Dynamic Valuation

LO 51.8: Describe the steps in valuing an MBS using Monte Carlo Simulation.

Any discussed earlier, mortgage borrowers have an option to prepay the underlying securities. The value of MBSs with embedded options to prepay cannot be determined using traditional option valuation techniques. Therefore, the Monte Carlo valuation methodology is used to value MBSs and other fixed-income securities with embedded options.

The **binomial model** is only applicable for securities where the decision to exercise a call option is not dependent on how interest rates evolve over time. While the binomial model is useful for callable agency debentures and corporate bonds, it is not applicable to valuing an MBS. The historical evolution of interest rates over time impacts prepayments and makes the binomial model inappropriate for MBSs.

Prepayments on mortgage pass-through securities are interest rate path-dependent. This means that a given month's prepayment rate depends on whether there were prior opportunities to refinance since the origination of the underlying mortgages. For example, if mortgage rates trend downward over a period of time, prepayment rates will increase at the beginning of the trend as homeowners refinance their mortgages, but prepayments will slow as the trend continues because many of the homeowners who can refinance will have already done so. As mentioned earlier, this prepayment pattern is called refinancing burnout. Another problem of the path-dependency of MBSs is related to the nature of structured securities, such as collateralized mortgage obligations (CMOs). The amount a CMO tranche receives in the form of cash flows for a specific month depends on the outstanding balances of other tranches in the deal. These outstanding balances are impacted by earlier principal and interest prepayments.

The **Monte Carlo methodology** is a simulation approach for valuing MBSs. Monte Carlo is actually a process of steps rather than a specific model. It is extremely useful when there are numerous variables with multiple outcomes. Monte Carlo is used to provide a probability distribution of the value of an MBS. The valuation of an MBS is influenced by future interest rates, the shape of the yield curve, future interest rate volatility, prepayment rates, default rates, and recovery rates.

Each of these variables or parameters of the Monte Carlo model could have multiple outcomes with different probabilities associated for each outcome. One valuation approach in these circumstances is the **best guess approach** where the expected value of each variable is used to estimate the value of the MBS. Unfortunately, this method is highly inaccurate. For example, suppose the probability of the best guess occurring for each variable is 70%. Then, with six different variables, the probability that the best guess MBS value will occur is only 11.8% (= 0.70^6).

The Monte Carlo approach provides a range of possible outcomes with a probability distribution for the value of a mortgage security. The mean or average value of this range of outcomes is then taken as the estimated value of the MBS. The other information, such as the range of possible outcomes and percentile information, is useful in gauging the value of the security.

The following steps are required to value a mortgage security using the Monte Carlo methodology:

Step 1: Simulate the interest rate path and refinancing path.
Step 2: Project cash flows for each interest rate path.
Step 3: Calculate the present value of cash flows for each interest rate path.
Step 4: Calculate the theoretical value of the mortgage security.

Step 1: Simulate the interest rate path and refinancing path.

The first step in applying the Monte Carlo approach is to estimate monthly interest rates for the entire life of the mortgage security. For example, a 30-year mortgage security would require 360 monthly interest rates. In equations to follow, the total number of months on an interest rate path will be denoted by T. Also, the total number of interest rate paths or *trials* that are simulated will be denoted by N. Random interest rate paths are generated using the term structure of interest rates and a volatility assumption. The term structure of interest rates is created using the theoretical spot rate (zero-coupon) curve for the market on the pricing date. The simulations are adjusted to ensure the average simulated price of a zero-coupon Treasury bond is equal to the actual price corresponding to the pricing date. Some models use LIBOR or swap rates instead of Treasury rates.

The dispersion of future interest rates in the simulation is determined by the volatility assumption. It is common practice to use more than one level of volatility. For example, with a short/long yield volatility approach, the volatility is specified based on maturities. One volatility number is used for shorter maturities (short yield volatility) and a second yield volatility is specified for longer maturities (long yield volatility). Short yield volatility is typically assumed to be greater than long yield volatility. When yield volatility is assumed for each maturity, it is referred to as **term structure yield volatility**.

The derivatives market is used to construct an arbitrage-free term structure of future interest rates. Short-term interest rate paths are used to discount the cash flows in Step 3 of the Monte Carlo process. These interest rate paths are also used to create the prepayment paths or *vectors*, which are cash flows for each interest rate path. The prepayment vector is computed based on refinancing rates that are available each month. The mortgagor has an incentive to refinance if the refinancing rate is low relative to the mortgagor's original coupon rate. The relationship between refinancing rates and short-term interest rates is an important assumption of the model.

