CSCE 420 : HW 3

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I did the last homework in Word and had a mental breakdown as a consequence. I'm doing this in latex this time just as God intended.

1. Translate the following sentences into First-Order Logic. Remember to break things down to simple concepts (with short predicate and function names), and make use of quantifiers. For example, "tasteDelicious(someRedTomatos)", is not broken down enough; instead we would be looking for a formulation such as:

"$\exists x$ tomato(x)$\wedge$red(x)$\wedge$taste(x,delicious)".

See the lecture slides for more examples and guidance.

(a) bowling balls are sporting equipment

(b) all domesticated horses have an owner

(c) the rider of a horse can be different than the owner

(d) horses move faster than frogs

(e) a finger is any digit on a hand other than the thumb

(f) an isosceles triangle is defined as a polygon with 3 edges, which are connected at 3 vertices, where 2 (but not 3) edges have the same length

Sol.

(a) $\forall x$ ball$(x) \wedge$ heavy$(x) \wedge$ hasThreeHoles$(x) \rightarrow$ sportingEquipment(x)

(b) $\forall x$ horse$(x) \wedge$ domesticated$(x) \rightarrow \exists y$ ownerOf(x, y)

(c) $\forall x$ horse$(x) \wedge \exists y$ riderOf$(x, y) \wedge \exists z$ ownerOf$(x, z) \rightarrow y \neq z$

(d) $\forall x$ horse$(x) \wedge \forall y$ frog$(y) \rightarrow$ movesFaster(x, y)

(e) $\forall x$ digit$(x) \wedge$ onHand$(x) \wedge$ notThumb$(x) \rightarrow$ finger(x)

(f) $\forall x$ isoscelesTriangle$(x) \Leftrightarrow$ polygon$(x) \wedge$hasEdges$(x, 3) \wedge$hasVertices$(x, 3) \wedge$ $\exists a, b, c$ lengthOf$(a, b, x) \wedge$lengthOf$(b, c, x) \wedge$lengthOf$(a, c, x) \wedge$(equals$(a, b) \vee$ equals$(b, c) \vee$ equals$(a, c)) \wedge \neg$(equals$(a, b) \wedge$ equals$(b, c) \wedge$ equals$(a, c))$

2. Convert the following first-order logic sentence into CNF:

$$\forall x \; \text{person}(x) \wedge [\exists z \; \text{petOf}(x,z) \wedge \forall y \; \text{petOf}(x,y) \rightarrow \text{dog}(y)] \rightarrow \text{doglover}(x)$$

Sol.

Remove implications

$\forall x \; [\neg\text{person}(x) \vee (\neg\exists z \; \text{petOf}(x,z) \vee \neg\forall y \; \text{petOf}(x,y) \vee \text{dog}(y)) \vee \text{doglover}(x)]$

Move $\neg$ inwards:

$\forall x \; [\neg\text{person}(x) \vee (\forall z \; \neg\text{petOf}(x,z) \vee \exists y \; \neg\text{petOf}(x,y) \vee \text{dog}(y)) \vee \text{doglover}(x)]$

Skolemization (removing existential quantifiers):

$\forall x \; [\neg\text{person}(x) \vee (\forall z \; \neg\text{petOf}(x,z) \vee \neg\text{petOf}(x,g(x)) \vee \text{dog}(g(x))) \vee \text{doglover}(x)]$

Drop universal quantifiers:

$\neg\text{person}(x) \vee (\neg\text{petOf}(x,z) \vee \neg\text{petOf}(x,g(x)) \vee \text{dog}(g(x))) \vee \text{doglover}(x)$

No distribution required, we are done.

3. Determine whether or not the following pairs of predicates are unifiable. If they are, give the most-general unifier and show the result of applying the substitution to each predicate. If they are not unifiable, indicate why. Capital letters represent variables; constants and function names are lowercase. For example, 'loves(A,hay)' and 'loves(horse,hay)' are unifiable, the unifier is u=A/horse, and the unified expression is 'loves(horse,hay)' for both.

(a) owes(owner(X), citibank, cost(X)), owes(owner(ferrari), Z, cost(Y))

(b) gives(bill, jerry, book21), gives(X, brother(X),Z)

(c) opened(X, result(open(X),s0))), opened(toolbox, Z)

Sol.

(a) The most general unifier is {X/ferrari, Z/citibank, Y/cost(ferrari)}. After applying the substitution, the predicates become:
owes(owner(ferrari), citibank, cost(ferrari))

(b) The predicates are not unifiable. The second predicate states that X gives something to brother(X). This implies that brother(X) refers to X's sibling, and therefore, X cannot simultaneously refer to bill and jerry. Hence, the predicates cannot be unified.

