The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Mechanics of Options Markets

Topic 41

Exam Focus

Stock options give the owner the right, but not the obligation, to buy or sell a stock at a specific price on or before a specific date. Call options give the owner the right to buy the stock, and put options give the owner the right to sell the stock. An option is exercised when the owner executes the right to buy or sell the stock. This topic covers the basic mechanics of option trading. You should understand the different kinds of options and the system by which exchange-traded options are bought and sold.

Option Types

LO 41.1: Describe the types, position variations, and typical underlying assets of options.

Option contracts have asymmetric payoffs. The buyer of an option has the right to exercise the option but is not obligated to exercise. Therefore, the maximum loss for the buyer of an option contract is the loss of the price (premium) paid to acquire the position, while the potential gains in some cases are theoretically infinite. Because option contracts are a zero-sum game, the seller of the option contract could incur substantial losses, but the maximum potential gain is the amount of the premium received for writing the option. *American options* may be exercised at any time up to and including the contract's expiration date, while *European options* can be exercised only on the contract's expiration date.

To understand the potential returns, we need to introduce the standard symbols used to represent the relevant factors:

X = strike price or exercise price specified in the option contract (a fixed value)
S_t = price of the underlying asset at time t
C_t = the market value of a call option at time t
P_t = the market value of a put option at time t
t = the time subscript, which can take any value between 0 and T, where T is the maturity or expiration date of the option

Call Options

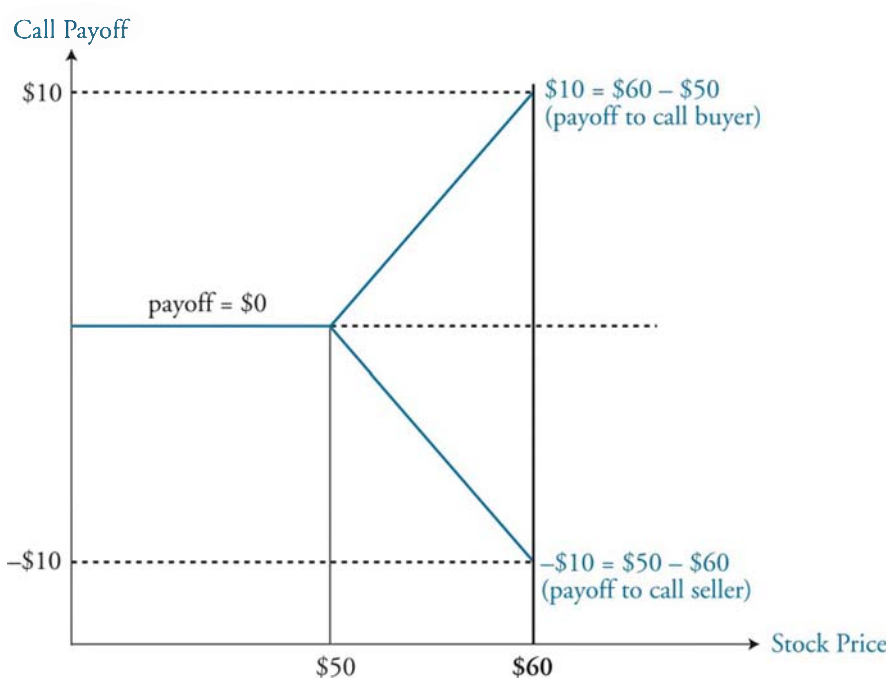
A *call option* gives the *owner* the right, but not the obligation, to buy the stock from the seller of the option. The owner is also called the *buyer* or the holder of the *long position*. The buyer benefits, at the expense of the option *seller*, if the underlying stock price is greater than the exercise price. The option *seller* is also called the *writer* or holder of the *short position*.

At maturity time *T*, if the price of the underlying stock is less than or equal to the strike price of a call option (i.e., $S_T \leq X$), the payoff is zero, so the option owner would not exercise the option. On the other hand, if the stock price is higher than the exercise price (i.e., $S_T > X$) at maturity, then the payoff of the call option is equal to the difference between the market price and the strike price ($S_T - X$). The "payoff" (at the option's maturity) to the call option seller, is the mirror image (opposite sign) of the payoff to the buyer.

Because of the linear relationships between the value of the option and the price of the underlying asset, simple graphs can clearly illustrate the possible value of option contracts at the expiration date. Figure 1 illustrates the payoff of a call with an exercise price equal to 50.

Professor's Note: An option payoff graph ignores the initial cost of the option.

Figure 1: Payoff of Call With Exercise Price Equal to $50



Example: Payoff of a call option

An investor writes an at-the-money call option on a stock with an exercise price of 50 (X = 50). If the stock price rises to \$60, what will be the *payoff* to the owner and seller of the call option?

Answer:

The call option may be exercised with the holder of the long position buying the stock from the writer at 50 for a \$10 gain. The payoff to the option buyer is \$10, and the payoff to the option writer is *negative* \$10. This is illustrated in Figure 1 and, as mentioned, does not include the premium paid for the option.

This example shows just how easy it is to determine option payoffs. At expiration time T (the option's maturity), the payoff to the option owner, represented by C_T, is:

$$C_T = S_T - X \quad \text{if} \quad S_T > X$$

$$C_T = 0 \quad \text{if} \quad S_T \leq X$$

Another popular way of writing this is with the "max (0, variable)" notation. If the variable in this expression is greater than zero, then max (0, variable) = variable; if the variable's value is less than zero, then max (0, variable) = 0. Thus, letting the variable be the quantity $S_0 - X$, we can write:

$$C_T = \max(0, S_T - X)$$

The payoff to the option seller is the negative value of these numbers. In what follows, we will always talk about payoff in terms of the option owner unless otherwise stated. We should note that max $(0, S_t - X)$, where $0 < t < T$, is also the payoff if the owner decides to exercise the call option early. In this topic, we will only consider time T in our analysis.