Step 2: Project cash flows for each interest rate path.

Cash flows for each month on each interest rate path are equal to the scheduled principal for the mortgage pool, the net interest, and prepayments. Scheduled principal payments are simply calculated based on the projected mortgage balance from the prior month. A prepayment model is used rather than a simple prepayment rate. A prepayment rate is specified for each month on a given interest rate path, and rates for a given month across all interest rate paths are not the same. In fact, there could actually be $T \times N$ different prepayment rates.

CMO deal structures dictate how principal and interest is to be paid. Therefore, it is necessary to reverse engineer the deal to determine the cash flows for a senior CMO. The cash flows for each month on an interest rate path are calculated using the scheduled principal, net interest, and prepayments for the collateral (i.e., the pool of agency passthroughs). The tranche's cash flows for each path are determined by the total principal and interest paid to the tranche, the interaction of the cash flow rules, and the prepayment model.

Step 3: Calculate the present value of cash flows for each interest rate path.

The present values of cash flows for each interest rate path are calculated by discounting the cash flows for each path by a discount rate. The discount rate is estimated using the simulated spot rates for each month on the interest rate path plus an appropriate spread. The simulated spot rates are determined from the simulated future monthly rates. The following equation quantifies the relationship that holds between the simulated spot rate, $z_T(n)$, for month T on path n, and the simulated future monthly rates, $f_j(n)$:

$$z_T(n) = \{[1 + f_1(n)][1 + f_2(n)] \ldots [1 + f_T(n)]\}^{1/T} - 1$$

where:
$z_T(n)$ = simulated spot rate for month T on path n
$f_j(n)$ = simulated future one-month rate for month j on path n

The interest rate paths for the simulated future one-month rates are converted to the interest rate paths for the simulated monthly spot rates. The present value of the cash flows for month T on interest rate path n discounted at the simulated spot rate for month T, $z_T(n)$, plus a spread, K, is:

$$PV[C_T(n)] = \frac{C_T(n)}{[1 + z_T(n) + K]^T}$$

where:
$PV[C_T(n)]$ = present value of cash flows for month T on path n
$C_T(n)$ = cash flow for month T on path n
$z_T(n)$ = spot rate for month T on path n
K = spread

The present value for path n is determined as the sum of the present values of the cash flows for each month on path n as follows:

$$PV[path(n)] = PV[C_1(n)] + PV[C_2(n)] + \ldots + PV[C_T(n)]$$

where:
$PV[path(n)]$ = present value of interest rate path n

Step 4: Calculate the theoretical value of the mortgage security.

The theoretical value for a specific interest rate path is thought of as the present value of all cash flows in that path, assuming that path was actually realized. The theoretical value of the mortgage security is calculated as the average present value of all theoretical values for each interest rate path as follows:

$$\text{theoretical value} = \frac{PV[path(1)] + PV[path(2)] + \ldots + PV[path(N)]}{N}$$

where:
N = number of interest rate paths

This average theoretical value is typically the only measurement that is evaluated when Monte Carlo simulations are used to value MBSs. It is unfortunate that other potentially valuable information, such as the distribution of the path present values, is usually ignored.

Option-Adjusted Spread

LO 51.9: Define Option Adjusted Spread (OAS), and explain its challenges and its uses.

The **option-adjusted spread** (OAS) is defined as the spread, *K*, that, when added to all the spot rates of all the interest rate paths, will make the average present value of the paths equal to the actual observed market price plus accrued interest. The OAS is mathematically determined by the following relationship:

$$\text{market price} = \frac{\text{PV[path(1)]} + \text{PV[path(2)]} + \ldots + \text{PV[path(N)]}}{\text{N}}$$

where:
N = number of interest rate paths

The left-hand side of the equation is the current market price of the MBS. The right-hand side of the equation is the Monte Carlo model's output of the average theoretical value of the MBS. The OAS is determined with an iterative process. If the average theoretical value determined by the model is higher (lower) than the MBS market value, the spread is increased (decreased).

The OAS can be interpreted as a measure of MBS returns that indicates the potential compensation after adjusting for prepayment risk. In other words, the OAS is *option adjusted* because the cash flows on the interest rate paths take into account the borrowers' option to prepay. An investor could estimate the value of a security using the OAS for comparable bonds to determine whether or not to invest in the security. A second approach is to compare the OAS generated at the market price to those available for comparable securities or an investment benchmark (such as a cost of funds).

