pays 10% semiannual coupons. This CTD bond has a conversion factor of 1.1 and a quoted bond price of 100. Assume that there are 180 days between coupons and the last coupon was paid 90 days ago. Also assume that Treasury bond futures contract is to be delivered 180 days from today, and the risk-free rate of interest is 3%. **Calculate** the theoretical price for this T-bond futures contract.

Answer:

The cash price of the CTD bond is equal to the quoted bond price plus accrued interest. Accrued interest is computed as follows:

$$AI = \text{coupon} \times \left(\frac{\text{number of days from last coupon to settlement date}}{\text{number of days in coupon period}} \right)$$

$$AI = 5 \times \frac{90}{180} = 2.5$$

$$\text{cash price} = 100 + 2.5 = 102.5$$

Since the next coupon will be received 90 days from today, that cash flow should be discounted back to the present using the familiar present value equation which discounts the cash flow using the risk-free rate:

$$5e^{-0.03 \times (90/365)} = \$4.96$$

Using the cost-of-carry model, the cash futures price (which expires 180 days from today) is then calculated as follows:

$$F_0 = (102.5 - 4.96)e^{(0.03)(180/365)} = 98.99$$

We are not done, however, since the futures contract expires 90 days after the last coupon payment. The quoted futures price at delivery is calculated after subtracting the amount of accrued interest (recall: QFP = cash futures price – AI).

$$98.99 - \left(5 \times \frac{90}{180} \right) = \$96.49$$

Finally, the conversion factor is utilized, producing a theoretical price for this T-bond futures contract of:

$$QFP = \frac{96.49}{1.1} = \$87.72$$

Eurodollar Futures

LO 39.8: Calculate the final contract price on a Eurodollar futures contract.

LO 39.9: Describe and compute the Eurodollar futures contract convexity adjustment.

The 3-month **eurodollar futures** contract trades on the Chicago Mercantile Exchange (CME) and is the most popular interest rate futures in the United States. This contract settles in cash and the minimum price change is one "tick," which is a price change of one basis point, or $25 per $1 million contract. Eurodollar futures are based on a eurodollar deposit (a eurodollar is a U.S. dollar deposited outside the United States) with a face amount of $1 million. The interest rate underlying this contract is essentially the 3-month (90-day) forward LIBOR. If Z is the quoted price for a eurodollar futures contract, the contract price is:

$$\text{eurodollar futures price} = \$10{,}000[100 - (0.25)(100 - Z)]$$

For example, if the quoted price, Z, is 97.8:

$$\text{contract price} = \$10{,}000[100 - (0.25)(100.0 - 97.8)] = \$994{,}500$$

Convexity Adjustment

The corresponding 90-day forward LIBOR (on an annual basis) for each contract is 100 – Z. For example, assume that the previous eurodollar contract was for a futures contract that matured in six months. Then the 90-day forward LIBOR six months from now is approximately 2.2% (100 – 97.8). However, the daily marking to market aspect of the futures contract can result in differences between actual forward rates and those implied by futures contracts. This difference is reduced by using the convexity adjustment. In general, long-dated eurodollar futures contracts result in implied forward rates larger than actual forward rates. The two are related as follows:

$$\text{actual forward rate} = \text{forward rate implied by futures} - (\tfrac{1}{2} \times \sigma^2 \times T_1 \times T_2)$$

where:

T_1 = the maturity on the futures contract
T_2 = the time to the maturity of the rate underlying the contract (90 days)
σ = the annual standard deviation of the change in the rate underlying the futures contract, or 90-day LIBOR

Notice that as T_1 increases, the convexity adjustment will need to increase. So as the maturity of the futures contract increases, the necessary convexity adjustment increases. Also, note that the σ and the T_2 are largely dictated by the specifications of the futures contract.

LO 39.10: Explain how Eurodollar futures can be used to extend the LIBOR zero curve.

Forward rates implied by convexity-adjusted eurodollar futures can be used to produce a LIBOR spot curve (also called a LIBOR zero curve since spot rates are sometimes referred to as zero rates). Recall the equation presented previously in Topic 37, which was used to generate the shape of the *futures* rate curve:

$$R_{Forward} = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

where:
R_i = spot rate corresponding with T_i periods
$R_{Forward}$ = the forward rate between T_1 and T_2

This forward rate equation can be rearranged to solve for the *spot* rate for the next time period (T_2):

$$R_2 = \frac{R_{Forward}(T_2 - T_1) + R_1 T_1}{T_2}$$

Given the first LIBOR spot rate (R_1) and the length of each forward contract period, we can calculate the next spot rate (R_2). The rate at T_2 can then be used to find the rate at T_3 and so on. The end result is a generated LIBOR spot (zero) curve.

Duration-Based Hedging

LO 39.11: Calculate the duration-based hedge ratio and create a duration-based hedging strategy using interest rate futures.

The objective of a **duration-based hedge** is to create a combined position that does not change in value when yields change by a small amount. In other words, a position that has a duration of zero needs to be produced. The combined position consists of our portfolio with a hedge horizon value of P and a futures position with a contract value of F. Denote the duration of the portfolio at the hedging horizon as D_P and the corresponding duration of the futures contract as D_F. Using this notation, the duration-based hedge ratio can be expressed as follows:

$$N = -\frac{P \times D_P}{F \times D_F}$$

where:
N = number of contracts to hedge

The minus sign suggests that the futures position is the opposite of the original position. In other words, if the investor is long the portfolio, he must short N contracts to produce a position with a zero duration.