Although our focus here is not to calculate C_t, we should clearly define it as the initial cost of the call when the investor purchases at time 0, which is T units of time before T. C_0 is also called the premium. Thus, we can write that the profit to the owner at t = T is:

$$\text{profit} = C_T - C_0$$

This says that at time T, the owner's profit is the option payoff minus the premium paid at time 0. Incorporating C_0 into Figure 1 gives us the profit diagram for a call at expiration, and this is Figure 2.

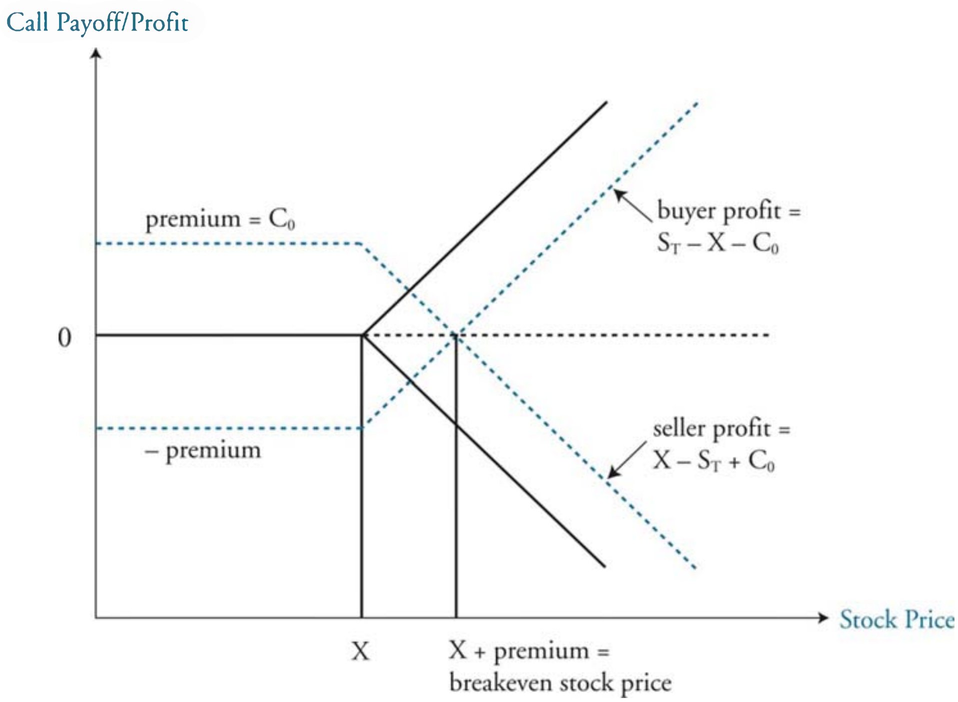
Figure 2 illustrates an important point, which is that the profit to the owner is negative when the stock price is less than the exercise price plus the premium. At expiration, we can say that:

if $S_T < X + C_0$ then: call buyer profit < 0 < call seller profit

if $S_T = X + C_0$ then: call buyer profit = 0 = call seller profit
if $S_T > X + C_0$ then: call buyer profit > 0 > call seller profit

The **breakeven price** is a very descriptive term that we use for $X + C_0$, or X + premium.

Figure 2: Profit Diagram for a Call at Expiration



Put Options

If you understand the properties of a call, the properties of a put should come to you fairly easily. A put option gives the owner the right to sell a stock to the seller of the put at a specific price. At expiration, the buyer benefits if the price of the underlying is less than the exercise price *X*:

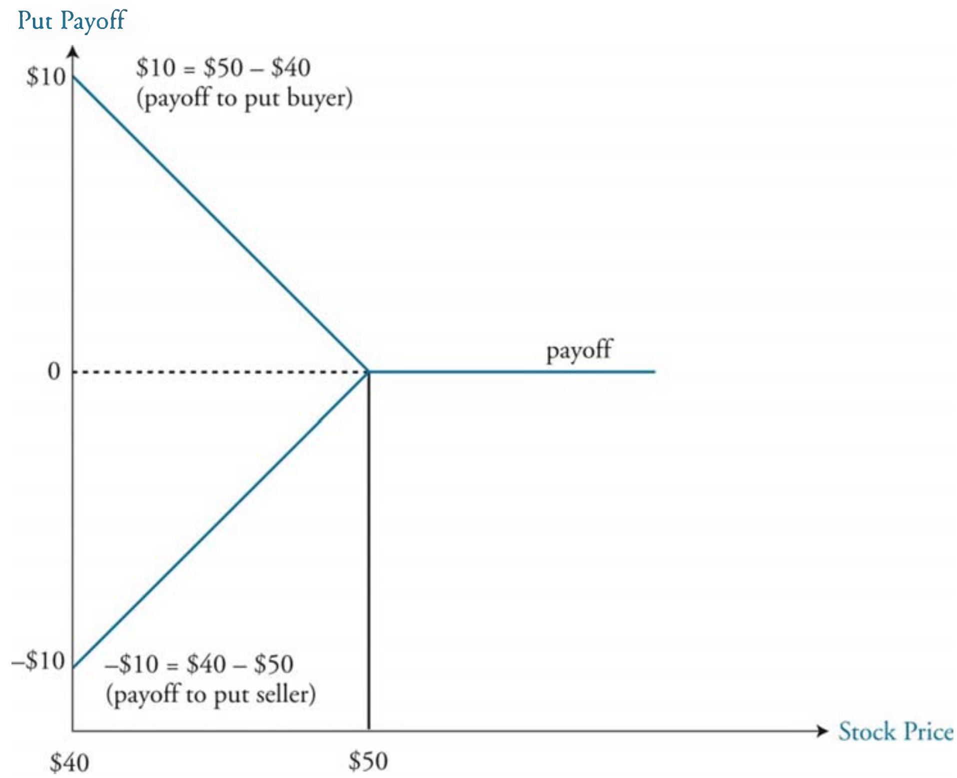
$P_T = X - S_T$ if $S_T < X$
$P_T = 0$ if $X \leq S_T$

or:

$P_T = \max(0, X - S_T)$

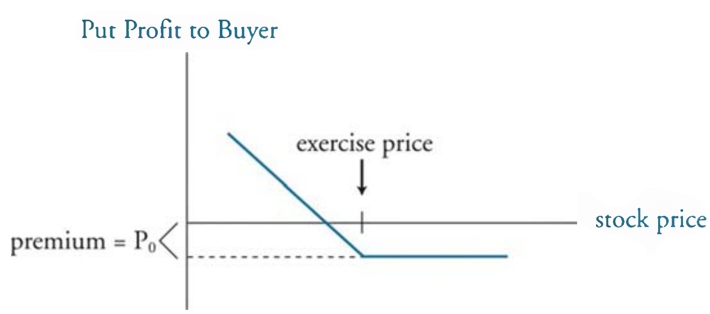
For example, an investor writes a put option on a stock with a strike price of X = 50. If the stock stays at $50 or above, the payoff of the put option is zero (because the holder may receive the same or better price by selling the underlying asset on the market rather than exercising the option). But if the stock price falls below $50, say to $40, the put option may be exercised with the option holder buying the stock from the market at $40 and selling it to the put writer at $50 for a $10 gain. The writer of the put option must pay the put price of $50 when it can be sold in the market at only $40, resulting in a $10 loss. The gain to the option holder is the same magnitude as the loss to the option writer. Figure 3 illustrates this example, excluding the initial cost of the put and transaction costs. Figure 4 includes the cost of the put (but not transaction costs) and illustrates the profit to the put owner.

Figure 3: Put Payoff to Buyer and Seller



Given the "mirror image quality" that results from the "zero-sum game" nature of options, we often just draw the profit to the buyer as shown in Figure 4. Then, we can simply remember that each positive (negative) value is a negative (positive) value for the seller.

Figure 4: Put Profit to Buyer



The breakeven price for a put position upon expiration is the exercise price minus the premium paid, $X - P_0$.

Underlying Assets

Exchange-traded options trade on four primary assets: individual stocks, foreign currency, stock indices, and futures.

Stock options. Stock options are typically exchange-traded, American-style options. Each option contract is normally for 100 shares of stock. For example, if the last trade on a call option occurred at $3.60, the option contract would cost $360. After issuance, stock option contracts are adjusted for stock splits but not cash dividends. The primary U.S. exchanges for stock options are the Chicago Board Options Exchange (CBOE), Boston Options Exchange, NYSE Euronext, and the International Securities Exchange.

Currency options. Investors holding currency options receive the right to buy or sell an amount of foreign currency based on a domestic currency amount. For calls, a currency option is going to pay off only if the actual exchange rate is above a specified exercise rate. For puts, a currency option is going to pay off only if the actual exchange rate is below a specified exercise rate. The majority of currency options are traded on the over-the-counter market, while the remainder are exchange traded. The NASDAQ OMX trades European-style options for several currencies. Note that the unit size for currency options is considerably larger than stock options (i.e., 1 million units for yen and 10,000 units for other currencies).

Index options. Options on stock indices are typically European-style options and are cash settled. Index options can be found on both the over-the-counter markets and the exchange-traded markets. The payoff on an index call is the amount (if any) by which the index level at expiration exceeds the index level specified in the option (the strike price), multiplied by the contract multiplier (typically 100).

Example: Index options

Assume you own a call option on an index with an exercise price equal to 950. The multiplier for this contract is 100. **Compute** the payoff on this option assuming that the index is 956 at expiration.

Answer:

The payoff on an index call (long) is the amount (if any) by which the index level at expiration exceeds the index level specified in the option (the exercise price), multiplied by the contract multiplier. An equal amount will be deducted from the account of the index call option writer. In this example, the expiration date payoff is $(956 - 950) \times \$100 = \$600$.

Futures options. American-style, exchange-traded options are most often utilized for futures contracts. Typically, the futures option expiration date is set to a date shortly before the expiration date of the futures contract. The market value of the underlying asset for futures options is the value of the underlying futures contract. The payoff for call options is calculated as the futures price less the strike price, while the payoff for put options is calculated as the strike price less the futures price.

Stock Options Specifications

LO 41.2: Explain the specification of exchange-traded stock option contracts, including that of nonstandard products.

Expiration

Options can be either American or European style. As mentioned previously, American options can be exercised throughout the life of the option, while European options can only be exercised on the expiration date of the option. For this reason, American options are always at least as valuable as corresponding European options. Exchange-traded stock options are typically American-style options. The expiration dates of these options dictate how the option is named. For example, a June put option on Intel means that the option expires in June. The actual day of expiration is the Saturday following the third Friday of the expiration month. Different expiration cycles dictate the actual expiration months of a stock option over a given year. **Long-term equity anticipation securities** (LEAPS®) are simply long-dated options with expirations greater than one year. All LEAPS have January expirations.

Strike Prices

Strike prices are dictated by the value of the stock. Low-value stocks have smaller strike increments than higher-value stocks. Typically, stocks that are priced around \$20 have increments of \$2.50, stocks that are priced around \$50 have increments of \$5.00, and so on. The strike price is usually denoted as *X* and the underlying stock as *S*.

Moneyness, Time Value, and Intrinsic Value

An *option class* refers to all options of the same type, whether calls or puts. An *option series* refers to an option class with the same expiration. For a call (put), when the underlying asset price is less (greater) than the strike price, the option is said to be out of the money. For both a call and put, when the underlying asset price is equal to the strike price, the option is said to be at the money. For a call (put), when the underlying asset is greater (less) than the strike price, the option is said to be in the money. An option price (or premium) prior to expiration has two components: the time value and the intrinsic value. The *intrinsic value* is the maximum of zero or the difference between the underlying asset and the strike price [i.e., intrinsic value of a call option = max(0, S – X) and intrinsic value of a put option = max(0, X – S)]. The *time value* is the difference between the option premium and the intrinsic value.

Nonstandard Products

Nonstandard option products include flexible exchange (FLEX) options, exchange-traded fund (ETF) options, weekly options, binary options, credit event binary options (CEBOs), and deep out-of-the-money (DOOM) options.