Cash flows for MBSs are monthly annuity payments, while Treasury securities pay semiannual interest-only payments and a large bullet payment. The **zero-volatility spread** (*z*-spread) is a spread measure that an investor realizes over the entire Treasury spot rate curve, assuming the mortgage security is held to maturity. It is a more accurate measure because it compares an MBS to a portfolio of Treasury securities. The zero-volatility spread is the yield that equates the present value of the cash flows from the MBS to the price of the MBS discounted at the Treasury spot rate plus the spread. Thus, an iterative process is required to determine the zero-volatility spread.

The zero-volatility spread accounts for variations in MBS principal payments at a given prepayment rate or speed. However, it does not consider the impact that prepayment risk or changing prepayment rates have on the value of the MBS.

The option cost measures the prepayment (or option) risk. It is the implied cost of the option embedded in the MBS. The option cost is calculated as the difference between the OAS at the assumed volatility of interest rates and the zero-volatility spread as follows:

option cost = zero-volatility spread – OAS

Therefore, the option cost is a by-product of the Monte Carlo analysis and is not determined using traditional option value approaches. As volatility declines, the option cost decreases, and the previously described relationship suggests that OAS increases as volatility declines, all other things equal.

OAS Challenges

There are four important limitations to consider when using OAS:

- Modeling risk associated with Monte Carlo simulations.
- Required adjustments to interest rate paths.
- An underlying assumption of a constant OAS over time in the model.
- The dependency of the underlying prepayment model.

The OAS is generated through Monte Carlo simulations. Therefore, the OAS is subject to all modeling risks associated with the simulation. Interest rate paths must be adjusted to ensure securities or rates making up the benchmark curve are properly valued when using Monte Carlo methods. This process of adjusting interest rate paths is subject to modeling error. If there is a term structure to the OAS, then this is not reflected in the Monte Carlo process because the OAS methodology assumes a constant OAS.

The prepayment model is very complex, given the amount of uncertainty regarding important variables. The behavior of both borrowers and lenders changes over time. Thus, the greatest weakness of using OAS valuation estimates generated from the Monte Carlo simulation is the dependence on the prepayment model.

Additionally, both *z*-spreads and OAS measures assume the securities are held to maturity. Some investors may hold a security to maturity, but many investors will only hold a security over a finite horizon. Thus, the investor should analyze the securities in a manner that is consistent with the investor's asset management horizon.

Key Concepts

LO 51.1

Key attributes that define mortgages are lien status, original loan term, credit classification, interest rate type, prepayments/prepayment penalties, and credit guarantees.

Agency MBSs are those that are guaranteed by government-sponsored enterprises (GSEs). Most of the MBSs are issued by these GSEs.

The GSEs have restrictions on which mortgages they can guarantee/securitize, which opened up the private label market (non-agency MBSs) for those participants willing to take on the risks inherent in nonconventional loans—jumbo loans and/or loans with high loan-to-value ratios.

LO 51.2

A mortgage is a loan that is collateralized with a specific piece of real property, either residential or commercial. A level-payment, fixed-rate conventional mortgage has a fixed term, a fixed interest rate, and a fixed monthly payment. Even though the term, rate, and payment are fixed, the cash flows are not known with certainty because the borrower has the right to repay all or any part of the mortgage balance at any time.

LO 51.3

Mortgage prepayments come in two forms: (1) increasing the frequency or amount of payments and (2) repaying/refinancing the entire outstanding balance. Prepayments are much more likely to occur when market interest rates fall and borrowers wish to refinance their existing mortgages at a new and lower rate.

Other factors that influence prepayments include seasonality, age of mortgage pool, personal, housing prices, and refinancing burnout.

LO 51.4

To reduce the risk from holding a potentially undiversified portfolio of mortgage loans, a number of financial institutions (originators) will work together to pool residential mortgage loans with similar characteristics into a more diversified portfolio. They will then sell the loans to a separate entity, called a special purpose vehicle (SPV), in exchange for cash. An issuer will purchase those mortgage assets in the SPV and then use the SPV to issue mortgage-backed securities (MBSs) to investors; the securities are backed by the mortgage loans as collateral.

Fixed-rate pass-through securities trade in one of the following ways:

- The specified pools market.
- The To Be Announced (TBA) market.

LO 51.5

The value of an MBS is a function of:

- Weighted average maturity (WAM).
- Weighted average coupon (WAC).
- Speed of prepayments.

