Key Concepts

LO 52.1

Three common deviations from normality that are problematic in modeling risk result from asset returns that are fat-tailed, skewed, or unstable. Fat-tailed refers to a distribution with a higher probability of observations occurring in the tails relative to the normal distribution. A distribution is skewed when the distribution is not symmetrical and there is a higher probability of outliers. Parameters of the model that vary over time are said to be unstable.

LO 52.2

The phenomenon of "fat tails" is most likely the result of the volatility and/or the mean of the distribution changing over time.

LO 52.3

If the mean and standard deviation are the same for asset returns for any given day, the distribution of returns is referred to as an unconditional distribution of asset returns. However, different market or economic conditions may cause the mean and variance of the return distribution to change over time. In such cases, the return distribution is referred to as a conditional distribution.

LO 52.4

A regime-switching volatility model assumes different market regimes exist with high or low volatility. The probability of large deviations from normality (such as fat tails) occurring are much less likely under the regime-switching model because it captures the conditional normality.

LO 52.5

Historical-based approaches of measuring VaR typically fall into three sub-categories: parametric, nonparametric, and hybrid.

- The parametric approach typically assumes asset returns are normally or lognormally distributed with time-varying volatility (i.e., historical standard deviation or exponential smoothing).
- The nonparametric approach is less restrictive in that there are no underlying assumptions of the asset returns distribution (i.e., historical simulation).
- The hybrid approach combines techniques of both parametric and nonparametric methods to estimate volatility using historical data.

LO 52.6

A major difference between the historical standard deviation approach and the two exponential smoothing approaches is with respect to the weights placed on historical returns. Exponential smoothing approaches give more weight to recent returns, and the historical standard deviation approach weights all returns equally.

LO 52.7

The RiskMetrics® and GARCH approaches are both exponential smoothing weighting methods. RiskMetrics® is actually a special case of the GARCH approach. Exponential smoothing methods are similar to the historical standard deviation approach because they are parametric, attempt to estimate conditional volatility, use recent historical data, and apply a set of weights to past squared returns.

LO 52.8

When a portfolio is comprised of more than one position using the RiskMetrics® or historical standard deviation approaches, a single VaR measurement can be estimated by assuming asset returns are all normally distributed. The historical simulation approach for calculating VaR for multiple portfolios aggregates each period's historical returns weighted according to the relative size of each position. The weights are based on the market value of the portfolio positions today, regardless of the actual allocation of positions K days ago in the estimation window. A third approach to calculating VaR for portfolios with multiple positions estimates the volatility of the vector of aggregated returns and assumes normality based on the strong law of large numbers.

LO 52.9

The implied-volatility-based approach for measuring VaR uses derivative pricing models such as the Black-Scholes-Merton option pricing model to estimate an implied volatility based on current market data rather than historical data.

LO 52.10

With an AR(1) model, long-run mean is computed as: $[a / (1 - b)]$. If b equals 1, the long-run mean is infinite (i.e., the process is a random walk). If b is less than 1, then the process is mean reverting.

LO 52.11

Under the context of mean reversion, the single-period conditional variance of the rate of change is σ^2, and the two-period variance is $(1 + b^2)\sigma^2$. Without mean reversion (i.e., $b = 1$), the two-period volatility is equal to the square root of $2\sigma^2$. With mean reversion (i.e., $b < 1$), the two-period volatility will be less than the volatility from no mean reversion.

LO 52.12

If mean reversion exists, the long horizon risk (and resulting VaR calculation) will be smaller than square root volatility.

Concept Checkers

1. Fat-tailed asset return distributions are most likely the result of time-varying:
   A. volatility for the unconditional distribution.
   B. means for the unconditional distribution.
   C. volatility for the conditional distribution.
   D. means for the conditional distribution.

2. The problem of fat tails when measuring volatility is least likely:
   A. in a regime-switching model.
   B. in an unconditional distribution.
   C. in a historical standard deviation model.
   D. in an exponential smoothing model.

3. Which of the following is not a disadvantage of nonparametric methods compared to parametric methods?
   A. Data is used more efficiently with parametric methods than nonparametric methods.
   B. Identifying market regimes and conditional volatility increases the amount of usable data as well as the estimation error for historical simulations.
   C. MDE may lead to data snooping or over-fitting in identifying required assumptions regarding an appropriate kernal function.
   D. MDE requires a large amount of data that is directly related to the number of conditioning variables used in the model.

4. The lowest six returns for a portfolio are shown in the following table.

Six lowest returns with hybrid weightings			
	Six Lowest Returns	*Hybrid Weight*	*Hybrid Cumulative Weight*
1	–4.10%	0.0125	0.0125
2	–3.80%	0.0118	0.0243
3	–3.50%	0.0077	0.0320
4	–3.20%	0.0098	0.0418
5	–3.10%	0.0062	0.0481
6	–2.90%	0.0027	0.0508

   What will the 5% VaR be under the hybrid approach? Assume each observation is a random event with 50% to the left and 50% to the right of each observation.
   A. –3.10%.
   B. –3.04%.
   C. –2.96%.
   D. –2.90%.

5. Which of the following statements is an advantage of the implied volatility method in estimating future volatility? The implied volatility:
   A. model reacts immediately to changing market conditions.
   B. model is not model dependent.
   C. is constant through time.
   D. is biased upward and is therefore more conservative.

Concept Checker Answers

1. **A** The most likely explanation for "fat tails" is that the second moment or volatility is time-varying for the unconditional distribution. For example, this explanation is much more likely given observed changes in volatility in interest rates prior to a much anticipated Federal Reserve announcement. Examining a data sample at different points of time from the full sample could generate fat tails in the unconditional distribution, even if the conditional distributions are normally distributed.

2. **A** The regime-switching model captures the conditional normality and may resolve the fat-tailed problem and other deviations from normality. A regime-switching model allows for conditional means and volatility. Thus, the conditional distribution can be normally distributed even if the unconditional distribution is not.

3. **B** The use of market regimes and identifying conditional means and volatility actually reduces—not increases—the amount of data from the full sample. The full sample of data is split into subgroups used to estimate conditional volatility. Therefore, the amount of data available for estimating future volatility is significantly reduced.

4. **C** The fifth and sixth lowest returns have cumulative weights of 4.81% and 5.08%, respectively. The point halfway between these two returns is interpolated as –3.00% with a cumulative weight of 4.945%, calculated as follows: (4.81% + 5.08%) / 2. Further interpolation is required to find the 5th percentile VaR level with a return somewhere between –3.00% and –2.90%. The 5% VaR using the hybrid approach is calculated as:

   $$3.00\% - (3.00\% - 2.90\%)[(0.05 - 0.04945) / (0.0508 - 0.04945)]$$
   $$= 3.00\% - 0.10\%(0.0005 / 0.00135) = 2.96\%$$

   Notice that the answer has to be between –2.90% and –3.00%, so –2.96% is the only possible answer.

5. **A** The only advantage listed is that the implied volatility model reacts immediately to changing market conditions. Forecast models based on historical data require time to adjust to market events. Disadvantages include the following: (1) implied volatility is model dependent; (2) a major assumption of the model is that asset returns follow a continuous time lognormal diffusion process and are assumed to be constant but that implied volatility varies through time; and (3) implied volatility is biased upward.

The following is a review of the Valuation and Risk Models principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Putting VaR to Work

Topic 53

Exam Focus

Derivatives and portfolios containing derivatives and other assets create challenges for risk managers in measuring value at risk (VaR). In this topic, risk measurement approaches are discussed for linear and non-linear derivatives. The advantages and disadvantages and underlying assumptions of the various approaches are presented. In addition, Taylor Series approximation is addressed, with examples of applying this theory to VaR approaches. Finally, structured Monte Carlo (SMC), stress testing, and worst case scenario (WSC) analysis are presented as useful methods in extending VaR techniques to more appropriately measure risk for complex derivatives and scenarios.

