Laminate Composite Example

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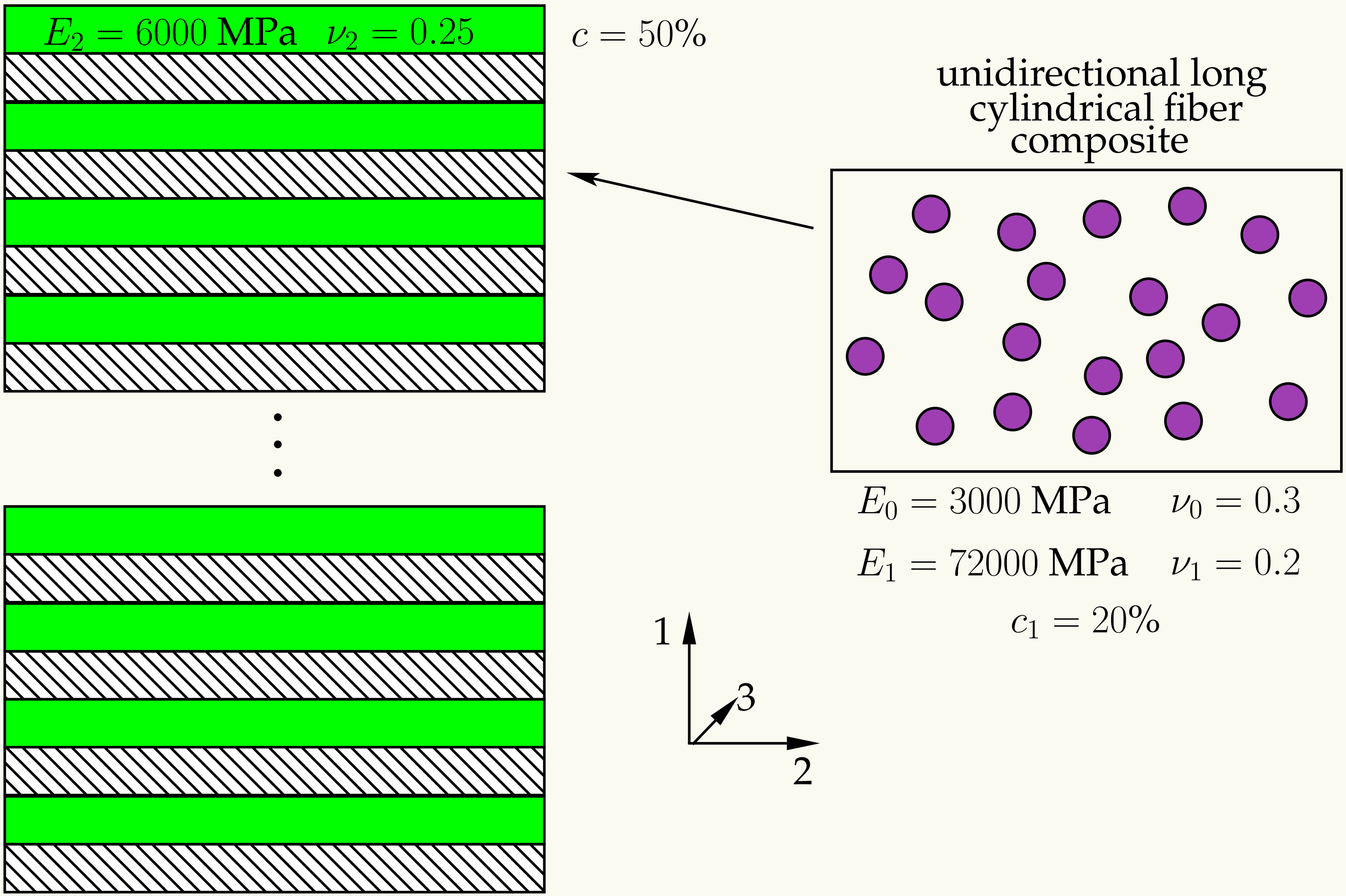
Laminate Composite Example



