

Induction and converting regular expressions to DFAs:

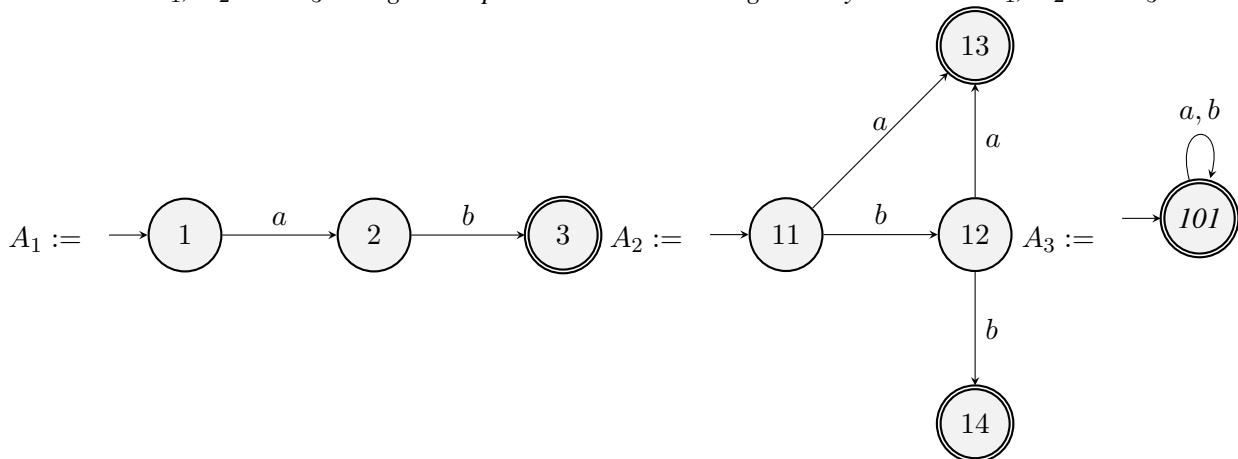
Problems for Week 2

Exercise 1 Show by induction on natural numbers that, for any natural number n , the sum of the first n squares is equal to $\frac{n(n+1)(2n+1)}{6}$.

Exercise 2 Consider the regular expression $(a^*(b|c)|(b^*b))$. In a step-by-step fashion, convert each sub-expression of the regular expression into an ϵ NFA and, by doing so, obtain a DFA that is equivalent to the overall regular expression.

Exercise 3 Show that every number $n > 1$ has a prime factor, by course-of-values induction.

Exercise 4 Let E_1 , E_2 and E_3 be regular expressions that are recognised by automata A_1 , A_2 and A_3 .



Using the rules given in the handout, convert the following regular expressions into equivalent DFAs:

- (i) $E_3|E_1$
- (ii) $E_1(E_2|E_3)$
- (iii) $E_3E_3^*$

Exercise 5 Andy has a grid of $2^n \times 2^n$ squares that are all white, except for one, which is red. A triomino is an L-shaped piece covering 3 squares. Show, by induction on natural numbers, that Andy can cover the white part of the grid by triominoes.