Linear vs. Non-Linear Derivatives

LO 53.1: Explain and give examples of linear and non-linear derivatives.

A derivative is described as *linear* when the relationship between an underlying factor and the derivative is linear in nature. For example, an equity index futures contract is a linear derivative, while an option on the same index is non-linear. The delta for a linear derivative must be constant for all levels of the underlying factor, but not necessarily equal to one.

For example, the rate on a foreign currency forward contract is defined as:

$$F_{t,T} = S_t(1+R_D)/(1+R_F)$$

Where $F_{t,T}$ is the forward rate at time t for the period $T\text{-}t$, S_t is the spot exchange rate, R_D is the domestic interest rate, and R_F is the foreign interest rate. The value at risk (VaR) of the forward is related to the spot rate, S_t, and the foreign and domestic interest rates. Assuming fixed interest rates for very short time intervals, we can approximate the forward rate, $F_{t,T}$, with the interest rate differential as a constant K that is not a function of time as follows:

$$F_{t,T} = S_t(1 + R_D)/(1 + R_F) \approx KS_t$$

Furthermore, the continuously compounded return on the foreign forward contract, $\Delta f_{t,t+1}$, is approximately equal to the return on the spot rate, $\Delta s_{t,t+1}$. This can be shown mathematically where the ln of the constant K is very close to zero and the approximate relationship is simplified as follows:

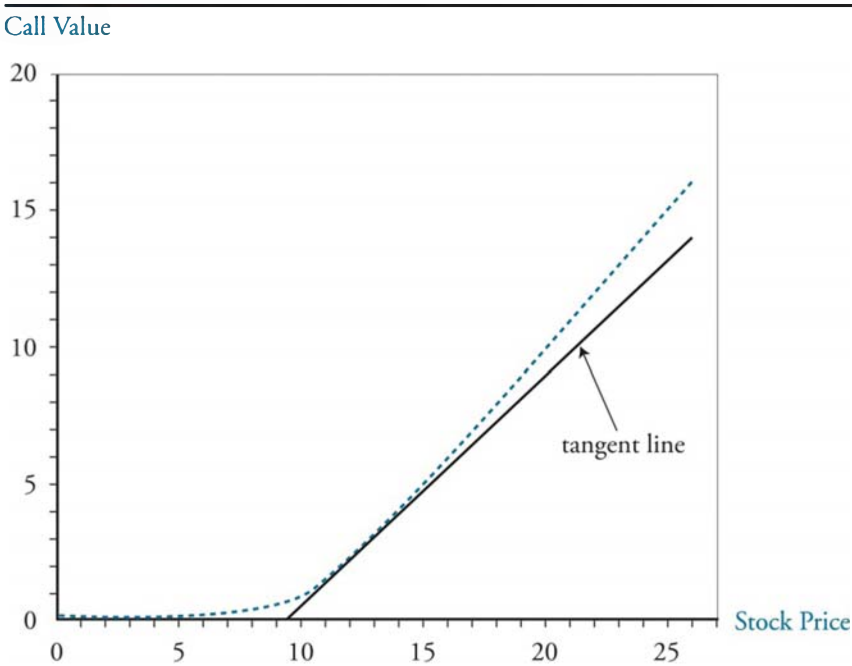
$$\Delta f_{t,t+1} = \ln(F_{t+1,T-1}/F_{t,T}) = \ln(S_{t+1}/S_t) + \ln(\Delta K) \approx \ln(S_{t+1}/S_t)$$

Changes in exchange rates can therefore be approximated by changes in spot rates. The VaR of a spot position is approximately equal to a forward position exchange rate if the only relevant underlying factor is the exchange rate. As this illustrates, many derivatives that are referred to as linear are actually only approximately linear. If we account for the changes in the two interest rates, the actual relationship would be nonlinear. Thus, the notion of linearity or nonlinearity is a function of the definition of the underlying risk factor.

The value of a *nonlinear* derivative is a function of the change in the value of the underlying asset and is dependent on the state of the underlying asset. A call option is a good example of a nonlinear derivative. The value of the call option does not increase (decrease) at a constant rate when the underlying asset increases (decreases) in value.

The change in the value of the call option is dependent in part on how far away the market value of the stock is from the exercise price. Thus, the relationship of the stock to the exercise price, *S/X*, captures the distance the option is from being in-the-money. Figure 1 illustrates how the value of the call option does not change at a constant rate with the change in the value of the underlying asset. The curved line represents the actual change in value of the call option based on the Black-Scholes-Merton model. The tangent line at any point on the curve illustrates how this is not a linear change in value. Furthermore, the slope of the line increases as the stock price increases. The percentage change in the call value given a change in the underlying stock price will be different for different stock price levels.

Figure 1: Call Option Value Given Underlying Stock Price



LO 53.2: Describe and calculate VaR for linear derivatives.

In general, the VaR of a long position in a linear derivative is $VaR_p = \Delta VaR_f$, where VaR_f is the VaR of the underlying factor and the derivative's delta, Δ, is the sensitivity of the derivative's price to changes in the underlying factor. Delta is assumed to be positive because we're modeling a long position. The local delta is defined as the percentage change in the derivative's price for a 1% change in the underlying asset. For small changes in the underlying price of the asset the change in price of the derivative can be extrapolated based on the local delta.

Example: Futures contract VaR

Determine how a risk manager could estimate the VaR of an equity index futures contract. Assume a 1-point increase in the index increases the value of a long position in the contract by $500.

Answer:

This relationship is shown mathematically as: $F_t = \$500S_t$, where F_t is the futures contract and S_t is the underlying index. The VaR of the futures contract is calculated as the amount of the index point movement in the underlying index, S_t, times the multiple, $500 as follows: $VaR(F_t) = \$500VaR(S_t)$.

Taylor Approximation

LO 53.3: Describe the delta-normal approach for calculating VaR for non-linear derivatives.

LO 53.4: Describe the limitations of the delta-normal method.

Suppose we create a table that shows the relationship of the call value to the stock price. The original stock price and call option value are $11.00 and $1.41, respectively. The Black-Scholes-Merton model is used to calculate the call value for different stock prices. Figure 2 summarizes some of the points.

Figure 2: Change in Call Value Given a Change in Stock Price (numbers reflect small rounding error)

Stock Price, *S*	$7.00	$8.00	$9.00	$10.00	$10.89	$11.00
Value of Call, *C*	$0.00	$0.05	$0.23	$0.69	$1.32	$1.41
Percentage Decrease in *S*	–36.36%	–27.27%	–18.18%	–9.09%	–1.00%	
Percentage Decrease in *C*	–100.00%	–96.76%	–83.31%	–51.06%	–6.35%	
Delta ($\Delta C\% / \Delta S\%$)	2.74	3.55	4.58	5.62	6.35	

The **delta** is calculated in Figure 2 by dividing the percentage change in the call value by the percentage change in the stock price (delta = $\Delta C\% / \Delta S\%$). The **local delta** is the slope of the line at any point of the nonlinear relationship for a 1% change in the stock price. The local delta can be used to estimate the change in the value of the call option given a *small* change in the value of the stock price.

Example: Call option VaR

Suppose a 6-month call option with a strike price, *X*, of $10 is currently trading for $1.41, when the market price of the underlying stock is $11. A 1% decrease in the stock price to $10.89 results in a 6.35% decrease in the call option with a value of $1.32. If the annual volatility of the stock is $\sigma = 0.1975$ and the risk-free rate of return is 5%, **calculate** the one day 5% VaR for this call option.