Laminate Composite Example

Questions

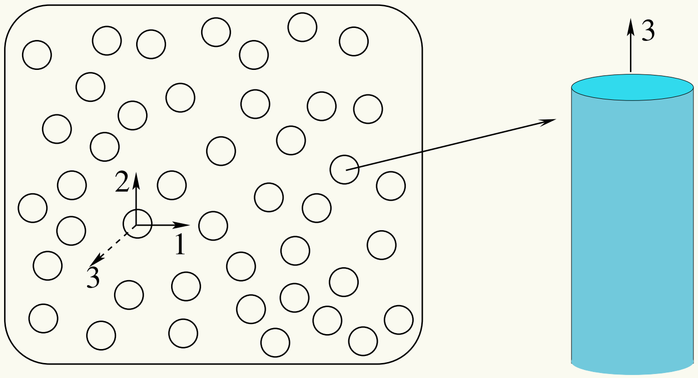
1. Identify the effective properties of the unidirectional long cylindrical fiber composite.
2. Identify the effective properties of the laminate composite.
3. Assume that the overall composite is subjected to the uniaxial macroscopic stress (in Voigt notation)

$$\bar{\sigma} = \begin{bmatrix} 50 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ MPa.}$$

Compute the average strains and stresses at every material phase.

Solution

Long Cylindrical Fiber Composite

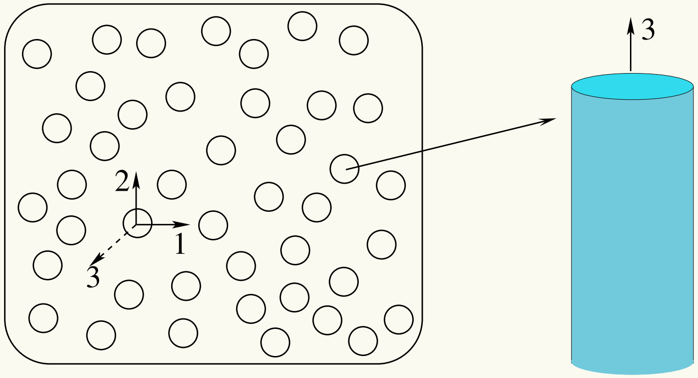


phases (i=0,1): isotropic

$$\boldsymbol{L}_i = \begin{bmatrix} K_i + \frac{4}{3}\mu_i & K_i - \frac{2}{3}\mu_i & K_i - \frac{2}{3}\mu_i & 0 & 0 & 0 \\ K_i - \frac{2}{3}\mu_i & K_i + \frac{4}{3}\mu_i & K_i - \frac{2}{3}\mu_i & 0 & 0 & 0 \\ K_i - \frac{2}{3}\mu_i & K_i - \frac{2}{3}\mu_i & K_i + \frac{4}{3}\mu_i & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_i & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_i & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_i \end{bmatrix} \qquad \tilde{\boldsymbol{S}}_1 = \begin{bmatrix} \frac{5-4\nu_0}{8[1-\nu_0]} & \frac{4\nu_0-1}{8[1-\nu_0]} & \frac{\nu_0}{2[1-\nu_0]} & 0 & 0 & 0 \\ \frac{4\nu_0-1}{8[1-\nu_0]} & \frac{5-4\nu_0}{8[1-\nu_0]} & \frac{\nu_0}{2[1-\nu_0]} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{3-4\nu_0}{4[1-\nu_0]} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$K_i = \frac{E_i}{3[1-2\nu_i]} \qquad \mu_i = \frac{E_i}{2[1+\nu_i]}$$

