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NE 155
HW 5

Problem 2.

The spectral radius can be defined as the largest ~~value~~ eigenvalue. We can define the convergence condition for when the spectral radius < 1. So when solving systems with iterative methods we should make the spectral radius as small as possible. Also we can relate the Rate of Convergence and the Spectral Radius. The smaller the Spectral Radius, the faster the solutions will go to 0.

Problem 3

$3x_1 - x_2 + x_3 = 1$
$3x_1 + 6x_2 + 2x_3 = 0$
$3x_1 + 3x_2 + 7x_3 = 4$

- Jacobi Method

$3x_1^{(k+1)} - x_2^{(k)} + x_3^{(k)} = 1$
$3x_1^{(k)} + 6x_2^{(k+1)} + 2x_3^{(k)} = 0$
$3x_1^{(k)} + 3x_2^{(k)} + 7x_3^{(k+1)} = 4$

$\vec{x}^{(0)} = \vec{0}$

$\vec{x}^{(0)} = \vec{0}$

$\boxed{\vec{x}^{(1)} = (1/3, 0, 4/7)}$

$3x_1^{(2)} - x_2^{(1)} + x_3^{(1)} = 1$
$3x_1^{(1)} + 6x_2^{(2)} + 2x_3^{(1)} = 0$
$3x_1^{(1)} + 3x_2^{(1)} + 7x_3^{(2)} = 4$

$3x_1^{(2)} - 0 + 4/7 = 1$
$1 + 6x_2^{(2)} + 8/7 = 0$
$1 + 0 + 7x_3^{(2)} = 4$

$x_1^{(2)} = 11/7/3 = 11/21$
$x_2^{(2)} = -5/14$
$x_3^{(2)} = -3/7$

$\boxed{\vec{x}^{(2)} = (1/7, -5/14, 3/7)}$

- Gauss-Seidel

$\vec{x}^{(0)} = \vec{0}$

$3x_1^{k+1} - x_2^{k} + x_3^{k} = 1$
$3x_1^{k+1} + 6x_2^{k+1} + 2x_3^{k} = 0$
$3x_1^{k+1} + 3x_2^{k+1} + 7x_3^{k+1} = 4$

$x_1^{(1)} = 1/3$
$x_2^{(1)} = 3x_1 / 6 = 1/6$
$x_3 = 4 - 3x_1 - 3x_2 = 5/2$

$\boxed{\vec{x}^{(1)} = (1/3, -1/6, 1/2)}$

$3x_1^{(2)} - 1/6 + 1/2 = 1$
$3x_1^{(2)} + 6x_2^{(2)} + 2x_3^{(1)} = 0$
$3x_1^{(2)} + 3x_2^{(2)} + 7x_3^{(2)} = 4$

$x_1 = -4/9 \quad x = -2/18 \quad x_3 = 13/21$

$\boxed{\vec{x}^{(2)} = (1/9, -2/9, 13/21)}$

• SOR Method ($\omega = 1.1$)

$$3x_1 - x_2 + x_3 = 1$$
$$3x_1 + 6x_2 + 2x_3 = 0$$
$$3x_1 + 3x_2 + 7x_3 = 4$$

$$\vec{x}^{(0)} = \vec{0}$$

$$x_1^{(1)} = (1-\omega)x_1^{(0)} + \omega(1 + x_2^{(0)} - x_3^{(0)})/3$$
$$= 0.36667$$

$$x_2^{(1)} = (1-\omega)x_2^{(0)} + \omega(0 - 3x_1^{(1)} - 2x_3^{(0)})/6$$
$$= -0.20167$$

$$x_3^{(1)} = (1-\omega)x_3^{(0)} + \omega(4 - 3x_1^{(1)} - 3x_2^{(1)})/7$$
$$= 0.55079$$

$$\boxed{\vec{x}^{(1)} = (0.36667, -0.20167, 0.55079)}$$

$$x_1^{(2)} = (1-\omega)x_1^{(1)} + \omega(1 + x_2^{(1)} - x_3^{(1)})/3$$
$$= 0.0541$$

$$x_2^{(2)} = (1-\omega)x_2^{(1)} + \omega(0 - 3x_1^{(2)} - 2x_3^{(1)})/6$$
$$= -0.2115$$

$$x_3^{(2)} = (1-\omega)x_3^{(1)} + \omega(4 - 3x_1^{(2)} - 3x_2^{(2)})/7$$
$$= 0.6477$$

$$\boxed{\vec{x}^{(2)} = (0.0541, -0.2115, 0.6477)}$$

It seems that Jacobi and G-S are extremelely similar. The only difference, however G-S requires only 1 storage vector which can be useful for large problems. While the SOR Method seems very dependent on the the weight (ω) value you choose.

Problem 5

a) Code Sent in

- Comparing the 2 we kan see that the absolute error is larger for the Jacobi + G-S method but not the SOR method
- The SOR method with relative error @ 8 iterations
- It takes more iterations

b) Code sent in

Explain in comments.

Best $\omega = 1.07$