ME40331
FORWARD AND INVERSE KINEMATICS REPORT

Robotics Matlab-Based Assignment

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Contents

Assignment Description

This report details the design and implementation of code to simulate a 6 degree of freedom (DoF) robot. The robot consists of six revolute joints. The robot has a layout as shown in the figure below, with three axes of rotation along the joint lengths and three axes of rotation perpendicular these.

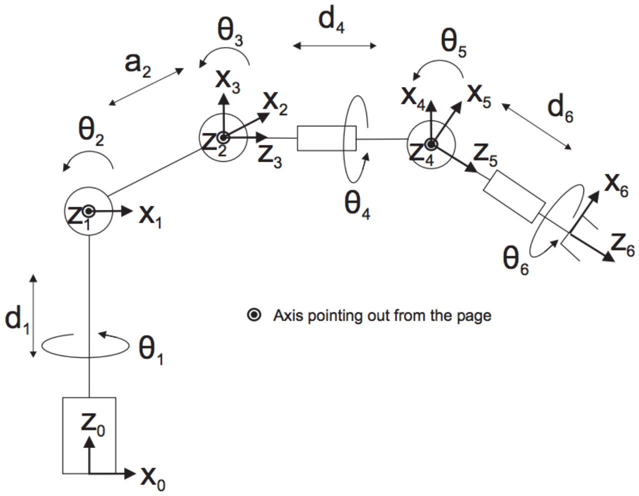

Figure 1 - 6 DoF Robot Diagram

The robot has dimensions as shown in the figure below. The figure also shows the direction of rotation of the revolute joints allowing the Denavit-Hartemberg (DH) parameters to be calculated.

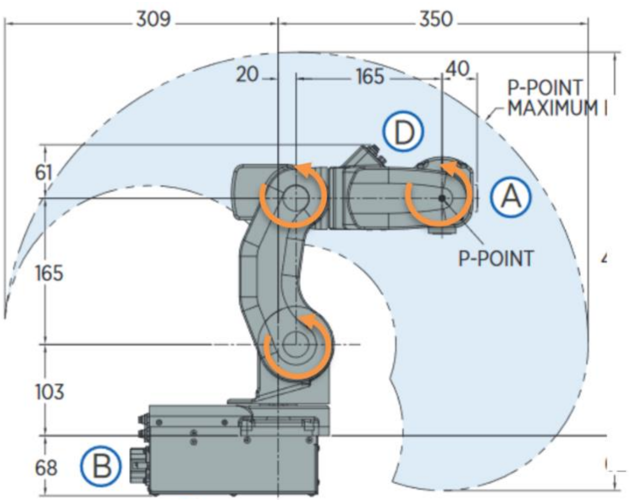

Figure 2 - 6 DoF Robot Measurements

The DH parameters have been calculated and can be seen summarised in the table below. Theta is the angle difference between the required joint angle and the initialisation position.

i	d_i	a_i	α_i	θ_i
1	103	20	90	Θ_1
2	0	165	0	Θ_2
3	0	0	90	Θ_3
4	165	0	-90	Θ_4
5	0	0	90	Θ_5
6	40	0	0	Θ_6

Table 1 - DH Parameters for 6 DoF Robot

Part 1 – Forward Kinematics

1.1 Method & Equations

The Forward Kinematics method uses given input angles to find the positions and rotation matrices of each joint reference frame. The Denavit-Hartemberg method can be used to find the transform matrices to each joint reference frame from the previous reference frame. The figure below shows the transform matrix calculations from the DH Parameter table (Table 1).

$$T = \begin{bmatrix} C\theta & -S\theta C\alpha & S\theta S\alpha & aC\theta \\ S\theta & C\theta C\alpha & -C\theta S\alpha & aS\theta \\ 0 & S\alpha & C\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 3 - Homogeneous Transform Matrix

Once the transform matrices from the previous reference frame have been found, the transform matrices from the robot base frame to each of the robot frames are found using the following equation:

Equation 1: $$T_n^0 = T_1^0 T_2^1 \ldots T_n^{n-1}$$

Once these transform matrices have been the rotation matrix and XYZ co-ordinates for each joint reference frame can be found in the transform matrix by using the following diagram:

$$T_n^0 = \begin{array}{c} \begin{array}{cc} \text{Rotation Matrix} & \quad \text{XYZ} \end{array} \\ \begin{bmatrix} \boxed{\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}} & \boxed{\begin{matrix} a_{14} \\ a_{24} \\ a_{34} \end{matrix}} \\ \begin{matrix} a_{41} & a_{42} & a_{43} \end{matrix} & a_{44} \end{bmatrix} \end{array}$$

Figure 4 - Transform Matrix Components

These XYZ co-ordinates and rotation matrices are then sent to the GUI to be plotted in the correct position. This process has been summarised in the following flowchart.