Example: Duration-based hedge

Assume there is a 6-month hedging horizon and a portfolio value of $100 million. Further assume that the 6-month T-bond contract is quoted at 105–09, with a contract size of $100,000. The duration of the portfolio is 10, and the duration of the futures contract is 12. **Outline** the appropriate hedge for small changes in yield.

Answer:

$$N = -\frac{100{,}000{,}000 \times 10}{105{,}281.25 \times 12} = -791.53$$

Rounding up to the nearest whole number means the manager should short 792 contracts.

Limitations of Duration

LO 39.12: Explain the limitations of using a duration-based hedging strategy.

The price/yield relationship of a bond is convex, meaning it is nonlinear in shape. Duration measures are linear approximations of this relationship. Therefore, as the change in yield increases, the duration measures become progressively less accurate. Moreover, duration implies that all yields are perfectly correlated. Both of these assumptions place limitations on the use of duration as a single risk measurement tool. When changes in interest rates are both large and nonparallel (i.e., not perfectly correlated), duration-based hedge strategies will perform poorly.

Key Concepts

LO 39.1

Day count conventions play a role when computing the interest that accrues on a fixed income security. When a bond is purchased, the buyer must pay any accrued interest earned through the settlement date. The most common day count conventions are Actual/Actual, 30/360, and Actual/360.

LO 39.2

T-bills are quoted on a discount rate basis. A T-bill with a $100 face value with *n* days to maturity and a cash price of *Y* is quoted as:

$$\text{T-bill discount rate} = \frac{360}{n}(100 - Y)$$

LO 39.3

For a U.S. Treasury bond, the dirty price is the price that the seller of the bond must be paid to give up ownership. It includes the present value of the bond plus the accrued interest. Conversely, the clean price is the dirty price less accrued interest.

LO 39.4

Since deliverable bonds have very different market values, the Chicago Board of Trade (CBOT) has created conversion factors. Conversion factors are supplied by the CBOT on a daily basis. They are calculated as:

(bond discounted price – accrued interest) / face value

LO 39.5

The conversion factor system is not perfect and often results in one bond that is the cheapest (or most profitable) to deliver. The cheapest-to-deliver (CTD) bond is the bond that minimizes the following:

quoted bond price – (quoted futures price × conversion factor)

LO 39.6

When the yield curve is not flat, there is a single bond that is the cheapest-to-deliver (CTD). When the yield curve is upward sloping, CTD bonds tend to have longer maturities. When the yield curve is downward sloping, CTD bonds tend to have shorter maturities.

LO 39.7

The theoretical price for a T-bond futures contract is calculated as:

(cash futures price – accrued interest) / conversion factor

LO 39.8

Eurodollar contracts are based on LIBOR and are quoted on a discount rate basis. If *Z* is the quoted price for a eurodollar futures contract, the contract price is:

eurodollar futures price = \$10,000 × [100 – (0.25) × (100 – Z)]

LO 39.9

Long-dated eurodollar contracts must be adjusted for convexity before being used to estimate the corresponding forward rates. As the maturity of the futures contract increases, the necessary convexity adjustment increases.

LO 39.10

Forward rates implied by convexity-adjusted eurodollar futures can be used to produce a LIBOR spot curve. The following equation is used to generate the shape of the futures rate curve:

$$R_{Forward} = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

where :
R_i = spot rate corresponding with T_i periods
$R_{Forward}$ = the forward rate between T_1 and T_2

LO 39.11

Duration can be used to compute the number of futures contracts needed to implement a duration-based hedging strategy. The duration-based hedge ratio can be expressed as follows:

$$\text{number of contracts} = -\frac{\text{portfolio value} \times \text{duration}_{\text{portfolio}}}{\text{futures value} \times \text{duration}_{\text{futures}}}$$

LO 39.12

The effectiveness of duration-based hedging strategies is limited when there are large changes in yield or nonparallel shifts in the yield curve.

Concept Checkers

1. Assume a 6-month hedging horizon and a portfolio value of $30 million. Further assume that the 6-month Treasury bond (T-bond) contract is quoted at 100–13, with a contract size of $100,000. The duration of the portfolio is 8, and the duration of the futures contract is 12. Which of the following is closest to the appropriate hedge for small changes in yield?
   A. Long 298 contracts.
   B. Short 298 contracts.
   C. Long 199 contracts.
   D. Short 199 contracts.

2. Which of the following items limits the use of duration as a risk metric?
   I. It assumes the price/yield relationship is linear.
   II. It assumes interest rate volatility is constant.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

3. Consider day count convention and, specifically, the following example: A semiannual bond with $100 face value has a 4% coupon. Today is August 3. Assume coupon dates of March 1 and September 1. Which of the following statements is true?
   A. Corporate bonds accrue more interest in July than T-bonds.
   B. Corporate bonds accrue more interest from March 1 to September 1 than September 1 to March 1.
   C. Corporate bonds accrue more interest than T-bonds for this period (March 1 to August 3).
   D. The T-bond accrued interest is $1.76 for this period (March 1 to August 3).