Long Cylindrical Fiber Composite



Composite: Transversely isotropic

$$
\bar{\boldsymbol{L}}_{\mathrm{MT}} = \begin{bmatrix}
\bar{K}^{\mathrm{tr}} + \bar{\mu}^{\mathrm{tr}} & \bar{K}^{\mathrm{tr}} - \bar{\mu}^{\mathrm{tr}} & \bar{l} & 0 & 0 & 0 \\
\bar{K}^{\mathrm{tr}} - \bar{\mu}^{\mathrm{tr}} & \bar{K}^{\mathrm{tr}} + \bar{\mu}^{\mathrm{tr}} & \bar{l} & 0 & 0 & 0 \\
\bar{l} & \bar{l} & \bar{n} & 0 & 0 & 0 \\
0 & 0 & 0 & \bar{\mu}^{\mathrm{tr}} & 0 & 0 \\
0 & 0 & 0 & 0 & \bar{\mu}^{\mathrm{ax}} & 0 \\
0 & 0 & 0 & 0 & 0 & \bar{\mu}^{\mathrm{ax}}
\end{bmatrix}
$$

Mori-Tanaka

$$\boldsymbol{T}_1 = \left[\boldsymbol{\mathcal{I}} + \boldsymbol{P}_1 : [\boldsymbol{L}_1 - \boldsymbol{L}_0]\right]^{-1}$$

$$\boldsymbol{P}_1 = \boldsymbol{S}_1 : \boldsymbol{L}_0^{-1}$$

$$\boldsymbol{A}_1 = \boldsymbol{T}_1 : \left[[1 - c_1]\boldsymbol{\mathcal{I}} + c_1 \boldsymbol{T}_1\right]^{-1}$$

$$\boldsymbol{A}_0 = \frac{1}{1 - c_1}\left[\boldsymbol{\mathcal{I}} - c_1 \boldsymbol{A}_1\right]$$

$$\begin{aligned}
\bar{\boldsymbol{L}} &= \boldsymbol{L}_0 + c_1[\boldsymbol{L}_1 - \boldsymbol{L}_0] : \boldsymbol{A}_1 \\
&= [1 - c_1]\boldsymbol{L}_0 : \boldsymbol{A}_0 + c_1 \boldsymbol{L}_1 : \boldsymbol{A}_1
\end{aligned}$$

Long Cylindrical Fiber Composite

Python script MT_cylinder.py

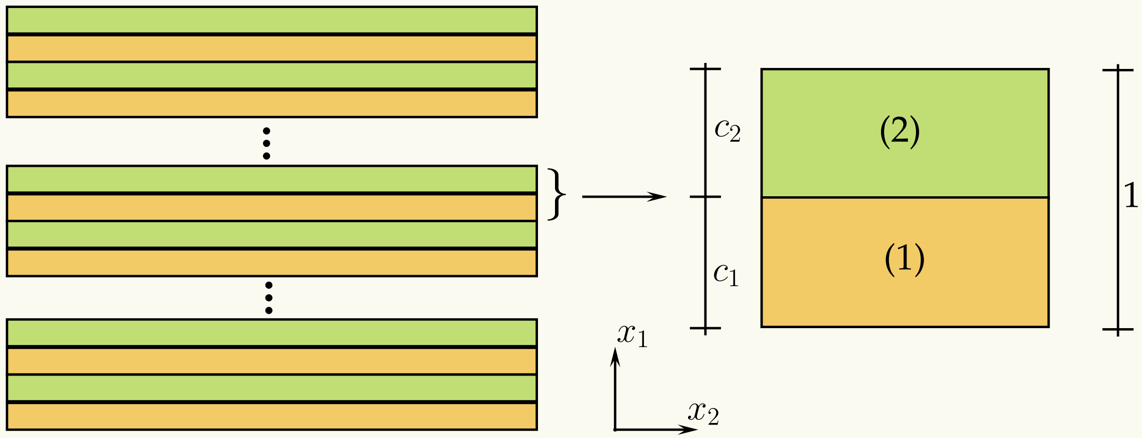
```python
import numpy as np
from elastic import isotropic
from polar_sc import polarization
# Data
E0=3000
v0=0.3
E1=72000
v1=0.2
c1=0.2
I=np.eye(6)

# Elasticity tensors
K0=E0/(3*(1-2*v0))
mu0=E0/(2*(1+v0))
K1=E1/(3*(1-2*v1))
mu1=E1/(2*(1+v1))
L0=np.zeros((6,6))
L1=np.zeros((6,6))
L0=isotropic(K0,mu0,L0)
L1=isotropic(K1,mu1,L1)

# polarization tensor
P1=np.zeros((6,6))
P1=polarization(K0,mu0,1.,"ec",P1)

# Interaction tensor
T1=(I+P1*(L1-L0)).I
# Mori-Tanaka strain concentration tensors
A1=T1*((1-c1)*I+c1*T1).I
A0=(I-c1*A1)/(1-c1)
# Effective elastic properties
Lmt=(1-c1)*L0*A0+c1*L1*A1
np.savetxt('Lmt.txt',Lmt,fmt='%1.2f')
```