1.2 Program Flowchart

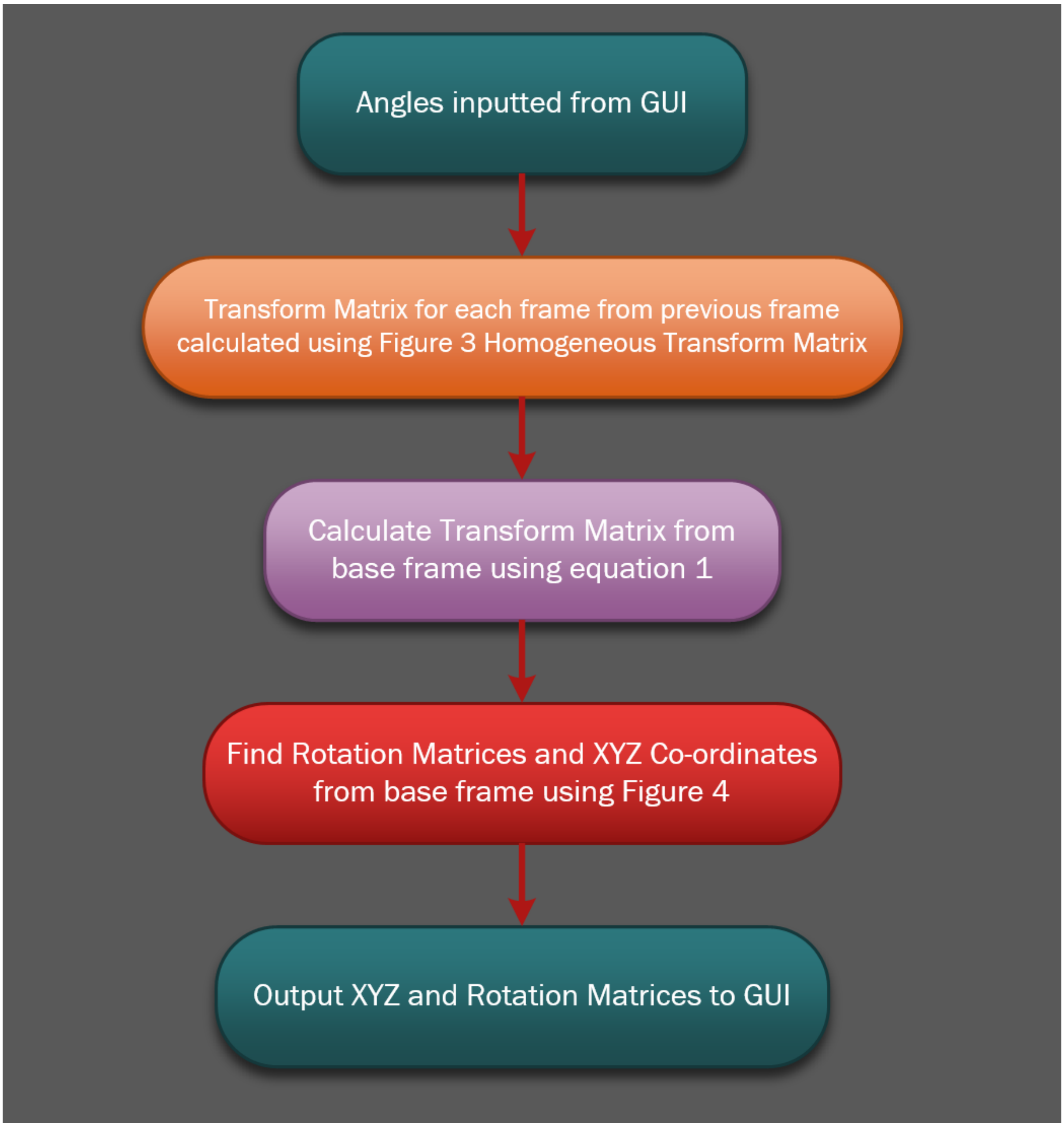

Figure 5 - Forward Kinematic Flowchart

1.3 Functions

The Forward Kinematics operation uses the following functions:

Name	Inputs	Outputs	Function Role
FK.m	Joint angles	End effector position XYZ Co-ordinates Rotation Matrices	To calculate the rotation matrices and XYZ co-ordinates from the inputted angles from the GUI.
DHConvention.m	Joint angles Frame number	Transform Matrix	To calculate the transform matrix from the selected frame and joint angles and DH-Parameters.