4. Assume an investor is about to deliver a short bond position and has four options to choose from which are listed in the following table. The settlement price is $92.50 (i.e., the quoted futures price). Determine which bond is the cheapest-to-deliver.

Bond	*Quoted Bond Price*	*Conversion Factor*
1	98	1.02
2	122	1.27
3	105	1.08
4	112	1.15

   A. Bond 1.
   B. Bond 2.
   C. Bond 3.
   D. Bond 4.

5. Assume the cash price on a 90-T-bill is quoted as 98.75. The discount rate is closest to:
   A. 4%.
   B. 7%.
   C. 6%.
   D. 5%.

Concept Checker Answers

1. D $N = -\frac{(\$30{,}000{,}000 \times 8)}{(\$100{,}406.25 \times 12)} = -199$

   The appropriate hedge is to short 199 contracts.

2. A The limitations of duration include: (1) that it is valid for only *small changes in yield*, (2) that it assumes the price/yield relationship is linear, and (3) it assumes that changes in yield are the same across all maturities and risk levels (i.e., they're perfectly correlated).

3. C July accrued T-bond interest is 31/184 = 0.1685; July accrued corporate bond interest is 30/180 = 0.1667. T-bonds accrue 155/184 = 0.8424 × \$2 = \$1.6848; C-bonds accrue 152/180 = 0.8444 × \$2 = \$1.6889.

4. A Cost of delivery:

   Bond 1: 98 – (92.50 × 1.02) = \$3.65

   Bond 2: 122 – (92.50 × 1.27) = \$4.53

   Bond 3: 105 – (92.50 × 1.08) = \$5.10

   Bond 4: 112 – (92.50 × 1.15) = \$5.63

   Bond 1 is the cheapest-to-deliver with a cost of delivery of \$3.65.

5. D The discount rate on a U.S. T-bill is calculated using the following equation:

$$\text{discount rate} = \frac{360}{n} \times (100 - \text{cash price})$$

$$\text{discount rate} = \frac{360}{90} \times (100 - 98.75) = 5\%$$

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Swaps

Topic 40

Exam Focus

An interest rate swap is an agreement between two parties to exchange interest payments based on a specified principal over a period of time. In a plain vanilla interest rate swap, one of the interest rates is floating, and the other is fixed. Swaps can be used to efficiently alter the interest rate risk of existing assets and liabilities. A currency swap exchanges interest rate payments in two different currencies. For valuation purposes, swaps can be thought of as a long and short position in two different bonds or as a package of forward rate agreements. Credit risk in swaps cannot be ignored.

Mechanics of Interest Rate Swaps

LO 40.1: Explain the mechanics of a plain vanilla interest rate swap and compute its cash flows.

The most common interest rate swap is the **plain vanilla interest rate swap.** In this swap arrangement, Company X agrees to pay Company Y a periodic fixed rate on a notional principal over the tenor of the swap. In return, Company Y agrees to pay Company X a periodic floating rate on the same notional principal. Both payments are in the same currency. Therefore, only the net payment is exchanged. Most interest rate swaps use the London Interbank Offered Rate (LIBOR) as the reference rate for the floating leg of the swap. Finally, since the payments are based in the same currency, there is no need for the exchange of principal at the inception of the swap. This is why it is called notional principal.

For example, companies X and Y enter into a 2-year plain vanilla interest rate swap. The swap cash flows are exchanged semiannually, and the reference rate is 6-month LIBOR. The LIBOR rates are shown in Figure 1. The fixed rate of the swap is 3.784%, and the notional principal is $100 million. We will compute the cash flows for Company X, the fixed payer of this swap.

Figure 1: 6-Month LIBOR

Beginning of Period	*LIBOR*
1	3.00%
2	3.50%
3	4.00%
4	4.50%
5	5.00%

The first cash flow takes place at the end of period one and uses the LIBOR at the beginning of that same period. In other words, at the beginning of each period, both payments for the end of the period are known. The gross cash flows for the end of the first period for both parties are calculated in the following manner:

floating = \$100 million × 0.03 × 0.5 = \$1.5 million

fixed = \$100 million × 0.03784 × 0.5 = \$1.892 million

Note that 0.5 is the semiannual day count. The net payment for Company X is an outflow of \$0.392 million. Note that we are ignoring the many day-count and business-day conventions associated with swaps. Figure 2 shows the other cash flows.

Figure 2: Swap Cash Flows

End of Period	*LIBOR at Beginning of Period*	*Floating Cash Flow*	*Fixed Cash Flow*	*Net X Cash Flow*
1	3.00%	\$1,500,000	\$1,892,000	–\$392,000
2	3.50%	\$1,750,000	\$1,892,000	–\$142,000
3	4.00%	\$2,000,000	\$1,892,000	\$108,000
4	4.50%	\$2,250,000	\$1,892,000	\$358,000

LO 40.2: Explain how a plain vanilla interest rate swap can be used to transform an asset or a liability and calculate the resulting cash flows.

Let's continue with companies X and Y. Suppose that X has a 2-year floating-rate liability, and Y has a 2-year fixed-rate liability. After they enter into the swap, interest rate risk exposure from their liabilities has completely changed for each party. X has transformed the floating-rate liability into a fixed-rate liability, and Y has transformed the fixed-rate liability to a floating-rate liability. Note that X pays fixed and receives floating, so X's liability becomes fixed.

