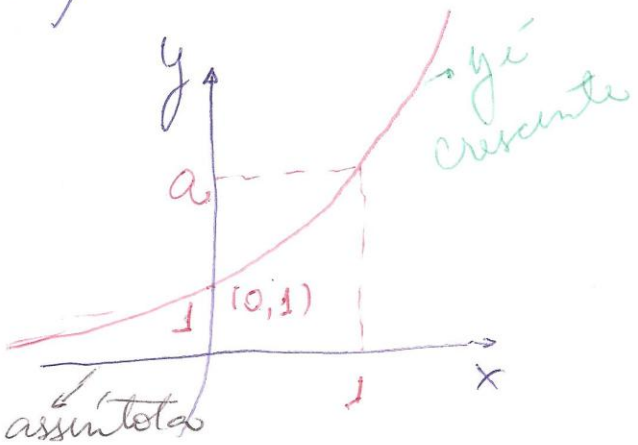
Função exponencial de base a

$$f: \mathbb{R} \longrightarrow (0, +\infty)$$

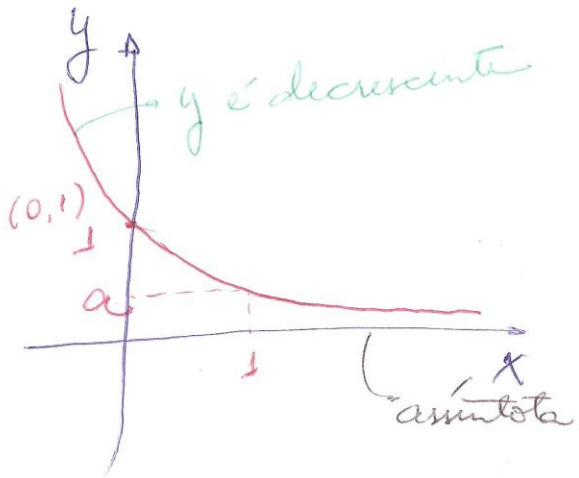
$$f(x) = a^x, \quad a \neq 1, \ a > 0, \ a \in \mathbb{R}$$

Gráficos:

