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CNC - Formal Languages
Assignment 1

1.

a) X U Y = {0,1,2,3,4,6}
b) X ∩ Y = {2,4}
c) X - Y = {1-0,2-2,3-4,4-6} = {1,0,-1,-2}
d) Y - X = {0-1,2-2,4-3,6-4} = {-1,0,1,2}
e) P(X) = {∅, {1}, {2}, {3}, {4}, {1,2}, {1,3}, {1,4}, {2,3}, {2,4}, {3,4}, {1,2,3}, {1,2,4}, {1,3,4}, {2,3,4}, {1,2,3,4}}

4. The first element of X and the first element of Y are equal:
x(0) = 0^3 + 3*0^2 + 3*0 = 0
y(1) = 1^3 - 1 = 0

The next element of X and Y are equal
x(1) = 1^3 + 3*1^2 + 3*1 = 7
y(2) = 2^3 - 1 = 7

x(n) = n^3 + 3n^2 +3n
y(n+1) = (n+1)^3 - 1

x(n) = y(n+1)
n^3 + 3n^2 +3n = (n+1)^3 - 1
RHS:
(n+1)(n+1)(n+1) - 1
(n+1)(n^2 + 2n + 1) - 1
(n^3 + n^2 + 2n^2 + 2n + n +1) - 1
(n^3 + 3n^2 + 3n) + 1 - 1
n^3 + 3n^2 + 3n

6.

a) reflexive and symmetric but not transitive.

S = {1,2,3}, R = {(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2)}

b) reflexive and transitive but not symmetric.

S = {1,2,3}, R = {(1,1), (1,2), (1,3), (2,2), (2,3), (3,3)}

c) symmetric and transitive but not reflexive.

S = {1,2,3}, R = { }

10.

22.

29. Give a recursive definition of the predecessor operation

Basis: pred(0) = 0, pred(1) = 0
Recursive Step: pred(n) = pred(n-1) + s(0)

34. Prove that $1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$ for all n > 0

This can be rewritten as $2^0 + 2^1 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$
Or the sum of 2^i from i=0 to n

P(0): n = 0, $2^0 = 2^{0+1} - 1$
$1 = 2^1 - 1$
$1 = 1$
P(k): $2^0 + 2^1 + \ldots 2^k = 2^{k+1} - 1$

P(k+1): $2^0 + 2^1 + \ldots + 2^k + 2^{k+1} = 2^{k+1+1} - 1$
$2^{k+1} - 1 + 2^{k+1} = 2^{k+2} - 1$, substitute the original summation with closed form
$2 * 2^{k+1} - 1 = 2^{k+2} - 1$
Note: $2 * 2^{k+1} = 2^1 * 2^{k+1} = 2^{1+k+1} = 2^{k+2}$ via the product rule
$2^{k+2} - 1 = 2^{k+2} - 1$

38.

42.

46.