FLEX options. FLEX options are exchange-traded options on equity indices and equities that allow some alteration of the options contract specifications. The nonstandard terms include alteration of the strike price, different expiration dates, or European-style (rather than the standard American-style). FLEX options were developed in order for the exchanges to better compete with the nonstandard options that trade over the counter. The minimum size for FLEX trades is typically 100 contracts.

ETF options. While similar to index options, ETF options are typically American-style options and utilize delivery of shares rather than cash at settlement.

Weekly options. *Weeklys* are short-term options that are created on a Thursday and have an expiration date on the Friday of the next week.

Binary options. Binary options generate discontinuous payoff profiles because they pay only one price ($100) at expiration if the asset value is above the strike price. The term binary means the option payoff has one of two states: the option pays $100 at expiration if the option is above the strike price or the option pays nothing if the price is below the strike price. Hence, a payoff discontinuity results from the fact that the payoff is only one value—it does not increase continuously with the price of the underlying asset as in the case of a traditional option.

CEBOs. A CEBO is a specific form of credit default swap. The payoff in a CEBO is triggered if the reference entity suffers a qualifying credit event (e.g., bankruptcy, missed debt payment, or debt restructuring) prior to the option's expiration date (which always occurs in December). Option payoff, if any, occurs on the expiration date. CEBOs are European options that are cash settled.

DOOM options. These put options are structured to only be in the money in the event of a large downward price movement in the underlying asset. Due to their structure, the strike price of these options is quite low. In terms of protection, DOOM options are similar to credit default swaps. Note that this option type is always structured as a put option.

The Effect of Dividends and Stock Splits

In general, options are not adjusted for cash dividends. This will have option pricing consequences that will need to be incorporated into a valuation model. Options are adjusted for *stock splits*. For example, if a stock has a 2-for-1 stock split, then the strike price will be reduced by one-half and the number of shares underlying the option will double. In general, if a stock experiences a b-for-a stock split, the strike price becomes (a/b) of its previous value and the number of shares underlying the option is increased by multiples of (b/a). Stock dividends are dealt with in the same manner. For example, if a stock pays a 25% stock dividend, this is treated in the same manner as a 5-for-4 stock split.

Position and Exercise Limits

The number of options a trader can have on one stock is limited by the exchange. This is called a position limit. Additionally, short calls and long puts are considered to be part of the same position. The exercise limit equals the position limit and specifies the maximum number of option contracts that can be exercised by an individual over any five consecutive business days.

Option Trading

LO 41.3: Describe how trading, commissions, margin requirements, and exercise typically work for exchange-traded options.

As mentioned, options are quoted relative to one underlying stock. To compute the actual option cost, the quote needs to be multiplied by 100. This is because an options contract represents an option on 100 shares of the underlying stock. The quotes will also include the strike, expiration month, volume, and the option class.

Market makers will quote bid and offer (or ask) prices whenever necessary. They profit on the bid-offer spread and add liquidity to the market. Floor brokers represent a particular firm and execute trades for the general public. The order book official enters limit orders relayed from the floor broker. An offsetting trade takes place when a long (short) option position is offset with a sale (purchase) of the same option. If a trade is not an offsetting trade, then open interest increases by one contract.

Commissions

Option investors must consider the commission costs associated with their trading activity. Commission costs often vary based on trade size and broker type (discount vs. full service). Brokers typically structure commission rates as a fixed amount plus a percentage of the trade amount. The following example provides an illustration on how commission costs affect an option trade's profitability.

Example: Commission costs

An investor buys a call contract with a strike price of $260. The current price of the underlying stock is $245. Assume the option price is $10 and the contract is settled with shares rather than cash. Using the commission schedule for a discount broker below, **calculate** (1) the commission costs incurred by the investor based on the initial trade and (2) the investor's net profit if the stock price increases to $280 prior to expiration. Assume the cost to exercise the option is 1% of the trade amount and the cost to sell stock is also 1% of the trade amount.

Figure 5: Commission Schedule

Trade Amount	*Commission Rate*
≤ $3,000	$30 + 0.8% of trade amount
$3,001 to $14,999	$30 + 0.6% of trade amount
≥ $15,000	$30 + 0.4% of trade amount

Other details:
Minimum charge per contract: $4
Maximum charge per contract: $35

Answer:

1. Contract cost = $10 × 100 = $1,000

   Initial commission costs = $30 + ($1,000 × 0.8%) = $38. Because this exceeds the maximum contract charge, $35 is charged (i.e., the maximum contract charge).

2. Gross profit: $280 – $260 = $20 per share. $20 × 100 shares = $2,000

   Additional commission costs = 1% × 2 × $280 × 100 = $560

   Total commission costs = $35 + $560 = $595

   Net profit = $2,000 – $1,000 – $595 = $405

Due to the costs associated with exercising the option and then selling the stock, some retail investors may find it more efficient to simply sell the option to another investor.

One final note on option commission costs is that they fail to account for the cost embedded in the bid-offer spread. The cost associated with this spread for options can be calculated by multiplying the spread by 50%. For example, if the bid price is $12 and the offer price is $12.20, the associated cost for both the option buyer and option seller would be $0.10 per contract [(= $12.20 – $12.00) × 50%]. This cost is also present in stock transactions.

Margin Requirements

Options with maturities nine months or fewer cannot be purchased on margin. This is because the leverage would become too high. For options with longer maturities, investors can borrow a maximum of 25% of the option value.

Investors who engage in writing options must have a margin account due to the high potential losses and potential default. The required margin for option writers is dependent on the amount and position of option contracts written.

Naked options (or *uncovered options*) refers to options in which the writer does not also own a position in the underlying asset. The size of the initial and maintenance margin for naked option writing is equal to the option premium plus a percentage of the underlying share price. Writing *covered calls* (selling a call option on a stock that is owned by the seller of the option) is far less risky than naked call writing.

