>

$with(LinearAlgebra);$

$A := Matrix(5, 5, [[0, -1, -2, 1, 1], [3, -7, -6, 0, 3], [-4, 9, 7, 0, -4], [1, -3, -3, 1, 2], [0, 1, 0, -1, -1]]);$

$$A := \begin{bmatrix} 0 & -1 & -2 & 1 & 1 \\ 3 & -7 & -6 & 0 & 3 \\ -4 & 9 & 7 & 0 & -4 \\ 1 & -3 & -3 & 1 & 2 \\ 0 & 1 & 0 & -1 & -1 \end{bmatrix} \tag{1}$$

$Id := IdentityMatrix(5);$

$$Id := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$p := Determinant(A - \text{lambda} \cdot Id); factor(p);$

$$p := -\lambda^5 + 3\lambda^3 + 2\lambda^2$$

$$-\lambda^2 (-2 + \lambda) (1 + \lambda)^2 \tag{3}$$

$J1 := JordanForm(A);$

$$J1 := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{4}$$

$NullSpace(A);$

$$\left\{ \begin{bmatrix} -1 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\} \tag{5}$$

$m1 := NullSpace((A)^2);$

$$m1 := \left\{ \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\} \tag{6}$$

$NullSpace(A + Id);$

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\} \tag{7}$$

$m2 := NullSpace\left( (A + Id)^2 \right);$

$$m2 := \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\} \tag{8}$$

$m3 := NullSpace(A - 2 \cdot Id);$

$$m3 := \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\} \tag{9}$$

$m1, m2, m3;$

$$\left\{ \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}, \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}, \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\} \tag{10}$$

$B := \langle \langle -1, 0, 0, 0, 1 \rangle | \langle 0, 0, 0, 1, 0 \rangle | \langle 1, 1, 0, 0, 1 \rangle | \langle 0, -1, 1, 0, 0 \rangle | \langle 1, 1, -1, 1, 0 \rangle \rangle;$

$$B := \begin{bmatrix} -1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \quad \mathbf{(11)}$$

$Binv := MatrixInverse(B);$

$$Binv := \begin{bmatrix} 0 & -1 & -1 & 0 & 1 \\ -1 & 2 & 2 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & -2 & -1 & 0 & 1 \\ 1 & -2 & -2 & 0 & 1 \end{bmatrix} \quad \mathbf{(12)}$$

$J := Binv \cdot A \cdot B;$

$$J := \begin{bmatrix} -1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix} \quad \mathbf{(13)}$$

$NullSpace(A);$

$$\left\{ \begin{bmatrix} -1 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\} \quad \mathbf{(14)}$$

$NullSpace\left( (A)^2 \right);$

$$\left\{ \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\} \quad \mathbf{(15)}$$

$v2 := \langle \langle -1, 0, 0, 0, 1 \rangle \rangle; v1 := (A) \cdot v2;$

$$v2 := \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$v1 := \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \tag{16}$$

$NullSpace(A - 2 \cdot Id);$

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\} \tag{17}$$

$v3 := \langle \langle 1, 1, -1, 1, 0 \rangle \rangle;$

$$v3 := \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \tag{18}$$

$NullSpace(A + Id);$

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\} \tag{19}$$

$NullSpace\left( (A + Id)^2 \right);$

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\} \tag{20}$$

$v5 := \langle\langle 1, 1, 0, 0, 1\rangle\rangle;\ v4 := (A + Id) \cdot v5;$

$$v5 := \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$v4 := \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \tag{21}$$

$B := \langle v1 | v2 | v3 | v4 | v5\rangle;$

$$B := \begin{bmatrix} 1 & -1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 1 \end{bmatrix} \tag{22}$$

$Binv := MatrixInverse(B);$

$$Binv := \begin{bmatrix} -1 & 2 & 2 & 1 & -1 \\ -1 & 1 & 1 & 1 & 0 \\ 1 & -2 & -2 & 0 & 1 \\ 1 & -2 & -1 & 0 & 1 \\ -1 & 3 & 2 & 0 & -1 \end{bmatrix} \tag{23}$$

$J := Binv \cdot A \cdot B;$

$$J := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{24}$$

$JordanForm(A);$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{25}$$