

1. a/ Tape	State	Instruction
␣ a ␣	0	Read a
␣ a ␣	1	Write X
␣ X a ␣	2	Right
␣ X a ␣	3	Read a
␣ X a ␣	2	Right
␣ X a ␣	3	Read ␣
␣ X a ␣	4	Return False

b/ Type	Score	Insurrection
u o b w	0	Read a
u a b w	1	Write x
u x b w	2	Right
u x b w	3	Read b
u x b w	10	Write x
u x x w	9	Left
u x x w	8	Read x
u x x w	9	Left
u x x w	8	Read w

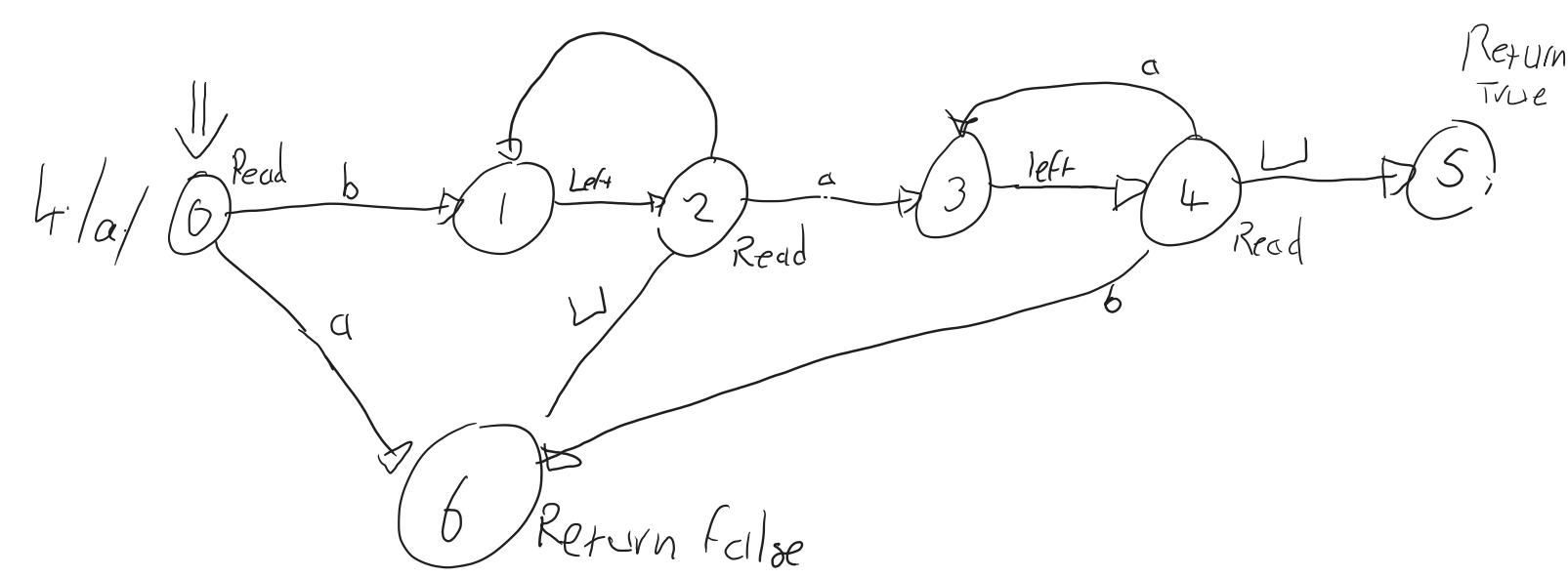
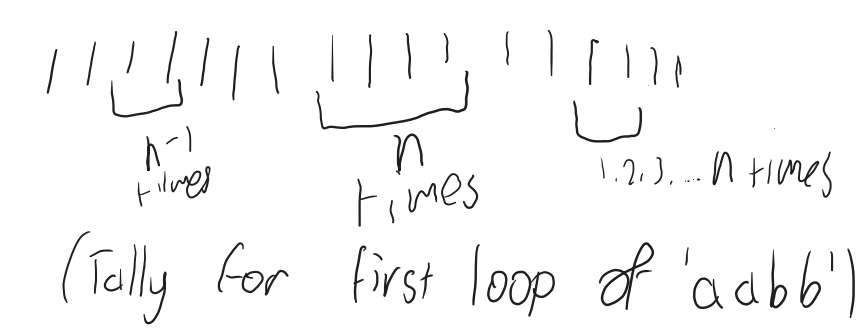
2/a) false, abaa is not in the language M

b/ True,  $abab$  is in the language  $M$

$$C: \{s \in (a, b)^* \mid |s|_a = |s|_b\}$$

3/d. / An example of a word that is linear  $\times 74$   
is 'a', or more generally 'a'. M reads 'a', then writes x,  
then iterates through Right  $\rightarrow$  Read  $\rightarrow$  n-1 times, reads 'a',  
then returns false.  $T(a) = 2(n-1) + 4 = 2n + 3$  Steps  
which is linear, and  $O(n)$

b. /An example of a  $O(n^3)$  word is 'aabb' or more generally  $a^n b^n$ . M reads a, writes x, then iterates through n-1 a's (like in 3a), but then reads b, writes x, and iterates back through n chr's, then reads 'x' and iterates back through the x's. This loop happens n times, but each loop the iteration through the x's grows smaller, which is  $\frac{1}{2}n(n-1)$ . The  $T(a^n b^n) = (n+9)n + \frac{1}{2}n(n-1) + \frac{1}{2}n(n-2)$   
 $= n^2 + 9n + \frac{1}{2}(n^2 - n) + \frac{1}{2}(n^2 - 3n + 2)$   
 $= 2n^2 + 7n + 1$  steps



b/ We start on the leftmost char of a non-empty block. We first need to iterate through to the first blank, and back one to get to the rightmost char to start with  $N$  (We start with  $IV$  as  $M$  moves all chars to 'x', which  $IV$  can't part on). We perform  $N$ , and if returned true, continue, otherwise return false. So we IV macro finishes on the  $L$  char to the leftmost char, so we first move right once, then perform  $M$ . The reason the checks for  $\{a^i b^j n \geq i\}$  is that  $N$  checks for  $a$ 's followed by exclusively  $b$ 's, and  $M$  checks for an equal amount of  $a$ 's and  $b$ 's.