Answer:

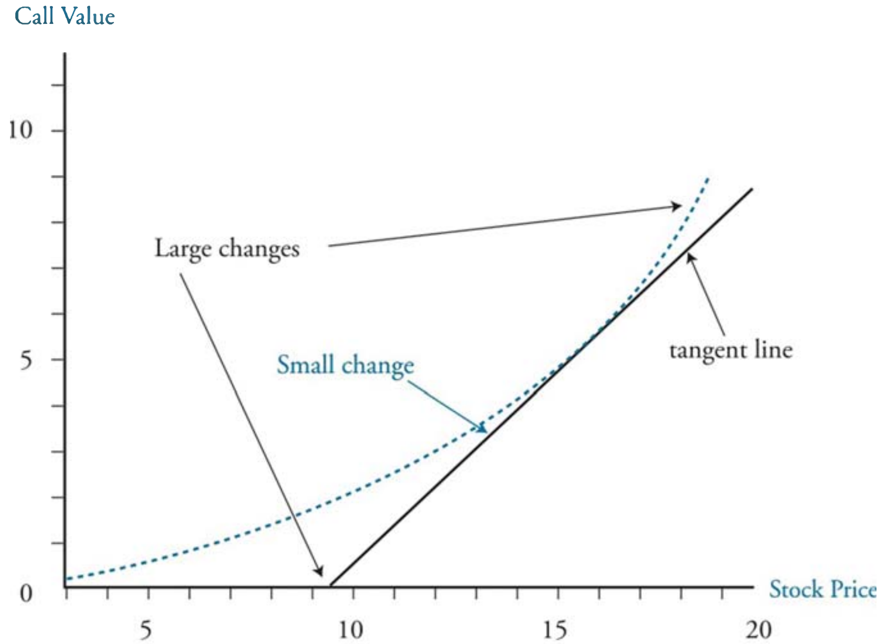
The daily volatility is approximately equal to 1.25% ($0.1975 / \sqrt{250}$). The 5% VaR for the stock price is equivalent to a one standard deviation move, or 1.65 for the normal curve. Assuming a random walk or 0 mean daily return, the 5% VaR of the underlying stock is 0 – 1.25%(1.65) = –2.06%. A 1% change in the stock price results in a 6.35% change in the call option value, therefore, the delta = 0.0635/0.01 = 6.35. For small moves, delta can be used to estimate the change in the derivative given the VaR for the underlying asset as follows: $VaR_{call} = \Delta VaR_{stock} = 6.35(2.06\%) = 0.1308$ or 13.1%. In words, the 5% VaR implies there is a 5% probability that the call option value will decline by 13.1% or more. Note this estimate is only an approximation for small changes in the underlying stock. The precise change can be calculated using the Black-Scholes-Merton model.

Figure 3 illustrates that the slope of the line is only useful in estimating the call value with small changes in the underlying stock value. The gap between the tangency line representing the delta or slope of the line at the tangency point widens the further away the estimate is from the point of tangency. The first derivative of a function tells us the slope of the line at any given point. The second derivative tells us the rate of change. This information is summarized mathematically in the **Taylor Series approximation** of the function f(x) as follows:

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2$$

The Taylor Series states that the change in value of any function can be expressed by adjusting the original function value, $f(x_0)$ plus the slope of the line, $f'(x_0)$, times the change in the *x* variable plus the rate of the change measured by the last term above. The last term captures the convexity or curvature. This is still an approximation, but it is much closer than the linear estimation.

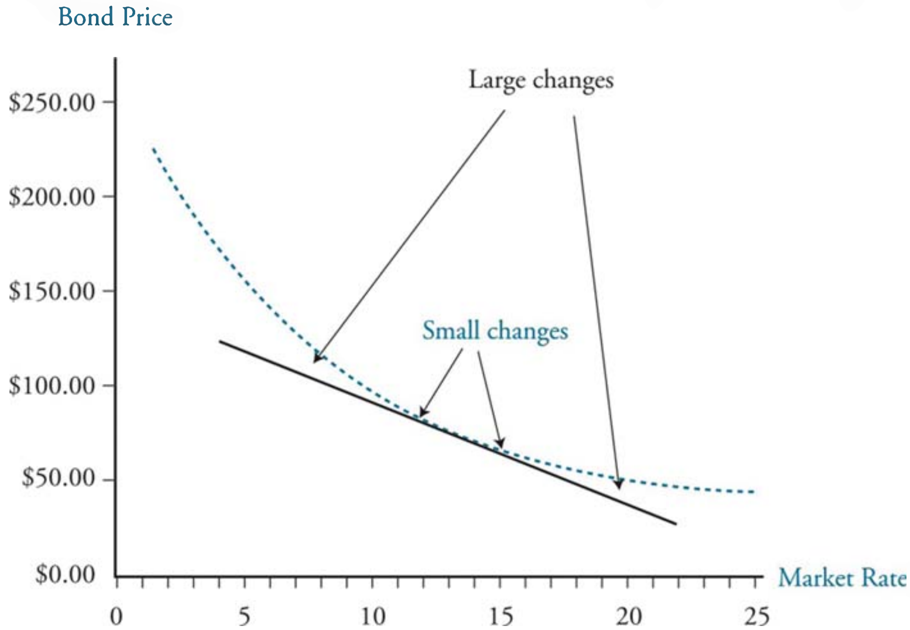
Figure 3: Call Option Example of Measurement Error Resulting from Convexity



Bond Example

As we will discuss in Topic 58, the **price-yield (P-Y) curve** depicts the change in the value of a bond as market rates of interest change. This is another example of a nonlinear relationship. Figure 4 illustrates the P-Y curve for a 20-year treasury bond with no embedded options. The straight line represents the duration of the bond. Duration is a linear estimation of the change in bond price given a change in interest rates and is only good for very small changes. Conversely, for large changes in interest rates, the gap between the P-Y curve and the tangent line represents the estimation error. Measuring the convexity in addition to the duration of the bond gives a much better approximation of the change in bond price for a given change in market rates. The use of duration and convexity to estimate bond prices as interest rates change is similar to the use of the delta and of the gamma approximation of the impact of fluctuations in the underlying factor on the value of an option. Both approximations are based on the Taylor Series that uses first and second derivatives of a known pricing model.

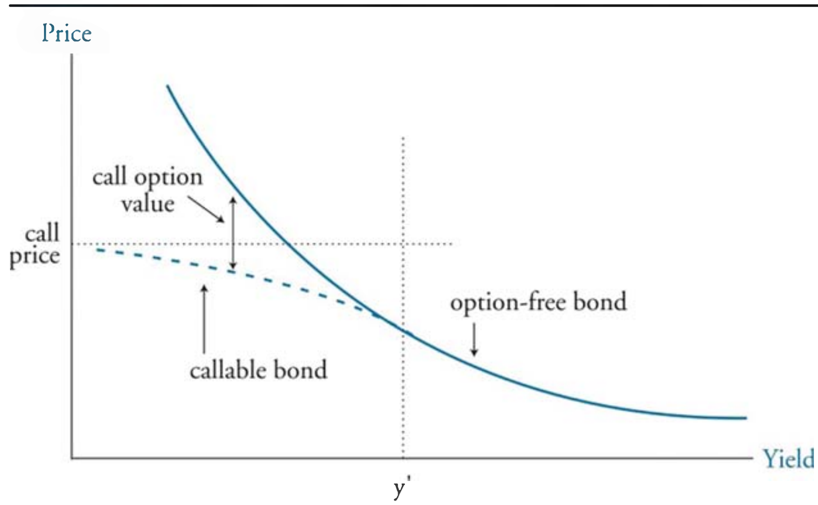
Figure 4: Measurement Error Resulting from Convexity in Bond Pricing



Consider a bond that is callable. The price-yield curve in Figure 5 illustrates that the call feature causes the P-Y curve to become concave as market interest rates approach the level where the bond will be called. Thus, the Taylor approximation is not useful because the callable bond is not a "well-behaved" function. In other words, the embedded call option causes the P-Y curve to deviate from the quadratic function that can be approximated by a polynomial of order two using the Taylor series.

