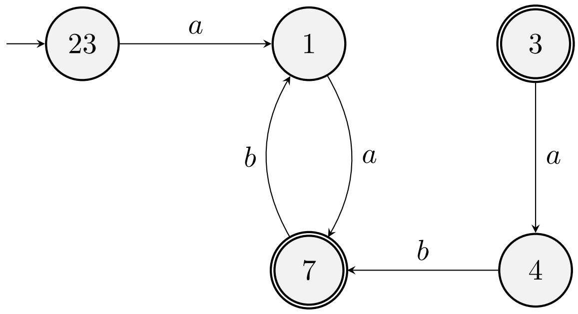
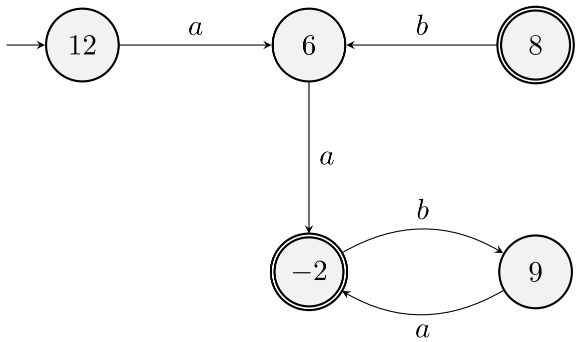
Exercise sheet for Week 3

Exercise 1. *Here are three automata over the alphabet $\Sigma = \{a, b\}$ are equivalent. Test algorithmically the equivalence of (1)–(2), and the equivalence of (1)–(3). If you obtain a negative answer, you should give a word that's accepted by one automaton but not the other.*

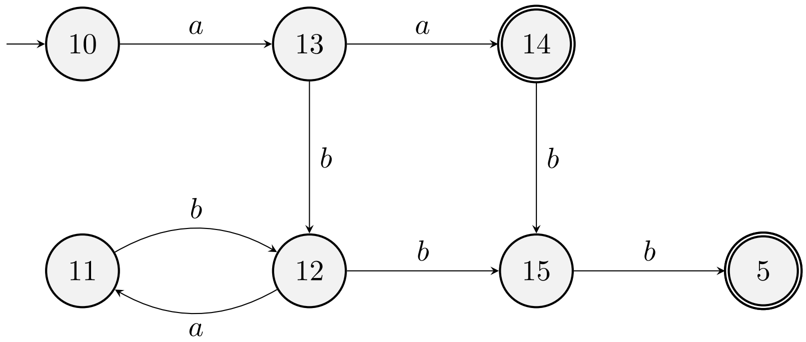
1.



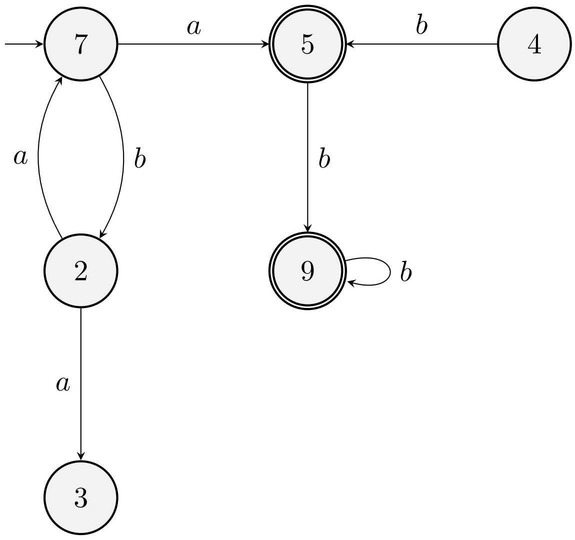
2.



3.



Exercise 2. *Minimize the following partial DFA, and then prove that the partial DFA you have obtained is minimal.*



Exercise 3. *The alphabet is $\{a, b\}$. Give a two-state partial DFA for the regex $(ab)^*$. Convert it into a DFA, then obtain a DFA for the set of all words that are* **not** *matched by $(ab)^*$.*

Exercise 4. *Show that the set $\mathbb{N} + \mathbb{N}$ is countably infinite.*

Exercise 5. *Consider the following language over the alphabet $\Sigma = \{a, b\}$:*

$$L = \{w | w \textit{ contains the same number of } a\text{'s and } b\text{'s}\}$$

Show that L is non-regular.

Exercise 6. *Are the following languages over $\Sigma = \{a, b\}$ regular ? Why (not)?*

1. $L = \{a^m b^n | m > n\}$
2. $L = \{a^m b^n | m < n\}$
3. $L = \{w | \textit{length}(w) \textit{ is a square number}\}$

Exercise 7. *For any string $w = w_1 w_2 \dots w_n$, the* ***reverse of*** *w, written w^R, is the string w in reverse order, $w_n \dots w_2 w_1$. For any language L, let $L^R = \{w^R | w \in L\}$. Show that if L is regular, so is L^R.*

Exercise 8. *Let $\Sigma = \{a, b\}$.*

1. *Let $L_1 = \{a^k u a^k | k \geq 1 \textit{ and } u \in \Sigma^*\}$. Show that L_1 is regular.*
2. *Let $L_2 = \{a^k b u a^k | k \geq 1 \textit{ and } u \in \Sigma^*\}$. Show that L_2 is not regular.*