Long Cylindrical Fiber Composite

Fiber composite properties

$$\bar{\boldsymbol{L}}_{\mathrm{MT}} = \begin{bmatrix} 5366.71 & 2226.34 & 2084.37 & 0 & 0 & 0 \\ 2226.34 & 5366.71 & 2084.37 & 0 & 0 & 0 \\ 2084.37 & 2084.37 & 17951.08 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1570.18 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1678.32 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1678.32 \end{bmatrix}$$

Laminate Composite



Strain per layer (Voigt notation)

$$\boldsymbol{\varepsilon}^{(k)} = \left[\begin{array}{cccccc} \varepsilon_{11}^{(k)} & \varepsilon_{22}^{(k)} & \varepsilon_{33}^{(k)} & 2\varepsilon_{12}^{(k)} & 2\varepsilon_{13}^{(k)} & 2\varepsilon_{23}^{(k)} \end{array}\right]^T \qquad k = 1,2$$

Stress per layer (Voigt notation)

$$\boldsymbol{\sigma}^{(k)} = \left[\begin{array}{cccccc} \sigma_{11}^{(k)} & \sigma_{22}^{(k)} & \sigma_{33}^{(k)} & \sigma_{12}^{(k)} & \sigma_{13}^{(k)} & \sigma_{23}^{(k)} \end{array}\right]^T \qquad k = 1,2$$

Laminate Composite

Connect micro-strain and micro-stess with macro-strain

$$\varepsilon_{ij}^{(k)} = \frac{1}{2}\left[\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} + \frac{\partial N_i^{kl(k)}}{\partial x_j} + \frac{\partial N_j^{kl(k)}}{\partial x_i}\right]\bar{\varepsilon}_{kl} = A_{ijkl}^{(k)}\bar{\varepsilon}_{kl}$$

$$\sigma_{ij}^{(k)} = L_{ijmn}^{(k)} A_{mnkl}^{(k)}\bar{\varepsilon}_{kl} = \left[L_{ijkl}^{(k)} + L_{ijmn}^{(k)}\frac{\partial N_m^{kl(k)}}{\partial x_n}\right]\bar{\varepsilon}_{kl}$$

Equilibrium equation inside a layer (k)

$$\frac{\mathrm{d}\sigma_{i1}^{(k)}}{\mathrm{d}x_1} = 0 \quad \Rightarrow \quad \frac{\mathrm{d}}{\mathrm{d}x_1}\left(L_{i1kl}^{(k)} + L_{i1m1}^{(k)}\frac{\mathrm{d}N_m^{kl(k)}}{\mathrm{d}x_1}\right)\bar{\varepsilon}_{kl} = 0$$

Continuity conditions at the interface

$$[\![N_m^{kl(k)}]\!] = 0, \quad [\![\sigma_{i1}^{(k)}]\!] = 0$$

Laminate Composite

Elementary macroscopic strain

$$kl = 11 \quad \rightarrow \quad \bar{\varepsilon}_{11} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

equilibrium equation inside a layer (k)

$$\frac{\mathrm{d}}{\mathrm{d}x_1}\left(L^{(k)}_{i111} + L^{(k)}_{i1m1}\frac{\mathrm{d}N^{11(k)}_m}{\mathrm{d}x_1}\right) = 0 \quad \Rightarrow \quad L^{(k)}_{i111} + L^{(k)}_{i1m1}\frac{\mathrm{d}N^{11(k)}_m}{\mathrm{d}x_1} = m^{(k)}_i \quad \Rightarrow$$

$$L^{(k)}_{i111}N^{11(k)}_1(x_1) + L^{(k)}_{i121}N^{11(k)}_2(x_1) + L^{(k)}_{i131}N^{11(k)}_3(x_1) = \left[m^{(k)}_i - L^{(k)}_{i111}\right]x_1 + e^{(k)}_i$$