Another example of a security with an embedded option are mortgage-backed securities (MBS). Borrowers will prepay loans early with significant drops in market interest rates. This causes the MBS to act similar to a bond that is called in. Unpredictable changes in duration due to early payoffs of MBS make the securities difficult to price and hedge. A convexity adjustment alone is not sufficient to estimate the change in the underlying security's value based on changes in market rates. The function explaining the relationship between the MBS value and market rates of interest does not behave similar at low and high levels of interest rates.

Figure 5: Price-Yield Curves for Callable and Noncallable Bonds



The Delta-Normal and Full Revaluation Methods

LO 53.5: Explain the full revaluation method for computing VaR.

LO 53.6: Compare delta-normal and full revaluation approaches for computing VaR.

Both the delta-normal and full revaluation methods measure the risk of nonlinear securities. The **full revaluation approach** calculates the VaR of the derivative by valuing the derivative based on the underlying value of the index after the decline corresponding to an *x*% VaR of the index. This approach is accurate, but can be highly computational. The revaluation of portfolios that include more complex derivatives (i.e., mortgage backed securities, or options with embedded features) are not easily calculated due to the large number of possible scenarios.

The **delta-normal approach** calculates the risk using the delta approximation ($VaR_p = \Delta VaR_f$), which is linear or the delta-gamma approximation, $f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2$, which adjusts for the curvature of the underlying relationship. This approach simplifies the calculation of more complex securities by approximating the changes based on linear relationships (delta).

The Monte Carlo Approach

LO 53.7: Explain structured Monte Carlo, stress testing, and scenario analysis methods for computing VaR, and identify strengths and weaknesses of each approach.

LO 53.8: Describe the implications of correlation breakdown for scenario analysis.

The **structured Monte Carlo (SMC) approach** simulates thousands of valuation outcomes for the underlying assets based on the assumption of normality. The VaR for the portfolio of derivatives is then calculated from the simulated outcomes. The general equation assumes the underlying asset has normally distributed returns with a mean of μ and a standard deviation of σ. An example of a simulation equation is as follows:

$$s_{t+1,i} = s_t e^{\mu + \sigma \times z}$$

where $s_{t+1,i}$ is the simulated value for a continuously compounded return, based on a random draw, z_i, from a normal distribution with the given first and second moments. Therefore, the draws from the normal distribution are denoted by $z_1, z_2, z_3, \ldots z_N$, and the N scenarios are $\mu + \sigma z_1, \mu + \sigma z_2, \mu + \sigma z_3, \ldots, \mu + \sigma z_N$. The N outcomes are then ordered and the $(1 - x/100) \times N$th value is the x% value.

An *advantage* of the SMC approach is that it is able to address multiple risk factors by assuming an underlying distribution and modeling the correlations among the risk factors. For example, a risk manager can simulate 10,000 outcomes and then determine the probability of a specific event occurring. In order to run the simulations, the risk manager just needs to provide parameters for the mean and standard deviation and assume all possible outcomes are normally distributed.

A *disadvantage* of the SMC approach is that in some cases it may not produce an accurate forecast of future volatility and increasing the number of simulations will not improve the forecast.

Example: SMC approach

Suppose a risk manager requires a VaR measurement of a long straddle position. **Demonstrate** how a SMC approach will be implemented to estimate the VaR for a long straddle position.

Answer:

The straddle represents a portfolio of a long call and long put that anticipates a large movement up or down in the underlying stock. The typical VaR measurement would require an estimate of the underlying stock moving more than one standard deviation. However, in a straddle position, the VaR occurs when the stock does not move in price or only moves a small amount. The SMC approach simulates thousands of possible movements in the underlying stock and then uses those outcomes to estimate the VaR for the straddle position.

Correlations During Crisis

The key point here is that in times of crisis, correlations increase (some substantially) and strategies that rely on low correlations fall apart in those times. Certain economic or crisis events can cause diversification benefits to deteriorate in times when the benefits are most needed. A contagion effect occurs with a rise in volatility and correlation causing a different return generating process. Some specific examples of events leading to the breakdown of historical correlation matrices are the Asian crisis, the U.S. stock market crash of October 1987, the events surrounding the failure of Long-Term Capital Management (LTCM), and the recent global credit crisis.

A simulation using the SMC approach is not capable of predicting scenarios during times of crisis if the covariance matrix was estimated during normal times. Unfortunately, increasing the number of simulations does not improve predictability in any way.

For example, the probability of a four or more standard deviation event occurring based on the normal curve is 6.4 out of 100,000 times. However, suppose the number of times the daily return for the equity index is four or more standard deviations based on historical returns is approximately 500 out of 100,000 times. Based on the historical data a four or more standard deviation event is expected to occur once every 0.8 years, not once every 62 years implied by the normal curve.

Stress Testing

During times of crisis, a contagion effect often occurs where volatility and correlations both increase, thus mitigating any diversification benefits. *Stressing* the correlation is a method used to model the contagion effect that could occur in a crisis event.

One approach for stress testing is to *examine historical crisis* events, such as the Asian crisis, October 1987 market crash, etc. After the crisis is identified, the impact on the current portfolio is determined. The *advantage* of this approach is that no assumptions of underlying asset returns or normality are needed. The biggest *disadvantage* of using historical events for stress testing is that it is limited to only evaluating events that have actually occurred.

The **historical simulation approach** does not limit the analysis to specific events. Under this approach, the entire data sample is used to identify "extreme stress" situations for different asset classes. For example, certain historical events may impact the stock market more than the bond market. The objective is to identify the five to ten worst weeks for specific asset classes and then evaluate the impact on today's portfolio. The *advantage* of this approach is that it may identify a crisis event that was previously overlooked for a specific asset class. The focus is on identifying extreme changes in valuation instead of extreme movements in underlying risk factors. The *disadvantage* of the historical simulation approach is that it is still limited to actual historical data.

An alternative approach is to analyze different predetermined *stress scenarios*. For example, a financial institution could evaluate a 200bp increase in short-term rates, an extreme inversion of the yield curve or an increase in volatility for the stock market. As in the previous method, the next step is then to evaluate the effect of the stress scenarios on the current portfolio.

An *advantage* to scenario analysis is that it is not limited to the evaluation of risks that have occurred historically. It can be used to address any possible scenarios. A *disadvantage* of the stress scenario approach is that the risk measure is deceptive for various reasons. For example, a shift in the domestic yield curve could cause estimation errors by overstating the risk for a long and short position and understating the risk for a long-only position. Asset-class-specific risk is another disadvantage of the stress scenario approach. For example, emerging market debt, mortgage-backed securities, and bonds with embedded options all have unique asset class specific features such that interest rate risk only explains a portion of total risk. Addressing asset class risks is even more crucial for financial institutions specializing in certain products or asset classes.

Worst Case Scenario Measure

LO 53.9: Describe worst-case scenario (WCS) analysis and compare WCS to VaR.

The **worst case scenario** (WCS) assumes that an unfavorable event will occur with certainty. The focus is on the distribution of worst possible outcomes given an unfavorable event. An expected loss is then determined from this worst case distribution analysis. Thus, the WCS information extends the VaR analysis by estimating the extent of the loss given an unfavorable event occurs.