The Options Clearing Corporation

Similar to a clearinghouse for futures, the **Options Clearing Corporation** (OCC) guarantees that buyers and sellers in the exchange-traded options market will honor their obligations and records all option positions. Exchange-traded options have no default risk because of the OCC, while over-the-counter options possess default risk. The OCC requires that all trades are cleared by one of its clearing members. OCC members must meet net capital requirements and help finance an emergency fund that is utilized in the event of a member default. Non-member brokers must contact a clearing member to clear their option trades. The OCC guarantees contract performance and therefore requires option writers to post margin as a means of supporting their obligation and option buyers to deposit required funds by the morning of the business day immediately following the day the option is purchased.

Exercising an Option

When an investor decides to exercise an option prior to contract expiration, her broker contacts the assigned OCC member responsible for clearing that broker's trades. This OCC member then submits an exercise order to the OCC which matches it with a clearing member who identifies an investor who has written a stock option. This assigned investor then must sell (if a call option) or buy (if a put option) the underlying at the specified strike price on the third business day after the order to exercise is received. Exercising an option results in the open interest being reduced by one. At contract expiration, unexercised options that are in the money after accounting for transaction costs will be exercised by brokers.

Other Option-Like Securities

Exchange-traded options are not issued by the company and delivery of shares associated with the exercise of exchange-traded options involves shares that are already outstanding. *Warrants* are often issued by a company to make a bond issue more attractive and will typically trade separately from the bond at some point. Warrants are like call options except that, upon exercise, the company receives the strike price and may issue new shares to deliver. The same distinction applies to *employee stock options,* which are issued as an incentive to company employees and provide a benefit if the stock price rises above the exercise price. When an employee exercises incentive stock options, any shares issued by the company will increase the number of shares outstanding.

Convertible bonds contain a provision that gives the bondholder the option of exchanging the bond for a specified number of shares of the company's common stock. At exercise, the newly issued shares increase the number of shares outstanding and debt is retired based on the amount of bonds exchanged for the shares. There is a potential for dilution of the firm's common shares from newly issued shares with warrants, employee stock options, and convertible bonds that does not exist for exchange-traded options.

Key Concepts

LO 41.1

A call (put) option gives the owner the right to purchase (sell) the underlying asset at a strike price. When the owner executes this right, the option is said to be exercised. Because long (buy, purchase) option positions give the owner the right to exercise, the seller (short, writer) of the option has the obligation to meet the terms of the option.

American options may be exercised at any time up to and including the contract's expiration date, while European options can be exercised only on the contract's expiration date. Exchange-traded options are typically American options.

Primary types of exchange-traded options include option on individual stocks, foreign currency, stock indices, and futures.

LO 41.2

For a call (put), when the underlying asset price is less (greater) than the strike price, the option is said to be out of the money. For both a call and put, when the underlying asset price is equal to the strike price, the option is said to be at the money. For a call (put), when the underlying asset price is greater (less) than the strike price, the option is said to be in the money. Options are not adjusted for cash dividends, but are adjusted for stock splits.

LEAPS are options with expiration dates greater than a year. Nonstandard option products include FLEX options, ETF options, weekly options, binary options, CEBOs, and DOOM options.

LO 41.3

Options with a maturity of nine months or fewer cannot be purchased on margin and must be paid in full due to the leverage effect of options. For options with longer maturities, investors can borrow up to 25% of the option value. Writers of options are required to have margin accounts with a broker.

Investors must account for commission costs when utilizing option. Commissions vary based on trade size and broker type. Commission rates typically are structured as a fixed dollar amount plus a percentage of the trade amount. In some instances, investors can earn higher profits by selling in-the-money options rather than exercising the options.

The Options Clearing Corporation (OCC) guarantees that buyers and sellers in the options market will honor their obligations and records all option positions. This minimizes default risk.

Warrants, employee stock options, and convertible bonds are option-like securities. Unlike options, these securities are issued by financial institutions or companies. The cost to the issuer of these securities is the possibility of increased dilution of the stock.

Concept Checkers

Use the following data to answer Questions 1 and 2.

An investor owns a stock option that currently has a strike price of $100.

1. If the stock experiences a 4-to-1 split, the strike price becomes:
   A. $20.
   B. $25.
   C. $50.
   D. $100.

2. The number of shares now covered by each option contract is:
   A. 100.
   B. 200.
   C. 300.
   D. 400.

3. If an option is quoted at $2.75, the cost of one contract to the potential buyer is closest to:
   A. $0.275.
   B. $2.75.
   C. $275.00.
   D. $2,750.00.

4. Which of the following statements regarding option value or expiration is correct?
   I. American-style options are less valuable than European options.
   II. All options expire on the third Wednesday of the expiration month.
   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

5. Which of the following option characteristics is correct?
   I. A put option is in the money when the asset price is less than the strike price.
   II. LEAPS are long-term (over one-year) options that expire in December of each year.
   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

Concept Checker Answers

1. B $\frac{a}{b} = \frac{1}{4} \times \$100 = \$25$

2. D $\frac{b}{a} = \frac{4}{1} \times \$100 = \$400$ (Each option contract is originally for 100 shares.)

3. C Multiply the quote by 100 because each option contract is for 100 shares. $2.75 × 100 = $275.00

4. D American-style options are at least as valuable as European-style options. Options expire on the Saturday after the third Friday.

5. A A put option is in the money when the asset price is less than the strike price. LEAPS expire in January.

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Properties of Stock Options

Topic 42

Exam Focus

Stock options have several properties relating both to their value and to the factors that affect their price. Six factors affect option prices: the current value of the stock; the strike price; the time to expiration; the volatility of the stock price; the risk-free rate; and dividends. The value of stock options have upper and lower pricing bounds. Be familiar with these pricing bounds as well as the relationships that exist between the value of European and American options.

Six Factors That Affect Option Prices

LO 42.1: Identify the six factors that affect an option's price and describe how these six factors affect the price for both European and American options.

The following six factors will impact the value of an option:

1. S_0 = current stock price.
2. X = strike price of the option.
3. T = time to expiration of the option.
4. r = short-term risk-free interest rate over T.
5. D = present value of the dividend of the underlying stock.
6. σ = expected volatility of stock prices over T.

When evaluating a change in any one of the factors, hold the other factors constant.

