

## Decidability and Computability: Problems for Week 9

**Exercise 1** The following program uses recursion to compute a binary function  $B$  on natural numbers.

```
B(0, n) = n+7
B(1, n) = n+5
B(m+2, 0) = m+17
B(m+2, 1) = m+B(m+1, 9)
B(m+2, n+2) = B(m, n+7) + B(m+2, n)
```

Show that it terminates for all  $m$  and  $n$ .

**Exercise 2** Write `nat k = max(j-i, 0)` in Primitive Java. You may use all the encodings listed in the handout.

### Exercise 3

Here is a unary program in Basic Java (using the encodings given in the handout).

```
nat i = 0;
nat j = 0;
while i != input0 {
    i++;
    i++;
    j++;
}
output = j;
```

What partial function from  $\mathbb{N}$  to  $\mathbb{N}$  does it compute? (Your answer should be 1–2 lines long.)

**Exercise 4** Complete the following sentences. Let's say that the alphabet  $\Sigma$  is  $\{a, b\}$ .

- A function  $f: \mathbb{N} \rightarrow \mathbb{N}$  is computable when ... If it is not computable, then by Church's thesis ...
- A subset  $A \subseteq \mathbb{N}$  is decidable when ... If it is not decidable, then by Church's thesis ...
- A language  $A \subseteq \Sigma^*$  is decidable when ... If it is not decidable, then by Church's thesis ...
- Ambiguity of a context free grammar over  $\Sigma$  is an undecidable property. This means ... By Church's thesis, this implies ...

**Exercise 5** Is ambiguity of a context free grammar a semidecidable property? What about non-ambiguity? Explain your answers. You may use facts that we have seen previously.