Table 2 - Forward Kinematics Functions

Part 2 – Reverse Kinematics

2.1 Method & Equations

2.1.1 Wrist Centre Position

As opposed to Forward Kinematics which uses given joint angles to find the end-effector position, Reverse Kinematics uses a given end-effector position and orientation to find the possible angles to satisfy the end-effector position and orientation. The end-effector position and rotation matrix from the base reference frame is inputted and the first step taken is to calculate the wrist centre position. The wrist centre is point where the elbow manipulator connects to the wrist manipulator shown in the figures below.

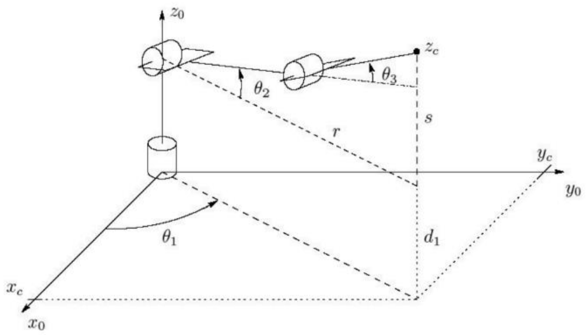

Figure 7 - Elbow Manipulator

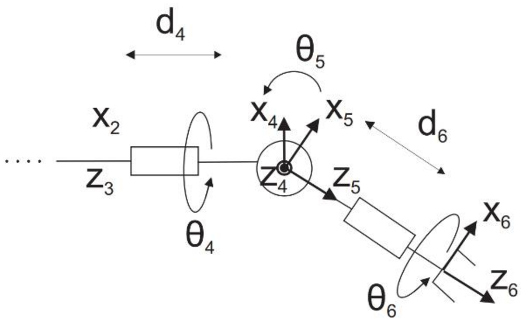

Figure 6 - Wrist Manipulator

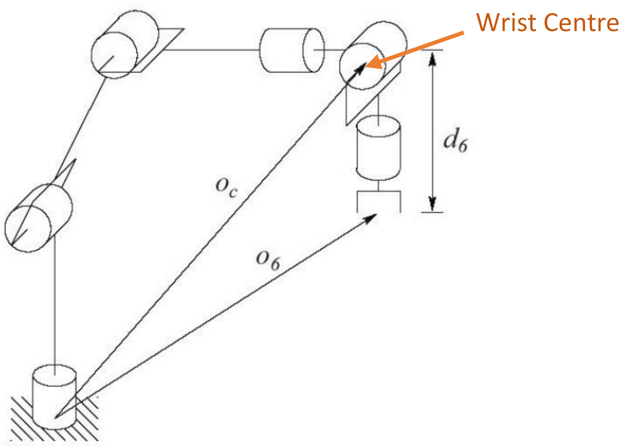

Figure 8 - Kinematic Decoupling

After the wrist centre position has been found using the following equation 2, the configurations of the first three angles θ_1, θ_2 & θ_3 from figure 8, can be found using the following method:

Equation 2:
$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} o_x - d_6 r_{13} \\ o_y - d_6 r_{23} \\ o_z - d_6 r_{33} \end{bmatrix}$$

2.1.2 Theta 1

θ_1:

Front

Equation 3: $$\theta_1 = \tan^{-1}\left(\frac{y_c}{x_c}\right)$$

Back

Equation 4: $$\theta_1 = \pi + \tan^{-1}\left(\frac{y_c}{x_c}\right)$$

The following two joints form an elbow which can be configured one of four ways; front elbow-up (FEU), front elbow-down (FED), back elbow-up (BEU) or back elbow-down (BED). These configurations can be seen on the figures below.

2.1.3 Theta 3

When calculating θ_3 the angle α is needed to be found first, this is done using the cosine rule on the triangle formed between joints 2, 3 and the wrist centre as shown on the diagram below. The distances S and r can be found from figure 7 and are used to calculate distance T.