Similarly, assume that X has a fixed-rate asset and Y has a floating-rate asset tied to LIBOR. After entering into the swap, X has transformed the fixed-rate asset into a floating-rate asset, and Y has transformed the floating-rate asset into a fixed-rate asset.

Financial Intermediaries

LO 40.3: Explain the role of financial intermediaries in the swaps market.

LO 40.4: Describe the role of the confirmation in a swap transaction.

In many respects, swaps are similar to forwards:

- Swaps typically require no payment by either party at initiation.
- Swaps are custom instruments.

- Swaps are not traded in any organized secondary market.
- Swaps are largely unregulated.
- Default risk is an important aspect of the contracts.
- Most participants in the swaps market are large institutions.
- Individuals are rarely swap market participants.

There are swap intermediaries who bring together parties with needs for the opposite side of a swap. Dealers, large banks, and brokerage firms, act as principals in trades just as they do in forward contracts. In many cases, a swap party will not be aware of the other party on the offsetting side of the swap since both parties will likely only transact with the intermediary. **Financial intermediaries**, such as banks, will typically earn a spread of about 3 to 4 basis points for bringing two nonfinancial companies together in a swap agreement. This fee is charged to compensate the intermediary for the risk involved. If one of the parties defaults on its swap payments, the intermediary is responsible for making the other party whole.

Confirmations, as drafted by the International Swaps and Derivatives Association (ISDA), outline the details of each swap agreement. A representative of each party signs the confirmation, ensuring that they agree with all swap details (such as tenor and fixed/floating rates) and the steps taken in the event of default.

Comparative Advantage

LO 40.5: Describe the comparative advantage argument for the existence of interest rate swaps and evaluate some of the criticisms of this argument.

Let's return to companies X and Y and assume that they have access to borrowing for two years as specified in Figure 3.

Figure 3: Borrowing Rates for X and Y

Company	*Fixed Borrowing*	*Floating Borrowing*
Y	5.0%	LIBOR + 10 bps
X	6.5%	LIBOR + 100 bps

Company Y has an **absolute advantage** in both markets but a comparative advantage in the fixed market. Notice that the differential between X and Y in the fixed market is 1.5%, or 150 basis points (bps), and the corresponding differential in the floating market is only 90 basis points. When this is the case, Y has a comparative advantage in the fixed market, and X has a comparative advantage in the floating market. When a **comparative advantage** exists, a swap arrangement will reduce the costs of both parties. In this example, the net potential borrowing savings by entering into a swap is the difference between the differences, or 60 bps. In other words, by entering into a swap, the total savings shared between X and Y is 60 bps.

To better understand where the 60 bps comes from, suppose Y borrows fixed at 5% for two years, X borrows floating for two years at LIBOR + 1%, and then X and Y enter into a swap to transform their liabilities. Specifically, X pays Y fixed and Y pays X floating based on LIBOR. If we assume the net savings is split evenly, the net borrowing costs for X are then 6.2% and LIBOR – 20 bps for Y. Each has saved 30 bps for a total of 60 bps. If an intermediary were used, part of the 60 bps would be used to pay the bid-ask spread.

Problems With Comparative Advantage

A problem with the comparative advantage argument is that it assumes X can borrow at LIBOR + 1% over the life of the swap. It also ignores the credit risk taken on by Y by entering into the swap. If X were to raise funds by borrowing directly in the capital markets, no credit risk is taken, so perhaps the savings is compensation for that risk. The same criticisms exist when an intermediary is involved.

Valuing Interest Rate Swaps

The Discount Rate

LO 40.6: Explain how the discount rates in a plain vanilla interest rate swap are computed.

Since a swap is nothing more than a sequence of cash flows, its value is determined by discounting each cash flow back to the valuation date. The question is, what is the appropriate *discount rate* to use? It turns out that the forward rates implied by either forward rate agreements (FRAs) or the convexity-adjusted Eurodollar futures are used to produce a LIBOR spot curve. The swap cash flows are then discounted using the corresponding spot rate from this curve. The following connection between forward rates and spot rates exists when continuous compounding is used:

$$R_{forward} = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$$

where:
R_i = spot rate corresponding with T_i years
$R_{forward}$ = forward rate between T_1 and T_2

We will utilize this equation later when we value an interest rate swap using a sequence of forward rate agreements.

Valuing an Interest Rate Swap With Bonds

LO 40.7: Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions.

Let's return to our two companies, X and Y, in our 2-year swap arrangement. From X's perspective, there are two series of cash flows—one fixed going out and one floating coming in. Essentially, X has a long position in a floating-rate note (since it is an inflow) and a short position in a fixed-rate note (since it is an outflow). From Y's perspective, it is exactly the opposite—Y has a short position in a floating-rate note (since it is an outflow) and a long position in a fixed-rate note (since it is an inflow).

If we denote the present value of the fixed-leg payments as B_{fix} and the present value of the floating-leg payments as B_{flt}, the value of the swap can be written for both X and Y as:

$$V_{swap}(X) = B_{flt} - B_{fix}$$

$$V_{swap}(Y) = B_{fix} - B_{flt}$$

Note that $V_{swap}(X) + V_{swap}(Y) = 0$. This is by design since the two positions are mirror images of one another. At inception of the swap, it is convention to select the fixed payment so that $V_{swap}(X) = V_{swap}(Y) = 0$. As expected floating rates in the future change, the swap value for each party is no longer zero.