Unknowns

3 displacements $N_i^{11(1)}$ at 0
3 displacements $N_i^{11(1)}$ at $c^{(1)}$
3 displacements $N_i^{11(2)}$ at $c^{(1)}$
3 displacements $N_i^{11(2)}$ at 1
6 constants $m_i^{(k)}$
6 constants $e_i^{(k)}$

Equations

3: equilibrium at 0 (material 1) for $i = 1, 2, 3$
3: equilibrium at $c^{(1)}$ (material 1) for $i = 1, 2, 3$
3: equilibrium at $c^{(1)}$ (material 2) for $i = 1, 2, 3$
3: equilibrium at 1 (material 2) for $i = 1, 2, 3$
6: continuity of displacements and tractions at the interface
6: At the boundary $N_i^{11(1)}(0) = N_i^{11(2)}(1) = 0$

Solution strategy

Similar technique for $kl = 22, 33$ etc. Combining all results yields

$$\frac{\mathrm{d}\boldsymbol{U}_n^{(k)}}{\mathrm{d}x_1} = \left[\boldsymbol{L}_{nn}^{(k)}\right]^{-1} \cdot \left[\boldsymbol{m}_n - \boldsymbol{L}_{nn}^{(k)}\right] \qquad \frac{\mathrm{d}\boldsymbol{U}_t^{(k)}}{\mathrm{d}x_1} = \left[\boldsymbol{L}_{nn}^{(k)}\right]^{-1} \cdot \left[\boldsymbol{m}_t - \boldsymbol{L}_{nt}^{(k)}\right]$$

$$\boldsymbol{U}_n^{(k)} = \begin{bmatrix} N_1^{11(k)} & N_1^{21(k)} & N_1^{31(k)} \\ N_2^{11(k)} & N_2^{21(k)} & N_2^{31(k)} \\ N_3^{11(k)} & N_3^{21(k)} & N_3^{31(k)} \end{bmatrix} \qquad \boldsymbol{U}_t^{(k)} = \begin{bmatrix} N_1^{22(k)} & N_1^{33(k)} & N_1^{23(k)} \\ N_2^{22(k)} & N_2^{33(k)} & N_2^{23(k)} \\ N_3^{22(k)} & N_3^{33(k)} & N_3^{23(k)} \end{bmatrix}$$

$$\boldsymbol{L}_{nn}^{(k)} = \begin{bmatrix} L_{1111}^{(k)} & L_{1121}^{(k)} & L_{1131}^{(k)} \\ L_{2111}^{(k)} & L_{2121}^{(k)} & L_{2131}^{(k)} \\ L_{3111}^{(k)} & L_{3121}^{(k)} & L_{3131}^{(k)} \end{bmatrix} \qquad \boldsymbol{L}_{nt}^{(k)} = \begin{bmatrix} L_{1122}^{(k)} & L_{1133}^{(k)} & L_{1123}^{(k)} \\ L_{2122}^{(k)} & L_{2133}^{(k)} & L_{2123}^{(k)} \\ L_{3122}^{(k)} & L_{3133}^{(k)} & L_{3123}^{(k)} \end{bmatrix}$$

$$\boldsymbol{m}_n = \left[\sum_{r=1}^{2} c^{(r)} \left[\boldsymbol{L}_{nn}^{(r)}\right]^{-1}\right]^{-1} \qquad \boldsymbol{m}_t = \boldsymbol{m}_n \cdot \left[\sum_{r=1}^{2} c^{(r)} \left[\boldsymbol{L}_{nn}^{(r)}\right]^{-1} \cdot \boldsymbol{L}_{nt}^{(r)}\right]$$

