1.

The following are the assignments determined by the auction algorithm and GLPK in the form of (agent: object)

0: 9

1: 8

2: 3

3: 1

4: 7

5: 4

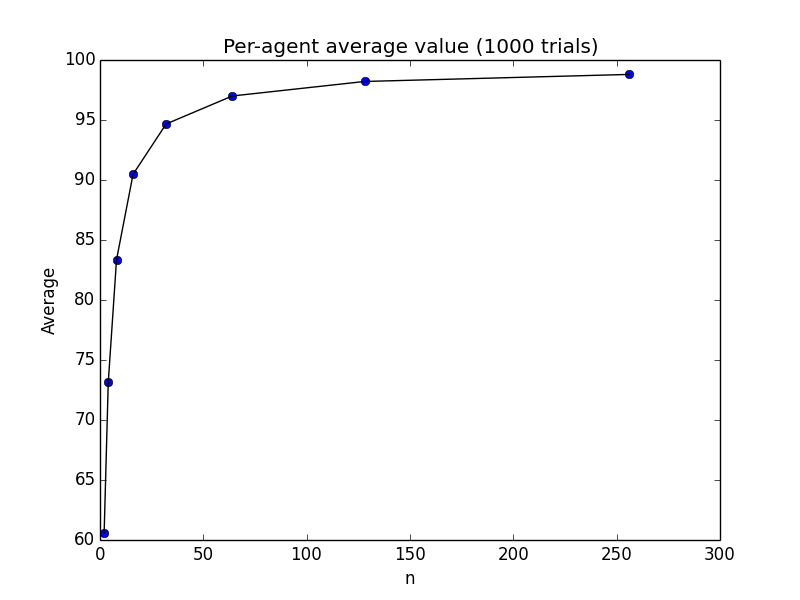
6: 6

7: 2

8: 0

9: 5

The total value of these assignments is 847



2. Possible Matches

Stable Match 1:

Equally Optimal, men and women each get their second choice

1,2

2,3

3,1

Stable Match 2:

Man Optimal, men get 1st choice and women get 3rd choice

1,1

2,2

3,3

Stable Match 3:

Woman Optimal, men get 3rd choice and women get 1st choice

1,3

2,1

3,2

3.

Let M be the matrix of male preferences

Let W be the matrix of female preferences

Let and is the value of the assignment of man i to woman j

Let be the assignment. Where

We want to Maximize

Subject to

4.

Match results with men proposing:

(1,1)

(2,2)

(3,3)

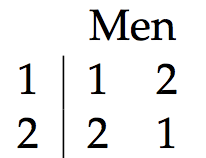
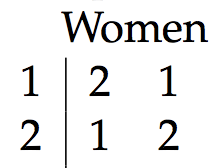
In the scenario presented for this questions, women cannot lie about their preferences and end up being better off. With men proposing the result is seen above. When women propose, the resulting assignments are:

2,1

1,2

3,3

In both man optimal and woman optimal assignments, man 3 and woman 3 are matched. This occurs because all of the men’s least favorite option is woman 3 and all of the women’s least favorite option is man 3. Due to this, we can effectively reduce the problem to only pay attention to the assignment of the first two men and women as seen below – accepting the assignment of man 3 to woman 3 as inevitable.

In this subproblem, it is easier to see that there is no way for the women to lie about their preferences and end up with a better assignment. If either of them switch their preferences, they will be proposed by their “fake” first option in the first iteration and will accept, leaving the other match with their preferred option.

5.

Preferences:

A1: h2 > h1 > h3 h1: a1 ~ a1 ~ a3

A2: h1 > h2 > h3 h2: a1 > a1 > a3

A3: h1 > h2 > h3 h3: a3 > a1 > a2

Agent proposing Gale-Shapley algorithm:

Where Red=Rejected, Blue=New Pair, Black=Old Pair

Iteration 1: Iteration 2: Iteration 3: Iteration 4 & Termination

A1 – h2 A1 – h2 A3 – h1 A1 – h1

A2 – h1 A3 – h1 A2 – h2 A2 – h2

A3 – h1 A2 – h2 A1 – h1 A3 – h3

Weakly stable:

A1 – h1: A1 prefers h1 but h1 has no strict preference

A2 – h2: A2 prefers h1 > h2 but h1 has no strict preference

A3 – h3: A3 prefers h1 > h3 but h1 has no strict preference

A3 prefers h2 > h3 but h2 prefers a2 > a3