In other words, the tail of the original return distribution is more thoroughly examined with another distribution that includes only probable extreme events. For example, within the lowest 5% of returns, another distribution can be formed with just those returns and a 1% WCS return can then be determined. Recall that VaR provides a value of the minimum loss for a given percentage, but says nothing about the severity of the losses in the tail. WCS analysis attempts to complement the VaR measure with analysis of returns in the tail.

Example: WCS approach

Suppose a risk manager simulates the data in Figure 6 using 10,000 random vectors for time horizons, *H*, of 20 and 100 periods. **Demonstrate** how a risk manager would interpret results for the 1% VaR and 1% WCS for a 100 period horizon.

Figure 6: Simulated Worst Case Scenario (WCS) Distribution

Time Horizon = H	*H = 20*	*H = 100*
Expected number of $Z_i < -2.33$	0.40	1.00
Expected number of $Z_i < -1.65$	1.00	4.00
Expected WCS	–1.92	–2.74
WCS 1 percentile	–3.34	–3.85
WCS 5 percentile	–2.69	–3.17

Answer:

Based on the simulation results in Figure 6, the 1% VaR assuming normality corresponds to –2.33 and over the next 100 trading periods a return worse than –2.33 is expected to occur one time. The 1% worst case scenario, denoted in this example by Z_i is –3.85. Thus, over the next 100 trading periods a return worse than –2.33 is expected to occur one time. More importantly, the size of that return is expected to be –2.74, with a 1% probability that the return will be –3.85 or lower.

Key Concepts

LO 53.1

A derivative is described as linear when the relationship between an underlying factor and the derivative's value is linear in nature (e.g., a forward currency contract). A nonlinear derivative's value is a function of the change in the value of the underlying asset and is dependent on the state of the underlying asset (e.g., a call option).

LO 53.2

In general, the VaR of a linear derivative is $VaR_p = \Delta VaR_f$, where the derivative's local delta, Δ, is the sensitivity of the derivative's price to a 1% change in the underlying asset's value.

LO 53.3

The last term of the following Taylor Series approximation adjusts for the curvature of the nonlinear derivative in addition to the slope or delta.

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2$$

LO 53.4

More complex derivatives such as mortgage backed securities or bonds with embedded options do not have "well-behaved" quadratic functions. The curvature of the function relating the nonlinear derivative's value to the underlying factor changes for different levels of the underlying factor. Thus, the Taylor Series approximation is not sufficient to capture the shift in curvature.

LO 53.5

The full revaluation approach calculates the VaR of the derivative by valuing the derivative based on the underlying value of the index after the decline corresponding to an *x*% VaR of the index. This approach is accurate, but can be highly computational.

LO 53.6

The delta-normal approach calculates the risk using the delta approximation ($VaR_p = \Delta VaR_f$) which is linear or the delta-gamma approximation, $f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2$, which adjusts for the curvature of the underlying relationship.

LO 53.7

The structured Monte Carlo (SMC) approach simulates thousands of possible movements in the underlying asset and then uses those outcomes to estimate the VaR for a portfolio of derivatives.

An advantage of the SMC approach is that it is able to address multiple risk factors by generating correlated scenarios based on a statistical distribution. A disadvantage of the SMC approach is that in some cases it may not produce an accurate forecast of future volatility and increasing the number of simulations will not improve the forecast.

LO 53.8

Crisis events cause diversification benefits to deteriorate due to a contagion effect that occurs when a rise in volatility and correlation result in a different return generating process for the underlying asset. This creates problems when using simulations for scenario analysis due to the fact that a simulation using the SMC approach cannot predict scenarios during times of crisis if the covariance matrix was estimated during normal times.

LO 53.9

The worst case scenario (WCS) extends VaR risk measurement by estimating the extent of the loss given an unfavorable event.

Concept Checkers

1. A call option and a mortgage backed security derivative are good examples of:
   A. a linear and nonlinear derivative, respectively.
   B. a nonlinear and linear derivative, respectively.
   C. linear derivatives.
   D. nonlinear derivatives.

2. Which of the following statements is(are) true?

   I. A linear derivative's delta must be constant for all levels of value for the underlying factor.
   II. A nonlinear derivative's delta must be constant for all levels of value for the underlying factor.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

3. Which of the following statements regarding the Taylor Series approximation is(are) true?

   I. The second derivative of the function for the relationship between the derivative and underlying asset estimates the rate of change in the slope.
   II. The Taylor Series approximation can be used to estimate the change in all nonlinear derivative values.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

4. Which of the following statements regarding the measurement of risk for non-linear derivatives is(are) true?

   I. A disadvantage of the delta-normal approach is that it is highly computational.
   II. The full revaluation approach is most appropriate for portfolios containing mortgage backed securities or options with embedded features.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

5. Which of the following statements is incorrect? A contagion effect:
   A. occurs with a rise in both volatility and correlation.
   B. causes a different return generating process in the underlying asset.
   C. results from a crisis event.
   D. increases diversification benefits.

Concept Checker Answers

1. **D** A *nonlinear* derivative's value is a function of the change in the value of the underlying asset and is dependent on the state of the underlying asset.

2. **A** The delta of a linear derivative must be constant. The delta, or slope, of a nonlinear derivative changes for different levels of the underlying factor.

3. **A** The Taylor Series of order two is represented mathematically as:

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2$$

The first derivative tells us the delta, or slope of the line. The second derivative tells us the rate of change. The last term including the second derivative captures the convexity or curvature. This approximation is only useful for "well-behaved" quadratic functions of order two.

4. **D** Both the delta-normal and full revaluation methods measure the risk of nonlinear securities. The *full revaluation approach* calculates the VaR of the derivative by valuing the derivative based on the underlying value of the index after the decline corresponding to an *x*% VaR of the index. This approach is accurate, but can be highly computational; therefore, it does not work well for portfolios of more complex derivatives such as mortgage-backed securities, swaptions, or options with embedded features. The *delta-normal approach* calculates the risk using the delta approximation, which is linear or the delta-gamma approximation, which adjusts for the curvature of the underlying relationship. This approach simplifies the calculation of more complex securities by approximating the changes.

5. **D** A *contagion effect* occurs with a rise in volatility and correlation causing a different return generating process. Some specific examples of events leading to the breakdown of historical correlation matrices causing a contagion effect are the Asian crisis and the U.S. stock market crash of October 1987. A contagion effect often occurs where volatility and correlations both increase, thus mitigating any diversification benefits.

The following is a review of the Valuation and Risk Models principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Measures of Financial Risk

Topic 54

Exam Focus

The assumption regarding the shape of the underlying return distribution is critical in determining an appropriate risk measure. The mean-variance framework can only be applied under the assumption of an elliptical distribution such as the normal distribution. The value at risk (VaR) measure can calculate risk measures when the return distribution is non-elliptical, but the measurement is unreliable and no estimate of the amount of loss is provided. Expected shortfall is a more robust risk measure that satisfies all the properties of a coherent risk measure with less restrictive assumptions. For the exam, focus your attention on the calculation of VaR, properties of coherent risk measures, and the expected shortfall methodology.

Mean-Variance Framework

LO 54.1: Describe the mean-variance framework and the efficient frontier.

The traditional mean-variance model estimates the amount of financial risk for portfolios in terms of the portfolio's expected return (i.e., mean) and risk (i.e., standard deviation or variance). Under the **mean-variance framework**, it is necessary to assume that return distributions for portfolios are elliptical distributions. The most commonly known elliptical probability distribution function is the normal distribution.