(c) The most general unifier is {X/toolbox, Z/result(open(toolbox),s0)}. After applying the substitution, the predicates become:
opened(toolbox, result(open(toolbox),s0))

4. Consider the following situation:

Marcus is a Pompeian.
All Pompeians are Romans.
Ceasar is a ruler.
All Romans are either loyal to Caesar or hate Caesar (but not both).
Everyone is loyal to someone.
People only try to assassinate rulers they are not loyal to.
Marcus tries to assassinate Caesar.

(a) Translate these sentences to First-Order Logic.

(b) Prove that Marcus hates Caesar using Natural Deduction. In the same style as the examples in the lecture slides, label all derived sentences with the ROI used, which prior sentences were used, and what unifier was used.

(c) Convert all the sentences to CNF

(d) Prove that Marcus hates Ceasar using Resolution Refutation

Sol.

(a)
  i. $\forall x(\text{Pompeian}(x) \rightarrow \text{Roman}(x))$
  ii. $\text{Ruler}(Ceasar)$
  iii. $\forall x(\text{Roman}(x) \rightarrow ((\text{Loyal}(x, Ceasar) \wedge \neg\text{Hate}(x, Ceasar)) \vee (\text{Hate}(x, Ceasar) \wedge \neg\text{Loyal}(x, Ceasar))))$
  iv. $\forall x \exists y(\text{Loyal}(x, y))$
  v. $\forall x \forall y((\text{Ruler}(y) \wedge \neg\text{Loyal}(x, y)) \rightarrow \text{Assassinate}(x, y))$
  vi. $\text{Pompeian}(Marcus)$
  vii. $\text{Assassinate}(Marcus, Ceasar)$

(b)
  1. $\forall x(\text{Pompeian}(x) \rightarrow \text{Roman}(x))$ (Premise)
  2. $\text{Ruler}(Ceasar)$ (Premise)
  3. $\forall x(\text{Roman}(x) \rightarrow ((\text{Loyal}(x, Ceasar) \wedge \neg\text{Hate}(x, Ceasar)) \vee (\text{Hate}(x, Ceasar) \wedge \neg\text{Loyal}(x, Ceasar))))$ (Premise)
  4. $\forall x \forall y((\text{Ruler}(y) \wedge \neg\text{Loyal}(x, y)) \rightarrow \text{Assassinate}(x, y))$ (Premise)
  5. $\text{Pompeian}(Marcus)$ (Premise)
  6. $\text{Assassinate}(Marcus, Ceasar)$ (Premise)
  7. $\text{Roman}(Marcus)$ (Modus Ponens of 1 and 5 using substitution $x/Marcus$)
  8. $(\text{Loyal}(Marcus, Ceasar) \wedge \neg\text{Hate}(Marcus, Ceasar)) \vee (\text{Hate}(Marcus, Ceasar) \wedge \neg\text{Loyal}(Marcus, Ceasar))$ (Modus Ponens of 3 and 7 using substitution $x/Marcus$)
  9. $\neg\text{Loyal}(Marcus, Ceasar) \wedge \text{Ruler}(Ceasar)$ (Modus Tollens of 4 and 6 using substitution $x/Marcus$, $y/Ceasar$)
  10. $\neg\text{Loyal}(Marcus, Ceasar)$ (Simplification of 9)
  11. $\text{Hate}(Marcus, Ceasar) \wedge \neg\text{Loyal}(Marcus, Ceasar)$ (Disjunctive Syllogism of 8 and 10)

12. Hate$(Marcus, Ceasar)$ (Simplification of 11)

(c)
 i. $\neg$Pompeian$(x) \vee$ Roman(x)
 ii. Ruler$(Ceasar)$
 iii. $(\neg\text{Roman}(x) \wedge \neg\text{Roman}(x)) \vee (\neg\text{Roman}(x) \wedge \text{Loyal}(x, Ceasar)) \vee (\neg\text{Roman}(x) \wedge \text{Hate}(x, Ceasar)) \vee (\neg\text{Hate}(x, Ceasar) \wedge \text{Loyal}(x, Ceasar)) \vee (\neg\text{Hate}(x, Ceasar) \wedge \text{Hate}(x, Ceasar))$
 iv. Loyal$(x, f(x))$
 v. $\neg$Ruler$(y) \vee$ Loyal$(x, y) \vee$ Assassinate(x, y)
 vi. Pompeian$(Marcus)$
 vii. Assassinate$(Marcus, Ceasar)$

5. Write a KB in First-Order Logic with rules/axioms for:

(a) Map-coloring – every state must be exactly 1 color, and adjacent states must be different colors. Assume possible colors are states are defined using unary predicate like color(red) or state(WA). To say a state has a color, use a binary predicate, e.g. 'color(WA,red)'.

