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QMentoring Coding Problem 1
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Using a Monte Carlo method to approximate π

Question 1. Using Python, take a Monte Carlo approach to approximate π.

1.1. Aim

The aim is to write a Python program to approximate the value of π, using a Monte Carlo method. Here the Monte Carlo method refers to generating a number of random Cartesian coordinates, $[x, y]$. The term "Monte Carlo" may make this seem like a complex method, however it's very simple, it just refers to using randomly generated inputs to solve problems.

1.2. Theory

First consider $\frac{1}{4}$ of a circle of radius 1, placed inside of a square, with sides also of length 1. A schematic of this is shown in 1

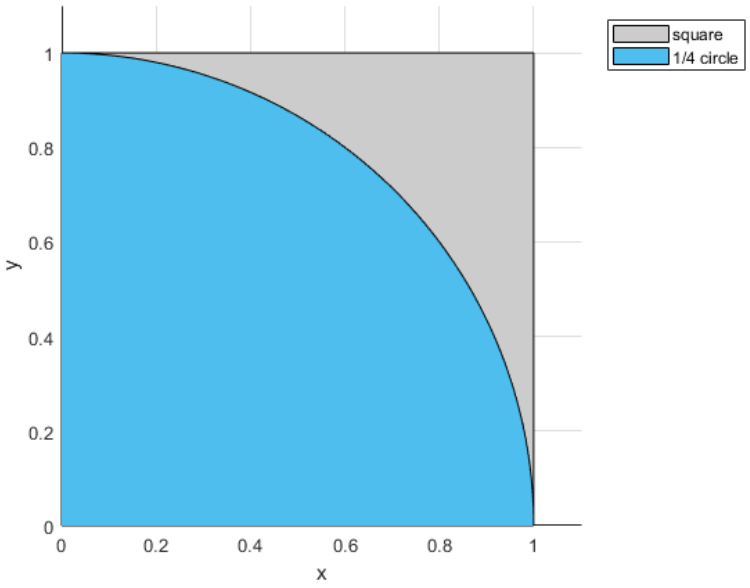

Figure 1. A $\frac{1}{4}$ circle placed in a square

First lets consider the area of the square which has a unitless side length of 1. We can then easily calculate the area of this square, $A_s = 1 \times 1 = 1$ squared unit. Similarly, we can calculate the area of the circle, $A_c = \pi(1 \times 1)/4 = \frac{\pi}{4}$ squared units (remember we only have $\frac{1}{4}$ of the circle and hence the division by four.

Now we can work out the ratio of the area of the circle to the area of the square, which is:

$$\frac{A_c}{A_s} = \frac{\frac{\pi}{4}}{1} = \frac{\pi}{4} \tag{1}$$

Now consider this: if we generate a random coordinate that falls within the square (grey shaded area), i.e. the x-coordinate $0.0 \leq x \leq 1.0$ and similarly for the y-coordinate, $0.0 \leq y \leq 1.0$, what is the probability the point lands within the quarter circle (the shaded blue area in Figure 1)?

The answer here is that the probability of the point landing within the circle is the same as the area ratio calculated in Equation 1, which is $\pi/4 \approx 0.785398...$. We can use this result to approximate the value of π, using the Monte Carlo method, by generating a number of random $[x, y]$ coordinates, then finding out

the proportion which land within the circle to total number of points, giving the approximation of $\pi/4$. Therefore, we can approimate π as follows:

$$\frac{4 \times \text{points inside circle}}{\text{total number of points}} \approx \pi \tag{2}$$

1.3. Task

- Generate a number of $[x, y]$ coordinate, where the number is an input and each coordinate is randomly generated.
- Count how many points land within the circle, then using Equation 2, find the approximation of π
- How does the approximation of π vary with the number of points generated? Run the program with the number of points varying between 100 and 1×10^8, in multiples of 10 (100, 1000, 10,000, ...) and record the results.