Current Price of the Stock

For call options, as S increases (decreases), the value of the call increases (decreases). For put options, as S increases (decreases), the value of the put decreases (increases). This simply states that as an option becomes closer to or more in-the-money, its value increases.

Strike Price of the Option

The effect of strike prices on option values will be exactly the opposite of the effect of the current price of the stock. For call options, as X increases (decreases), the value of the call decreases (increases). For put options, as X increases (decreases), the value of the put increases (decreases). This is the same as the logic for the current price of the stock: the option's value will increase as it becomes closer to or more in-the-money.

The Time to Expiration

For American-style options, increasing time to expiration will increase the option value. With more time, the likelihood of being in-the-money increases. A general statement cannot be made for European-style options. Suppose we have a 1-month and 3-month call option on the same underlying with the same exercise price. Also suppose a large dividend is expected to be paid in two months. Because the stock price and 3-month option price will fall when the dividend is paid in two months, the 1-month option may be worth more than the 3-month option.

The Risk-Free Rate Over the Life of the Option

As the risk-free rate increases, the value of the call (put) will increase (decrease). The intuition behind this property involves arbitrage arguments that require the use of synthetic securities.

Dividends

The option owner does not have access to the cash flows of the underlying stock, and the stock price decreases when a dividend is paid. Thus, as the dividend increases, the value of the call (put) will decrease (increase).

Volatility of the Stock Price Over the Life of the Option

Volatility is the friend of all options. As volatility increases, option values increase. This is due to the asymmetric payoff of options. Since long option positions have a maximum loss equal to the premium paid, increased volatility only increases the chances that the option will expire in-the-money. Many consider volatility to be the most important factor for option valuation.

Figure 1 summarizes the factors' effects on option prices: "+" indicates a positive effect on option price from an increase in the factor, and "–" is a negative effect on option price.

Figure 1: Summary of Effects of Increasing a Factor on the Price of an Option

Factor	*European Call*	*European Put*	*American Call*	*American Put*
S	+	–	+	–
X	–	+	–	+
T	?	?	+	+
σ	+	+	+	+
r	+	–	+	–
D	–	+	–	+

Upper and Lower Pricing Bounds

LO 42.2: Identify and compute upper and lower bounds for option prices on non-dividend and dividend paying stocks.

In addition to those previously introduced, consider the following variables:

- c = value of a European call option.
- C = value of an American call option.
- p = value of a European put option.
- P = value of an American put option.
- S_T = value of the stock at expiration.

Also, assume in the following examples that there are no transaction costs, all profits are taxed at the same rate, and borrowing and lending can be done at the risk-free rate.

Upper Pricing Bounds for European and American Options

A call option gives the right to purchase one share of stock at a certain price. Under no circumstance can the option be worth more than the stock. If it were, everyone would sell the option and buy the stock and realize an arbitrage profit. We express this as:

$$c \leq S_0 \text{ and } C \leq S_0$$

Similarly, a put option gives the right to sell one share of stock at a certain price. Under no circumstance can the put be worth more than the sale or strike price. If it were, everyone would sell the option and invest the proceeds at the risk-free rate over the life of the option. We express this as:

$$p \leq X \text{ and } P \leq X$$

For a European put option, we can further reduce the upper bound. Since it cannot be exercised early, it can never be worth more than the present value of the strike price:

$$p \leq Xe^{-rT}$$

Lower Pricing Bounds for European Calls on Nondividend-Paying Stocks

Consider the following two portfolios:

- Portfolio P_1: one European call, *c*, with exercise price *X* plus a zero-coupon risk-free bond that pays *X* at *T*.
- Portfolio P_2: one share of the underlying stock, *S*.

At expiration, *T*, Portfolio P_1 will always be the greater of *X* (when the option expires out-of-the-money) or S_T (when the option expires in-the-money). Portfolio P_2, on the other hand, will always be worth S_T. Therefore, P_1 is always worth at least as much as P_2 at expiration. If we know that at T, $P_1 \geq P_2$, then it always has to be true because if it were not, arbitrage would be possible. Therefore, we can state the following:

$$c + Xe^{-rT} \geq S_0$$

Since the value of a call option cannot be negative (if the option expires out-of-the-money, its value will be zero), the lower bound for a European call on a nondividend-paying stock is:

$$c \geq \max\,(S_0 - Xe^{-rT}, 0)$$

Lower Pricing Bounds for European Puts on Nondividend-Paying Stocks

Consider the following two portfolios:

- Portfolio P_3: one European put, p, plus one share of the underlying stock, S.
- Portfolio P_4: zero-coupon risk-free bond that pays X at T.

At expiration, T, Portfolio P_3 will always be the greater of X (when the option expires in-the-money) or S_T (when the option expires out-of-the-money). Portfolio P_4, on the other hand, will always be worth X. Therefore, P_3 is always worth at least as much as P_4 at expiration. If we know that at T, $P_3 \geq P_4$, it has to be true always because if it were not, arbitrage would be possible. Therefore, we can state the following:

$$p + S_0 \geq Xe^{-rT}$$

Since the value of a put option cannot be negative (if the option expires out-of-the-money, its value will be zero), the lower bound for a European put on a nondividend-paying stock is:

$$p \geq \max\,(Xe^{-rT} - S_0, 0)$$

Computing Option Values Using Put-Call Parity

LO 42.3: Explain put-call parity and apply it to the valuation of European and American stock options.

The derivation of **put-call parity** is based on the payoffs of two portfolio combinations, a fiduciary call and a protective put.

A *fiduciary call* is a combination of a pure-discount (i.e., zero coupon), riskless bond that pays X at maturity and a call with exercise price X. The payoff for a fiduciary call at expiration is X when the call is out of the money, and $X + (S - X) = S$ when the call is in the money.

A *protective put* is a share of stock together with a put option on the stock. The expiration date payoff for a protective put is $(X - S) + S = X$ when the put is in the money, and S when the put is out of the money.

Professor's Note: When working with put-call parity, it is important to note that the exercise prices on the put and the call and the face value of the riskless bond are all equal to X.

