

Chris Fenton
CSF - Discrete Math
Homework #3

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Chapter 2.1
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2. Use set builder notation to give a description of each of these sets.

a) $\{0, 3, 6, 9, 12\}$

$\{ x \in \mathbb{R} : 0 \leq x \leq 4 \}$

5. Determine whether each of these pairs of sets are equal.

a) $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}, \{5, 3, 1\}$

True. $\{1, 3, 3, 3, 5, 5, 5, 5, 5\} == \{5, 3, 1\}$

6. Suppose that $A = \{2, 4, 6\}$, $B = \{2, 6\}$, $C = \{4, 6\}$, and $D = \{4, 6, 8\}$. Determine which of these sets are subsets of which other of these sets.

$B \subseteq A$

$C \subseteq A, C \subseteq D$

10. Determine whether these statements are true or false.

a) $\emptyset \in \{\emptyset\}$, True

b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$, True

c) $\{\emptyset\} \in \{\emptyset\}$, False

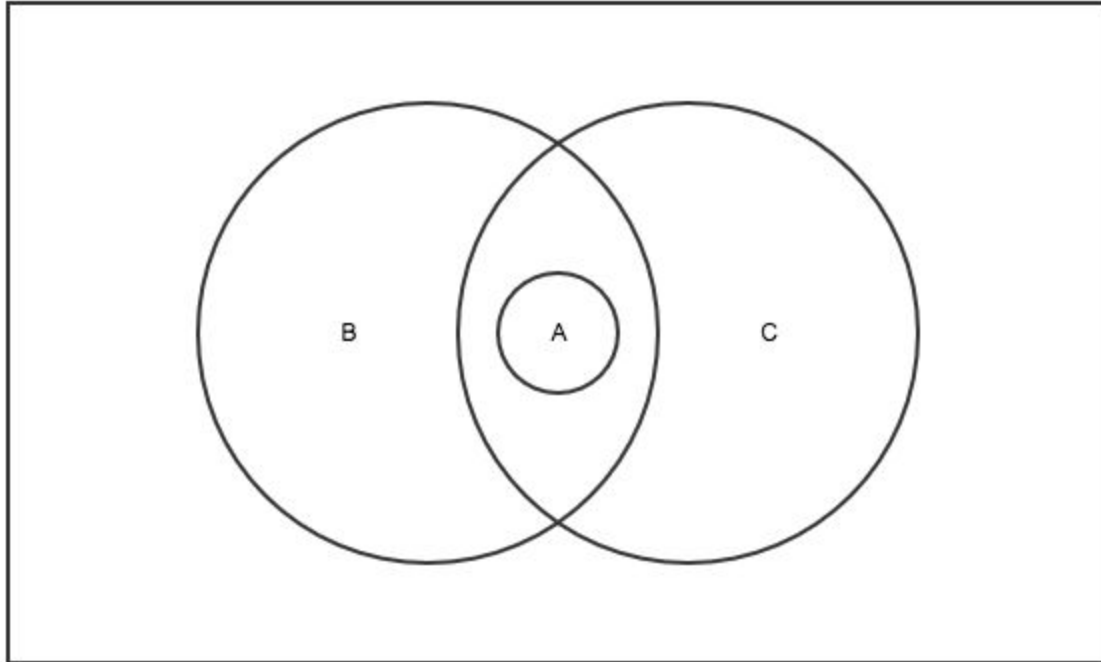
d) $\{\emptyset\} \in \{\{\emptyset\}\}$, True

e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$, True

f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$, True

g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$, False

16. Use a Venn diagram to illustrate the relationships $A \subset B$ and $A \subset C$.



20. What is the cardinality of each of these sets?

- a) $\emptyset = 0$
- b) $\{\emptyset\} = 1$
- c) $\{\emptyset, \{\emptyset\}\} = 2$
- d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\} = 3$

36. How many different elements does $A \times B \times C$ have if A has m elements, B has n elements, and C has p elements?

$$|A \times B \times C| = m * n * p$$

44. Find the truth set of each of these predicates where the domain is the set of integers.

- a) $P(x): x^3 \geq 1, \{1, 2, 3, \dots, \text{infinity}\}$
- b) $Q(x): x^2 = 2, \{\}$
- c) $R(x): x < x^2, \{0, 1\}$

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Chapter 2.2

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4. Let $A = \{a,b,c,d,e\}$ and $B = \{a,b,c,d,e,f,g,h\}$. Find

- a) $A \cup B$, $\{a,b,c,d,e,f,g,h\}$
- b) $A \cap B$, $\{A\}$
- c) $A - B$, $\{\}$
- d) $B - A$, $\{f,g,h\}$

6. Prove the identity laws in Table 1 by showing that

a) $A \cup \emptyset = A$.

$x \in A \mid \mid x \in \emptyset$
 $x \in A$ // x is not an element of the empty set!

b) $A \cap U = A$.

$A \cap U \subseteq A$ // The intersection is a subset of both sets
 $x \in A \ \&\& \ x \in A$ // If x is an element of A it's also an element of the universe
 $x \in A \cap U$ // x is an element of the intersection
 $A \subseteq A \cap U$ // A is a subset of the intersection $A \cap U$
 $A \cap U = A$

8. Prove the idempotent laws in Table 1 by showing that

- a) $A \cup A = A$.
- b) $A \cap A = A$.

A	$A \cup A$	$A \cap A$
0	0	0
1	1	1

14. Find the sets A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$, and $A \cap B = \{3, 6, 9\}$.

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{1,5,7,8} ⊆ A      // From A - B we know that A has these elements
{1,5,7,8} ⊄ B      // We also know that B doesn't contain these elements
{2,10} ⊆ B         // From B - A we know that B has these elements
{2,10} ⊄ A         // And A doesn't have these elements
{3,6,9} ⊆ A and B  // The intersection is a subset of of both sets
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A = {1,3,5,6,7,8,9}
B = {2,3,6,9,10}

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18. Let A, B, and C be sets. Show that

d) $(A - C) \cap (C - B) = \emptyset$.

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x ∈ (A - C) && x ∈ (C - B)           // intersection
(x ∈ A) && (x ∉ C) && (x ∈ C) && (x ∉ B) // difference
x = ∅
// If x is both an element of and not an element of C then x is ∅

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32. Find the symmetric difference of $\{1, 3, 5\}$ and $\{1, 2, 3\}$.

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{1,3,5} Δ {1,2,3} = ({1,3,5} ∪ {1,2,3}) - ({1,3,5} ∩ {1,2,3})
{1,2,3,5} - {1,3} = {2,5}

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34. Draw a Venn diagram for the symmetric difference of the sets A and B.

