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CNC - formallang
Assignment 2

Chapter 7

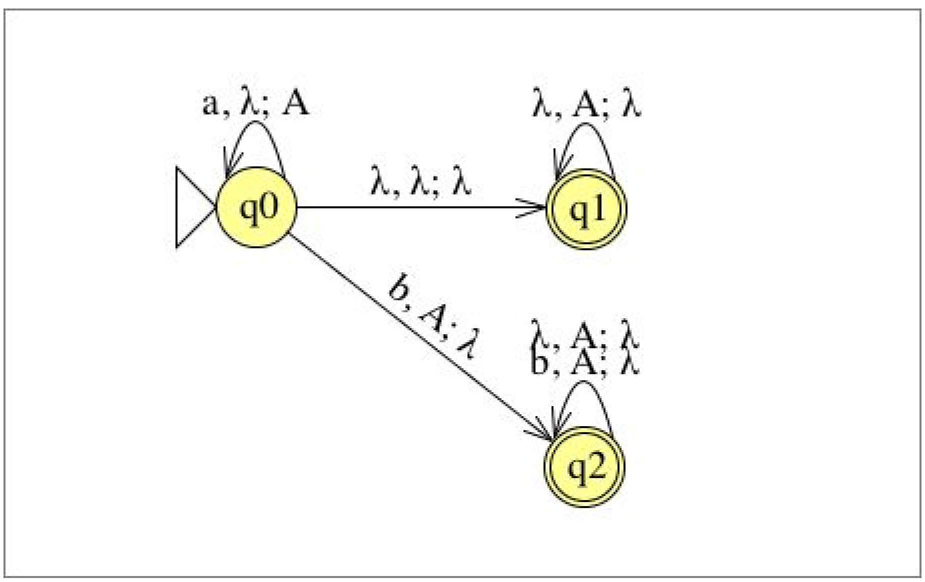
1. Let M be the PDA defined by

$Q = \{q0, q1, q2\}$	$\delta(q0, a, \lambda) = \{[q0, A]\}$
$\Sigma = \{a, b\}$	$\delta(q0, \lambda, \lambda) = \{[q1, \lambda]\}$
$\Gamma = \{A\}$	$\delta(q0, b, A) = \{[q2, \lambda]\}$
$F = \{q1, q2\}$	$\delta(q1, \lambda, A) = \{[q1, \lambda]\}$
	$\delta(q1, b, A) = \{[q2, \lambda]\}$
	$\delta(q2, \lambda, A) = \{[q2, \lambda]\}$

(a) Describe the language accepted by M.

$L(M) = \{ a^i b^j \mid i \geq j \}$

(b) Give the state diagram of M.



(c) Trace all computations of the strings aab, abb, aba in M.

aab:

$[q0, aab, \lambda]$
$\vdash [q0, ab, A]$
$\vdash [q0, b, AA]$
$\vdash [q2, \lambda, A]$
$\vdash [q2, \lambda, \lambda]$

abb:

$[q0, abb, \lambda]$
$\vdash [q0, bb, A]$
$\vdash [q2, b, \lambda]$
$\vdash$ tries to pop an A from the stack, but it's empty -- rejected

aba:

$[q0, aba, \lambda]$
$\vdash [q0, ba, A]$
$\vdash [q2, a, \lambda]$
$\vdash$ tries to pop an A from the stack, but it's empty -- rejected

(d) Show that aabb, aaab $\in$ L(M).

A string is in the language if there is a computation of the PDA that accepts the string.

aabb:

$[q0, aabb, \lambda]$
$\vdash [q0, abb, A]$
$\vdash [q0, bb, AA]$
$\vdash [q2, b, A]$
$\vdash [q2, \lambda, \lambda]$ accepted

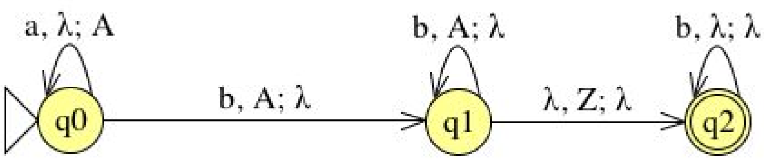
aaab:

$[q0, aaab, \lambda]$
$\vdash [q0, aab, A]$
$\vdash [q0, ab, AA]$
$\vdash [q0, b, AAA]$
$\vdash [q2, \lambda, AA]$
$\vdash [q2, \lambda, A]$
$\vdash [q2, \lambda, \lambda]$ accepted

3. Construct PDAs that accept each of the following languages

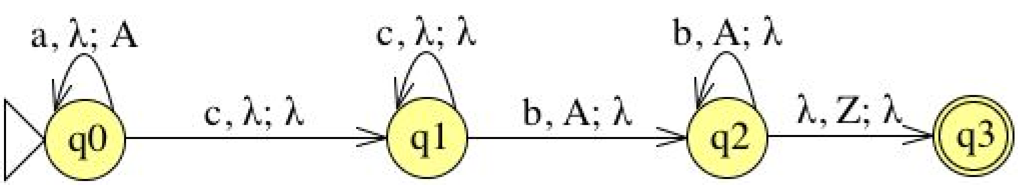
(a) { $a^i b^j$ | $0 \le i \le j$ }

Q = {q0,q1,q2}	δ(q0, b, A) = {[q1, λ]}
Σ = {a, b}	δ(q0, a, λ) = {[q0, A]}
Γ = {A}	δ(q1, b, A) = {[q1, λ]}
F = {q1,q2}	δ(q1, λ, Z) = {[q2, λ]}
S = q0	δ(q2, b, λ) = {[q2, λ]}
z = Z	



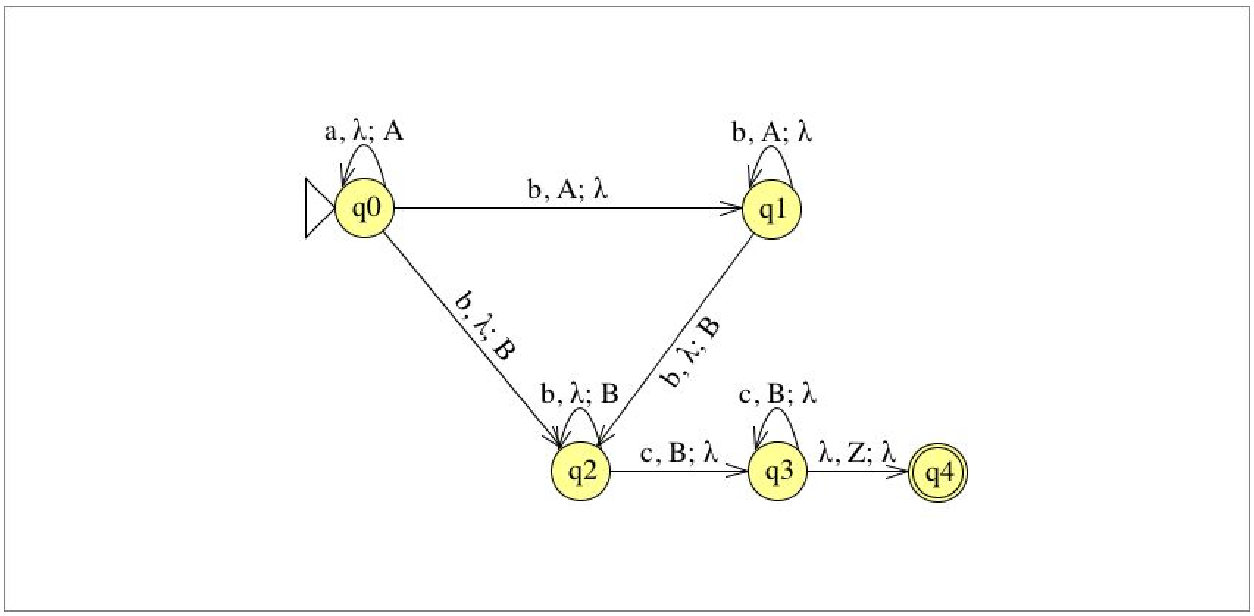
(b) { $a^i c^j b^i$ | i, j ≥ 0 }

Q = {q0,q1,q2,q3}	δ(q0, a, λ) = {[q1, A]}
Σ = {a, b, c}	δ(q0, c, λ) = {[q1, λ]}
Γ = {A,B,Z}	δ(q1, c, λ) = {[q1, λ]}
F = {q3}	δ(q1, b, A) = {[q2, λ]}
S = q0	δ(q2, b, A) = {[q2, λ]}
z = Z	δ(q2, λ, Z) = {[q3, λ]}