Laminate Composite

Concentration tensor for each layer (Voigt notation)

$$\widetilde{\boldsymbol{A}}^{(k)} = \begin{bmatrix} 1+\dfrac{\mathrm{d}N_1^{11(k)}}{\mathrm{d}x_1} & \dfrac{\mathrm{d}N_1^{22(k)}}{\mathrm{d}x_1} & \dfrac{\mathrm{d}N_1^{33(k)}}{\mathrm{d}x_1} & \dfrac{\mathrm{d}N_1^{21(k)}}{\mathrm{d}x_1} & \dfrac{\mathrm{d}N_1^{31(k)}}{\mathrm{d}x_1} & \dfrac{\mathrm{d}N_1^{23(k)}}{\mathrm{d}x_1} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \dfrac{\mathrm{d}N_2^{11(k)}}{\mathrm{d}x_1} & \dfrac{\mathrm{d}N_2^{22(k)}}{\mathrm{d}x_1} & \dfrac{\mathrm{d}N_2^{33(k)}}{\mathrm{d}x_1} & 1+\dfrac{\mathrm{d}N_2^{21(k)}}{\mathrm{d}x_1} & \dfrac{\mathrm{d}N_2^{31(k)}}{\mathrm{d}x_1} & \dfrac{\mathrm{d}N_2^{23(k)}}{\mathrm{d}x_1} \\ \dfrac{\mathrm{d}N_3^{11(k)}}{\mathrm{d}x_1} & \dfrac{\mathrm{d}N_3^{22(k)}}{\mathrm{d}x_1} & \dfrac{\mathrm{d}N_3^{33(k)}}{\mathrm{d}x_1} & \dfrac{\mathrm{d}N_3^{21(k)}}{\mathrm{d}x_1} & 1+\dfrac{\mathrm{d}N_3^{31(k)}}{\mathrm{d}x_1} & \dfrac{\mathrm{d}N_3^{23(k)}}{\mathrm{d}x_1} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Laminate Composite

Python script PH_layered.py

```python
import numpy as np
from elastic import isotropic,orthoinv
from MT_cylinder import Lmt
# Data
E2=6000
v2=0.25
c=0.5

# Elasticity tensors
L1=Lmt
K2=E2/(3*(1-2*v2))
mu2=E2/(2*(1+v2))
L2=np.zeros((6,6))
L2=isotropic(K2,mu2,L2)

# mn and mt
Lnn1=np.matrix([[L1[0,0], L1[0,3], L1[0,4]],\
                [L1[0,3], L1[3,3], L1[3,4]],\
                [L1[0,4], L1[3,4], L1[4,4]]])
Lnt1=np.matrix([[L1[0,1], L1[0,2], L1[0,5]],\
                [L1[1,3], L1[2,3], L1[3,5]],\
                [L1[1,4], L1[2,4], L1[4,5]]])
Lnn2=np.matrix([[L2[0,0], L2[0,3], L2[0,4]],\
                [L2[0,3], L2[3,3], L2[3,4]],\
                [L2[0,4], L2[3,4], L2[4,4]]])
Lnt2=np.matrix([[L2[0,1], L2[0,2], L2[0,5]],\
                [L2[1,3], L2[2,3], L2[3,5]],\
                [L2[1,4], L2[2,4], L2[4,5]]])
mn=((1-c)*Lnn1.I+c*Lnn2.I).I
mt=mn*((1-c)*Lnn1.I*Lnt1+c*Lnn2.I*Lnt2)

# Strain concentration tensors
dUndx1=Lnn1.I*(mn-Lnn1)
dUtdx1=Lnn1.I*(mt-Lnt1)
dUndx2=Lnn2.I*(mn-Lnn2)
dUtdx2=Lnn2.I*(mt-Lnt2)
```

```python
A1=np.matrix([[1.+dUndx1[0,0], dUtdx1[0,0], dUtdx1[0,1],\
               dUndx1[0,1], dUndx1[0,2], dUtdx1[0,2]],\
              [0., 1., 0., 0., 0., 0.],\
              [0., 0., 1., 0., 0., 0.],\
              [dUndx1[1,0], dUtdx1[1,0], dUtdx1[1,1],\
               1.+dUndx1[1,1], dUndx1[1,2], dUtdx1[1,2]],\
              [dUndx1[2,0], dUtdx1[2,0], dUtdx1[2,1],\
               dUndx1[2,1], 1.+dUndx1[2,2], dUtdx1[2,2]],\
              [0., 0., 0., 0., 0., 1.]])
A2=np.matrix([[1.+dUndx2[0,0], dUtdx2[0,0], dUtdx2[0,1],\
               dUndx2[0,1], dUndx2[0,2], dUtdx2[0,2]],\
              [0., 1., 0., 0., 0., 0.],\
              [0., 0., 1., 0., 0., 0.],\
              [dUndx2[1,0], dUtdx2[1,0], dUtdx2[1,1],\
               1.+dUndx2[1,1], dUndx2[1,2], dUtdx2[1,2]],\
              [dUndx2[2,0], dUtdx2[2,0], dUtdx2[2,1],\
               dUndx2[2,1], 1.+dUndx2[2,2], dUtdx2[2,2]],\
              [0., 0., 0., 0., 0., 1.]])

# Effective properties
Lper=(1-c)*L1*A1+c*L2*A2
np.savetxt('Lper.txt',Lper,fmt='%1.2f')
Vprop=np.zeros(9)
Vprop=orthoinv(Lper,Vprop)
print('Ex=',Vprop[0])
print('Ey=',Vprop[1])
print('Ez=',Vprop[2])
print('vxy=',Vprop[3])
print('vxz=',Vprop[4])
print('vyz=',Vprop[5])
print('muxy=',Vprop[6])
print('muxz=',Vprop[7])
print('muyz=',Vprop[8])
```