When the put is in the money, the call is out of the money, and both portfolios pay X at expiration.

Similarly, when the put is out of the money and the call is in the money, both portfolios pay S at expiration.

Put-call parity holds that portfolios with identical payoffs must sell for the same price to prevent arbitrage. We can express the put-call parity relationship as:

$$c + Xe^{-rT} = S + p$$

Equivalencies for each of the individual securities in the put-call parity relationship can be expressed as:

$$\begin{aligned} S &= c - p + Xe^{-rT} \\ p &= c - S + Xe^{-rT} \\ c &= S + p - Xe^{-rT} \\ Xe^{-rT} &= S + p - c \end{aligned}$$

The single securities on the left-hand side of the equations all have exactly the same payoffs as the portfolios on the right-hand side. The portfolios on the right-hand side are the "synthetic" equivalents of the securities on the left. Note that the options must be European-style and the puts and calls must have the same exercise price for these relations to hold.

For example, to synthetically produce the payoff for a long position in a share of stock, you use the relationship:

$$S = c - p + Xe^{-rT}$$

This means that the payoff on a long stock can be synthetically created with a long call, a short put, and a long position in a risk-free discount bond.

The other securities in the put-call parity relationship can be constructed in a similar manner.

Professor's Note: After expressing the put-call parity relationship in terms of the security you want to synthetically create, the sign on the individual securities will indicate whether you need a long position (+ sign) or a short position (– sign) in the respective securities.

Example: Call option valuation using put-call parity

Suppose that the current stock price is \$52 and the risk-free rate is 5%. You have found a quote for a 3-month put option with an exercise price of \$50. The put price is \$1.50, but due to light trading in the call options, there was not a listed quote for the 3-month, \$50 call. **Estimate** the price of the 3-month call option.

Answer:

Rearranging put-call parity, we find that the call price is:

$$\text{call} = \text{put} + \text{stock} - Xe^{-rT}$$

$$\text{call} = \$1.50 + \$52 - \$50e^{-0.0125} = \$4.12$$

This means that if a 3-month, \$50 call is available, it should be priced at \$4.12 per share.

Lower Pricing Bounds for an American Call Option on a Nondividend-Paying Stock

LO 42.4: Explain the early exercise features of American call and put options.

Recall the following equation from our earlier discussion of the lower pricing bounds for a *European* call option:

$$c \geq \max(S_0 - Xe^{-rT}, 0)$$

Since the only difference between an American option and a European option is that the American option can be exercised early, American options can always be used to replicate their corresponding European options simply by choosing not to exercise them until expiration. Therefore, it follows that:

$$C \geq c \geq \max(S_0 - Xe^{-rT}, 0)$$

Note that when an American call is exercised, it is only worth $S_0 - X$. Since this value is never larger than $S_0 - Xe^{-rT}$ for any r and $T > 0$, it is never optimal to exercise early. In other words, the investor can keep the cash equal to X, which would be used to exercise the option early, and invest that cash to earn interest until expiration. Since exercising the American call early means that the investor would have to forgo this interest, it is never optimal to exercise an American call on a nondividend-paying stock before the expiration date (i.e., c = C).

Lower Pricing Bounds for an American Put Option on a Nondividend-Paying Stock

While it is never optimal to exercise an American call on a nondividend-paying stock, American puts are optimally exercised early if they are sufficiently in-the-money. If an option is sufficiently in-the-money, it can be exercised, and the payoff ($X - S_0$) can be invested to earn interest. In the extreme case when S_0 is close to zero, the future value of the exercised cash value, Xe^{rT}, is always worth more than a later exercise, *X*. We know that:

$$P \geq p \geq \max(Xe^{-rT} - S_0, 0) \text{ for the same reasons that } C \geq c$$

However, we can place an even stronger bound on an American put since it can always be exercised early:

$$P \geq \max(X - S_0, 0)$$

Figure 2 summarizes what we now know regarding the boundary prices for American and European options.

Figure 2: Lower and Upper Bounds for Options

Option	*Minimum Value*	*Maximum Value*
European call	$c \geq \max(0, S_0 - Xe^{-rT})$	S_0
American call	$C \geq \max(0, S_0 - Xe^{-rT})$	S_0
European put	$p \geq \max(0, Xe^{-rT} - S_0)$	Xe^{-rT}
American put	$P \geq \max(0, X - S_0)$	X

Professor's Note: For the exam, know the price limits in Figure 2. You will not be asked to derive them, but you may be expected to use them.

Example: Minimum prices for American vs. European puts

Compute the lowest possible price for 4-month American and European 65 puts on a stock that is trading at 63 when the risk-free rate is 5%.

Answer:

$$P \geq \max(0, X - S_0) = \max(0, 2) = \$2$$

$$p \geq \max(0, Xe^{-rT} - S_0) = \max(0, 65e^{-0.0167} - 63) = \$0.92$$

Example: Minimum prices for American vs. European calls

Compute the lowest possible price for 3-month American and European 65 calls on a stock that is trading at 68 when the risk-free rate is 5%.

Answer:

$$C \geq \max(0, S_0 - Xe^{-rT}) = \max(0, 68 - 65e^{-0.0125}) = \$3.81$$

$$c \geq \max(0, S_0 - Xe^{-rT}) = \max(0, 68 - 65e^{-0.0125}) = \$3.81$$

Relationship Between American Call Options and Put Options

Put-call parity only holds for European options. For American options, we have an inequality. This inequality places upper and lower bounds on the difference between the American call and put options.

$$S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$$

Example: American put option bounds

Consider an American call and put option on stock XYZ. Both options have the same 1-year expiration and a strike price of \$20. The stock is currently priced at \$22, and the annual interest rate is 6%. What are the upper and lower bounds on the American put option if the American call option is priced at \$4?

