

Chapter 8

3. Construct a Turing machine with input alphabet $\{a, b\}$ to perform each of the following operations. Note that the tape head is scanning position zero in state q_f whenever a computation terminates.

(b) Concatenate a copy of the reversed input string to the input. Input configuration $q_0\text{BuB}$, result $q_f\text{Buu}^R\text{B}$.

```
; Concatenate a copy of the reversed input string to the input.  
; Input configuration  $q_0\text{BuB}$ , result  $q_f\text{Buu}^R\text{B}$ .  
; '<current state> <current symbol> <new symbol> <direction> <new state>'
```

```
; Algorithm:  
; convert letter to uppercase and copy to the right  
; go back to input char in original tape, check for done  
; if not done, keep copying  
; if done, cleanup
```

```
; start state  
 $0\ a\ * \ r\ 0$   
 $0\ b\ * \ r\ 0$   
 $0\ \_ \ * \ l\ ci$ 
```

```
; copy initial tape  
 $ci\ a\ A\ r\ mra$   
 $ci\ b\ B\ r\ mrb$   
 $ci\ \_ \ * \ r\ cp$ 
```

```
; move right to paste a  
 $mra\ a\ * \ r\ mra$   
 $mra\ b\ * \ r\ mra$   
 $mra\ A\ * \ r\ mra$   
 $mra\ B\ * \ r\ mra$   
 $mra\ \_ \ * \ r\ pa$ 
```

```
; move right to paste b  
 $mrb\ a\ * \ r\ mrb$   
 $mrb\ b\ * \ r\ mrb$   
 $mrb\ A\ * \ r\ mrb$   
 $mrb\ B\ * \ r\ mrb$   
 $mrb\ \_ \ * \ r\ pb$ 
```

```
; paste a
pa a * r pa
pa b * r pa
pa _ a l mlc
```

```
; move left to copy
mlc _ * l mlc
mlc A * l mlc
mlc B * l mlc
mlc a * l cfd
mlc b * l cfd
```

```
; paste b
pb a * r pb
pb b * r pb
pb _ b l mlc
```

```
; check for done
cfd _ * l cfdd
cfdd A * * cl
cfdd B * * cl
cfdd _ * r cfddd
cfddd _ * r cfddd
cfddd a * * ci
cfddd b * * ci
```

```
; cleanup
cl A _ r cla
cl B _ r clb
cl _ * r finish
```

```
; cleanup paste a
cla _ a l sl
```

```
; cleanup paste b
clb _ b l sl
```

```
; shift left
sl * * l cl
```

```
; shift right
finish * * r halt-accept
```

5. Construct a Turing machine with input alphabet {a, b} to accept each of the following languages by final state.

(b) $\{ a^i b^j a^i b^j \mid i, j > 0 \}$

```
; Accept  $L = \{ a^i b^j a^i b^j \mid i, j > 0 \}$ 
; Example strings = {"abbabb", "aabaab", "aaabbbbaabbb"}
; '<current state> <current symbol> <new symbol> <direction> <new state>'

; Algorithm:
; Check first group of a's
; - see an a, mark it, move right till b's, move right till next a, mark it
; - - if you don't find the a, and come to a blank - fail
; - move to beginning
; Check first group of b's
; - move right till first b, mark it, move right till first A, move right till next b,
mark it
; - - if you don't find the b, and come to a blank - fail
; Move to beginning
; Convert char to lowercase
; - if anything is already lowercase, fail
; Reach end of tape
; Goto beginning of tape
; Halt

; Initial state
0 * * * cfa

; Check First A's (cfa)
cfa a A r mrb
cfa A * r cfa
cfa b * * cfb
mrb a * r mrb
mrb b * r mra
mra b * r mra
mra A * r mra
mra a A r mre
mre b * r meb
mre a * * gta
mre _ * * gta
meb b * r meb
meb _ * l gta
meb a * * halt
mra _ * * halt

; Goto Beginning after a's (gta)
gta a * l gta
gta b * l gta
```

```
gta A * 1 gta
gta B * 1 gta
gta _ * r cfa
```

```
; Check B's (cfb)
```

```
cfb b B r mrr
cfb A * r cfb
cfb a * * halt
cfb B * r cfb
cfb _ * 1 gtc
mrr a * r mrr
mrr A * r mrr
mrr B * r mrr
mrr b B 1 gtb
mrr _ * * halt
```

```
; Goto Beginning after b's (gtb)
```

```
gtb a * 1 gtb
gtb b * 1 gtb
gtb A * 1 gtb
gtb B * 1 gtb
gtb _ * r cfb
```

```
; Goto Beginning to clean (gtc)
```

```
gtc A a 1 gtc
gtc B b 1 gtc
gtc _ * r halt-accept
```

24.

25.