(b) Sammy's Sport Shop – include sentences for specific facts like obs(1,W) or label(2,B), as well as for general constraints about the boxes and colors. Use binary predicate 'contains(x,q)' to represent that box x contains tennis balls of color q (where q could be W, Y, or B).

(c) Wumpus World - Write rules for which rooms are 'stenchy', 'breezy', and 'safe'. (hint: define a helper concept called 'adjacent(x,y,p,q)' which defines when a room at coordinates (x,y) is adjacent to another room at (p,q), and x,y,p,q are integers 1-4.

(d) 4-Queens – assume row(1)...row(4) and col(1)...col(4) are facts; write rules that describe configurations of 4 queens such that none can attack each other, using 'queen(r,c)' to represent that there is a queen in row r and col c.

Sol.

(a)
  i. adjacent(WA, NT) $\vee$ adjacent(WA, SA) $\vee$ adjacent(NT, Q) $\vee$ adjacent(SA, Q) $\vee$ adjacent(SA, NSW) $\vee$ adjacent(SA, V) $\vee$ adjacent(Q, NSW) $\vee$ adjacent(V, NSW)
  ii. $\forall x\ \forall y\ \forall z$ (adjacent(x,y) $\wedge$ color(x,z) $\rightarrow \neg$ color(y,z))
  iii. $\forall x\ \exists y$ color(x,y)

(b)
  i. $\forall x$ (label(x, W) $\rightarrow \neg$contains(x, W))
  ii. $\forall x$ (label(x, Y) $\rightarrow \neg$contains(x, Y))
  iii. $\forall x$ (label(x, B) $\rightarrow \neg$contains(x, B))
  iv. $\forall x$ (contains(x, W) $\rightarrow \neg$contains(x, Y))
  v. $\forall x$ (contains(x, Y) $\rightarrow \neg$contains(x, W))
  vi. $\forall x$ (contains(x, B) $\rightarrow$ (contains(x, W) $\wedge$ contains(x, Y)))
  vii. $\forall x$ (obs(x, W) $\rightarrow$ (contains(x, W) $\vee$ contains(x, B)))
  viii. $\forall x$ (obs(x, Y) $\rightarrow$ (contains(x, Y) $\vee$ contains(x, B)))
  ix. $\exists x\ \exists y\ \exists z$ (contains(x, W) $\wedge$ contains(y, Y) $\wedge$ contains(z, B) $\wedge$ $(x \neq y) \wedge (x \neq z) \wedge (y \neq z)$)

(c)
  i. $adjacent(x, y, a, b) \leftrightarrow (x = a \wedge (y = b+1 \vee y = b-1)) \vee (y = b \wedge (x = a+1 \vee x = a-1))$
  ii. $stench(x, y) \leftrightarrow \exists a, b(adjacent(x, y, a, b) \wedge wumpus(a, b))$
  iii. $breeze(x, y) \leftrightarrow \exists a, b(adjacent(x, y, a, b) \wedge pit(a, b))$
  iv. $safe(x, y) \leftrightarrow \neg stench(x, y) \wedge \neg breeze(x, y)$

(d)
  i. $row(1) \wedge row(2) \wedge row(3) \wedge row(4)$
  ii. $col(1) \wedge col(2) \wedge col(3) \wedge col(4)$
  iii. $\forall x(queen(x, 1) \rightarrow \neg \exists y(queen(y, 1) \wedge y \neq x))$

iv. $\forall x(queen(x, 2) \rightarrow \neg\exists y(queen(y, 2) \wedge y \neq x))$

v. $\forall x(queen(x, 3) \rightarrow \neg\exists y(queen(y, 3) \wedge y \neq x))$

vi. $\forall x(queen(x, 4) \rightarrow \neg\exists y(queen(y, 4) \wedge y \neq x))$

vii. $\forall x\forall y\forall a\forall b((queen(x, y) \wedge queen(a, b)) \rightarrow (x \neq a \wedge y \neq b \wedge x + y \neq a + b \wedge x - y \neq a - b))$