14.3.13 $\oint_C \frac{z+2}{z-2}\,dz, \quad C: |z-1| = 2$

$= 2\pi i\,(2+2) = 8\pi i$

14.3.18 $\oint_C \frac{\sin z}{4z^2 - 8iz}\,dz \overset{\div 4z}{=} \oint_C \frac{\frac{1}{4z}\sin z}{z-2i}\,dz = 2\pi i \cdot \frac{1}{4i \cdot 2}\sin(2i)$

$= \underline{\frac{\pi}{4}\sin(2i)} = \frac{\pi}{4}\left(e^{i(2i)} - e^{-i(2i)}\right)\cdot\frac{1}{2i} = \frac{\pi}{8i}\left(e^{-2} - e^{2}\right)$

$= \frac{\pi i}{8}\left(e^{2} - e^{-2}\right) = \underline{\underline{2.85i}}$

14.4.2 $\oint_C \frac{z^6}{(2z-1)^6}\,dz = \frac{2\pi i}{n!}\,f^{(5)}(z_0)$

$z_0 = 1/2, \quad f(z) = 2^{-6} z^6 \cancel{/2^6}$

$f^{(5)} = 6\cdot5\cdot4\cdot3\cdot2\,z = 720\,z \cdot 2^{-6}$

$\Rightarrow \cancel{I = \frac{2\pi i}{5!}\cdot\frac{720}{\sqrt[6]{2}}\cdot\frac{1}{2} = 8.397i}$

$I = \frac{2\pi i}{5!}\cdot\frac{6\cdot5\cdot4\cdot3\cdot2}{2^6}\cdot\frac{1}{2} = \underline{0.295i}$

14.4.7 $\oint_C \frac{\cos z}{z^{2n+1}}\,dz, \quad n \in \mathbb{N} \cup \{0\} = \frac{2\pi i}{(2n)!}\,f^{(2n)}(z_0)$

$z_0 = 0, \quad f(z) = \cos z$

$I = \frac{2\pi i}{(2n)!}\underbrace{\left(\frac{d}{dz}\right)^{2n}\cos z\,\Big|_{z=0}}_{\pm 1,\ +1 \text{ siden } 2n \text{ like}} = \frac{2\pi i}{(2n)!}$

14.4.16 $\oint_C \cancel{\cosh(4z)}\,\frac{e^{4z}}{z(z-2i)^2}\,dz, \quad C: \begin{matrix}|z-i| = 3 & \text{pos.}\\ |z| = 1 & \text{neg.}\end{matrix}$

$n=1, \quad z_0 = 2i, \quad f = \frac{1}{z}e^{4z}, \quad f' = 4\,\tfrac{1}{z}\,e^{4z} - \frac{1}{z^2}e^{4z}$

$f'(2i) = 4e^{4\cdot2i} + \frac{1}{4}e^{4\cdot2i} = \frac{5}{4}e^{8i} = \frac{5}{4}\cdot(-0.1455 + i\cdot0.989)$

$I = 2\pi i \cdot f'(z_0) = \frac{5\pi i}{2}(\cos 8 + i\sin 8)$

$= -\frac{5}{2}\pi\sin 8 + \frac{5}{2}\pi i\cos 8$

$= \underline{-7.77 - 1.14i}$

15.1.17 $\displaystyle\sum_{n=2}^{\infty} \frac{(-i)^n}{\ln n}$

F.h.t: $\displaystyle\lim_{n\to\infty}\left|\frac{z_{n+1}}{z_n}\right| = \lim_{n\to\infty}\left|\frac{(-i)^{n+1}}{\ln(n+1)}\cdot\frac{\ln n}{(-i)^n}\right| = \lim_{n\to\infty}\left|-i\,\frac{\ln n}{\ln(n+1)}\right| \cancel{\to 0}$ ~~div.~~ $= 1$

siden $\ln n \approx \ln(n+1)$ for høye n

S.m.l test: $b_n = 1/n^2$

$$\frac{|z_n|}{b_n} = \frac{1/\ln n}{1/n^2} = \frac{n^2}{\ln n} \xrightarrow[(L'H)]{n\to\infty} \infty \Rightarrow \text{div.}$$

$$\frac{2n}{1/n} = 2n^2$$

15.1.18 $\displaystyle\sum_{n=1}^{\infty} n^2\left(\frac{i}{4}\right)^n$

$$\left|\frac{z_{n+1}}{z_n}\right| = \left|\frac{(n+1)^2\cdot i^{n+1}}{4^{n+1}}\cdot\frac{4^n}{n^2\, i^n}\right| = \frac{1}{4}\,\frac{(n+1)^2}{n^2} \xrightarrow{n\to\infty} \frac{1}{4} < 1 \quad \text{abs. konv.}$$

15.2.5 $\displaystyle f(z) = \sum_{n=0}^{\infty} a_n z^{2n}, \quad g(z) = \sum_{n=0}^{\infty} a_n z^n$

$R_g = R$. Vis at $R_f = \sqrt{R}$. $R < \infty$.

f og ~~g~~ g har samme senter og koeff. Videre er $z^{2n} = (z^2)^n$. Hvis g konv. i $R_f = R$ har vi ~~konv. at~~ $\sum_n a_n R^n$ som konv. Det er herfra klart at $R_f = \sqrt{R}$ slik at vi k. $\sum_n a_n\left[(\sqrt{R})^2\right]^n$ også konv.

15.2.10 $\displaystyle\sum_{n=0}^{\infty} \frac{(z-2i)^n}{n^n}$, $z_0 = 2i$, $a_n = \frac{1}{n^n}$

$$\cancel{\left|\frac{a_n}{a_{n+1}}\right| = \left|\frac{(z-2i)^n}{n^n}\cdot\frac{(n+1)^{n+1}}{(z-2i)^{n+1}}\right| = \left|\frac{(n+1)^{n+1}}{n^n}\cdot\frac{1}{(z-2i)}\right| = 1}$$

$$L^* = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{1}{(n+1)^{n+1}}\cdot n^n\right| \quad \left|\frac{n^n}{(n+1)(n+1)^n}\right| \xrightarrow{n\to\infty} 0$$

rottest heller?

$R = 1/L^* = \underline{\underline{\infty}}$.

15.2.14 $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(n!)^2} z^{2n}$, $z_0 = 0$, $a_n = \dfrac{(-1)^n}{2^{2n}(n!)^2}$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\cancel{(-1)^{n+1}}}{2^{2n+2}((n+1)!)^2}\cdot\frac{2^{2n}(n!)^2}{\cancel{(-1)^n}}\right| = \left|\frac{1}{2^2}\cdot\left[\frac{n!}{(n+1)!}\right]^2\right| = \left|\frac{1}{4}\,\frac{1}{(n+1)^2}\right|$$

$$\xrightarrow{n\to\infty} 0 \Rightarrow R = \underline{\underline{\infty}}$$