

## Chapter 6

*14. Use the pumping lemma to show that each of the following sets is not regular.*

**(a) The set of palindromes over  $\{a, b\}$ .**

To Prove: the set of palindromes over  $\{a, b\}$  is not regular

Let  $L$  = the set of palindromes over  $\{a, b\}$ , assume  $L$  is regular

Let  $k$  be the number from the pumping lemma

Let  $s = a^{k+1}ba^{k+1}$

By the pumping lemma  $s = uvw$  where  $v \neq \lambda$  and  $|uv| \leq k$

Since  $|uv| \leq k$ ,  $uv$  must consist of  $a$

Since  $v \neq \lambda$ ,  $v$  must consist of one or more  $a$  and  $u$  is the empty string

Suppose we pump

By the pumping lemma

$s = uvvw$ ,  $u = \lambda$ ,  $vv = aa$ ,  $w = ba$ ,  $s = aaba$

$aaba$  IS NOT a palindrome

Contradiction!  $L$  is not regular

**(b)  $\{a^n b^m \mid n < m\}$**

To Prove:  $\{a^n b^m \mid n < m\}$  is not regular

Let  $L = \{a^n b^m \mid n < m\}$ , assume  $L$  is regular

Let  $k$  be the number from the pumping lemma

Let  $s = a^k b^m$  where  $k < m$

By the pumping lemma  $s = uvw$  where  $v \neq \lambda$  and  $|uv| \leq k$

Let  $u = \lambda$  and  $v = a^k$  and  $w = b^m$  so that  $|uv| \leq k$  and  $v \neq \lambda$

Suppose we pump once starting from  $s = abb$

By the pumping lemma

$s = aabb$

The number of  $a$ 's is not less than the numbers of  $b$ 's. Contradiction!  $L$  is not regular.

(c)  $\{a^i b^j c^{2j} \mid i \geq 0, j \geq 0\}$

To Prove:  $\{a^i b^j c^{2j} \mid i \geq 0, j \geq 0\}$  is not regular

Let  $L = \{a^i b^j c^{2j} \mid i \geq 0, j \geq 0\}$ , assume  $L$  is regular

Let  $k$  be the number from the pumping lemma

Let  $s = abcc$

By the pumping lemma  $s = uvw$  where  $v \neq \lambda$  and  $|uv| \leq k$

Let  $v = a^i b^j c^{2j}$

Suppose we pump once

By the pumping lemma

$s = abccabcc$

Contradiction!  $L$  is not regular.