Regarding prepayment speeds, the single monthly mortality (SMM) rate is derived from the conditional prepayment rate and is used to estimate monthly prepayments for a mortgage pool:

$$SMM = 1 - (1 - CPR)^{1/12}$$

LO 51.6

A dollar roll transaction occurs when an MBS market maker is buying positions for one settlement month and, at the same time, selling those same positions for another month.

LO 51.7

Borrowers may prepay a mortgage due to the sale of the property or a desire to refinance at lower prevailing rates. In addition, prepayments may occur when the borrower has defaulted on the mortgage or when the borrower has cash available to make partial prepayments (curtailment).

LO 51.8

The Monte Carlo methodology is a simulation approach for valuing MBSs. The binomial model is not appropriate for valuing MBSs because MBSs have embedded prepayment options and the historical evolution of interest rates over time impacts prepayments.

A mortgage security is valued using the Monte Carlo methodology by simulating the interest rate path and refinancing path, projecting cash flows for each interest rate path, calculating the present value of cash flows for each interest rate path, and calculating the theoretical value of the mortgage security.

LO 51.9

The option-adjusted spread (OAS) is the spread that, when added to all the spot rates of all the interest rate paths, will make the average present value of the paths equal to the actual observed market price plus accrued interest. The zero-volatility spread (*z*-spread) is the spread that an investor realizes over the entire Treasury spot rate curve, assuming the mortgage security is held to maturity. The option cost is the implied cost of the embedded prepayment option and is calculated as the difference between the *z*-spread and OAS.

Four major limitations of OASs are related to: (1) modeling risk associated with Monte Carlo simulations, (2) required adjustments to interest rate paths, (3) model assumption of a constant OAS over time, and (4) dependency on the underlying prepayment model.

Concept Checkers

1. Which of the following factors is least likely to influence the level of residential mortgage prepayments?
   A. Seasonality.
   B. Inflation.
   C. Housing prices.
   D. Age of mortgage pool.

2. If the conditional prepayment rate (CPR) for a pool of mortgages is assumed to be 5% on an annual basis and the weighted average maturity of the underlying mortgages is 15 years, which of the following amounts is closest to the constant maturity mortality?
   A. 0.333%.
   B. 0.405%.
   C. 0.427%.
   D. 0.5%.

3. Which of the following factors would not cause a dollar roll to trade special?
   A. Decrease in the back month price.
   B. Increase in the front month price.
   C. Surplus of securities in the market used for settlement.
   D. Shortage of securities in the market used for settlement.

4. When using the Monte Carlo approach to estimate the value of mortgage-backed securities (MBSs), the model should:
   A. use one consistent volatility measure for all interest rate paths.
   B. use a short/long yield volatility approach.
   C. use annual interest rates over the entire life of the mortgage security.
   D. ignore the distribution of the interest rate paths used to determine the theoretical value.

5. All of the following describe limitations of using option-adjusted spreads (OASs) for valuing mortgage-backed securities (MBSs) except:
   A. modeling risk is associated with Monte Carlo simulations.
   B. model requires making adjustments to interest rate paths.
   C. model assumes a dynamic OAS over time.
   D. prepayment model influences the model valuation.

Concept Checker Answers

1. B Seasonality does impact the level of prepayments—they are noticeably higher in the summertime. Increases in housing prices may spur an increase in prepayments caused by refinancing mortgages stemming from borrowers wanting to take out some of the increased equity for personal use. The lower the age of the mortgage pool, the less likely the risk of prepayment.

2. C The constant maturity mortality (or single monthly mortality rate) is a monthly measure. Its relationship to CPR is as follows:

   $SMM = 1 - (1 - CPR)^{1/12} = 1 - (1 - 0.05)^{1/12} = 1 - 0.95^{1/12} = 0.43\%$

3. C When the drop is large enough to result in financing at less than the implied cost of funds, then the dollar roll is trading special. It could be caused by:
   - A decrease in the back month price (due to an increased number of sale/settlement transactions on the back month date by originators).
   - An increase in the front month price (due to an increased demand in the front month for deal collateral).
   - Shortages of certain securities in the market that require the dealer to suddenly purchase the security for delivery in the front month, which would increase the front month price.

4. B When using the Monte Carlo approach to estimate the value of MBSs, the model should use more than one volatility measure for all interest rate paths. It is very common to use a short/long yield volatility approach to estimate monthly rates. Although the information regarding the distributions of interest rate paths is oftentimes ignored, it contains valuable information and should be considered.