Valuing an interest rate swap in terms of bond positions involves understanding that the value of a floating rate bond will be equal to the notional amount at any of its periodic settlement dates when the next payment is set to the market (floating) rate. Since $V_{swap} = \text{Bond}_{fixed} - \text{Bond}_{floating}$, we can value the fixed-rate bond using the spot rate curve and then discount the next (known) floating-rate payment plus the notional amount at the current discount rate. The following example illustrates this method.

Example: Valuing an interest rate swap

Consider a \$1 million notional swap that pays a floating rate based on 6-month LIBOR and receives a 6% fixed rate semiannually. The swap has a remaining life of 15 months with pay dates at 3, 9, and 15 months. Spot LIBOR rates are as follows: 3 months at 5.4%; 9 months at 5.6%; and 15 months at 5.8%. The LIBOR at the last payment date was 5.0%. **Calculate** the value of the swap to the fixed-rate receiver using the bond methodology.

Answer:

$$B_{fixed} = \left(PMT_{fixed,\,3\text{ months}} \times e^{-(r \times t)}\right) + \left(PMT_{fixed,\,9\text{ months}} \times e^{-(r \times t)}\right) + \left[\left(\text{notional} + PMT_{fixed,\,15\text{ months}}\right) \times e^{-(r \times t)}\right]$$

$$B_{fixed} = \left(\$30{,}000 \times e^{-(0.054 \times 0.25)}\right) + \left(\$30{,}000 \times e^{-(0.056 \times 0.75)}\right) + \left[\left(\$1{,}000{,}000 + \$30{,}000\right) \times e^{-(0.058 \times 1.25)}\right]$$
$$= \$29{,}598 + \$28{,}766 + \$957{,}968 = \$1{,}016{,}332$$

$$B_{floating} = \left[\text{notional} + \left(\text{notional} \times \frac{r_{floating}}{2}\right)\right] \times e^{-(r \times t)}$$
$$= \left[\$1{,}000{,}000 + \left(\$1{,}000{,}000 \times \frac{0.05}{2}\right)\right] \times e^{-(0.054 \times 0.25)} = \$1{,}011{,}255$$

$$V_{swap} = \left(B_{fixed} - B_{floating}\right) = \$1{,}016{,}332 - \$1{,}011{,}255 = \$5{,}077$$

Figure 4 sums up the payments and present value factors.

Figure 4: Valuing an Interest Rate Swap With Two Bond Positions

Time	*Fixed Cash Flow*	*Floating Cash Flow*	*Present Value Factor*	*PV Fixed CF*	*PV Floating CF*
0.25 (3 months)	30,000	1,025,000	0.9866*	29,598	1,011,255
0.75 (9 months)	30,000		0.9589*	28,766	
1.25 (15 months)	1,030,000		0.9301*	957,968	
Total				1,016,332	1,011,255

* Note that some rounding has occurred.

Again we see that the value of the swap = 1,016,332 – 1,011,255 = $5,077.

Valuing an Interest Rate Swap With FRAs

LO 40.8: Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs).

At settlement, the payment made on a forward rate agreement is the notional amount multiplied by the difference between a market (floating) rate such as LIBOR and the contract (fixed) rate specified in the FRA. This is identical to a periodic payment on an interest rate swap when the reference floating rates and notional principal amounts are the same and the swap fixed rate is equal to the contract rate specified in the FRA. Viewed this way, we can see that an interest rate swap is equivalent to a series of FRAs. One way to value a swap would be to use expected forward rates to forecast the expected net cash flows and then discount these expected cash flows at the corresponding spot rates, consistent with forward rate expectations.

Example: Valuing an interest rate swap with FRAs

Consider the previous example on valuing an interest rate swap with two bond positions. An investor has a $1 million notional swap that pays a floating rate based on 6-month LIBOR and receives a 6% fixed rate semiannually. The swap has a remaining life of 15 months with pay dates at 3, 9, and 15 months. Spot LIBOR rates are as follows: 3 months at 5.4%; 9 months at 5.6%; and 15 months at 5.8%. The LIBOR at the last payment date was 5.0%. **Calculate** the value of the swap to the fixed-rate receiver using the FRA methodology.

Answer:

To calculate the value of the swap, we'll need to find the floating rate cash flows by calculating the expected forward rates via the LIBOR based spot curve.

The first floating rate cash flow is calculated in a similar fashion to the previous example.

LIBOR rate (last payment date): 5%.

Floating rate cash flow in 3 months: 1,000,000 × 0.05 / 2 = $25,000.

