CSCE 420 : HW 4

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A Bayesian Inference

Consider two factors that influence whether a student passes a given test: a) being smart, and b) studying. Suppose 30% of students believe they are intrinsically smart. But since students do not know a priori whether they are smart enough to pass a test, suppose 40% of will study for it anyway. (assume Smart and Study are independent). The causal relationship of these variables on the probability of actually passing the test can be expressed in a conditional probability table (CPT) as follows:

P(pass|Smart, Study)	¬smart	smart
¬study	0.2	0.7
study	0.6	0.95

prior probabilities: P(smart)=0.3, P(study)=0.4

(a) Write out the equation for calculating joint probabilities, P(Smart,Study,Pass).

(b) Calculate all the entries in the full joint probability table (JPT) [a 4x2 matrix, like Fig 12.3 in the textbook; [Note: names of variables are capitalized, lower-case indicates truth value, e.g. 'pass' means Pass=T, and '-pass' means Pass=F.]

(c) From the JPT, compute the probability that a student is smart, given that they pass the test but did not study.

(d) From the JPT, compute the probability that a student did not study, given that they are smart but did not pass the test.

(e) Compute the marginal probability that a student will pass the test given that they are smart.

(f) Compute the marginal probability that a student will pass the test given that they study.

Sol.

(a) The equation for calculating joint probabilities, P(Smart,Study,Pass) is:

$$P(Smart, Study, Pass) = P(Pass \mid Smart, Study) \cdot P(Smart) \cdot P(Study)$$

Smart	**Study**	**Pass = False**	**Pass = True**
False	False	0.336	0.084
False	True	0.112	0.168
True	False	0.054	0.126
True	True	0.006	0.114

(b)

(c) We will use the Bayes' theorem:

$$\begin{aligned} P(smart \mid pass, -study) &= \frac{P(pass, -study \mid smart) \cdot P(smart)}{P(pass, -study)} \\ &= \frac{P(smart, -study, pass)}{P(smart, -study, pass) + P(-smart, -study, pass)} \\ &= \frac{0.126}{0.126 + 0.084} = 0.6 \end{aligned}$$

(d) From the JPT,

$$\begin{aligned} P(-study \mid smart, -pass) &= \frac{P(smart, -pass \mid -study) \cdot P(-study)}{P(smart, -pass)} \\ &= \frac{P(smart, -study, -pass)}{P(smart, study, -pass) + P(smart, -study, -pass)} \\ &= \frac{0.054}{0.006 + 0.054} = 0.9 \end{aligned}$$

(e) To compute the marginal probability that a student will pass the test given that they are smart, we sum over all possible values of Study:

$$\begin{aligned} P(pass \mid smart) &= \sum_{study} P(smart, study, pass) \\ &= P(smart, study, pass) + P(smart, -study, pass) \\ &= 0.114 + 0.126 = 0.24 \end{aligned}$$

(f) To compute the marginal probability that a student will pass the test given that they study, we sum over all possible values of Smart:

$$\begin{aligned} P(pass \mid study) &= \sum_{smart} P(smart, study, pass) \\ &= P(smart, study, pass) + P(-smart, study, pass) \\ &= 0.114 + 0.168 = 0.282 \end{aligned}$$

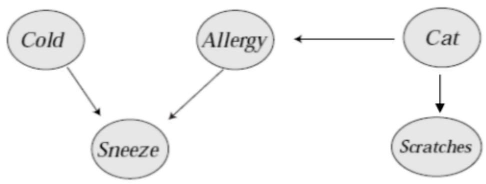
B 2. Bayesian Networks.

Here is a probabilistic model that describes what it might mean when a person sneezes, e.g. depending on whether they have a cold, or whether a cat is present and they are allergic. Scratches on the furniture would be evidence that a cat had been present.

a) Using Equation 13.2 in the textbook (p. 415), write out the expression for the joint probability for any state (combination of truth values for the 5 variables). [Note: Use capital letters for names of variables, and lower-case to indicate truth value, e.g. 'cold' means Cold=T, and '-cold' means Cold=F.]

b) Use the joint prob. equation to calculate the probability for all 32 entries in the JPT. (you might want to write a little script to do this)

c) Calculate the probability that someone is allergic, given that they sneezed but do not have a cold, and it is unknown whether a cat has been present but there are



Cold = False	Cold = True
.95	.05

Cat Presence = False	Cat Presence = True
.98	.02

Cat Presence	Allergic Reaction = False	Allergic Reaction = True
False	.95	.05
True	.25	.75

Cold	Allergic Reaction	Sneeze = False	Sneeze = True
False	False	.99	.01
False	True	.3	.7
True	False	.2	.8
True	True	.1	.9

Cat Presence	Scratches = False	Scratches = True
False	.95	.05
True	.5	.5

Figure 1: Caption

scratches on the furniture, $P(allergic \mid sneeze, \neg cold, scratches)$. Do this calculation numerically using entries in the JPT. (hint: you will have to marginalize over cat presence.)