5. C When using OAS to value MBS, the model assumes a constant OAS over time. This is problematic if there is a term structure to the OAS because this is not reflected in the Monte Carlo process.

Self-Test: Financial Markets and Products

15 Questions: 36 Minutes

1. An investor enters a short position in a gold futures contract at $318.60. Each futures contract controls 100 troy ounces. The initial margin is $5,000 and the maintenance margin is $4,000. At the end of the first day the futures price rises to $329.22. Which of the following is the amount of the variation margin at the end of the first day?
   A. $0.
   B. $62.
   C. $1,000.
   D. $1,062.

2. A large-cap U.S. equity portfolio manager is concerned about near-term market conditions and wishes to reduce the systematic risk of her portfolio from 1.2 to 0.90. Her portfolio value is $56 million, and the S&P 500 futures index is currently trading at 1,050 and has a multiplier of 250. How can the portfolio manager's objective be achieved?
   A. Sell 47 contracts.
   B. Buy 47 contracts.
   C. Sell 64 contracts.
   D. Buy 64 contracts.

3. Suppose you observe a 1-year (zero-coupon) Treasury security trading at a yield to maturity of 5% (price of 95.2381% of par). You also observe a 2-year T-Note with a 6% coupon trading at a yield to maturity of 5.5% (price of 100.9232). And, finally, you observe a 3-year T-Note with a 7% coupon trading at a yield to maturity of 6.0% (price of 102.6730). Assume annual coupon payments and discrete compounding. Use the bootstrapping method to determine the 2-year and 3-year spot rates.

	2-year spot rate	3-year spot rate
A.	5.51%	5.92%
B.	5.46%	5.92%
C.	5.51%	6.05%
D.	5.46%	6.05%

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4. Former Treasury Secretary Robert Rubin decided to stop issuing 30-year Treasury bonds in 2001 and to replace them by borrowing more with shorter-maturity Treasury bills and notes (although the U.S. Treasury has since resumed issuing 30-year bonds). Which of the following statements concerning this decision is most accurate?
   A. If the expectations theory of the term structure is correct, this decision will reduce the government's borrowing cost.
   B. If the liquidity theory of the term structure is correct, this decision will reduce the government's borrowing cost.
   C. If the liquidity theory of the term structure is correct, this decision will not change the government's borrowing cost.
   D. If the expectations theory of the term structure is correct, this decision will increase the government's borrowing cost.

5. A portfolio manager owns Macrogrow, Inc., which is currently trading at $35 per share. She plans to sell the stock in 120 days but is concerned about a possible price decline. She decides to take a short position in a 120-day forward contract on the stock. The stock will pay a $0.50 per share dividend in 35 days and $0.50 again in 125 days. The risk-free rate is 4%. The value of the trader's position in the forward contract in 45 days, assuming in 45 days the stock price is $27.50 and the risk-free rate has not changed, is closest to:
   A. $7.16.
   B. $7.50.
   C. $7.92.
   D. $7.00.

6. A 6-month futures contract on an equity index is currently priced at 1,276. The underlying index stocks are valued at 1,250 and pay dividends at a continuously compounded rate of 1.70%. The current continuously compounded risk-free rate is 5%. The potential arbitrage is closest to:
   A. 5.20.
   B. 8.32.
   C. 16.58.
   D. 26.00.

7. Company J and Company K enter into a 2-year plain vanilla interest rate swap. Company J agrees to pay Company K a periodic fixed rate on a notional principal over the swap's tenor. In exchange, Company K agrees to pay Company J a periodic floating rate on the same notional principal. Assume currency is the same, so the net payment will be exchanged. The exchanges will be made semi-annually. The reference rate is the 6-month LIBOR. The fixed rate of the swap is 1.1%, and the notional principal is $100 million. 6-month LIBOR rates are as follows:

Beginning of Period	*LIBOR*
1	0.5%
2	0.75%
3	1.00%
4	1.25%
5	1.50%

What is the net payment for the end of the first period?
A. Company J pays Company K $300,000.
B. Company J pays Company K $550,000.
C. Company K pays Company J $250,000.
D. Company K pays Company J $50,000.

Use the following information to answer Questions 8 and 9.

Stock ABC trades for $60 and has 1-year call and put options written on it with an exercise price of $60. The annual standard deviation estimate is 10%, and the continuously compounded risk-free rate is 5%. The value of the call is $4.09.

Chefron, Inc. common stock trades for $60 and has a 1-year call option written on it with an exercise price of $60. The annual standard deviation estimate is 10%, the continuous dividend yield is 1.4%, and the continuously compounded risk-free rate is 5%.