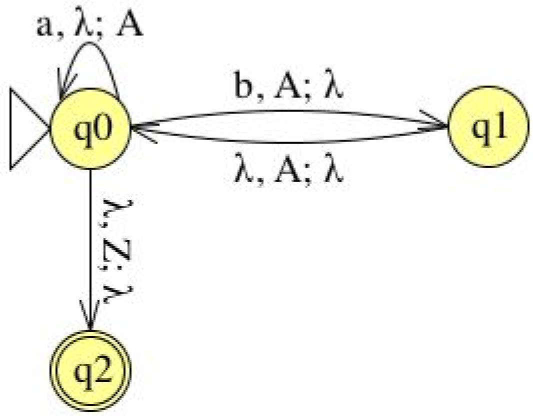
(c) { $a^i b^j c^k$ | i + k = j }

Q = {q0,q1,q2,q3,q4}	$\boldsymbol{\delta}$(q0, a, λ) = {[q1, A]}
Σ = {a, b}	$\boldsymbol{\delta}$(q0, b, A) = {[q0, λ]}
Γ = {A,B,Z}	$\boldsymbol{\delta}$(q0, b, λ) = {[q1, B]}
F = {q4}	$\boldsymbol{\delta}$(q1, b, A) = {[q1, λ]}
S = q0	$\boldsymbol{\delta}$(q1, b, λ) = {[q2, B]}
z = Z	$\boldsymbol{\delta}$(q2, b, λ) = {[q2, B]}
	$\boldsymbol{\delta}$(q2, c, B) = {[q3, λ]}
	$\boldsymbol{\delta}$(q3, c, B) = {[q3, λ]}
	$\boldsymbol{\delta}$(q3, λ, Z) = {[q4, λ]}



(d) { w | w ∈ {a,b}* and w has twice as many a's as b's }

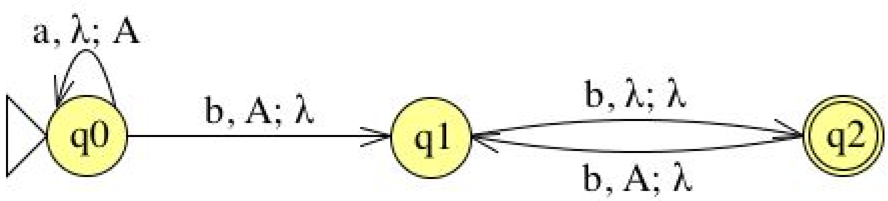
Q = {q0,q1,q2}	$\boldsymbol{\delta}$(q0, a, λ) = {[q1, A]}
Σ = {a, b}	$\boldsymbol{\delta}$(q0, b, A) = {[q1, λ]}
Γ = {A,Z}	$\boldsymbol{\delta}$(q0, λ, Z) = {[q2, λ]}
F = {q2}	$\boldsymbol{\delta}$(q1, λ, A) = {[q0, λ]}
S = q0	
z = Z	



14. Let M be the PDA

Q = {q0,q1,q2}	δ(q0, a, λ) = {[q0, A]}
Σ = {a, b}	δ(q0, b, A) = {[q1, λ]}
Γ = {A}	δ(q1, b, λ) = {[q2, λ]}
F = {q2}	δ(q2, b, A) = {[q1, λ]}

(a) Give the state diagram of M.



(b) Give a set-theoretic definition of L(M).

(c) Using the technique from Theorem 7.3.2, build a context-free grammar G that generates L(M).

New Transitions:

δ'(q0, a, A) = {[q0, AA]}
δ'(q0, b, A) = {[q1, λ]}
δ'(q1, b, A) = {[q2, A]}
δ'(q2, b, A) = {[q1, λ]}

Still need create the grammar rules from the transitions

(d) Trace the computation of aabbbb in M.

$[q0, aabbbb, \lambda]$
$\vdash [q0, abbbb, A]$
$\vdash [q0, bbbb, AA]$
$\vdash [q1, bbb, A]$
$\vdash [q2, bb, A]$
$\vdash [q1, b, \lambda]$
$\vdash [q2, \lambda, \lambda]$ accepted

(e) Give the derivation of aabbbb in G.

17.

(a)

(c)

(d)

19.

(a)

(b)

(c)