FYS-MEK1110 - Mandatory assignment 4

William Dugan

March 28, 2022

a

We are going to sketch the function

$$U(x) = \begin{cases} U_0, & |x| \geq x_0 \\ U_0 \frac{|x|}{x_0}, & |x| < x_0 \end{cases} \tag{1}$$

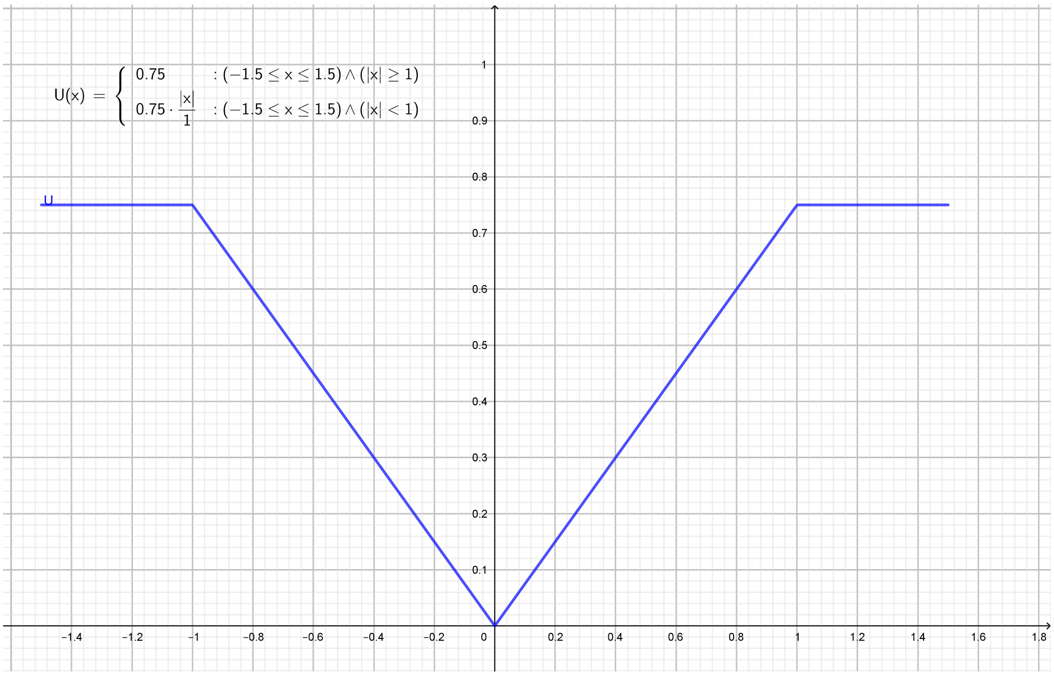

Figure 1: U(x) plotted for $x \in [-1.5, 1.5]$ with $U_0 = 0.75, x_0 = 1$.

We have three equilibrium points, at $x = \pm x_0$ and $x = 0$. The point in $x = 0$ is a stable equilibrium point as the potential is at a minimum. The other are unstable as the potential is at maximum. We can use the relation $E_{atom,total} = KE_{atom} + PE_{atom}$. If $E_{atom,total} \geq PE_{atom}$ the atom will escape the trap, else it will stay inside the trap.

b

To find the force on the atom due to the field we use

$$F = -\nabla U \tag{2}$$

In one dimension, we get $F = -\frac{dU}{dx}$. Hence

$$F(x) = \begin{cases} 0, & x = 0, |x| \geq x_0 \\ -\frac{U_0}{x_0}\frac{x}{|x|}, & 0 < |x| < x_0 \end{cases} \tag{3}$$

Since the force on the atom due to the field can be written as a gradient of a potential, it is per definition a conservative force.

c

Since $\sum F_{ext} = 0$ we can use conservation of energy to find the velocity of an atom with mass m and $v_0 = \sqrt{4U_0/m}$ at $x = x_0/2$ and $x = 2x_0$.

$$\begin{aligned} E_1 &= E_0 \\ \frac{1}{2}mv^2 + U(x) &= \frac{1}{2}mv_0^2 \\ v^2 &= \frac{4U_0}{m} - \frac{2}{m}U(x) \\ v &= \pm\sqrt{\frac{2}{m}(2U_0 - U(x))} \end{aligned}$$

for $x = x_0/2$ we get $v = \sqrt{3U_0/m}$ and for $x = 2x_0$ we get $v = \sqrt{2U_0/m}$.

d

If we change the sign of our x-value, we simply change the sign of the velocity. Hence, for $x = -x_0/2$ we get $v = -\sqrt{3U_0/m}$ and for $x = -2x_0$ we get $v = -\sqrt{2U_0/m}$.

e

If $KE = 0$ at $x = 0$ the electrostatic force F_0 must be greater or equal to $F(x_0)$ in order for the atom to escape. This gives $|F_0| > U_0/x_0$.

f

We introduce the force

$$F = -\alpha v \tag{4}$$

for $|x| < x_0$. The force depends on the velocity of the atom, which means that it depends on the path the atom takes. This makes it non-conservative.