8. The value of the put on ABC stock is closest to:
A. $1.16.
B. $3.28.
C. $4.09.
D. $1.00.

9. The value of the call on Chefron stock is closest to:
   A. $3.51.
   B. $4.16.
   C. $5.61.
   D. $6.53.

10. One of your clients, Christopher Stachowski, realizes that the market prices of options must take into account the beliefs of the market participants. He thinks he will be able to make significant profits because he believes that there will be a large movement in the direction of stock prices but is unsure which direction. Such a belief is completely different from the other market participants. As a result, Christopher would like you to implement an options trading strategy to generate him those profits. Which of the following combination option strategies is likely to benefit the least amount from a large positive or negative movement in the price of the underlying?
   A. Strip.
   B. Strap.
   C. Collar.
   D. Long strangle.

11. Consider a bearish option strategy of buying one $50 put for $7, selling two $42 puts for $4 each, and buying one $37 put for $2. All the options have the same maturity. Calculate the final profit per share of the strategy if the underlying is trading at $33 at expiration.
   A. $1 per share.
   B. $2 per share.
   C. $3 per share.
   D. $4 per share.

12. You believe that a stock will increase in price and would like to buy a call option. You would like to choose the date during the option's term when the option payoff is determined. However, if the option payoff is greater at the option's maturity, you want to be paid this value. What type of option should you buy?
   A. Chooser option.
   B. Compound option.
   C. Shout option.
   D. Asian option.

13. Suppose the spot rate is 0.7102 USD/CHF. Swiss and U.S. interest rates are 7.6% and 5.2%, respectively. If the 1-year forward rate is 0.7200 USD/CHF, an investor could:
   A. not earn arbitrage profits.
   B. earn arbitrage profits by investing in USD.
   C. earn arbitrage profits by investing in CHF.
   D. only earn arbitrage profits by investing in a third currency.

14. Consider a U.K.-based company that exports goods to the EU. The U.K. company expects to receive payment on a shipment of goods in 60 days. Because the payment will be in euros, the U.K. company wants to hedge against a decline in the value of the euro against the pound over the next 60 days. The U.K. risk-free rate is 3% and the EU risk-free rate is 4%. No change is expected in these rates over the next 60 days. The current spot rate is 0.9230 £ per €. To hedge the currency risk, the U.K. company should take a short position in a Euro contract at a forward price of:
A. 0.9205.
B. 0.9215.
C. 0.9244.
D. 0.9141.

15. A level-payment, fixed-rate mortgage has the following characteristics:
- Term 30 years.
- Mortgage rate 9.0%.
- Servicing fee 0.5%.
- Original mortgage loan balance $150,000.

The monthly mortgage payment is:
A. $416.67.
B. $1,125.00.
C. $1,206.93.
D. $1,216.70.

Self-Test Answers: Financial Markets and Products

1. D The short position loses when the price rises.

   (\$329.22 – \$318.60) × 100 = 1,062 loss

   Margin account will change as follows: \$5,000 – \$1,062 = \$3,938

   Variation margin of \$1,062 is required because the balance has fallen below the maintenance margin level. This variation margin payment is required in order to restore the account back to the initial level.

   (See Topic 35)

2. C The portfolio manager wants to reduce exposure to systematic risk so she will want to sell S&P index futures. This will reduce the current beta to her target beta of 0.90.

   number of contracts = (target beta – current beta) × (portfolio value / futures value)

   number of contracts = (0.9 – 1.2) × [\$56 million / (1,050 × 250)]

   number of contracts = –64 (i.e., sell 64 contracts)

   (See Topic 36)

3. C Here are the cash flows associated with the three bonds:

	0	1	2	3
1-year	–\$95.2381	+\$100		
2-year	–\$100.9232	+\$6	+\$106	
3-year	–\$102.6730	+\$7	+\$7	+\$107

   To find Z_2, the 2-year spot rate:

$$\$100.9232 = \frac{\$6}{1.05^1} + \frac{\$106}{(1+Z_2)^2} \Rightarrow Z_2 = 5.51\%$$

   To find Z_3, the 3-year spot rate:

$$\$102.6730 = \frac{\$7}{1.05^1} + \frac{\$7}{1.0551^2} + \frac{\$107}{(1+Z_3)^3} \Rightarrow Z_3 = 6.05\%$$

   (See Topic 37)

4. B If the expectations theory of the term structure is correct, altering the maturity of the government's borrowing will not affect the government's borrowing cost (i.e., borrowing once for 30 years is the same as borrowing 30 times for one year at a time). If the liquidity theory is correct, the government's borrowing cost will go down, as it no longer has to compensate lenders with the liquidity premium for borrowing long term.