The second floating rate cash flow is calculated by finding the forward rate that corresponds to the period between 3 months and 9 months. To calculate forward rate for the period between 3 and 9 months, use the previously mentioned forward rate formula:

$$R_{\text{forward}} = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$$

$$R_{\text{forward}} = 0.056 + (0.056 - 0.054)\frac{0.25}{0.75 - 0.25} = 0.057 = 5.7\%$$

This rate is a continuously compounded rate, so we need to find the equivalent forward rate with semiannual compounding:

$R_{\text{forward (SC)}} = 2 \times [e^{(0.057/2)} - 1] = 0.05782 = 5.782\%$

Floating rate cash flow in 9 months: 1,000,000 × 0.05782 / 2 = $28,910

The third floating rate cash flow is calculated by finding the forward rate that corresponds to the period between 9 months and 15 months.

$$R_{\text{forward}} = 0.058 + (0.058 - 0.056)\frac{0.75}{1.25 - 0.75} = 0.061 = 6.1\%$$

$R_{\text{forward (SC)}} = 2 \times [e^{(0.061/2)} - 1] = 0.06194 = 6.1939\%$

Floating rate cash flow in 15 months: 1,000,000 × 0.061939 / 2 = $30,969

Figure 5: Valuing an Interest Rate Swap Based on a Sequence of FRAs

Time	*Fixed Cash Flow*	*Floating Cash Flow*	*Present Value Factor*	*PV Fixed CF*	*PV Floating CF*
0.25 (3 months)	30,000	25,000	0.9866*	29,598	24,665
0.75 (9 months)	30,000	28,910	0.9589*	28,766	27,721
1.25 (15 months)	30,000	30,969	0.9301*	27,902	28,803
Total				86,266	81,189

* Note that some rounding has occurred.

The value of the swap based on a sequence of FRAs = 86,266 – 81,189= $5,077.

As you can see from the previous two examples, valuing a swap based on a sequence of forward rate agreements produces the same result as valuing a swap based on two simultaneous bond positions.

Currency Swaps

LO 40.9: Explain the mechanics of a currency swap and compute its cash flows.

LO 40.11: Calculate the value of a currency swap based on two simultaneous bond positions.

A **currency swap** exchanges both principal and interest rate payments with payments in different currencies. The exchange rate used in currency swaps is the spot exchange rate. The valuation and application of currency swaps is similar to the interest rate swap. However, since the principals in a currency swap are not the same currency, they are exchanged at the inception of the currency swap so that they have equal value using the spot exchange rate. Also, the periodic cash flows throughout the swap are not netted as they are in the interest rate swap.

Suppose we have two companies, A and B, that enter into a fixed-for-fixed currency swap with periodic payments annually. Company A pays 6% in Great Britain pounds (GBP) to Company B and receives 5% in U.S. dollars (USD) from Company B. Company A pays a principal amount to B of USD175 million, and B pays GBP100 million to A at the outset of the swap. Notice that A has effectively borrowed GBP from B and so it must pay interest on that loan. Similarly, B has borrowed USD from A. The cash flows in this swap are actually more easily computed than in an interest rate swap since both legs of the swap are fixed. Every period (12 months), A will pay GBP6 million to B, and B will pay USD8.75 million to A. At the end of the swap, the principal amounts are re-exchanged.

From Company A's perspective, there are two series of cash flows: one fixed GBP cash flow stream going out and one fixed USD cash flow stream coming in. Essentially, A has a long position in a USD-denominated note (since it's an inflow) and a short position in a GBP-denominated note (since it's an outflow).

If we denote the present value of the GBP-denominated payments as B_{GBP} and the present value of the USD payments as B_{USD}, the value of the swap in USD to Company A is:

$$V_{swap}(USD) = B_{USD} - (S_0 \times B_{GBP})$$

where:
S_0 = spot rate in USD per GBP

Example: Calculate the value of a currency swap

Suppose the yield curves in the United States and Great Britain are flat at 2% and 4%, respectively, and the current spot exchange rate is USD1.50 = GBP1. **Value** the currency swap just discussed assuming the swap will last for three more years.

Answer:

$$B_{USD} = 8.75e^{-0.02\times1} + 8.75e^{-0.02\times2} + 183.75e^{-0.02\times3} = \text{USD}190.03 \text{ million}$$

$$B_{GBP} = 6e^{-0.04\times1} + 6e^{-0.04\times2} + 106e^{-0.04\times3} = \text{GBP}105.32 \text{ million}$$

$$V_{swap} \text{ (to A in USD)} = 190.03 - (1.5 \times 105.32) = \text{USD}32.05 \text{ million}$$

LO 40.12: Calculate the value of a currency swap based on a sequence of FRAs.

The value of a currency swap can also be calculated based on a sequence of FRAs.

Example: Value of a currency swap with FRAs

Suppose the yield curves in the United States and Great Britain are flat at 2% and 4%, respectively, and the current spot exchange rate is USD1.50 = GBP1.

Compute the value of the currency swap discussed previously using a sequence of FRAs to Company A. Assume the swap will last for three more years.

The corresponding forward rates are as follows:

Figure 6: Forward Rates

Year 1	$1.47/£
Year 2	$1.44/£
Year 3	$1.41/£

Professor's Note: The year 1 forward rate is calculated as follows: $F_1 = 1.5e^{(0.02-0.04)\times1} = \$1.47/£$. Interest rate parity suggests that the dollar will appreciate relative to the pound, so the \$/£ forward rate will decline (i.e., it will take fewer USD to buy 1 GBP). We will discuss interest rate parity in the foreign exchange risk topic (Topic 49).

Answer:

Figure 7 denotes the cash flows and forward rates for this currency swap.

