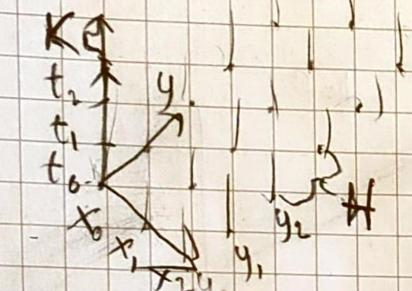
Numerisk løsning av varmelikningen i to romlige dimensjoner (Eksplisitt)

Utgangspunkt i (alle konstanter = 0

$$u_t = \Delta u = u_{xx} + u_{yy}$$

$$\frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d^2 f}{dx^2} = \lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$



$$t_k = t_0 + kK \qquad x_i = x_0 + iH \qquad y_j = y_0 + jH$$

$$u_t = u_{xx} + u_{yy}$$

$$\Rightarrow \frac{u(x_i, y_j, t_k + K) - u(x_i, y_j, t_k)}{K} = \frac{u(x_i + H, y_j, t_k) - 2u(x_i, y_j, t_k) + u(x_i - H, y_j, t_k)}{H^2} + \frac{u(x_i, y_j + H, t_k) - 2u(x_i, y_j, t_k) + u(x_i, y_j - H, t_k)}{H^2}$$

$u(x_i, y_j, t_k) = u_{i,j,k}$

$$\Rightarrow \frac{u_{i,j,k+1} - u_{i,j,k}}{K} = \frac{u_{i+1,j,k} + u_{i-1,j,k} - 4u_{i,j,k} + u_{i,j+1,k} + u_{i,j-1,k}}{H^2}$$

$$\Rightarrow u_{i,j,k+1} = \frac{K}{H^2}\left(u_{i+1,j,k} + u_{i-1,j,k} - 4u_{i,j,k} + u_{i,j+1,k} + u_{i,j-1,k}\right) + u_{i,j,k}$$

Kan bruke python for å beregne utviklingen med denne formelen og klar $\partial\Omega = 0$ og $u(x,y,0) = f(x,y)$