(See Topic 37)

5. A The dividend in 125 days is irrelevant because it occurs after the forward contract matures.

$$\text{PVD} = \$0.50e^{-0.04\times(35/365)} = \$0.4981$$
$$\text{FP} = (\$35 - \$0.4981)\times 1.04^{120/365} = \$34.95$$
$$V_{45}(\text{short position}) = -(\$27.50 - \$34.95e^{-0.04\times(75/365)}) = \$7.16$$

(See Topic 38)

6. A $F = S \times e^{(\text{risk-free rate} - \text{dividend yield}) \times t}$

$F = 1{,}250 \times e^{(0.05 - 0.017) \times 0.5}$

$F = 1{,}270.80$

The actual futures price is 1,276, so selling the futures and buying the underlying index nets a profit of 1,276 – 1,270.80 = 5.20.

(See Topic 38)

7. A Floating = \$100 million × 0.005 × 0.5 = \$250,000

Fixed = \$100 million × 0.011 × 0.5 = \$550,000

(See Topic 40)

8. A According to put/call parity, the put's value is:

$$p_0 = c_0 - S_0 + \left(X \times e^{-R_c^f \times T}\right) = \$4.09 - \$60.00 + \left[\$60.00 \times e^{-(0.05 \times 1.0)}\right] = \$1.16$$

(See Topic 42)

9. A ABC and Chefron stock are identical in all respects except Chefron pays a dividend. Therefore, the call option on Chefron stock must be worth less than the call on ABC (i.e., less than \$4.09). \$3.51 is the only possible answer.

(See Topic 42)

10. C A collar is the combination of a protective put and a covered call. Ignoring transaction costs, at levels below the put strike price or above the call strike price, the profit from a collar levels off. Between the put strike price and the call strike price, the profit level is gradually rising.

(See Topic 43)

11. **B** Consider each option separately:

$50 long put:	$50 – $33 = +$17
$42 short put:	$42 – $33 = –$9 × 2 = –$18
$37 long put:	$37 – $33 = +$4
Net cost of options:	(–7 + 8 – 2) = –$1
Overall profit per share:	$2 per share

(See Topic 43)

12. **C** The shout option allows the buyer to choose the date when he "shouts" to the option seller that the intrinsic value should be determined. At expiration, the option buyer receives the maximum of the shout value or the intrinsic value at expiration.

(See Topic 44)

13. **C** Note that while the USD has the lower interest rate, it is also trading at a forward discount relative to the CHF. Since the USD will earn less interest *and* depreciate in value, we definitely want to invest in CHF (not in USD), and no calculation is necessary.

As an illustration of covered interest arbitrage, we have:

$$(1+R_A) < \frac{(1+R_B)(\text{forward rate})}{\text{spot rate}}$$

$$1.052 < \frac{(1.076)(0.72)}{0.7102} = 1.0908$$

Today:
(1) Borrow USD1 at 5.2% and purchase CHF at $0.7102 to get $1 / 0.7102 = 1.408 CHF at spot rate.
(2) Lend the purchased CHF at 7.6% and sell forward 1.5150 CHF at the forward rate of 0.7200 USD/CHF.

In one year:
(1) Use the proceeds of the savings account [(1.408)(1.076) = 1.5150 CHF] to purchase USD1.0908 at the forward rate (1.515 CHF × 0.72 USD/CHF).
(2) Pay off the loan of USD1 × 1.052 = USD1.052 and earn a riskless profit = USD1.0908 – USD1.052 = USD0.0388.

(See Topic 49)

14. **B** The U.K. company will be receiving euros in 60 days, so it should short the 60-day forward on the euro as a hedge. The no-arbitrage forward price is:

$$F_T = £0.923 \times \frac{1.03^{60/365}}{1.04^{60/365}} = 0.9215$$

(See Topic 49)

15. **C** N = 360; I = 9/12 = 0.75; PV = 150,000; CPT → PMT = $1,206.93

(See Topic 51)

Formulas

Topic 32

combined ratio: loss ratio + expense ratio

combined ratio after dividends: combined ratio + dividends

operating ratio: combined ratio after dividends – investment income

Topic 33

net asset value: $\text{NAV} = \dfrac{\text{fund assets} - \text{fund liabilities}}{\text{total shares outstanding}}$