Laminate Composite

Laminate composite properties

$$\bar{\boldsymbol{L}}_{\text{per}} = \begin{bmatrix} 6149.63 & 2300.5 & 2219.16 & 0 & 0 & 0 \\ 2300.5 & 6282.15 & 2240 & 0 & 0 & 0 \\ 2219.16 & 2240 & 12571.58 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1898.37 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1975.31 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2039.16 \end{bmatrix}$$

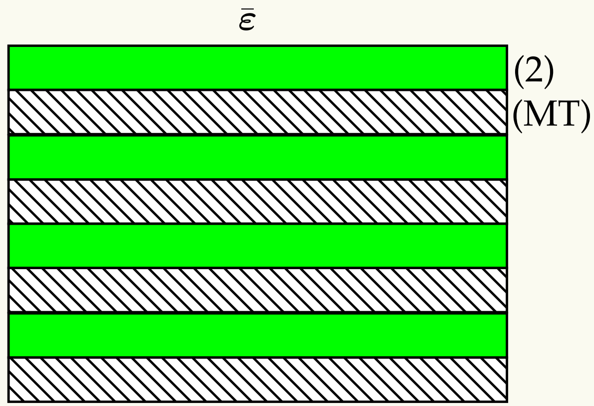
Engineering constants

E_x	5140.97	MPa
E_y	5252.7	MPa
E_z	11404.15	MPa
ν_{xy}	0.3238	
ν_{xz}	0.1188	
ν_{yz}	0.1198	
μ_{xy}	1898.37	MPa
μ_{xz}	1975.31	MPa
μ_{yz}	2039.16	MPa

Macroscopic scale

$$\bar{\boldsymbol{\varepsilon}} = \overline{\boldsymbol{L}}_{\text{per}}^{-1} : \overline{\boldsymbol{\sigma}}$$

