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CNC - Formal Languages
Assignment 2

14. Let X_1, \dots, X_n be a partition of a set X . Define an equivalence relation \equiv on X , whose equivalence classes are precisely the sets X_1, \dots, X_n .

If each element is related to itself

$$X_n R X_n \text{ if } X_n = X_n$$

$[X_n] = \{X_n\}$. This relation partitions X into X_n equivalence classes.

20. Prove that there are an uncountable number of total functions from N to $\{0, 1\}$

Using Cantor's diagonalization argument.

Let S = all of the functions from N to $\{0, 1\}$

$$S = \{f_1, f_2\}$$

$$f(n) = 0 \text{ if } f_n(n) = 1$$

$$f(n) = 1 \text{ if } f_n(n) = 0$$

$$f_n \neq f$$

30. Give a recursive definition of the relation greater than on $N \times N$ using the successor operator `s`.

> gt' a b

> | a == 0 = False

> | a == s b = True

> | gt a (s b) = True

> | otherwise = False

> s n = n + 1

33. Give a recursive definition of multiplication of natural numbers using the operations `s` and addition.

$$x * 0 = 0$$

$$x * S(y) = (x * y) + x$$

As close as I can come in Haskell:

> mult a 0 = 0

> mult a b = (mult a (pred b)) + a

> pred 1 = 0

> pred n = n - s(0)

40.

47.