Sol.

a) $P(Cold, Cat, Allergy, Sneeze, Scratches) = P(Cold) \cdot P(Cat) \cdot P(Allergy \mid Cat) \cdot P(Sneeze \mid Cold, Allergy) \cdot P(Scratches \mid Cat)$

b)

Cold	Allergy	Cat	Sneeze	Scratches	Joint Probability
False	False	False	False	False	0.83183
False	False	False	False	True	0.04378
False	False	False	True	False	0.00840
False	False	False	True	True	0.00044
False	False	True	False	False	0.00235
False	False	True	False	True	0.00235
False	False	True	True	False	0.00002
False	False	True	True	True	0.00002
False	True	False	False	False	0.01327
False	True	False	False	True	0.00070
False	True	False	True	False	0.03096
False	True	False	True	True	0.00163
False	True	True	False	False	0.00214
False	True	True	False	True	0.00214
False	True	True	True	False	0.00499
False	True	True	True	True	0.00499
True	False	False	False	False	0.00884
True	False	False	False	True	0.00047
True	False	False	True	False	0.03538
True	False	False	True	True	0.00186
True	False	True	False	False	0.00003
True	False	True	False	True	0.00003
True	False	True	True	False	0.00010
True	False	True	True	True	0.00010
True	True	False	False	False	0.00023
True	True	False	False	True	0.00001
True	True	False	True	False	0.00209
True	True	False	True	True	0.00011
True	True	True	False	False	0.00004
True	True	True	False	True	0.00004
True	True	True	True	False	0.00034
True	True	True	True	True	0.00034

c) We can use the formula: First, we need to find the joint probability for the given conditions:

$$P(allergic, sneeze, \neg cold, scratches) = \sum_{cat} P(\neg cold, cat, allergic, sneeze, scratches)$$

From the table, this is equal to 0.00163 + 0.00499 = 0.00662.
Next, we need to calculate the marginal probability for the given evidence:

$$P(sneeze, \neg cold, scratches) = \sum_{cat, allergic} P(\neg cold, cat, allergic, sneeze, scratches)$$

From the table, this is equal to 0.00044 + 0.00002 + 0.00163 + 0.00499 = 0.00708.
Therefore, we get $P(allergic|sneeze, cold, scratches)$ = 0.00662 / 0.00708 = 0.935.

C PDDL and Situation Calculus

Starting a car: You have to be at the car and have the key, and the car has to have a charged battery and the tank has to have gas. Afterwards, the car will be running, and you will still be at the car and have the key after starting the engine.

a. Write a PDDL operator to describe this action. (note: you can express this ego-centrically – you don't have to refer explicitly to the person starting the car; but the operator should take the car being started as an argument)

b. Describe the same operator using Situation Calculus (remember to add a situation argument to your predicates).

c. Add a Frame Axiom that says that starting this car will not changes whether any other car is out of gas (tank empty).

Sol.

a. startCar(car):

- pre-conds: atCar(), hasKey(), chargedBattery(car), hasGas(car)
- effects: carRunning(car), atCar(), hasKey()

b.
$\forall s$ Poss(startCar(car), s) $\Leftrightarrow$ (Holds(AtCar(), s) $\wedge$ Holds(HasKey(), s) $\wedge$ Holds(ChargedBattery(car), s) $\wedge$ Holds(HasGas(car), s))

$\forall s$ Holds(CarRunning(car), do(startCar(car), s)) $\Leftrightarrow$ (Holds(AtCar(), s) $\wedge$ Holds(HasKey(), s) $\wedge$ Holds(ChargedBattery(car), s) $\wedge$ Holds(HasGas(car), s))

c. $\forall$ car1, car2, s (car1 car2) $\rightarrow$ (Holds(OutOfGas(car1), do(startCar(car2), s)) $\Leftrightarrow$ Holds(OutOfGas(car1), s))