1. $3\sum_{i=1}^{n-1} 4^i = 4^n - 4$
   Base case: we can verify the summation formula for n = 1: 12 = 12. This is true.
   Inductive step: let us assume we have proved the summation for some arbitrary k, i.e.
   $3\sum_{i=i}^{k-1} 4^i = 4^k - 4$.
   Now we consider the left-hand-side to find the next term in the sequence,
   $3\sum_{i=i}^{k} 4^i = 3\sum_{i=i}^{k-1} 4^i + 3(4^k)$
   Add the next term of the sequence to the right-hand-side
   $4^k - 4 + 3(4^k) = 4^k(1+3) - 4 = 4^k(4) - 4 = 4^{k+1} - 4$ which verifies that if the formula holds for one n, then it also holds for the next, completing the proof by induction.

2. **Proof by induction:** Base Case: for n = 1, $a_n = n2^{n-1} = 1$ which by the given recursive definition is true. Inductive step: Suppose that we have already proved that the statement is true for some arbitrary $k \geq 1$, i.e. $a_{k+1} = 2a_k + 2^k$. The goal is now to show that that the statement is true for n = k + 1. We can do this by substituting our base case:
   $a_{k+1} = 2(k2^{k-1}) + 2^k =$
   $a_{k+1} = (k2^k) + 2^k = a_{k+1} = (k+1)2^k$. Which verifies that if the formula holds for one n, then it also holds for the next, completing the proof by induction.

3. **Proof by induction:** Base case: We can verify the base case n = 1, A(1, 1) = $2^1$ = 2 is true.
   Inductive step: Let us assume we have proved the Ackermann function for some arbitrary n = k $\geq$ 1, i.e. A(1, k). The goal is now to show that statement is true for n = k + 1 by plugging in k + 1 for n. A(1, k + 1) = A (1 – 1, A(1, k + 1 – 1)) = A(0, A(1,k)) = A(1, $2^k = 2(2^k)$. Therefore, A(1, k + 1) = $2(2^k)$ = $2^{k+1}$. By induction we can conclude that A(1, n) = $2^n$.

4. (a)
   S = {1, 2, 3, 5, …}
   $S_n = S_{n-3} + 4$ for all $n \geq 4$
   i.e
   $S_5 = 6 = S_{5-3} + 4$,
   $S_6 = 7 = S_{6-3} + 4$

   (b)
   S = {1, 2, 4, 8, 16, …}
   $S_n = (2)S_{n-1}$ for all $n \geq 2$
   i.e.
   $S_2 = 4 = (2)S_1$,
   $S_3 = 8 = (2)S_2$

5. Using the division algorithm, we can verify that any positive integer divided by 10 that has a remainder of 7 has a last decimal digit 7. We can generalize this statement: if for an arbitrary positive integer x, (10x + 7)mod(10) = 7, then the values last digit is 7. We can also prove this by laws of modulo: (10x + 7)mod(10) = ((10x)mod(10) + (7)mod(10))mod(10) = (0 + 7)mod(10) = 7.

   Next, we can prove that the base case, $7 \in S$ follows the guidelines as said above:
   For x = 0, (10x + 7)mod(10) = 7.
   Finally we can prove our other cases:
   i. $2x + 3$ for x = 7 = 2(14) + 3 = 17.
      $17 = 10x + 7$ with x = 1
      17 ∈ S
   ii. $x^2 + 8$ = 49 + 8 = 57
      $57 = 10x + 7$ with x = 5
      57 ∈ S

   Therefore, by structural induction, if a value is in S, then its last digit is 7.

6. For i:
   2x + 3 = 27
   x = 12.
   The last digit of 12 is not 7, therefore not in S

   For ii:
   Again pick 27 therefore
   $x^2 + 8 = 27$
   $x^2 = 19$
   $\sqrt{19}$ is not in S.

   Therefore, 27 is not in S.

7. Not sure how to start this one…
8. n/a

9. EXTRA CREDIT:

main.py

```python
print("Name: William Jedynak")
print("ID  : 1227139214")

import math

count = 100000

def sequencef(n):
    if (n <= 1):
        return n
    else:
        return sequencef(math.floor(n / 2)) + sequencef(math.floor(n / 3))

print("Value of sequencef(100000) = " + str(sequencef(count)))
```

main

```
"C:\Users\WillJedynak\OneDrive\ASU Online\Fall 2022\Session B\MAT 243\Homewor
Name: William Jedynak
ID  : 1227139214
Value of sequencef(100000) = 5503

Process finished with exit code 0
```