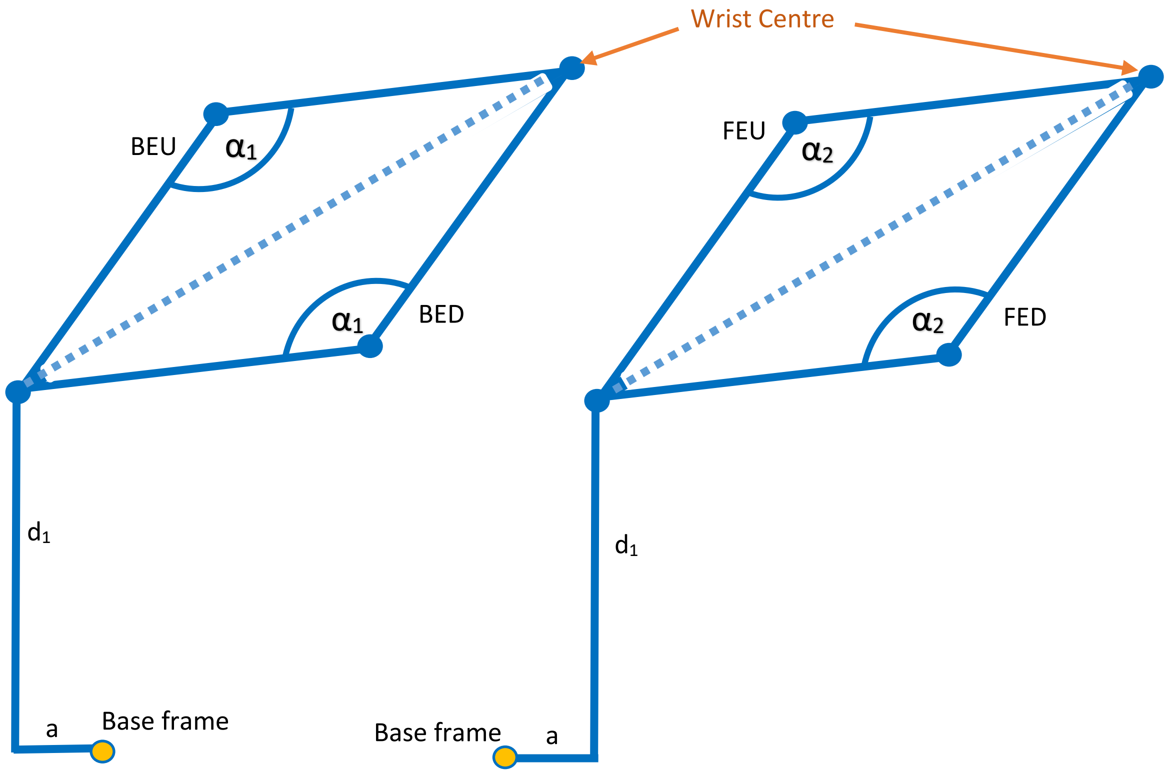

Figure 10 - Elbow Configurations

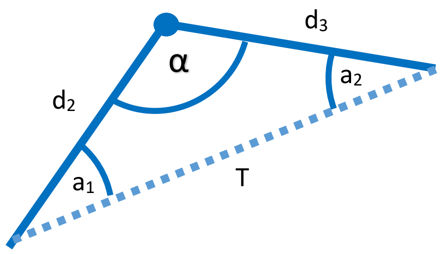

Figure 9 - Elbow Angle Diagram

Equation 8: S = $z_c - d_1$

Equation 8: r = $\sqrt{{y_c}^2 + {x_c}^2} \pm a$

Equation 8: $T^2 = d_2^2 + d_3^2 - 2.a_1.a_2.\text{Cos}(\alpha)$

Equation 8: T = $\sqrt{S^2 + r^2}$

Angle α is found combining equations 4,5,6 & 7 to produce the following equation:

Equation 9: $$\alpha = \cos^{-1}\left(\frac{T^2 - d_2^2 - d_3^2}{-2.d_2^2.d_3^2}\right)$$

Θ_3:

Elbow up

Equation 10: $$\theta_3 = \alpha - \frac{\pi}{2}$$

Elbow down

Equation 11: $$\theta_3 = \alpha + \frac{\pi}{2}$$

2.1.4 Theta 2

When calculating Θ_2 the angles γ & β need to be calculated first,

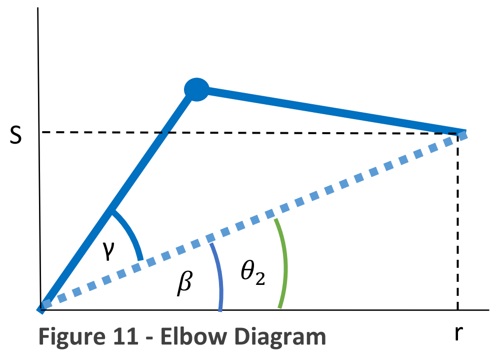

Figure 11 - Elbow Diagram

Equation 17: $$\beta = \tan^{-1}\frac{s}{r}$$

Equation 12: $$\gamma = \pi - \frac{\theta_3}{2} - \frac{\pi}{2}$$

Hence Θ_2:

Front Elbow up

Equation 16: $$\theta_2 = \beta + \gamma$$

Front Elbow down

Equation 16: $$\theta_2 = \beta - \gamma$$

Back Elbow up

Equation 16: $$\theta_2 = \pi - \beta + \gamma$$

Back Elbow down

Equation 16: $$\theta_2 = \pi + \beta - \gamma$$

2.1.5 Theta 4,5 & 6

Once these first three θ angles have been calculated the next three can then be calculated. Using θ_1, θ_2 & θ_3 and the DH Convention the rotation matrix to the wrist centre can be found and then using the following equation where R is the rotation matrix to the end-effector from the base reference frame, the rotation matrix from the wrist centre to the end-effector can be found.

Equation 18: $$R_6^3 = (R_3^0)^T R$$

Then using the ZYZ Euler rotation matrix shown in the figure below the two variations of the last three angles can be found.

$$R_{ZYZ}(\alpha,\beta,\gamma) = \begin{bmatrix} C\alpha C\beta C\gamma - S\alpha S\gamma & -C\alpha C\beta S\gamma - S\alpha C\gamma & C\alpha S\beta \\ S\alpha C\beta C\gamma + C\alpha S\gamma & -S\alpha C\beta S\gamma + C\alpha C\gamma & S\alpha S\beta \\ -S\beta C\gamma & S\beta S\gamma & C\beta \end{bmatrix}$$

Figure 12 - ZYZ Rotation Matrix

The two variations of the angles α, β & γ from the above matrix are found using the following equations:

First Config

Equation 21: $\beta = atan2(\sqrt{1 - r_{33}^2}, r_{33})$

Equation 21: $\alpha = atan2(r_{23}/S\beta, r_{13}/S\beta)$

Equation 21: $\gamma = atan2(r_{32}/S\beta, -r_{31}/S\beta)$

Second Config

Equation 24: $\beta = atan2(-\sqrt{1 - r_{33}^2}, r_{33})$

Equation 24: $\alpha = atan2(-r_{23}/S\beta, -r_{13}/S\beta)$

Equation 24: $\gamma = atan2(-r_{32}/S\beta, r_{31}/S\beta)$

The angles α, β & γ correspond respectively to θ_4, θ_5 & θ_6, once the variations of these angles have been found they can be outputted with the first three angles in the solution matrix for the four different elbow configurations. The flow diagram for the Inverse Kinematics operation can be found below.

