Trabalho 3 - Modelos Markovianos

Maike W. Martins - GRR20159111

10/08/2020

Exercicio 1

Mostrar que se o estado x é recorrente e do não se comunica com o estado y então $\gamma_{x,y} = 0$.

Respota:

Seja S(x,y) onde x é um estado recorrente, logo $\{\rho_{x,x} = 1 \ e \ \rho_{x,y} = 0\}$. Seja C_n uma cadeia de markov com espaço de estados S onde $N(y) =$ n° de vezes que a cadeia esta no espaço y, então:

$$N(y) = \sum_{n=1}^{\infty} l_y(C_n)$$

Também observamos que $N(y) \geq 1$ é o mesmo que $T_y < \infty$ logo:

$$P_x(N(y) \geq 1) = P_x(T_y < \infty) = \rho_{x,y}$$

Sejam m e n números inteiros positivos.Sabemos que a probabilidade com a qual a cadeia começando em x visitar a primeira vez y no tempo m e visitar novamente no tempo n é:

$$P_x(T_y = m)P_y(T_y = n)$$

Logo:

$$P_x(N(y) \geq 2) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} P_x(T_y = m).P_y(T_y = n) = \left[\sum_{m=1}^{\infty} P_x(T_y = m)\right].\left[\sum_{n=1}^{\infty} P_y(T_y = m)\right] = \rho_{x,x}\rho_{x,y} = 1{*}0 = 0$$

Exercicio 3

Mostre que se o estado x se comunica com y e y se comunica com z, então x se comunica com z.

Resposta:

Se x se comunica com y, então temos $P_x(T_y = n) > 0$ para algum n finito.Partindo do mesmo princípio, se y se comunica com z temos $P_y(T_z = m) > 0$ para algum m finito.Portanto $P_x(T_z = m + n) > 0$ o que demonstra que x se comunica com z

Exercicio 4

Considere uma cadeia de marvok com espaço de estados *{1,2,...,9}* e matriz de probabilidade de transição:

$$\Gamma = \begin{bmatrix} 0 & 0.5 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 \\ 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Esta cadeira é irredutível? ou seja, prove que o conjunto de estados erredutíveis F satisfaz $F = S$, sendo $S = \{1, .., 9\}$. Prove também que esta cadeia é recorrente, ou seja, prove que cada estado em S é recorrente.

Resposta:

```r
library(markovchain)
mat4 <- matrix(c(0.0,0.0,0.0,1.0,0.0,0.0,0.0,0.0,1.0,
                 0.5,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,
                 0.0,1.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,
                 0.0,0.0,1.0,0.0,0.0,0.0,0.0,0.0,0.0,
                 0.5,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,
                 0.0,0.0,0.0,0.0,1.0,0.0,0.0,0.0,0.0,
                 0.0,0.0,0.0,0.0,0.0,1.0,0.0,0.0,0.0,
                 0.0,0.0,0.0,0.0,0.0,0.0,1.0,0.0,0.0,
                 0.0,0.0,0.0,0.0,0.0,0.0,0.0,1.0,0.0), nrow=9)
estados4 = c("0","1","2","3","4","5","6","7","8")
mc4 <- new("markovchain", states=estados4, transitionMatrix=mat4, name="Exercicio 4")
mc4
```

```
## Exercicio 4
##  A  9 - dimensional discrete Markov Chain defined by the following states:
##  0, 1, 2, 3, 4, 5, 6, 7, 8
##  The transition matrix  (by rows)  is defined as follows:
##   0   1 2 3   4 5 6 7 8
## 0 0 0.5 0 0 0.5 0 0 0 0
## 1 0 0.0 1 0 0.0 0 0 0 0
## 2 0 0.0 0 1 0.0 0 0 0 0
## 3 1 0.0 0 0 0.0 0 0 0 0
## 4 0 0.0 0 0 0.0 1 0 0 0
## 5 0 0.0 0 0 0.0 0 1 0 0
## 6 0 0.0 0 0 0.0 0 0 1 0
## 7 0 0.0 0 0 0.0 0 0 0 1
## 8 1 0.0 0 0 0.0 0 0 0 0
```

```r
transientStates(mc4)
```

```
## character(0)
```

```r
steadyStates(mc4)
```

```
##        0   1   2   3   4   5   6   7   8
## [1,] 0.2 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1
```

```r
steadyStates(mc4^1000)
```

```
##        0   1   2   3   4   5   6   7   8
## [1,] 0.0 0.2 0.0 0.2 0.2 0.0 0.2 0.0 0.2
## [2,] 0.4 0.0 0.2 0.0 0.0 0.2 0.0 0.2 0.0
```

```r
transientStates(mc4)
```

```
## character(0)
```

```r
summary(mc4)
```

```
## Exercicio 4  Markov chain that is composed by:
## Closed classes:
## 0 1 2 3 4 5 6 7 8
## Recurrent classes:
```

```
## {0,1,2,3,4,5,6,7,8}
## Transient classes:
## NONE
## The Markov chain is irreducible
## The absorbing states are: NONE
```

Exercicio 6

A Fiscalía de Mídia identificou seis estados associados à televisão: 0(nunca assiste TC), 1(assiste apenas noticias), 2(assiste TC com bastante frequência), 3(viciado), 4(em modificação de comportamento) e 5(morte encefálica). As transições de estado para estado podem ser modeladas com uma cadeira de Markov com a seguinte matriz de transição:

$$\Gamma = \begin{bmatrix} 1.0 & 0 & 0 & 0 & 0.5 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 & 0 \\ 0.1 & 0 & 0.5 & 0.3 & 0 & 0.1 \\ 0 & 0 & 0 & 0.7 & 0.1 & 0.2 \\ 1/3 & 0 & 0 & 1/3 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

a)Quais estados são recorrentes e quais transientes?

Resposta:

```
#Resposta
```

b)Començando do estado 1, qual é a probabilidade de o estado 5 ser atingido antes do estado 0, ou seja, qual é a probabilidade de um isualizador de notícias acabar com morte cerebral?

Resposta:

$$f(x) = \sum_{y \epsilon S_R} \gamma_{x,y} + \sum_{y \epsilon S_T} \gamma_{x,y} f(y) \ x \epsilon S_T f(1) = (\gamma_{1,0} + \gamma_{1,5}) + (\gamma_{1,1} f(1) + \gamma(1,2) f(2) + \gamma_{1,3} f(3) + \gamma_{1,4} f(4)) = 0,5 + 0,5 f(2) f(2) = (\gamma_2$$

$$\begin{cases} f(1) - 1/2 f(2) + 0 f(3) + 0 f(4) & = 1/2 \\ 0 f(1) + 1/2 f(2) - 0,3 f(3) + 0 f(4) & = 1/5 \\ 0 f(1) + 0 f(2) + 0,3 f(3) - 0,1 f(4) & = 1/5 \\ 0 f(1) + 0 f(2) - 1/3 f(3) + 2/3 f(4) & = 1/3 \end{cases} \begin{bmatrix} 1 & -0,5 & 0 & 0 \\ 0 & 0,5 & -0,3 & 0 \\ 0 & 0 & 0,3 & -0,1 \\ 0 & 0 & -1/3 & 2/3 \end{bmatrix} * \begin{bmatrix} 0,5 \\ 0,2 \\ 0,2 \\ 1/3 \end{bmatrix}$$