436-432 THERMOFLUIDS 4: Fluid Mechanics Unit
The Turbulent Boundary-Layer Prac

(Last modified: February 1, 2006)

1 Aim

To measure the mean-velocity profile of a turbulent boundary layer developing on a flat surface, and to study the resulting wall, logarithmic, and wake regions in the plotted profiles.

(Your GROUP report is to be submitted within three weeks of your prac.)

Nomenclature

Symbols

A	universal constant (for inner-scaled mean velocity profile)
B	constant (for outer-scaled mean velocity profile)
C_f	skin-friction coefficient: $\tau_w = \frac{1}{2}\rho U_1^2 C_f$
d_p	Pitot-tube diameter (external)
H	form factor: $H = \delta^*/\theta$
U_1	mean free-stream velocity
U_τ	friction velocity: $\rho U_\tau^2 = \tau_w$
u	instantaneous local streamwise velocity
$\bar{u}$	temporal-mean component of u; time-averaged streamwise velocity
x^*, y^*	non-dimensional Preston-tube calibration variables
y	wall-normal distance
Δp_p	Preston-tube reading
δ	boundary-layer thickness
δ^*	displacement thickness
κ	universal Karman constant
ρ	density
θ	momentum thickness
ν	kinematic viscosity
τ_w	wall shear stress

2 Background Theory

Scaling laws are general semi-empirical relations that enable the simplification of the analysis of experimental data. They can thereby offer the experimentalist valuable insight into the turbulent phenomena in the absence of a detailed solution to the governing Navier–Stokes equations. For the turbulent boundary layer, there arose two such scaling laws that are generally regarded for their universality: one for the inner-flow, or wall, region and one for the outer-flow region.

Due to its proximity to the wall, it is found that the most important influencing parameters for the flow in the inner region are the wall shear stress via the friction velocity, $U_\tau = \sqrt{\tau_w/\rho}$, and the fluid viscosity, ν:

$$\bar{u} = f(y, \tau_w, \rho, \nu), \tag{1}$$

where f represents some unknown function. In non-dimensional form for the inner-flow region, we have the (Prandtl's) law of the wall:

$$\frac{\bar{u}}{U_\tau} = f\!\left(\frac{yU_\tau}{\nu}\right). \tag{2}$$

It could then be surmised that for the outer-flow region the influencing parameters would merely require the addition of the boundary-layer thickness, δ, to those above to account for the effects of the large-scale geometry on the gross structure of the turbulence:

$$U_1 - \bar{u} = g(y, U_\tau, \delta, \nu), \tag{3}$$

where g represents some unknown function.

Townsend's Reynolds number similarity hypothesis, however, states:

> "…the mean [relative] motion and the motion of the energy-containing components of the turbulence are determined by the boundary conditions of the flow alone, and are independent of the fluid viscosity, except so far as a change in the fluid viscosity may change the boundary conditions." [1] (p.89)

That is, the effect of the viscosity enters consideration in the outer flow only through the influence of wall shear stress. It is found that U_τ, and not ν, is the critical parameter for the velocity defect. Hence, Eq. (3) can be modified so that, in non-dimensional form for the outer-flow region, we have the (von Kármán's) velocity-defect law:

$$\frac{U_1 - \bar{u}}{U_\tau} = g\left(\frac{y}{\delta}\right). \tag{4}$$

The region in which the inner- and outer-flow regions overlap such that both laws apply is called the turbulent wall (or overlap or logarithmic) region. As was shown by Millikan in reference [2], this turbulent wall region is described in terms of inner-scaled and outer-scaled variables, respectively, by:

$$\frac{\bar{u}}{U_\tau} = \frac{1}{\kappa} \ln\left(\frac{yU_\tau}{\nu}\right) + A \tag{5}$$

and

$$\frac{U_1 - \bar{u}}{U_\tau} = -\frac{1}{\kappa} \ln\left(\frac{y}{\delta}\right) + B, \tag{6}$$

where A and κ are universal constants, and B is dependent on large-scale flow geometry.