The **normal distribution** is a continuous distribution that illustrates all possible outcomes for random variables. Recall that the standard normal distribution has a mean of zero and a standard deviation of one. If returns are normally distributed, approximately 66.7% of returns will occur within plus or minus one standard deviation of the mean. Approximately 95% of the observations will occur within plus or minus two standard deviations of the mean. Thus, given this type of distribution, returns are more likely to occur closer to the mean return.

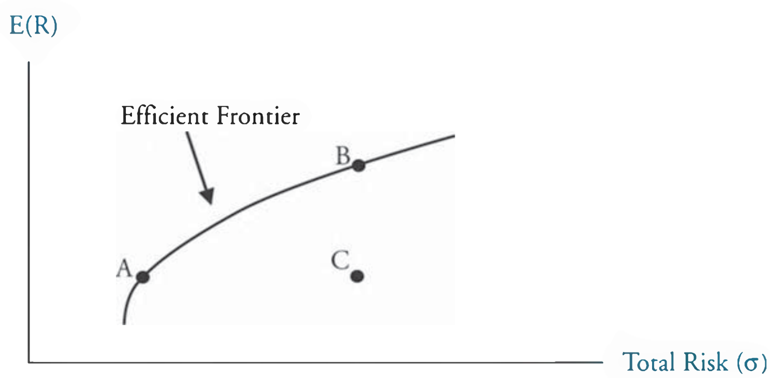
Portfolio managers are concerned with measuring downside risk and therefore are particularly interested in measuring the possibility of outcomes to the left or below the expected mean return. If the return distribution is symmetrical (like the normal distribution), then the standard deviation is an appropriate measure of risk when determining the probability that an undesirable outcome will occur.

If we assume that return distributions for all risky securities are normally distributed, then we can choose portfolios based on the expected returns and standard deviations of all possible combinations of risky securities. Figure 1 illustrates the concept of the **efficient frontier.**

In theory, all investors prefer securities or portfolios that lie on the efficient frontier. Consider portfolios A, B, and C in Figure 1. If you had to choose between portfolios A and C, which one would you prefer and why? Since portfolios A and C have the same expected return, a risk-averse investor would choose the portfolio with the least amount of risk (which would be Portfolio A). Now if you had to choose between portfolios B and C, which one would you choose and why? Because portfolios B and C have the same amount of risk, a risk-averse investor would choose the portfolio with the higher expected return (which would be Portfolio B). We say that Portfolio B dominates Portfolio C with respect to expected return, and that Portfolio A dominates Portfolio C with respect to risk. Likewise, all portfolios on the efficient frontier dominate all other portfolios in the investment universe of risky assets with respect to either risk, return, or both.

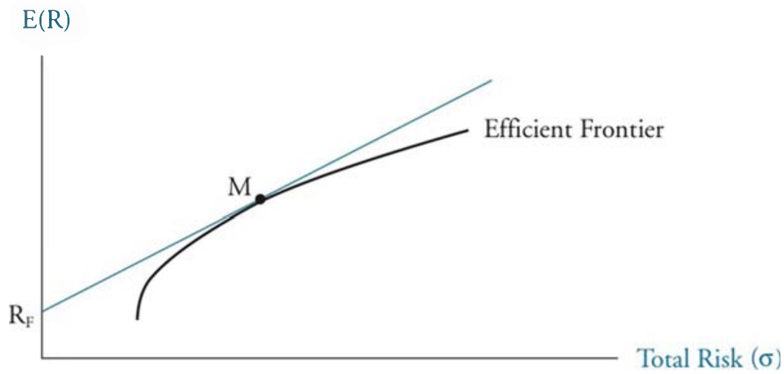
There are an almost unlimited number of combinations of risky assets to the right and below the efficient frontier. However, in the absence of a risk-free security, portfolios to the left and above the efficient frontier are not possible. Therefore, all investors will choose some portfolio on the efficient frontier. If an investor is more risk-averse, she may choose a portfolio on the efficient frontier closer to Portfolio A. If an investor is less risk-averse, she will choose a portfolio on the efficient frontier closer to Portfolio B.

Figure 1: The Efficient Frontier



If we now assume that there is a risk-free security, then the mean-variance framework is extended beyond the efficient frontier. Figure 2 illustrates that the optimal set of portfolios now lie on a straight line that runs from the risk-free security through the **market portfolio**, *M*. All investors will now seek investments by holding some portion of the risk-free security and the market portfolio. To achieve points on the line to the right of the market portfolio, an investor who is very aggressive will borrow money (at the risk-free rate) and invest in more of the market portfolio. More risk-averse investors will hold some combination of the risk-free security and the market portfolio to achieve portfolios on the line segment between the risk-free security and the market portfolio.

Figure 2: The Efficient Frontier with the Risk-Free Security



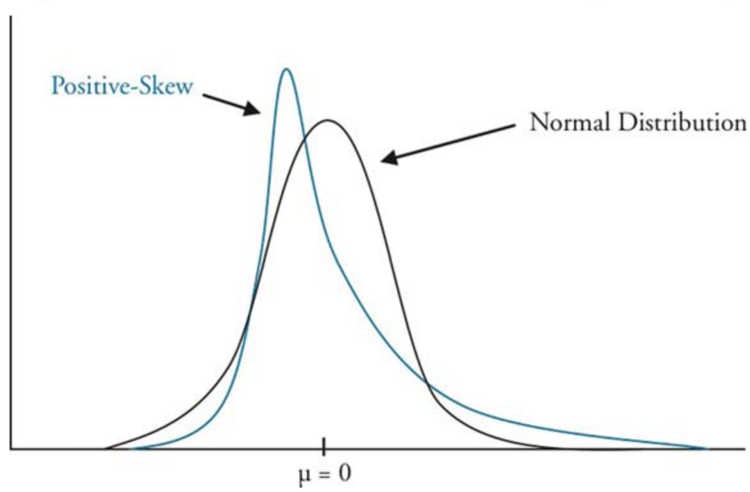
Mean-Variance Framework Limitations

LO 54.2: Explain the limitations of the mean-variance framework with respect to assumptions about the return distributions.

The use of the standard deviation as a risk measurement is not appropriate for non-normal distributions. If the shape of the underlying return density function is not symmetrical, then the standard deviation does not capture the appropriate probability of obtaining undesirable return outcomes.

Figure 3 illustrates two probability distribution functions. One probability distribution function is the normal distribution with a mean of zero. The other probability distribution is positively skewed. This positively skewed distribution has the same mean and standard deviation as the normal distribution. The degree of skewness alters the entire distribution. For the positively skewed distribution, outcomes below the mean are more likely to occur closer to the mean. Clearly normality is an important assumption when using the mean-variance framework. Thus, the mean-variance framework is unreliable when the assumption of normality is not met.

Figure 3: Normal Distribution vs. Positively-Skewed Distribution



Value at Risk

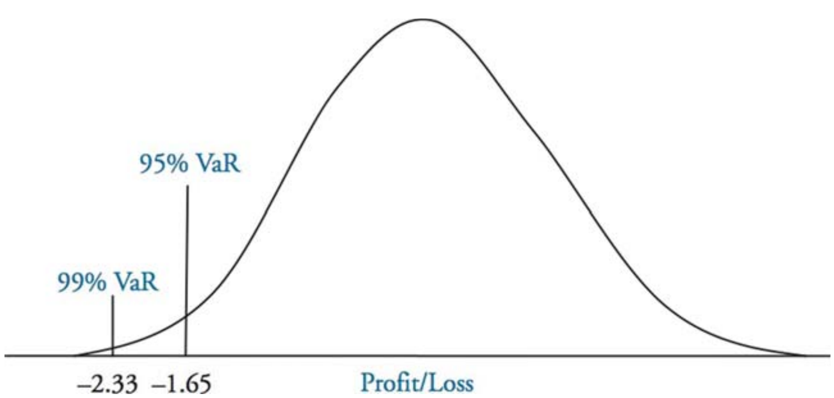
LO 54.3: Define the Value-at-Risk (VaR) measure of risk, describe assumptions about return distributions and holding period, and explain the limitations of VaR.

