Exercise MODIS. MODIS along-scan ΔS and along-track ΔT ground pixel sizes can be derived as functions of θ from the following relationships:

$$\Delta S = R_e s \left(\frac{\cos\theta}{\sqrt{\left(\frac{R_e}{r}\right)^2 - \sin^2\theta}} - 1 \right) \tag{1}$$

$$\Delta T = rs \left(\cos\theta - \sqrt{\left(\frac{R_e}{r}\right)^2 - \sin^2\theta} \right) \tag{2}$$

where $R_e = 6378.137$ km (radius of the Earth), $h = 705$ km (altitude of the satellite), $r = R_e + h$, $s = \frac{px}{h}$, px is the pixel nadir resolution in kilometers, and θ is the scan angle for the given pixel.

Express θ in terms of ΔS and ΔT.

Proof. Starting here

Step 1: Let $\gamma = \sqrt{\left(\frac{R_e}{r}\right)^2 - \sin^2\theta}$

Step 2: Substitute γ into Eq. (2), and rearrange for γ

$$\Delta T = rs \left(\cos\theta - \sqrt{\left(\frac{R_e}{r}\right)^2 - \sin^2\theta} \right)$$

$$\Delta T = rs\left(\cos\theta - \gamma\right)$$

$$\frac{\Delta T}{rs} = \cos\theta - \gamma$$

$$\gamma = \left(\cos\theta - \frac{\Delta T}{rs}\right) \tag{3}$$

Step 3: Substitute γ and Eq. (3) into Eq. (1).

$$\Delta S = R_e s \left(\frac{\cos\theta}{\sqrt{\left(\frac{R_e}{r}\right)^2 - \sin^2\theta}} - 1 \right)$$

$$\Delta S = R_e s \left(\frac{\cos\theta}{\gamma} - 1 \right)$$

$$\Delta S = R_e s \left(\frac{\cos\theta}{\left(\cos\theta - \frac{\Delta T}{rs}\right)} - 1 \right)$$

$$\frac{\Delta S}{R_e s} = \frac{\cos\theta}{\left(\cos\theta - \frac{\Delta T}{rs}\right)} - 1$$

Step 4: Let $\alpha = \frac{\Delta T}{rs}$ and $\beta = \frac{\Delta S}{R_e s}$, and solve for θ.

$$\begin{aligned}
\frac{\Delta S}{R_e s} &= \frac{\cos\theta}{\left(\cos\theta - \frac{\Delta T}{rs}\right)} - 1 \\
\beta &= \frac{\cos\theta}{(\cos\theta - \alpha)} - 1 \\
\beta + 1 &= \frac{\cos\theta}{(\cos\theta - \alpha)} \\
(\beta + 1)(\cos\theta - \alpha) &= \cos\theta \\
\beta\cos\theta - \alpha\beta + \cos\theta - \alpha &= \cos\theta \\
\beta\cos\theta - \alpha\beta - \alpha &= 0 \\
\beta\cos\theta &= \alpha\beta + \alpha \\
\beta\cos\theta &= \alpha(\beta + 1) \\
\cos\theta &= \frac{\alpha(\beta + 1)}{\beta} \\
\therefore \theta &= \cos^{-1}\left(\frac{\alpha(\beta + 1)}{\beta}\right)
\end{aligned}$$

where $\alpha = \frac{\Delta T}{rs}$ and $\beta = \frac{\Delta S}{R_e s}$.

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Exercise VIIRS. VIIRS along-scan ΔS and along-track ΔT ground pixel sizes can be derived as functions of θ from the following relationships:

$$\Delta S = R_e s \left(\frac{\cos\theta}{\sqrt{\left(\frac{R_e}{r}\right)^2 - \sin^2\theta}} - 1 \right) \tag{3}$$

$$\Delta T = rs \left(\cos\theta - \sqrt{\left(\frac{R_e}{r}\right)^2 - \sin^2\theta} \right) \tag{4}$$

where $R_e = 6378.137$ km (radius of the Earth), $h = 834$ km (altitude of the satellite), $r = R_e + h$, $s = \frac{px}{h}$, px is the pixel nadir resolution in kilometers, and θ is the scan angle for the given pixel.

Unlike MODIS, VIIRS has an automatic Bowtie adaption that automatically changes the size of the along-scan ΔS dimensions, rendering it unusable in the estimation of θ. Given this, derive θ from ΔT alone.

Proof. Starting here

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\begin{aligned}
\Delta T &= rs\left(\cos\theta - \sqrt{\left(\frac{R_e}{r}\right)^2 - \sin^2\theta}\right) \\
\frac{\Delta T}{rs} &= \cos\theta - \sqrt{\left(\frac{R_e}{r}\right)^2 - \sin^2\theta} \\
\frac{\Delta T}{rs} - \cos\theta &= -\sqrt{\left(\frac{R_e}{r}\right)^2 - \sin^2\theta} \\
\cos\theta - \frac{\Delta T}{rs} &= \sqrt{\left(\frac{R_e}{r}\right)^2 - \sin^2\theta} \\
\left(\cos\theta - \frac{\Delta T}{rs}\right)^2 &= \left(\sqrt{\left(\frac{R_e}{r}\right)^2 - \sin^2\theta}\right)^2 \\
\left(\cos\theta - \frac{\Delta T}{rs}\right)\left(\cos\theta - \frac{\Delta T}{rs}\right) &= \left(\frac{R_e}{r}\right)^2 - \sin^2\theta \\
\cos^2\theta - \cos\theta\frac{\Delta T}{rs} - \cos\theta\frac{\Delta T}{rs} + \left(\frac{\Delta T}{rs}\right)^2 &= \left(\frac{R_e}{r}\right)^2 - \sin^2\theta \\
\cos^2\theta - 2\cos\theta\frac{\Delta T}{rs} + \left(\frac{\Delta T}{rs}\right)^2 &= \left(\frac{R_e}{r}\right)^2 - \sin^2\theta \\
\cos^2\theta - 2\cos\theta\frac{\Delta T}{rs} + \left(\frac{\Delta T}{rs}\right)^2 + \sin^2\theta &= \left(\frac{R_e}{r}\right)^2 \\
\left(\cos^2\theta + \sin^2\theta\right) - 2\cos\theta\frac{\Delta T}{rs} + \left(\frac{\Delta T}{rs}\right)^2 &= \left(\frac{R_e}{r}\right)^2 \\
1 - 2\cos\theta\frac{\Delta T}{rs} + \left(\frac{\Delta T}{rs}\right)^2 &= \left(\frac{R_e}{r}\right)^2 \\
1 - \left(\frac{R_e}{r}\right)^2 + \left(\frac{\Delta T}{rs}\right)^2 &= 2\cos\theta\frac{\Delta T}{rs} \\
\frac{rs\left(1 - \left(\frac{R_e}{r}\right)^2 + \left(\frac{\Delta T}{rs}\right)^2\right)}{2\Delta T} &= \cos\theta \\
\therefore \theta &= \cos^{-1}\left(\frac{rs\left(1 - \left(\frac{R_e}{r}\right)^2 + \left(\frac{\Delta T}{rs}\right)^2\right)}{2\Delta T}\right)
\end{aligned}
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