3 Methods of Measuring Wall Shear Stress

3.1 The Clauser Chart

By multiplying Eq. (5) by U_τ/U_1, the logarithmic law of the wall can be rewritten to describe a family of logarithmic curves as a function of the friction coefficient, C_f:

$$\frac{\bar{u}}{U_1} = \frac{1}{\kappa}\sqrt{\frac{C_f}{2}} \ln\left(\frac{yU_1}{\nu}\right) + \frac{1}{\kappa}\sqrt{\frac{C_f}{2}} \ln\left(\sqrt{\frac{C_f}{2}}\right) + A\sqrt{\frac{C_f}{2}}. \tag{7}$$

The appropriate value of C_f, and hence τ_w and U_τ, is determined by matching a given set of data to a member of the family.

As the Clauser-chart method is based on the assumption of the existence of a universal logarithmic region [3], the values of the universal constants A and κ used in the above expression *must be assumed.*

The above Clauser-chart method is described in references [3] and [4].

3.2 The Preston Tube

Preston's method correlates a simple one-point Pitot-tube reading at the surface of interest to the wall shear stress via a calibration curve. The calibration used in Preston's method rests on the assumption of a universal law of the wall for all boundary layers and pipe flows alike [5].

The determination of turbulent skin friction by the Preston-tube method, using the calibration of V.C. Patel [5][1], first requires the measurement of the dynamic pressure when the Pitot tube is just resting on the wall (i.e., the first point in your velocity profile). The calibration curve relating this Preston tube reading, Δp_p, and the wall shear stress, τ_w, is given in terms of non-dimensional variables y^* versus x^*, where

$$x^* = \log_{10}\left(\frac{\Delta p_p\ d_p^2}{4\rho\nu^2}\right) \qquad \text{and} \qquad y^* = \log_{10}\left(\frac{\tau_w\ d_p^2}{4\rho\nu^2}\right). \tag{8}$$

Patel's results show the existence of three different calibration regions. The appropriate region (each defined by a different empirical curve fit equation) depends on your value of x^*:

If $0 < x^* < 2.9$, then

$$y^* = \frac{1}{2}x^* + 0.037. \tag{9}$$

If $2.9 \leq x^* < 5.6$, then

$$y^* = 0.8287 - 0.1381x^* + 0.1437x^{*2} - 0.0060x^{*3}. \tag{10}$$

If $5.6 \leq x^* < 7.6$, then

$$y^* + 2\log_{10}(1.95y^* + 4.10) = x^*. \tag{11}$$

It is then possible to calculate U_τ from τ_w according to its definition.

The above calibration for the Preston-tube method is described in reference [5].

3.3 The Momentum-Integral Equation

The momentum-integral equation, first derived by von Kármán, is given for steady, incompressible flow by:

$$\frac{d\theta}{dx} + \frac{\theta}{U_1}\frac{dU_1}{dx}(H + 2) = \frac{C_f}{2}. \tag{12}$$

The determination of turbulent skin friction using the above momentum-integral equation relies on graphical differentiation of the streamwise momentum-thickness distribution from a series of velocity profile measurements.

Von Kármán's momentum equation can be found in many standard fluid dynamics and boundary-layer text books.

[1] Preston's original calibration was shown to be incorrect [5].

References

[1] A.A. Townsend. *The Structure of Turbulent Shear Flow*. Cambridge University Press, Cambridge, 1956.

[2] C.B. Millikan. A critical discussion of turbulent flows in channels and circular tubes. In *Proceedings of the Fifth Congress of Applied Mechanics*, Cambridge, Massachusetts, 1938.

[3] F.H. Clauser. Turbulent boundary layers in adverse pressure gradients. *Journal of Aeronautical Sciences*, 21:91–108, February 1954.

[4] F.H. Clauser. The turbulent boundary layer. In *Advances in Applied Mechanics*, volume IV. Academic Press, New York, 1956.

[5] V.C. Patel. Calibration of the preston tube and limitations on its use in pressure gradients. *Journal of Fluid Mechanics*, 23:185–208, 1965.