Figure 7: Valuing a Currency Swap Based on a Sequence of FRAs

Time	*USD Cash Flow*	*GBP Cash Flow*	*Forward Rate*	*\$ Value of £*	*Net Cash Flows*	*PV of Net CF*
1	8.75	6	1.47	8.82	–0.07	–0.069
2	8.75	6	1.44	8.64	0.11	0.106
3	8.75	6	1.41	8.46	0.29	0.273
3	175	100	1.41	141	34	32.02
Total						32.33*

* Note some rounding has occurred.

Ignoring the rounding differences, we see that the value of the currency swap to Company A is 32 million using both the two simultaneous bond positions and the forward rate agreements.

Using a Currency Swap to Transform Existing Positions

LO 40.10: Explain how a currency swap can be used to transform an asset or liability and calculate the resulting cash flows.

Currency swaps can be combined with existing positions to completely alter the risk of a liability or an asset. For example, suppose that Company A has a dollar-based liability. By entering into a currency swap, the liability has become a pound-based liability at the GBP fixed (or floating) rate.

Comparative Advantage

Comparative advantage is also used to explain the success of currency swaps. Typically, a domestic borrower will have an easier time borrowing in his own currency. This often results in comparative advantages that can be exploited by using a currency swap. The argument is directly analogous to that used for interest rate swaps. Suppose A and B have the 5-year borrowing rates in the United States and Germany (EUR) shown in Figure 8.

Figure 8: Borrowing Rates

Borrowing Rates for A and B		
Company	*USD Borrowing*	*EUR Borrowing*
A	5.0%	7.0%
B	6.0%	7.5%

Company A needs EUR, and Company B needs USD. Company A has an absolute advantage in both markets but a comparative advantage in the USD market. Notice that the differential between A and B in the USD market is 1%, or 100 basis points (bps), and the corresponding differential in the EUR market is only 50 basis points. When this is the case, A has a comparative advantage in the USD market, and B has a comparative advantage in the EUR market. The net potential borrowing savings by entering into a swap is the difference between the differences, or 50 bps. In other words, by entering into a currency swap, the savings for both A and B totals 50 bps.

Swap Credit Risk

LO 40.13: Describe the credit risk exposure in a swap position.

Because $V_{swap}(A) + V_{swap}(B) = 0$, whenever one side of a swap has a positive value, the other side must be negative. For example, if $V_{swap}(A) > 0$, $V_{swap}(B) < 0$. As $V_{swap}(A)$ increases in value, $V_{swap}(B)$ must become more negative. This results in increased credit risk to A since the likelihood of default increases as B has larger and larger payments to make to A. However, the potential losses in swaps are generally much smaller than the potential losses from defaults on debt with the same principal. This is because the value of swaps is generally much smaller than the value of the debt.

Other Types of Swaps

LO 40.14: Identify and describe other types of swaps, including commodity, volatility and exotic swaps.

In an **equity swap**, the return on a stock, a portfolio, or a stock index is paid each period by one party in return for a fixed-rate or floating-rate payment. The return can be the capital appreciation or the total return including dividends on the stock, portfolio, or index.

In order to reduce equity risk, a portfolio manager might enter into a 1-year quarterly pay S&P 500 index swap and agree to receive a fixed rate. The percentage increase in the index each quarter is netted against the fixed rate to determine the payment to be made. If the index return is negative, the fixed-rate payer must also pay the percentage decline in the index to the portfolio manager. Uniquely among swaps, equity swap payments can be floating on both sides and the payments are not known until the end of the quarter. With interest rate swaps, both the fixed and floating payments are known at the beginning of the period for which they will be paid.

A swap on a single stock can be motivated by a desire to protect the value of a position over the period of the swap. To protect a large capital gain in a single stock, and to avoid a sale for tax or control reasons, an investor could enter into an equity swap as the equity-returns payer and receive a fixed rate in return. Any decline in the stock price would be paid to the investor at the settlement dates, plus the fixed-rate payment. If the stock appreciates, the investor must pay the appreciation less the fixed payment.

A **swaption** is an option which gives the holder the right to enter into an interest rate swap. Swaptions can be American- or European-style options. Like any option, a swaption is purchased for a premium that depends on the strike rate (the fixed rate) specified in the swaption.

Firms may enter into **commodity swap** agreements where they agree to pay a fixed rate for the multi-period delivery of a commodity and receive a corresponding floating rate based on the average commodity spot rates at the time of delivery. Although many commodity swaps exist, the most common use is to manage the costs of purchasing energy resources such as oil and electricity.

A **volatility swap** involves the exchanging of volatility based on a notional principal. One side of the swap pays based on a pre-specified volatility while the other side pays based on historical volatility.

As you can see, many different types of swaps exist. Some additional examples include: accrual swaps, cancelable swaps, index amortizing rate swaps, and constant maturity swaps. Swaps are also sometimes created for exotic structures. An example of an **exotic swap** was between Procter and Gamble and Banker's Trust where P&G's payments were based on the commercial paper rate.

Key Concepts

LO 40.1

A plain vanilla interest rate swap exchanges floating-rate payments (LIBOR) for fixed-rate payments over the life of the swap. The floating rate payments at time t in a plain vanilla interest rate swap are computed using the floating rate at time t – 1.

LO 40.2

Interest rate swaps can be combined with existing asset and liability positions to drastically change the interest rate risk.

LO 40.3

A swap dealer or financial intermediary facilitates the ability to enter into swaps.