Value at risk (VaR) is interpreted as the worst possible loss under normal conditions over a specified period. Another way to define VaR is as an estimate of the maximum loss that can occur with a given confidence level. If an analyst says, "for a given month, the VaR is $1 million at a 95% level of confidence," then this translates to mean "under normal conditions, in 95% of the months (19 out of 20 months), we expect the fund to either earn a profit or lose no more than $1 million." Analysts may also use other standard confidence levels (e.g., 90% and 99%). Recall that delta-normal VaR can be computed using the following expression: $[\mu - (z)(\sigma)]$.

A major limitation of the VaR measure for risk is that two arbitrary parameters are used in the calculation—the confidence level and the holding period. The confidence level indicates the likelihood or probability that we will obtain a value greater than or equal to VaR. The holding period can be any pre-determined time period measured in days, weeks, months, or years.

Figure 4 illustrates VaR parameters for a confidence level of 95% and 99%. As you can see, the level of risk is dependent on the degree of confidence chosen. VaR increases when the confidence level increases. In addition, VaR will increase at an increasing rate as the confidence level increases.

Figure 4: VaR Measurements for a Normal Distribution



The second arbitrary parameter is the holding period. VaR will increase with increases in the holding period. The rate at which VaR increases is determined in part by the mean of the distribution. If the return distribution has a mean, μ, equal to 0, then VaR rises with the square root of the holding period (i.e., the square root of time). If the return distribution has a $\mu > 0$, then VaR rises at a lower rate and will eventually decrease. Thus, the mean of the distribution is an important determinant for estimating how VaR changes with changes in the holding period.

VaR estimates are also subject to both model risk and implementation risk. Model risk is the risk of errors resulting from incorrect assumptions used in the model. Implementation risk is the risk of errors resulting from the implementation of the model.

Another major limitation of the VaR measure is that it does not tell the investor the amount or magnitude of the actual loss. VaR only provides the maximum value we can lose for a given confidence level. Two different return distributions may have the same VaR, but very different risk exposures. A practical example of how this can be a serious problem is when a portfolio manager is selling out-of-the-money options. For a majority of the time, the options will have a positive return and, therefore, the expected return is positive. However, in the unfavorable event that the options expire in-the-money, the resulting loss can be very large. Thus, different strategies focusing on lowering VaR can be very misleading since the magnitude of the loss is not calculated.

To summarize, VaR measurements work well with elliptical return distributions, such as the normal distribution. VaR is also able to calculate the risk for non-normal distributions; however, VaR estimates may be unreliable in this case. Limitations in implementing the VaR model for determining risk result from the underlying return distribution, arbitrary confidence level, arbitrary holding period, and the inability to calculate the magnitude of losses. The measure of VaR also violates the coherent risk measure property of subadditivity when the return distribution is not elliptical. This property is further explained in the next LO.

Coherent Risk Measures

LO 54.4: Define the properties of a coherent risk measure and explain the meaning of each property.

In order to properly measure risk, one must first clearly define what is meant by a measure of risk. If we allow R to be a set of random events and $\rho(R)$ to be the risk measure for the random events, then **coherent risk measures** should exhibit the following properties:

1. **Monotonicity**: a portfolio with greater future returns will likely have less risk:
   $R_1 \geq R_2$, then $\rho(R_1) \leq \rho(R_2)$

2. **Subadditivity**: the risk of a portfolio is at most equal to the risk of the assets within the portfolio: $\rho(R_1 + R_2) \leq \rho(R_1) + \rho(R_2)$

3. **Positive homogeneity**: the size of a portfolio, β, will impact the size of its risk:
   for all $\beta > 0$, $\rho(\beta R) = \beta\rho(R)$

4. **Translation invariance**: the risk of a portfolio is dependent on the assets within the portfolio: for all constants c, $\rho(c + R) = \rho(R) - c$

The first, third, and fourth properties are more straightforward properties of well-behaved distributions. Monotonicity infers that if a random future value R_1 is always greater than a random future value R_2, then the risk of the return distribution for R_1 is less than the risk of the return distribution for R_2. Positive homogeneity suggests that the risk of a position is proportional to its size. Positive homogeneity should hold as long as the security is in a liquid market. Translation invariance implies that the addition of a sure amount reduces the risk at the same rate as the cash needed to make the position acceptable.

Subadditivity is the most important property for a coherent risk measure. The property of subadditivity states that a portfolio made up of sub-portfolios will have equal or less risk than the sum of the risks of each individual sub-portfolio. This assumes that when individual risks are combined, there may be some diversification benefits or, in the worst case, no diversification benefits and no greater risk. This implies grouping or adding risks does not increase the overall aggregate risk amount.

Expected Shortfall

LO 54.5: Explain why VaR is not a coherent risk measure.

LO 54.6: Explain and calculate expected shortfall (ES), and compare and contrast VaR and ES.

Value at risk is the minimum percent loss, equal to a pre-specified worst case quantile return (typically the 5th percentile return). **Expected shortfall** (ES) is the expected loss given that the portfolio return already lies below the pre-specified worst case quantile return (i.e., below the 5th percentile return). In other words, expected shortfall is the mean percent loss among the returns falling below the q-quantile. Expected shortfall is also known as **conditional VaR** or **expected tail loss** (ETL).

For example, assume an investor is interested in knowing the 5% VaR (the 5% VaR is equivalent to the 5th percentile return) for a fund. Further, assume the 5th percentile return for the fund equals –20%. Therefore, 5% of the time, the fund earns a return less than –20%. The value at risk is –20%. However, VaR does not provide good information regarding the expected size of the loss if the fund performs in the lower 5% of the possible outcomes. That question is answered by the expected shortfall amount, which is the expected value of all returns falling below the 5th percentile return (i.e., below –20%). Therefore, expected shortfall will equal a larger loss than the VaR. In addition, unlike VaR, ES has the ability to satisfy the property of subadditivity.

The ES method provides an estimate of how large of a loss is expected if an unfavorable event occurs. VaR did not provide any estimate of the magnitude of losses, only the probability that they might occur. The property of subadditivity under the ES framework is also beneficial in eliminating another problem for VaR. When adjusting both the holding period and confidence level at the same time, an ES surface curve showing the interactions of both adjustments is convex. This implies that the ES method is more appropriate than the VaR method in solving portfolio optimization problems.

ES is similar to VaR in that both provide a common consistent risk measure across different positions. ES can be implemented in determining the probability of losses the same way that VaR is implemented as a risk measure, and they both appropriately account for correlations.

However, ES is a more appropriate risk measure than VaR for the following reasons:

- ES satisfies all of the properties of coherent risk measurements including subadditivity. VaR only satisfies these properties for normal distributions.
- The portfolio risk surface for ES is convex because the property of subadditivity is met. Thus, ES is more appropriate for solving portfolio optimization problems than the VaR method.
- ES gives an estimate of the magnitude of a loss for unfavorable events. VaR provides no estimate of how large a loss may be.
- ES has less restrictive assumptions regarding risk/return decision rules.

LO 54.7: Describe spectral risk measures, and explain how VaR and ES are special cases of spectral risk measures.

