CSCE 420 : HW 3

William Allen

April 4, 2023

I did the last homework in Word and had a mental breakdown as a consequence. I'm doing this in latex this time just as God intended.

1. Translate the following sentences into First-Order Logic. Remember to break things down to simple concepts (with short predicate and function names), and make use of quantifiers. For example, "tasteDelicious(someRedTomatos)", is not broken down enough; instead we would be looking for a formulation such as:

"$\exists x$ tomato(x)$\wedge$red(x)$\wedge$taste(x,delicious)".

See the lecture slides for more examples and guidance.

(a) bowling balls are sporting equipment

(b) all domesticated horses have an owner

(c) the rider of a horse can be different than the owner

(d) horses move faster than frogs

(e) a finger is any digit on a hand other than the thumb

(f) an isosceles triangle is defined as a polygon with 3 edges, which are connected at 3 vertices, where 2 (but not 3) edges have the same length

Sol.

(a) $\forall x$ ball$(x) \wedge$ heavy$(x) \wedge$ hasThreeHoles$(x) \rightarrow$ sportingEquipment(x)

(b) $\forall x$ horse$(x) \wedge$ domesticated$(x) \rightarrow \exists y$ ownerOf(x, y)

(c) $\forall x$ horse$(x) \wedge \exists y$ riderOf$(x, y) \wedge \exists z$ ownerOf$(x, z) \rightarrow y \neq z$

(d) $\forall x$ horse$(x) \wedge \forall y$ frog$(y) \rightarrow$ movesFaster(x, y)

(e) $\forall x$ digit$(x) \wedge$ onHand$(x) \wedge$ notThumb$(x) \rightarrow$ finger(x)

(f) $\forall x$ isoscelesTriangle$(x) \Leftrightarrow$ polygon$(x) \wedge$hasEdges$(x, 3) \wedge$hasVertices$(x, 3) \wedge$ $\exists a, b, c$ lengthOf$(a, b, x) \wedge$lengthOf$(b, c, x) \wedge$lengthOf$(a, c, x) \wedge$(equals$(a, b) \vee$ equals$(b, c) \vee$ equals$(a, c)) \wedge \neg$(equals$(a, b) \wedge$ equals$(b, c) \wedge$ equals$(a, c))$

2. Convert the following first-order logic sentence into CNF:

$$\forall x \text{ person}(x) \wedge [\exists z \text{ petOf}(x, z) \wedge \forall y \text{ petOf}(x, y) \rightarrow \text{dog}(y)] \rightarrow \text{doglover}(x)$$

Sol.

Remove implications

$\forall x\ [\neg\text{person}(x) \vee (\neg\exists z \text{ petOf}(x, z) \vee \neg\forall y \text{ petOf}(x, y) \vee \text{dog}(y)) \vee \text{doglover}(x)]$

Move $\neg$ inwards:

$\forall x\ [\neg\text{person}(x) \vee (\forall z\ \neg\text{petOf}(x, z) \vee \exists y\ \neg\text{petOf}(x, y) \vee \text{dog}(y)) \vee \text{doglover}(x)]$

Skolemization (removing existential quantifiers):

$\forall x\ [\neg\text{person}(x) \vee (\forall z\ \neg\text{petOf}(x, z) \vee \neg\text{petOf}(x, g(x)) \vee \text{dog}(g(x))) \vee \text{doglover}(x)]$

Drop universal quantifiers:

$\neg\text{person}(x) \vee (\neg\text{petOf}(x, z) \vee \neg\text{petOf}(x, g(x)) \vee \text{dog}(g(x))) \vee \text{doglover}(x)$

No distribution required, we are done.

3. Determine whether or not the following pairs of predicates are unifiable. If they are, give the most-general unifier and show the result of applying the substitution to each predicate. If they are not unifiable, indicate why. Capital letters represent variables; constants and function names are lowercase. For example, 'loves(A,hay)' and 'loves(horse,hay)' are unifiable, the unifier is u=A/horse, and the unified expression is 'loves(horse,hay)' for both.