a) $a > 1$



b) $0 < a < 1$



$D(f) = \mathbb{R}$

$Im(f) = (0, +\infty)$

Propriedades: $a > 0, b > 0$

1) $a^x \cdot a^y = a^{x+y}$

2) $a^x \cdot b^x = (ab)^x$

3) $\frac{a^x}{a^y} = a^{x-y}$

4) $\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$

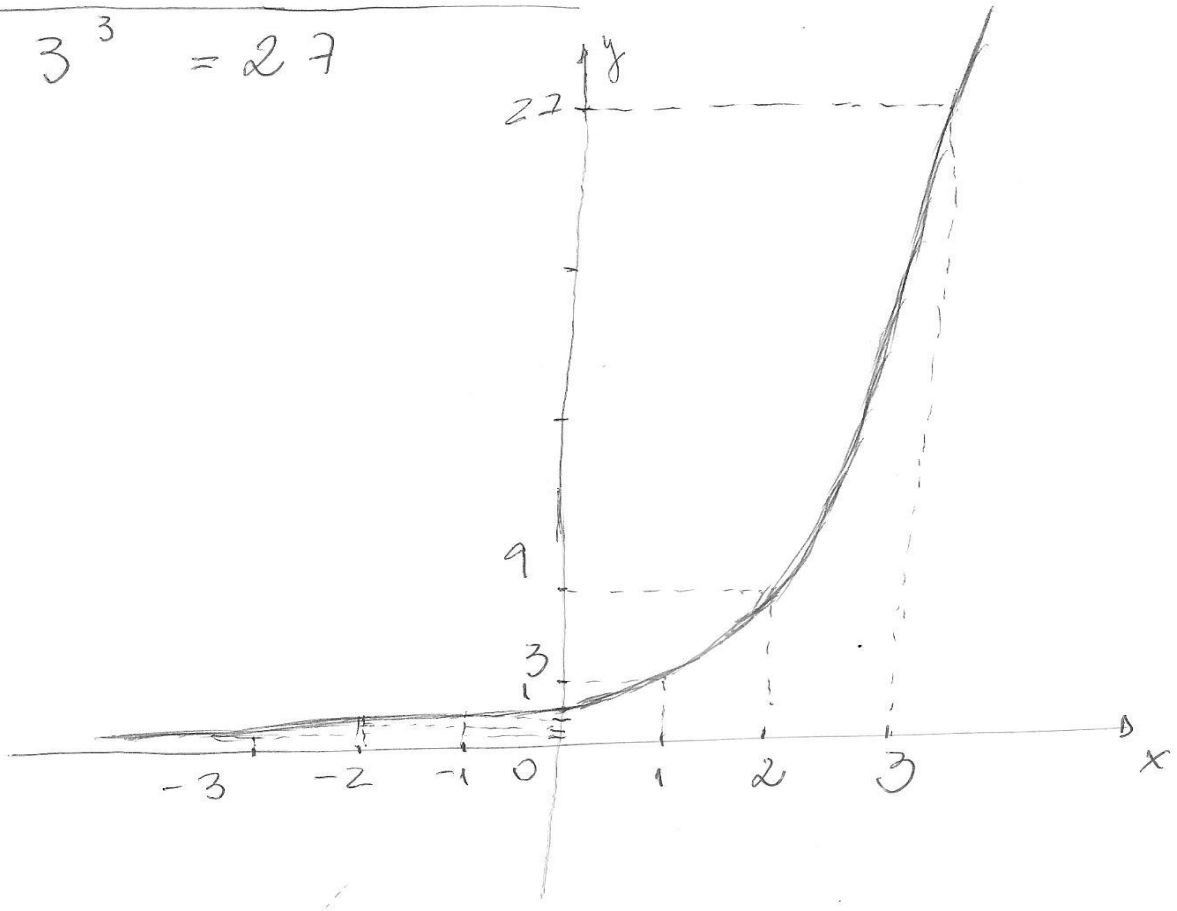
5) $(a^x)^y = (a^y)^x = a^{xy}$

Obs: $e = 2,718\,28\ldots$ (irracional)
A constante de Euler ($e \in \mathbb{I}$)

$y = e^x$: função exponencial

Exemplo: $y = 3^x$, $a = 3 > 1$

x	$y = 3^x$
-3	$3^{-3} = \frac{1}{3^3} = \frac{1}{27} \cong 0{,}04$
-2	$3^{-2} = \frac{1}{3^2} = \frac{1}{9} \cong 0{,}1$
-1	$3^{-1} = \frac{1}{3} \cong 0{,}3$
0	$3^0 = 1$
1	$3^1 = 3$
2	$3^2 = 9$
3	$3^3 = 27$



Função logarítmo de base a

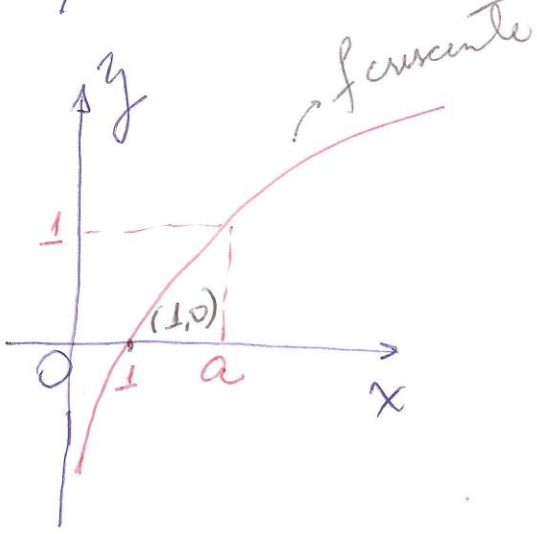
$$f: \mathbb{R}_+^* \longrightarrow \mathbb{R}$$

$$y = f(x) = \log_a x \quad , \; x > 0, \; a > 0, \; a \neq 1.$$

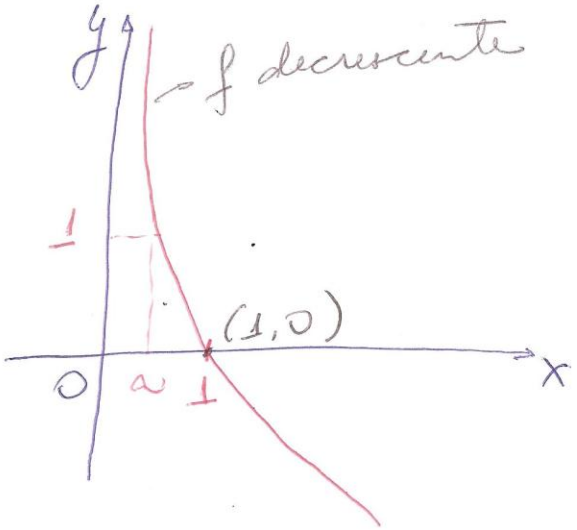
$$\log_a x = y \iff a^y = x$$

Gráficos:

a) $a > 1$



b) $0 < a < 1$



$D(f) = \mathbb{R}_+^*$

$Im(f) = \mathbb{R}$

Propriedades: $x > 0, y > 0$

1) $\log_a (x \cdot y) = \log_a x + \log_a y$

2) $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

3) $\log_a (y^x) = x \cdot \log_a y$

4) Mudança de base: $\log_a M = \frac{\log_c M}{\log_c a}$

Obs: $\ln x$: logaritmo na base e

Exemplo: $y = \log_2 x \longrightarrow 2^y = x$

x	$y = \log_2 x$	
$\frac{1}{4}$	$2^y = \frac{1}{4} = \frac{1}{2^2} = 2^{-2} \Rightarrow y = -2$	$(\frac{1}{4}, -2)$
$\frac{1}{2}$	$2^y = \frac{1}{2} = 2^{-1} \Rightarrow y = -1$	$(\frac{1}{2}, -1)$
1	$2^y = 1 \Rightarrow 2^y = 2^0 \Rightarrow y = 0$	$(1, 0)$
2	$2^y = 2^1 \Rightarrow y = 1$	$(2, 1)$
4	$2^y = 4 = 2^2 \Rightarrow y = 2$	$(4, 2)$