Problem 1 *worth 6 points*
Consider $a = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, find their **outer** product ab^T

Solution:

$$ab^T = \begin{bmatrix} a_1b_1 & a_1b_2 & \cdots & a_1b_n \\ a_2b_1 & a_2b_2 & \cdots & a_2b_n \\ \vdots & \vdots & \ddots & \vdots \\ a_mb_1 & a_mb_2 & \cdots & a_mb_n \end{bmatrix}$$
$$ab^T = \begin{bmatrix} 1(0) & 1(1) & 1(0) \\ 0(0) & 0(1) & 0(0) \\ 1(0) & 1(1) & 1(0) \end{bmatrix}$$
$$ab^T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Problem 2a *worth 4 points*
Show that $(ABC)^T = C^TB^TA^T$. (Hint: assume $D = AB$, what do we know about $(DC)^T$?)

Solution: We know from our properties of transpose, $(AB)^T = B^TA^T$. If we let $D = AB$, then $D^T = B^TA^T$.

$$\begin{aligned}(DC)^T &= C^TD^T \\ &= C^TB^TA^T\end{aligned}$$

Problem 2b *worth 6 points*
Consider linear functions $f(x)$ and $g(x)$ from vectors to vectors represented as matrices A and B, find the matrix C which represents $f(g(x) + x)$.

Solution: If the matrix A represents a function of vectors to vectors, then $f(x) = Ax$. Similarly, $g(x) = Bx$. If we are looking for a matrix C such that $f(g(x) + x) = Cx$, then

$$Cx = A(Bx + x)$$

Problem 3 *worth 4 points*

Find the matrix multiplication of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Solution:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{bmatrix}$$

$$C_{1,1} = a_1 \cdot b_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1(0) + 2(1) = 2$$

$$C_{1,2} = a_1 \cdot b_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1(1) + 2(0) = 1$$

$$C_{2,1} = a_2 \cdot b_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 3(0) + 4(1) = 4$$

$$C_{2,2} = a_2 \cdot b_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 3(1) + 4(0) = 3$$

$$C = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$