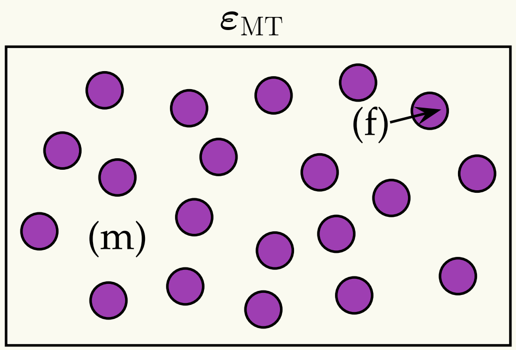
Mesoscopic scale



$$\boldsymbol{\varepsilon}_2 = \boldsymbol{A}_2 : \bar{\boldsymbol{\varepsilon}} \qquad \boldsymbol{\sigma}_2 = \boldsymbol{L}_2 : \boldsymbol{\varepsilon}_2$$

$$\boldsymbol{\varepsilon}_{\text{MT}} = \boldsymbol{A}_{\text{MT}} : \bar{\boldsymbol{\varepsilon}}$$

Microscopic scale



$$\boldsymbol{\varepsilon}_{\text{f}} = \boldsymbol{A}_{\text{f}} : \boldsymbol{\varepsilon}_{\text{MT}} \qquad \boldsymbol{\sigma}_{\text{f}} = \boldsymbol{L}_{\text{f}} : \boldsymbol{\varepsilon}_{\text{f}}$$

$$\boldsymbol{\varepsilon}_{\text{m}} = \boldsymbol{A}_{\text{m}} : \boldsymbol{\varepsilon}_{\text{MT}} \qquad \boldsymbol{\sigma}_{\text{m}} = \boldsymbol{L}_{\text{m}} : \boldsymbol{\varepsilon}_{\text{m}}$$

Python script fields.py

```
import numpy as np
from MT_cylinder import Lmt, L0 as Lm, L1 as Lf, A0 as Am, A1 as Af, c1 as cf
from PH_layered import Lper, L2, A2, A1 as Amt, c as c2
# Data
Ms=np.matrix([[50.], [0.], [0.], [0.], [0.], [0.]])
# Macroscale
Me=Lper.I*Ms
# Mesoscale
e2=A2*Me
s2=L2*e2
emt=Amt*Me
# Microscale
ef=Af*emt
sf=Lf*ef
em=Am*emt
sm=Lm*em
# Write results
f = open('fields.txt', 'w')
f.write("e2 s2 ef sf em sm\n")
for i in range(6):
    f.write("%1.5f,%1.2f,%1.5f,%1.2f,%1.5f,%1.2f\n" %\
           (e2[[i]],s2[[i]],ef[[i]],sf[[i]],em[[i]],sm[[i]]))
f.close()
```

Average fields

$$\varepsilon_2 = \begin{bmatrix} 0.00838 \\ -0.00315 \\ -0.00116 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \sigma_2 = \begin{bmatrix} 50 \\ -5.34 \\ 4.23 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{MPa}$$

$$\varepsilon_{\mathrm{f}} = \begin{bmatrix} 0.0011 \\ 0.00007 \\ -0.00116 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \sigma_{\mathrm{f}} = \begin{bmatrix} 66.25 \\ 4.67 \\ -69.02 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{MPa}$$

$$\varepsilon_{\mathrm{m}} = \begin{bmatrix} 0.01357 \\ -0.00396 \\ -0.00116 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \sigma_{\mathrm{m}} = \begin{bmatrix} 45.94 \\ 5.51 \\ 11.97 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{MPa}$$

Useful Python Script Functions

file	function	description
elastic.py	isotropic	isotropic $\boldsymbol{L}$ for known K and μ
elastic.py	orthoinv	Engineering constants for known orthotropic $\boldsymbol{L}$
polar_sc.py	polarization	$\boldsymbol{P} = \boldsymbol{S} : \boldsymbol{L}^{-1}$ for various types of inclusions

Use of Functions

L=isotropic(K,mu,L)

P=polarization(K,mu,r,typ,P)

inclusion shape	typ	r
prolate spheroid	“ps"	a_3/a_1
oblate spheroid	“os"	a_3/a_1
elliptic cylinder	“ec"	a_1/a_2
spherical	“si"	1
disk-like	“dl"	1

Vprop=orthoinv(L,Vprop)

i of Vprop[i]	constant
0	E_x
1	E_y
2	E_z
3	v_{xy}
4	v_{xz}
5	v_{yz}
6	μ_{xy}
7	μ_{xz}
8	μ_{yz}