LO 40.4

Confirmations outline the details of each swap agreement. A representative of each party signs the confirmation, ensuring that they agree with all swap details and the steps taken in the event of default.

LO 40.5

The comparative advantage argument suggests that when one of two borrowers has a comparative advantage in either the fixed- or floating-rate market, both borrowers will be better off by entering into a swap to exploit the advantage. The comparative advantage argument is flawed in that it assumes rates can be borrowed for the life of the swap. It also ignores the credit risk associated with the swap that does not exist if funds were raised directly in the capital markets.

LO 40.6

Since a swap is nothing more than a sequence of cash flows, its value is determined by discounting each cash flow back to the valuation date. The cash flows are discounted using the corresponding spot rate from the LIBOR spot curve.

LO 40.7

The value of a swap to the fixed-rate receiver at a point in time is the difference between the present value of the remaining fixed-rate payments and the present value of the remaining floating-rate payments.

LO 40.8

Valuing a swap based on a sequence of forward rate agreements (FRAs) produces the same result as valuing a swap based on two simultaneous bond positions.

LO 40.9

A currency swap exchanges interest rate payments in two different currencies. The exchange rate used in currency swaps is the spot exchange rate.

LO 40.10

Currency swaps can be combined with existing positions to completely alter the risk of a liability or an asset.

LO 40.11

Since the principals in a currency swap are not the same currency, they are exchanged at the inception of the currency swap so that they have equal value using the spot exchange rate. Also, the periodic cash flows throughout the swap are not netted as they are in an interest rate swap.

LO 40.12

In addition to valuing a currency swap based on two simultaneous bond positions, the value of a currency swap can also be calculated based on a sequence of FRAs.

LO 40.13

Credit risk is an important factor in existing swap positions, although potential losses are usually smaller than that with debt agreements.

LO 40.14

Many different types of swaps exist. Examples of swaps, in addition to interest rate swaps and currency swaps, include: equity swaps, commodity swaps, and volatility swaps.

Concept Checkers

Use the following data to answer Question 1.

Two companies, C and D, have the borrowing rates shown in the following table.

Borrowing Rates for C and D

Company	*Fixed Borrowing*	*Floating Borrowing*
C	10%	LIBOR + 50 bps
D	12%	LIBOR + 100 bps

1. According to the comparative advantage argument, what is the total potential savings for C and D if they enter into an interest rate swap?
   A. 0.5%.
   B. 1.0%.
   C. 1.5%.
   D. 2.0%.

2. Which of the following is most accurate regarding the credit risk of a currency swap? As the value of the:
   I. domestic currency leg increases, so does the credit risk of the domestic currency payer.
   II. foreign currency leg increases, so does the credit risk of the foreign currency payer.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

3. Which of the following would properly transform a floating-rate liability to a fixed-rate liability? Enter into a pay:
   A. foreign currency swap.
   B. fixed interest rate swap.
   C. domestic currency swap.
   D. floating interest rate swap.

4. Use the following information to determine the value of the swap to the floating rate payer using the bond methodology. Assume we are at the floating rate reset date.
   - $1 million notional value, semiannual, 18-month maturity.
   - Spot LIBOR rates: 6 months, 2.6%; 12 months, 2.65%; 18 months, 2.75%.
   - The fixed rate is 2.8%, with semiannual payments.

   A. –$66.
   B. $476.
   C. $3,425.
   D. $5,077.

5. Suppose Company X pays 5% annually (in euros) to Company Y and receives 4% annually (in dollars). Company X pays a principal amount of $150 million to Y, and Y pays a €100 million to X at the inception of the swap. Assume the yield curve is flat in the United States and in Germany (Europe). The U.S. rate is 3%, and the German rate is 5%. The current spot exchange rate is $1.45/€. What is the value of the currency swap to Company X using the bond methodology if it is expected to last for two more years?
   A. $3.53 million.
   B. $52.98 million.
   C. $8.09 million.
   D. $12.74 million.

Concept Checker Answers

1. C The difference of the differences is (12% – 10%) – [LIBOR + 1% – (LIBOR + 0.5%)] = 1.5%.

2. D As one currency (A) appreciates relative to another currency (B), the value of a currency swap increases on behalf of the currency A payer. As a result, the credit risk of the currency B payer increases.

3. B The fixed interest rate swap will allow for the conversion of a floating-rate liability to a fixed-rate liability.

4. B $B_{fix} = [\$14{,}000 \times e^{-(0.026\times0.5)}] + [\$14{,}000 \times e^{-(0.0265\times1.0)}] + [(\$1{,}000{,}000 + \$14{,}000) \times e^{-(0.0275\times1.5)}] = \$13{,}819 + \$13{,}634 + \$973{,}023 = \$1{,}000{,}476$

   Note that we are at a (semiannual) reset date, so the floating rate portion has a value equal to the notional amount.

   $V_{swap} = (B_{fix} - B_{floating}) = \$1{,}000{,}476 - \$1{,}000{,}000 = \$476$

5. C $B_{\$} = 6e^{-0.03\times1} + 156e^{-0.03\times2} = \$5.82 + \$146.92 = \$152.74$

   $B_{€} = 5e^{-0.05\times1} + 105e^{-0.05\times2} = €4.76 + €95.00 = €99.76$

   V_{swap} (to X) = 152.74 – (1.45 × 99.76) = \$8.09 million