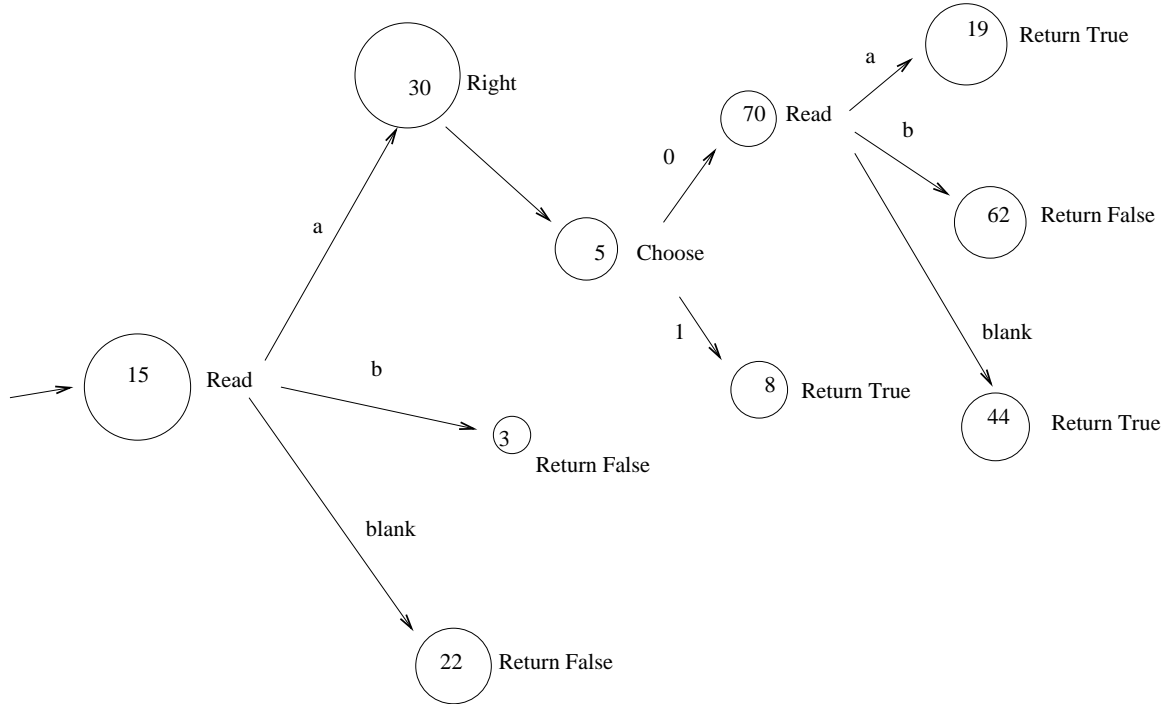


Nondeterministic Turing machines: Problems for Week 8

Exercise 1 What is the language of the following NDTM, over the alphabet $\Sigma = \{a, b\}$? (For a word w , the machine starts with just w on the tape, and the head on the leftmost character, or on a blank if $w = \varepsilon$.)



Exercise 2 Are these formulas satisfiable? Justify your answer.

1. $(\neg p \vee \neg q \vee r) \wedge (\neg p \vee q) \wedge p \wedge \neg r$.
2. $(p \vee q \vee \neg r) \wedge (\neg p \vee q) \wedge p \wedge \neg r$

Exercise 3 Consider the following problem: given a set of integers, say whether it has a subset that adds up to 0. For example, if we're given the set $\{12, 2, -7, -8, 3, 14, -5, 1\}$, we could return $\{-7, -8, 1, 14\}$. That's not the only solution, but we're only asked to find one. If we're given the set $\{3, 9, -55, -2\}$ we return "Impossible". Show this problem is in **NP**.

Hint: You are not expected to give a full Turing machine. Just

- say what a certificate is for this problem,
- explain why it has length polynomial in the input,
- explain why it takes polynomially many steps to check that it is indeed a certificate.

Exercise 4

Suppose there are three boxes numbered 0, 1, 2 and three bottles, one red, one green and one brown. Each box can accommodate at most two bottles. Let $\phi_{R,i}$ indicate that the red bottle is in space i , and let $\phi_{G,i}$ indicate that the green bottle is in box i , and let $\phi_{B,i}$ indicate that the brown bottle is in box i .

1. Write a formula saying that each bottle is in precisely one box.
2. Write a formula saying that no box contains all three bottles.

First, we want you to write your answers in full, using $\vee, \wedge, \neg, \Rightarrow$. Then, abbreviate your answers using \bigvee and \bigwedge . For example,

$$\bigvee_{i \in \{0,1,2,3\}} \phi_i,$$

is an abbreviation for

$$\phi_0 \vee \phi_1 \vee \phi_2 \vee \phi_3.$$

Exercise 5 For the alphabet Σ , let L and L' be languages in **NP**. Show that the language $L \cap L'$ is also in **NP**. *Hint:* use the “checking machine” definition of **NP**. You need only describe the machines in outline.