A more general risk measure than either VaR or ES is known as the **risk spectrum** or risk aversion function. The risk spectrum measures the weighted averages of the return quantiles from the loss distributions. ES is a special case of this risk spectrum measure. When modeling the ES case, the weighting function is set to [1 / (1 – confidence level)] for tail losses. All other quantiles will have a weight of zero.

VaR is also a special case of spectral risk measure models. The weighting function with VaR assigns a probability of one to the event that the p-value equals the level of significance (i.e., $p = \alpha$), and a probability of zero to all other events where $p \neq \alpha$. Thus, the ES measure places equal weights on tail losses while VaR places no weight on tail losses.

In order for a risk measure to be coherent, it must give higher losses at least the same weight as lower losses. In the ES case, all losses are given the same weight. This suggests that investors are risk-neutral with respect to losses. This is contradictory to the common notion that investors are risk-averse. In the VaR case, only the loss associated with a p-value equal to α is given any weight. Greater losses are given no weight at all. This implies that investors are risk-seekers. Thus, the ES and VaR measures are inadequate in that the weighting function is not consistent with risk aversion.

Scenario Analysis

LO 54.8: Describe how the results of scenario analysis can be interpreted as coherent risk measures.

The results of scenario analysis can be interpreted as coherent risk measures by first assigning probabilities to a set of loss outcomes. These losses can be thought of as tail drawings of the relevant distribution function. The expected shortfall for the distribution can then be computed by finding the arithmetic average of the losses. Therefore, the outcomes of scenario analysis must be coherent risk measurements, because ES is a coherent risk measurement.

Scenario analysis can also be applied in situations where there are numerous distribution functions involved. It can be shown that the ES, the highest ES from a set of comparable expected shortfalls based on different distribution functions, and the highest expected shortfall from a set of highest losses are all coherent risk measures. For example, assume you are considering a set of n loss outcomes out of a family of distribution functions. The

ES is obtained from each distribution function. If there is a set of *m* comparable expected shortfalls, that each have a different corresponding loss distribution function, then the maximum of these expected shortfalls is a coherent risk measure. Thus, in cases where n = 1, the ES is the same as the probable maximum loss because there is only one tail loss in each scenario. If *m* equals one, then the highest expected loss from a single scenario analysis is a coherent measure. In cases where *m* is greater than one, the highest expected of *m* worst case outcomes is a coherent risk measure.

Key Concepts

LO 54.1

The traditional mean-variance model estimates the amount of financial risk for portfolios in terms of the portfolio's expected return (mean) and risk (standard deviation or variance). A necessary assumption for this model is that return distributions for the portfolios are elliptical distributions.

The efficient frontier is the set of portfolios that dominate all other portfolios in the investment universe of risky assets with respect to risk and return. When a risk-free security is introduced, the optimal set of portfolios consists of a line from the risk-free security that is tangent to the efficient frontier at the market portfolio.

LO 54.2

The mean-variance framework is unreliable when the underlying return distribution is not normal or elliptical. The standard deviation is not an accurate measure of risk and does not capture the probability of obtaining undesirable return outcomes when the underlying return density function is not symmetrical.

LO 54.3

Value at risk (VaR) is a risk measurement that determines the probability of an occurrence in the left-hand tail of a return distribution at a given confidence level. VaR is defined as: $[\mu - (z)(\sigma)]$. The underlying return distribution, arbitrary choice of confidence levels and holding periods, and the inability to calculate the magnitude of losses result in limitations in implementing the VaR model when determining risk.

LO 54.4

The properties of a coherent risk measure are:

- Monotonicity: $Y \geq X \Rightarrow \rho(Y) \leq \rho(X)$
- Subadditivity: $\rho(X + Y) \leq \rho(X) + \rho(Y)$
- Positive homogeneity: $\rho(hX) = h\rho(X)$ for $h > 0$
- Translation invariance: $\rho(X + n) = \rho(X) - n$

LO 54.5

Subadditivity, the most important property for a coherent risk measure, states that a portfolio made up of sub-portfolios will have equal or less risk than the sum of the risks of each individual sub-portfolio. VaR violates the property of subadditivity.

LO 54.6

Expected shortfall is a more accurate risk measure than VaR for the following reasons:

- ES satisfies all the properties of coherent risk measurements including subadditivity.
- The portfolio risk surface for ES is convex since the property of subadditivity is met. Thus, ES is more appropriate for solving portfolio optimization problems than the VaR method.
- ES gives an estimate of the magnitude of a loss for unfavorable events. VaR provides no estimate of how large a loss may be.
- ES has less restrictive assumptions regarding risk/return decision rules.

LO 54.7

ES is a special case of the risk spectrum measure where the weighting function is set to 1 / (1 – confidence level) for tail losses that all have an equal weight, and all other quantiles have a weight of zero. The VaR is a special case where only a single quantile is measured, and the weighting function is set to one when *p*-value equals the level of significance, and all other quantiles have a weight of zero.

LO 54.8

The outcomes of scenario analysis are coherent risk measurements, because expected shortfall is a coherent risk measurement. The ES for the distribution can be computed by finding the arithmetic average of the losses for various scenario loss outcomes.

Concept Checkers

1. The mean-variance framework is inappropriate for measuring risk when the underlying return distribution:
   A. is normal.
   B. is elliptical.
   C. has a kurtosis equal to three.
   D. is positively skewed.

2. Assume an investor is very risk-averse and is creating a portfolio based on the mean-variance model and the risk-free asset. The investor will most likely choose an investment on the:
   A. left-hand side of the efficient frontier.
   B. right-hand side of the efficient frontier.
   C. line segment connecting the risk-free rate to the market portfolio.
   D. line segment extending to the right of the market portfolio.

3. $\rho(X + Y) \leq \rho(X) + \rho(Y)$ is the mathematical equation for which property of a coherent risk measure?
   A. Monotonicity.
   B. Subadditivity.
   C. Positive homogeneity.
   D. Translation invariance.

4. Which of the following is not a reason that expected shortfall (ES) is a more appropriate risk measure than value at risk (VaR)?
   A. For normal distributions, only ES satisfies all the properties of coherent risk measurements.
   B. For non-elliptical distributions, the portfolio risk surface formed by holding period and confidence level is more convex for ES.
   C. ES gives an estimate of the magnitude of a loss.
   D. ES has less restrictive assumptions regarding risk/return decision rules than VaR.

5. If the weighting function in the general risk spectrum measure is set to 1 / (1 – confidence level) for all tail losses, then the risk spectrum is a special case of:
   A. value at risk.
   B. mean-variance.
   C. expected shortfall.
   D. scenario analysis.

Concept Checker Answers

1. D The mean-variance framework is only appropriate when the underlying distribution is elliptical. The normal distribution is a special case of elliptical distributions where skewness is equal to zero and kurtosis is equal to three. If there is any skewness, the distribution function will not be symmetrical, and standard deviation will not be an appropriate risk measure.

2. C Under the mean-variance framework, when a risk-free security is included in the analysis, the optimal set of portfolios lies on a straight line that runs from the risk-free security to the market portfolio. All investors will hold some portion of the risk-free security and the market portfolio. More risk-averse investors will hold some combination of the risk-free security and the market portfolio to achieve portfolios on the line segment between the risk-free security and the market portfolio.

3. B The property of subadditivity states that a portfolio made up of sub-portfolios will have equal or less risk than the sum of the risks of each individual sub-portfolio.

4. A VaR and ES both satisfy all the properties of coherent risk measures for normal distributions. However, only ES satisfies all the properties of coherent risk measures when the assumption of normality is not met.

5. C Expected shortfall is a special case of the risk spectrum measure that is found by setting the weighting function to 1 / (1 – confidence level) for tail losses that all have an equal weight.