Brushing the Hairs of Transcendental Functions

Willie Rush Lim

Graduate Student Seminar

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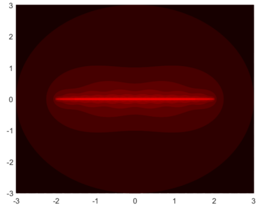
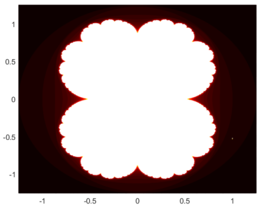
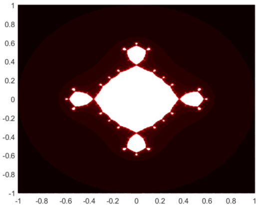
Escaping set of polynomials (in black)



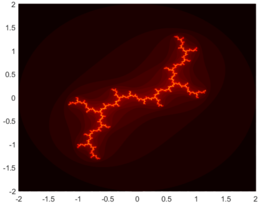
$z^2 - 2$



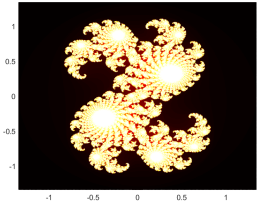
$z^2 + z$



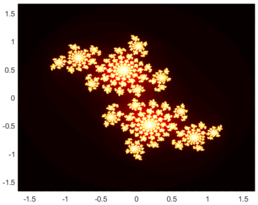
$8z^4 - 0.5$



$iz^4 + 2z^2 - 1 - i$



$z^2 + 0.33 + 0.06i$



$z^2 + 0.02 + 0.66i$

Escaping set of a transcendental entire function

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- $I(f) \neq \emptyset$.
- If $f \in \mathcal{B}$, then $I(f)$ is nowhere dense.
- If $f \in \mathcal{B}$, then $J(f) = \partial I(f) = \overline{I(f)}$.

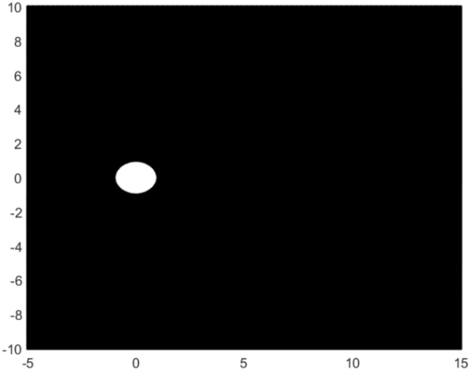
Escaping set of a transcendental entire function

Theorem (Rempe-Gillen, Schleicher, '11)

If $f \in \mathcal{B}$ has finite order, then

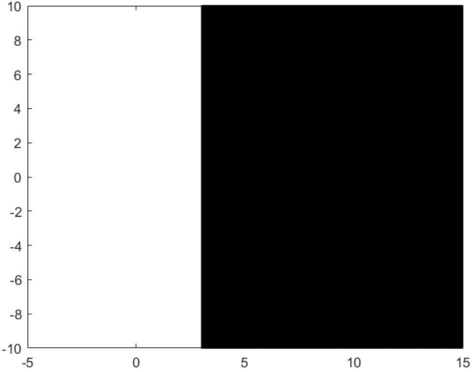
- *any $z \in I(f)$ can be joined to ∞ by a path in $I(f)$,*
- *each component of $J(f)$ is homeomorphic to an infinite ray.*

$f(z) = e^{z-2}$



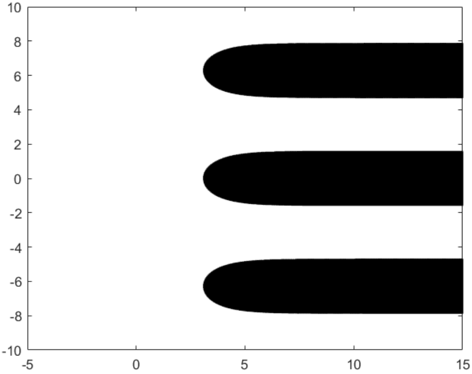
$$\bar{W} = \{|z| > 1\}$$

$f(z) = e^{z-2}$



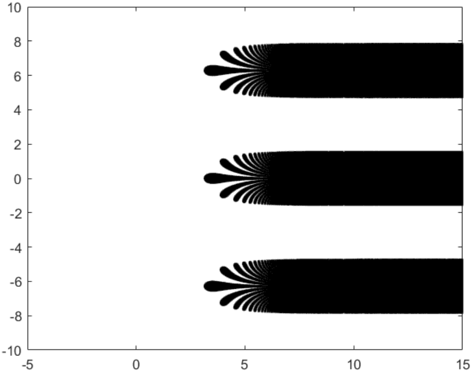
$$f^{-1}(\bar{W}) = \{\text{Re} z \geq 3\}$$