Answer:

The upper and lower bounds on the difference between the American call and American put options are:

$$S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$$

$$S_0 - X = 22 - 20 = \$2$$

$$S_0 - Xe^{-rT} = 22 - 20e^{-0.06(1)} = 22 - 18.84 = \$3.16$$

$$\$2 \leq C - P \leq \$3.16$$

or

$$-\$2 \geq P - C \geq -\$3.16$$

Therefore, when the American call is valued at \$4, the upper and lower bounds on the American put option will be:

$$\$2 \geq P \geq \$0.84$$

The Impact of Dividends on Option Pricing Bounds

Since most stock options have an expiration of less than a year, dividends can be estimated fairly accurately. Recall that to prevent arbitrage, when a stock pays a dividend, its value must decrease by the amount of the dividend. This increases the value of a put option and decreases the value of a call option.

Consider the following portfolios:

- Portfolio P_6: one European call option, *c*, plus cash equal to $D + Xe^{-rT}$.
- Portfolio P_7: one share of the underlying stock, *S*.

Similar to the development of the $c \geq \max(S_0 - Xe^{-rT}, 0)$ equation, Portfolio P_6 is always at least as large as P_7, or:

$$c \geq S_0 - D - Xe^{-rT}$$

All else equal, the payment of a dividend will reduce the lower pricing bound for a call option.

For put options:

- Portfolio P_8: one European put, *p*, plus one share of the underlying stock, *S*.
- Portfolio P_9: cash equal to $D + Xe^{-rT}$.

Using the same development as the $p \geq \max(Xe^{-rT} - S_0, 0)$ equation:

$$p \geq D + Xe^{-rT} - S_0$$

All else equal, the payment of a dividend will increase the lower pricing bound for a put option.

Impact of Dividends on Early Exercise for American Calls and Put-Call Parity

When the dividend is large enough, American calls might be optimally exercised early. This will be the case if the amount of the dividend exceeds the amount of interest that is forgone as a result of the early exercise. Note that if a large dividend makes early exercise optimal, exercise should take place immediately before the ex-dividend date. Put-call parity is adjusted for dividends in the following manner:

$$p + S_0 = c + D + Xe^{-rT}$$

This equation is verified using the same development as was used to derive the $p + S_0 = c + Xe^{-rT}$ equation. The $S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$ equation that we used to show the relationship between American call and put options is modified as follows:

$$S_0 - X - D \leq C - P \leq S_0 - Xe^{-rT}$$

Key Concepts

LO 42.1

Six factors influence the value of an option: current value of the underlying asset (stock); the strike price; the time to expiration of the option; the volatility of the stock price; the risk-free rate; and dividends.

With the exception of time to expiration, all of these factors affect European- and American-style options in the same way.

LO 42.2

Call options cannot be worth more than the underlying security, and put options cannot be worth more than the strike price.

When the stock does not pay a dividend, European call options cannot be worth less than the difference between the current stock price and the present value of the strike price. European put options cannot be worth less than the difference between the present value of the strike price and the current stock price.

LO 42.3

Put-call parity is a no-arbitrage relationship for European-style options with the same characteristics. It states that a portfolio consisting of a call option and a zero-coupon bond with a face value equal to the strike must have the same value as a portfolio consisting of the corresponding put option and the stock:

$$p + S_0 = c + Xe^{-rT}$$

LO 42.4

It is never optimal to exercise an American call option on nondividend-paying stock prior to expiration.

American put options on nondividend-paying stocks can be optimally exercised prior to expiration if the put is sufficiently in-the-money.

Call options are always worth more than corresponding put options prior to expiration when both are at-the-money.

The difference between prices of an American call and corresponding put is bounded below by the difference between the current stock price and strike price, and above by the difference between the current stock price and the present value of the strike price.

Concept Checkers

1. Which of the following will not cause a decrease in the value of a European call option position on XYZ stock?
   A. XYZ declares a 3-for-1 stock split.
   B. XYZ raises its quarterly dividend from $0.15 per share to $0.17 per share.
   C. The Federal Reserve lowers interest rates by 0.25% in an effort to stimulate the economy.
   D. Investors believe the volatility of XYZ stock has declined.

2. Consider a European put option on a stock trading at $50. The put option has an expiration of six months, a strike price of $40, and a risk-free rate of 5%. The lower bound and upper bound on the put are:
   A. $10, $40.00.
   B. $10, $39.01.
   C. $0, $40.00.
   D. $0, $39.01.

3. Consider a 1-year European put option that is currently valued at $5 on a $25 stock and a strike of $27.50. The 1-year risk-free rate is 6%. Which of the following is closest to the value of the corresponding call option?
   A. $0.00.
   B. $3.89.
   C. $4.10.
   D. $5.00.

4. Consider an American call and put option on the same stock. Both options have the same 1-year expiration and a strike price of $45. The stock is currently priced at $50, and the annual interest rate is 10%. Which of the following could be the difference in the two option values?
   A. $4.95.
   B. $7.95.
   C. $9.35.
   D. $12.50.

5. According to put-call parity for European options, purchasing a put option on ABC stock would be equivalent to:
   A. buying a call, buying ABC stock, and buying a zero-coupon bond.
   B. buying a call, selling ABC stock, and buying a zero-coupon bond.
   C. selling a call, selling ABC stock, and buying a zero-coupon bond.
   D. buying a call, selling ABC stock, and selling a zero-coupon bond.

Concept Checker Answers

1. **A** After a stock split, both the price of the stock and the strike price of the option will be adjusted, so the value of the option position will be the same. An increase in the dividend, a lower risk-free interest rate, and lower volatility of the price of the underlying stock, will all decrease the value of a European call option.

2. **D** The upper bound is the present value of the exercise price: $\$40 \times e^{-0.05 \times 0.5} = \$39.01$. Since the put is out-of-the-money, the lower bound is zero.

3. **C** $c = p - Xe^{-rT} + S_0 = \$5 - \$27.50e^{-0.06 \times 1} + \$25 = \$4.10$

4. **B** The upper and lower bounds are: $S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$ or $\$5 \leq C - P \leq \$9.28$. Only \$7.95 falls within the bounds.

5. **B** The formula for put-call parity is $p + S_0 = c + Xe^{-rT}$. Rearranging to solve for the price of a put, we have $p = c - S_0 + Xe^{-rT}$.