2.2 Program Flowchart

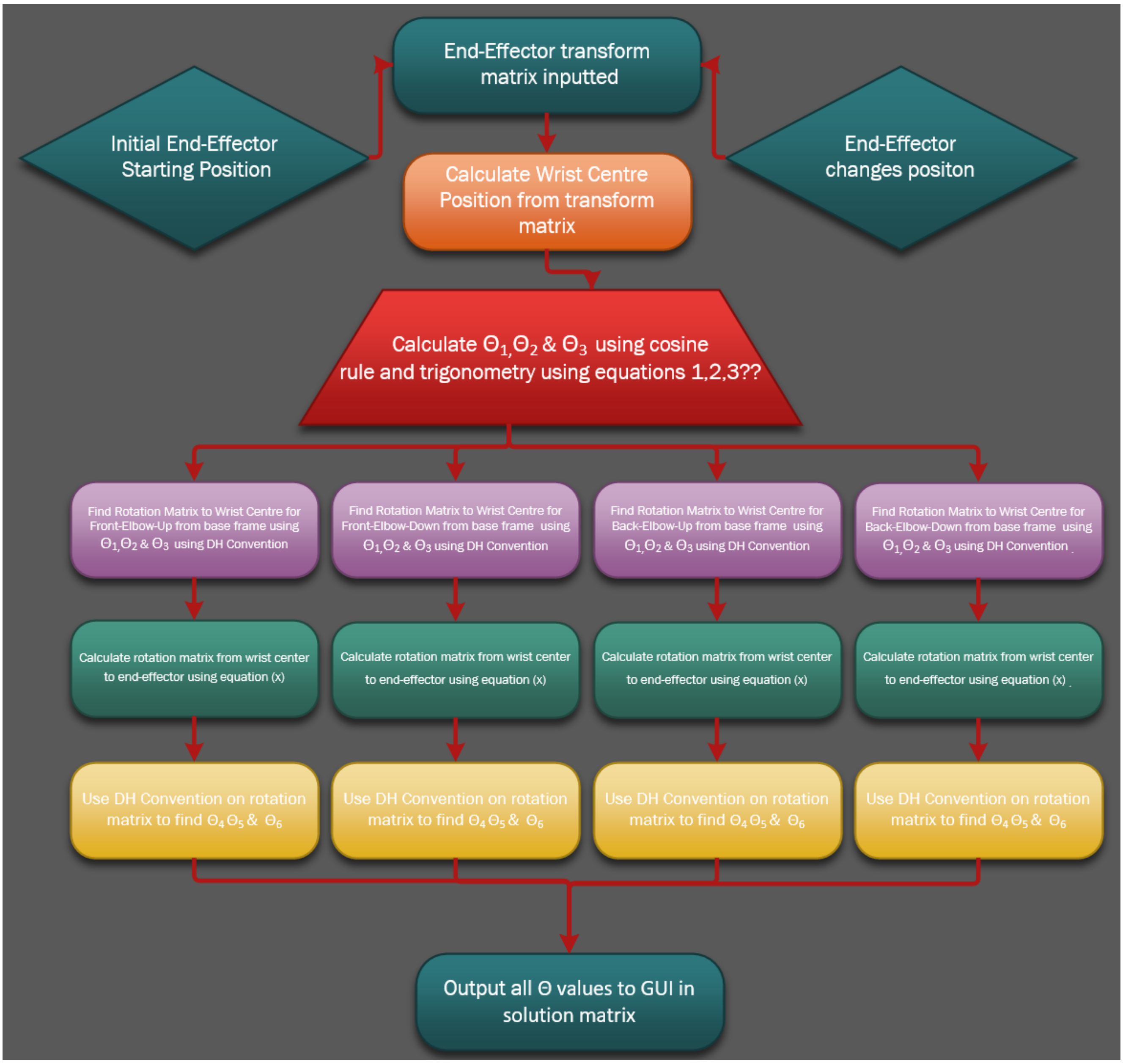

Figure 13 - Inverse Kinematics Flowchart

2.3 Functions

The functions used to perform the Inverse Kinematics operation can be found in the table below:

Name	Inputs	Outputs	Function Role
IK.m	Effector position Rotation Matrix R_6^0	End effector position XYZ Co-ordinates Rotation Matrices	To calculate the rotation matrices and XYZ co-ordinates from the inputted angles from the GUI.
DHConvention.m	Joint angles Frame number	Transform Matrix	To calculate the transform matrix to the selected frame from the previous frame using joint angles and DH-Parameters.
WcCalc.m	Rotation Matrix R_6^0 Effector position	Wrist Centre (XYZ)	This function calculates the wrist centre co-ordinates from the inputs and last link length.

Elbow.m	Wrist Centre (XYZ)	FEU θ_1, θ_2 & θ_3 FED θ_1, θ_2 & θ_3 BEU θ_1, θ_2 & θ_3 BED θ_1, θ_2 & θ_3	This function calculates the first three joint angles from the wrist centre position, using the method outlined in 2.1.2-4
R03Calc.m	θ_1, θ_2 & θ_3 Angles	Rotation Matrix R_3^0	This function calculates the rotation matrix to the wrist centre from the robot base frame, using DH-Convention.
Wrist.m	Rotation Matrix R_3^0 Rotation Matrix R_6^0	Angles θ_4, θ_5 & θ_6	Calculates the two configurations of the last three angles using the method outlined in 2.1.5.

Table 3 - Inverse Kinematics Functions

Figures

Tables

Bibliography

Bois, P. I. (2018). *Robotics Engineering Lecture Notes.* Bath: Unversity of Bath.

EECS, U. (2018, 11 10). *IK closedform*. Retrieved from eecs.umich: http://web.eecs.umich.edu/~ocj/courses/autorob/autorob_10_ik_closedform.pdf