(a) owes(owner(X), citibank, cost(X)), owes(owner(ferrari), Z, cost(Y))

(b) gives(bill, jerry, book21), gives(X, brother(X),Z)

(c) opened(X, result(open(X),s0))), opened(toolbox, Z)

Sol.

(a) The most general unifier is {X/ferrari, Z/citibank, Y/cost(ferrari)}. After applying the substitution, the predicates become:
owes(owner(ferrari), citibank, cost(ferrari))

(b) The predicates are not unifiable. The second predicate states that X gives something to brother(X). This implies that brother(X) refers to X's sibling, and therefore, X cannot simultaneously refer to bill and jerry. Hence, the predicates cannot be unified.

(c) The most general unifier is {X/toolbox, Z/result(open(toolbox),s0)}. After applying the substitution, the predicates become:
opened(toolbox, result(open(toolbox),s0))

4. Consider the following situation:

Marcus is a Pompeian.
All Pompeians are Romans.
Ceasar is a ruler.
All Romans are either loyal to Caesar or hate Caesar (but not both).
Everyone is loyal to someone.
People only try to assassinate rulers they are not loyal to.
Marcus tries to assassinate Caesar.

(a) Translate these sentences to First-Order Logic.

(b) Prove that Marcus hates Caesar using Natural Deduction. In the same style as the examples in the lecture slides, label all derived sentences with the ROI used, which prior sentences were used, and what unifier was used.

(c) Convert all the sentences to CNF

(d) Prove that Marcus hates Ceasar using Resolution Refutation

Sol.

(a) i. $\forall x(\text{Pompeian}(x) \rightarrow \text{Roman}(x))$

ii. $\text{Ruler}(Ceasar)$

iii. $\forall x(\text{Roman}(x) \rightarrow ((\text{Loyal}(x, Ceasar) \wedge \neg\text{Hate}(x, Ceasar)) \vee (\text{Hate}(x, Ceasar) \wedge \neg\text{Loyal}(x, Ceasar))))$

iv. $\forall x \exists y(\text{Loyal}(x, y))$

v. $\forall x \forall y((\text{Ruler}(y) \wedge \neg\text{Loyal}(x, y)) \rightarrow \text{Assassinate}(x, y))$

vi. $\text{Pompeian}(Marcus)$

vii. $\text{Assassinate}(Marcus, Ceasar)$

(b) 1. $\forall x(\text{Pompeian}(x) \rightarrow \text{Roman}(x))$ (Premise)

2. $\text{Ruler}(Ceasar)$ (Premise)

3. $\forall x(\text{Roman}(x) \rightarrow ((\text{Loyal}(x, Ceasar) \wedge \neg\text{Hate}(x, Ceasar)) \vee (\text{Hate}(x, Ceasar) \wedge \neg\text{Loyal}(x, Ceasar))))$ (Premise)

4. $\forall x \exists y(\text{Loyal}(x, y))$ (Premise)

5. $\exists y(\text{Loyal}(Marcus, y))$ (Universal instantiation of 4)

6. $\text{Roman}(Marcus)$ (Modus Ponens of 1 and 5 using substitution $x/Marcus$)

7. $\text{Loyal}(Marcus, Ceasar) \vee (\text{Hate}(Marcus, Ceasar) \wedge \neg\text{Loyal}(Marcus, Ceasar))$ (Modus Ponens of 3 and 6 using substitution $x/Marcus$)

8. $\neg\text{Hate}(Marcus, Ceasar)$ (Modus Tollens of 2 and 7 using substitution $x/Marcus$)

9. $\text{Hate}(Marcus, Ceasar) \wedge \neg\text{Loyal}(Marcus, Ceasar)$ (Conjunction of 7 using substitution $x/Marcus$)

10. $\text{Hate}(Marcus, Ceasar)$ (Simplification of 9)