Topic 34

call option payoff: $C_T = \max(0, S_T - X)$

put option payoff: $P_T = \max(0, X - S_T)$

forward contract payoff: payoff = $S_T - K$

where:
S_T = spot price at maturity
K = delivery price

Topic 36

basis = $S_t - F_0$

where:
S_t = cash (or spot) price of the underlying asset at time t
F_0 = current price of the futures contract

hedge ratio: $\text{HR} = \rho_{S,F}\dfrac{\sigma_S}{\sigma_F}$

beta: $\dfrac{\text{Cov}_{S,F}}{\sigma_F{}^2} = \beta_{S,F}$

correlation: $\rho = \dfrac{\text{Cov}_{S,F}}{\sigma_S\sigma_F}$

hedging with stock index futures:

$$\text{number of contracts} = \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{value of futures contract}}\right)$$

$$= \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}}\right)$$

adjusting the portfolio beta: $\text{number of contracts} = (\beta^* - \beta)\frac{P}{A}$

Topic 37

discrete compounding: $FV = A\left(1 + \frac{R}{m}\right)^{m \times n}$

continuous compounding: $FV = Ae^{R \times n}$

forward rate agreement: $\text{cash flow (if receiving } R_K) = L \times (R_K - R) \times (T_2 - T_1)$

$\text{cash flow (if paying } R_K) = L \times (R - R_K) \times (T_2 - T_1)$

Topic 38

forward price: $F_0 = S_0 e^{rT}$

forward price with carrying costs: $F_0 = (S_0 - I)\, e^{rT}$

forward price when the underlying asset pays a dividend: $F_0 = S_0 e^{(r-q)T}$

Topic 39

$$\text{accrued interest} = \text{coupon} \times \frac{\text{\# of days from last coupon to the settlement date}}{\text{\# of days in coupon period}}$$

cash price of a bond: cash price = quoted price + accrued interest

annual rate on a T-Bill: T-bill discount rate = $\frac{360}{n}(100 - Y)$

cheapest-to-deliver bond: quoted bond price – (QFP × CF)

Eurodollar futures price = \$10,000[100 – (0.25)(100 – Z)]

convexity adjustment:

actual forward rate = forward rate implied by futures – $(0.5 \times \sigma^2 \times t_1 \times t_2)$

duration-based hedge ratio: $N = -\frac{P \times D_P}{F \times D_F}$

Topic 40

forward rate between T_1 and T_2: $R_{forward} = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$

Topic 42

put-call parity:

$S = c - p + Xe^{-rT}$
$p = c - S + Xe^{-rT}$
$c = S + p - Xe^{-rT}$
$Xe^{-rT} = S + p - c$

lower and upper bounds for options:

Option	*Minimum Value*	*Maximum Value*
European call	$c \geq \max(0, S_0 - Xe^{-rT})$	S_0
American call	$C \geq \max(0, S_0 - Xe^{-rT})$	S_0
European put	$p \geq \max(0, Xe^{-rT} - S_0)$	Xe^{-rT}
American put	$P \geq \max(0, X - S_0)$	X

Topic 43

bull call spread: profit $= \max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0}$

bear put spread: profit $= \max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}$

butterfly spread with calls:

$$\text{profit} = \max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0}$$

straddle: profit $= \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$

strangle: profit $= \max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$

Topic 45

pricing a commodity forward with a lease payment: $F_{0,T} = S_0 e^{(r - \delta_l)T}$

commodity forward pricing with storage costs: $F_{0,T} = S_0 e^{(r+\lambda)T}$

commodity forward pricing with convenience yield: $F_{0,T} = S_0 e^{(r-c)T}$

Topic 49

interest rate parity: $\text{forward} = \text{spot}\left[\frac{(1+r_{DC})}{(1+r_{FC})}\right]^T$

$$\text{forward} = \text{spot} \times e^{(r_{DC}-r_{FC})T}$$

nominal interest rate:

exact methodology: $(1+r) = (1+\text{real } r)[1+E(i)]$

linear approximation: $r \approx \text{real} + E(i)$

Topic 50

original-issue discount (OID) = face value – offering price

dollar default rate:

$$\frac{\text{cumulative dollar value of all defaulted bonds}}{(\text{cumulative dollar value of all issuance}) \times (\text{weighted average \# of years outstanding})}$$

Topic 51

single monthly mortality rate: $SMM = 1 - (1 - CPR)^{1/12}$

option cost = zero-volatility spread – OAS