$f(z) = e^{z-2}$



$f^{-2}(\bar{W})$

$f(z) = e^{z-2}$



$f^{-3}(\bar{W})$

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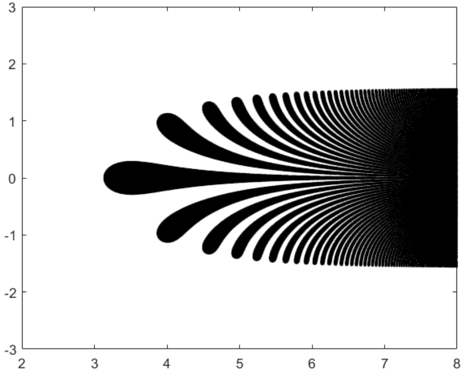

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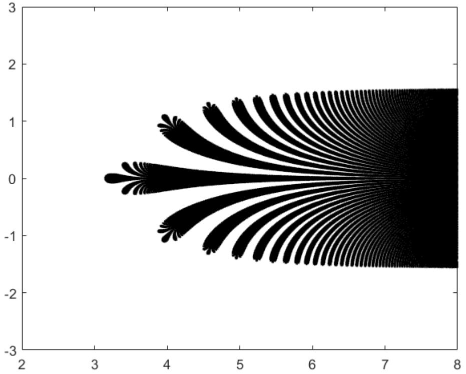

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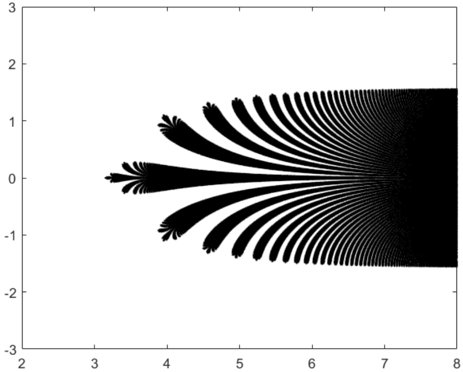
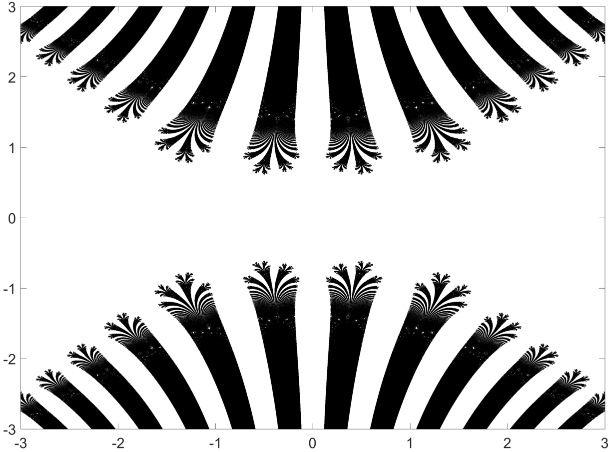
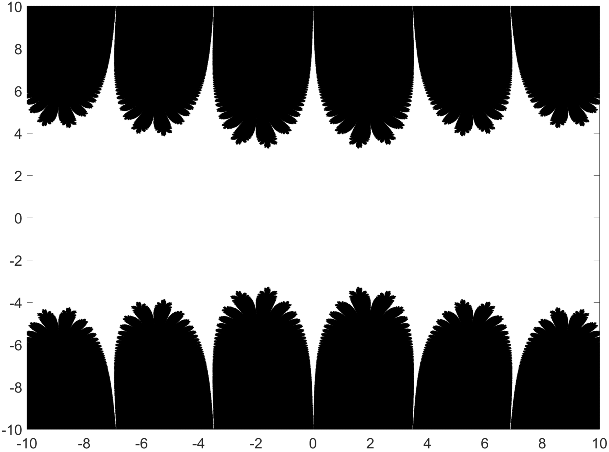


$f^{-50}(\bar{W})$

$f(z) = z^2 e^{-z^2}$



$f(z) = \frac{\sin z}{z}$



Escaping set of a transcendental entire function

Theorem (Baranski, Jacque, Rempe-Gillen, '12)

If a disjoint type $f \in \mathcal{B}$ has finite order, then $J(f)$ is ambiently homeomorphic to a straight brush.

Escaping set of a transcendental entire function

Theorem (Baranski, Jacque, Rempe-Gillen, '12)

If a disjoint type $f \in \mathcal{B}$ has finite order, then $J(f)$ is ambiently homeomorphic to a straight brush.

Corollary

The set of endpoints E of $J(f)$ is totally separated, but $E \cup \{\infty\}$ is connected.

Thank you.