# Context Free Languages: Problems for Week 4

**Exercise 1** *Consider the language generated by the grammar*

$$\Rightarrow S \quad ::= \quad \texttt{b}SS \quad | \quad \texttt{a}S \quad | \quad \texttt{a}$$

*and the string* `aabbaaa`.

1. *Find a leftmost derivation for this string.*
2. *Draw the derivation tree.*

**Exercise 2** *Let's look at a "Natural Language" example. The alphabet is*

{ *the, a, cat, dog, happy, tired, slept, died, ate, dinner, and, .* }

*The grammar is*

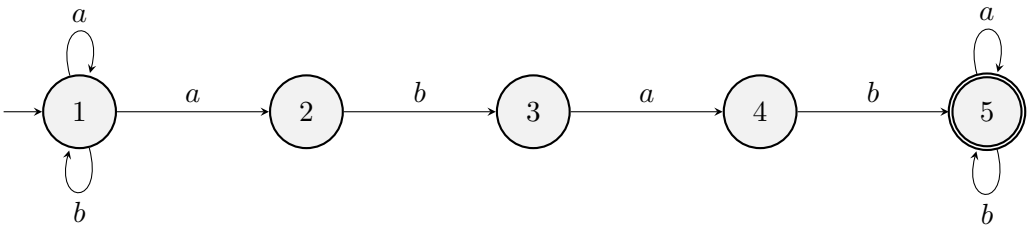
| | | | |
|---|---|---|---|
| *Sentence* | $\Rightarrow S$ | ::= | *C .* |
| *Clause* | *C* | ::= | *NP VP* \| *C and C* |
| *Noun phrase* | *NP* | ::= | *Art N* \| *dinner* |
| *Noun* | *N* | ::= | *Adj N* \| *cat* \| *dog* |
| *Adjective* | *Adj* | ::= | *happy* \| *tired* |
| *Verb phrase* | *VP* | ::= | *VI* \| *VT NP* |
| *Intransitive verb* | *VI* | ::= | *slept* \| *died* |
| *Transitive verb* | *VT* | ::= | *ate* |
| *Article* | *Art* | ::= | *a* \| *the* |

*This grammar accepts "words" such as*

*the happy tired happy dog died and the cat slept.*
*the tired tired cat ate dinner.*
*dinner ate a happy dog.*

*Try writing derivations and derivation trees for these sentences.*

**Exercise 3** *Let's consider an NFA that accepts any string that contains the substring "abab".*




1. *Convert the above NFA into its equivalent total DFA.*
2. *Convert the resultant DFA in an equivalent CFG. It is suggested to minimize the DFA before writing CFG.*

**Exercise 4** *Give a context free grammar for the set of palindromes over the alphabet* {`a`, `b`}.

**Exercise 5** *Try deriving the string* $3 + 5 \times 3$ *in two different ways using leftmost derivation only, using the grammar given below:*

$$\begin{aligned} \Rightarrow A \quad &::= \quad A + B \mid B \\ B \quad &::= \quad B \times C \mid C \\ C \quad &::= \quad (A) \mid 3 \mid 5 \end{aligned}$$

**Exercise 6** *Show that the following grammar is ambiguous. The alphabet is* $\{\mathtt{a}, \mathtt{b}\}$.

$$\begin{array}{rrl} \Rightarrow P & ::= & \varepsilon \mid Q\mathtt{a} \mid \mathtt{a}Q \\ Q & ::= & \mathtt{aa}P \mid \mathtt{b}R \\ R & ::= & Q\mathtt{a} \end{array}$$

**Exercise 7** *Convert the following CFG into an equivalent CFG in Chomsky normal form*

$$\begin{array}{rrl} \Rightarrow A & ::= & BAB \mid B \mid \varepsilon \\ B & ::= & 00 \mid \varepsilon \end{array}$$

**Exercise 8** *Give grammars for the following two languages:*

1. *All binary strings with both an even number of zeroes and an even number of ones.*
2. *All strings of the form* $0^a1^b0^c$ *where* $a + c = b$.