g

To find the acceleration:

$$\sum F = ma$$
$$F_0 + F(x) = ma$$
$$a = \frac{F_0 + F(x)}{m}$$

If we insert our expressions we get

$$a(x) = \begin{cases} 0, & x = 0, |x| \geq x_0 \\ -\frac{U_0}{x_0}\frac{x}{|x|} - \alpha v, & 0 < |x| < x_0 \end{cases} \tag{5}$$

To simulate the movement of the atom we need U_0, x_0, and α.

h

```python
import numpy as np
import matplotlib.pyplot as plt

u0 = 150
m = 23
x0 = 2
alpha = 39.48

n = 100001
t = np.linspace(0, 6, n)
x = np.zeros_like(t)
dt = t[1] - t[0]

for v_, x_ in ((8.0, -5), (10.0, -5)):
    v = v_       # v0
    x[0] = x_    # x0

    for i in range(n-1):
        a = (-np.sign(x[i])*u0/x0 - alpha*v)/m * (np.abs(x[i])<x0)
        v += a*dt
        x[i+1] = x[i] + v*dt

    plt.plot(t, x, label='x(t)')
    plt.xlabel('Time')
    plt.ylabel('Distance')
    plt.legend()
    plt.savefig(f'plot_integrated_v0_{v_:g}.pdf')
    plt.show()
```

i

The conditions used are $U_0 = 150, m = 23, x_0 = 2, \alpha = 39.48$. We simulate with $v_0 = 8.0$ and $x = -5$.

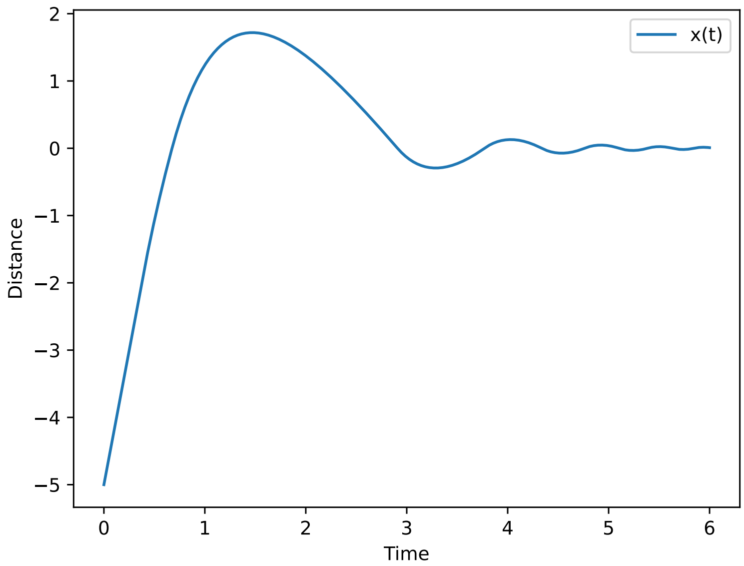

Figure 2: Plot of $x(t)$ for $t \in [0, 6]$

We observe that the atom moves into the trap and continues past $x = 0$. It will then experience a force in negative x-direction, which accelerates it towards $x = 0$ again. Since F_0 is not conservative, the motion observed follows damped harmonic motion. It ends in $x = 0$ as this is the only stable equilibrium point.

j

Now, we simulate with $v_0 = 10.0$ and $x = -5$.

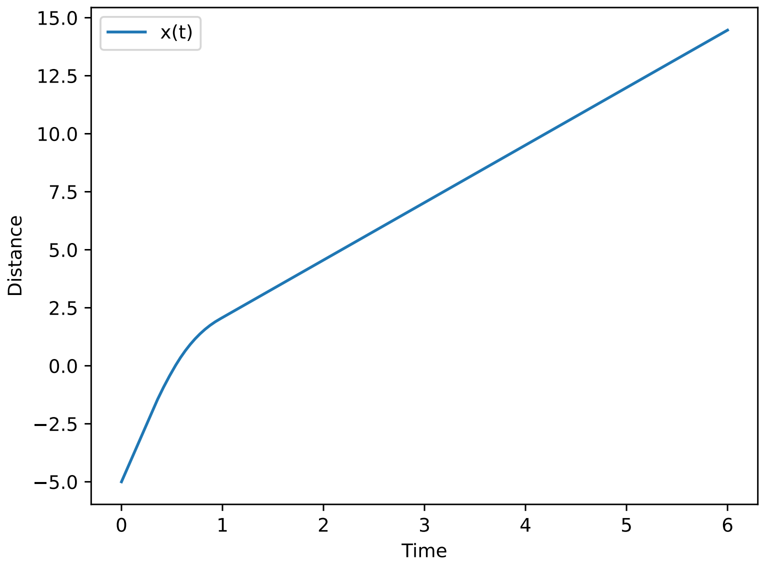

Figure 3: Plot of $x(t)$ for $t \in [0, 6]$

In this simulation, the atom has enough energy to escape the trap on the other side. When it reaches $x = x_0 = 2$ the force on the atom will be 0, and it will continue forever with constant velocity.