

$\Lambda_b \rightarrow p, \Lambda$ transition form factors from QCD light-cone sum rulesYu-Ming Wang^a, Yue-Long Shen^b and Cai-Dian Lü^a^a *Institute of High Energy Physics and Theoretical Physics Center
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(Dated: February 21, 2013)

Light-cone sum rules for the $\Lambda_b \rightarrow p, \Lambda$ transition form factors are derived from the correlation functions expanded by the twist of the distribution amplitudes of Λ_b baryon. In terms of the Λ_b three-quark distribution amplitudes models constrained by the QCD theory, we calculate the form factors at small momentum transfers and compare the results with that estimated in the conventional light-cone sum rules (LCSR) and perturbative QCD approaches. Our results indicate that the two different version of sum rules can lead to the consistent numbers of form factors responsible for $\Lambda_b \rightarrow p$ transition. The $\Lambda_b \rightarrow \Lambda$ transition form factors from LCSR with the asymptotic Λ baryon distribution amplitudes are found to be almost one order larger than that obtained in the Λ_b -baryon LCSR, implying that the pre-asymptotic corrections to the baryonic distribution amplitudes are of great importance. Moreover, SU(3) symmetry breaking effect between the form factors $f_1^{\Lambda_b \rightarrow p}$ and $f_1^{\Lambda_b \rightarrow \Lambda}$ are computed as $28_{-8}^{+14}\%$ in the framework of Λ_b -baryon LCSR.

PACS numbers: 14.20.Mr, 13.30.Eg, 11.55.Hx

I. INTRODUCTION

The priority to investigate b quark decays can be attributed to their sensitivity of the flavor structure of nature, which serves as a touch-stone in the ongoing effort to explore the standard model (SM) describing the interactions between elementary particles. Weak decays of heavy baryons containing a b quark may provide important clues on flavor-changing currents beyond the SM in a complementary fashion to B meson decays. $\Lambda_b \rightarrow p, \Lambda$ transition form factors are the essential hadronic objects in the exclusive semileptonic $\Lambda_b \rightarrow pl\bar{\nu}_l$, $\Lambda_b \rightarrow \Lambda l\bar{l}$ and radiative $\Lambda_b \rightarrow \Lambda\gamma$ decays. Such form factors can also be employed to describe the nonleptonic charmless Λ_b decays in terms of factorization approach. There is no doubt that reliable estimation of the transition form factors in QCD is indispensable to an accurate determination of the flavor-changing couplings of the quarks (which is known as Cabibbo-Kobayashi-Maskawa (CKM) matrix elements) and to a deep understanding of the underlying structure and dynamics of hadrons and currents.

There has been continuous interest concentrated on the theoretical analysis of form factors from the underlying field theory, the main challenge of which is to deal with the nonperturbative effect in the hadron as a bound state properly. Several theoretical tools going beyond the realm of perturbation theory have been developed in this aspect, such as lattice QCD (LQCD), perturbative QCD (PQCD) approach, QCD sum rules (QCDSR) and light-cone sum rules (LCSR) approaches. LQCD is only applicable to the computations of heavy-to-light transition form factors with large momentum transfer. Phenomenologically, PQCD approach has been applied to the investigations of various baryonic transitions including proton Dirac form factor [1, 2], semileptonic charmless decay $\Lambda_b \rightarrow pl\bar{\nu}$ [3], semileptonic charming decay $\Lambda_b \rightarrow \Lambda_c l\bar{\nu}$ [4, 5] and radiative decay $\Lambda_b \rightarrow \Lambda\gamma$ [6]. Three-point QCDSR approach based on the short-distance operator product expansion (OPE) and double dispersion relations have also been employed to calculate the weak transition form factors, such as $\Lambda_b \rightarrow pl\bar{\nu}$ [7] and $\Lambda_b \rightarrow \Lambda\gamma, \Lambda l\bar{l}$ [8], where the nonperturbative contributions are embedded in the vacuum condensates of quarks and gluons.

Alternative sum rule approach to hadronic form factors is to perform the OPE of a dedicated correlation function near the light-cone and the light-cone distribution amplitudes are employed to describe the long-distance dynamics in the correlator. As a marriage of the standard QCDSR technique and the theory of hard exclusive process, the LCSR procedure involves a partial resummation of local operators and cure the problem of QCDSR applied to large momentum transfer. An attractive advantage of the LCSR is that it offers a systematic way to take into account both hard scattering and soft (end-point) contributions to the transition form factors almost model independently [9]. Utilizing the distribution amplitudes of proton and Λ baryon, the form factors responsible for $\Lambda_b \rightarrow pl\bar{\nu}$ [10] and $\Lambda_b \rightarrow \Lambda\gamma, \Lambda l\bar{l}$ [11] transitions have been investigated in the LCSR approach. The main uncertainties in the standard LCSR approach originate from the less known nonperturbative parameters involved in the distribution amplitudes of light hadrons, apart from the systematic uncertainty brought by the quark-hadron duality assumption in the heavy hadron channel.

Another version of LCSR approach (B meson LCSR) proposed in Ref. [12, 13] starts with the B -to-vacuum correlation function, where the light hadron is interpolated by an appropriate current and the B meson is put on the mass shell. The on-shell B meson can be well described in the heavy quark effective theory (HQET) and the correlation function is found to be light-cone dominance. In this context, the distribution amplitudes of B meson are treated as universal nonperturbative inputs, which embody the long-distance dynamics of all the $B \rightarrow L$ (L being a light meson) transition form factors. Different from the LCSR with light-meson distribution amplitudes (light-

meson LCSR), systematic uncertainty in B meson LCSR is from the quark-hadron duality in the channels of light mesons. Along this line, LCSR with distribution amplitudes of heavy hadrons have been employed to compute the form factors of $B \rightarrow D^{(*)}$ [14], $B \rightarrow a_1(1260)$ [15], and $\Lambda_b \rightarrow \Lambda_c$ [16] very recently. In the present work, we would like to follow the same prescription to analyze the $\Lambda_b \rightarrow p$, Λ transition form factors, which provides an independent way to test predictions of the standard LCSR with light-hadron distribution amplitudes.

The fundamental nonperturbative functions in the Λ_b baryon LCSR are the distribution amplitudes of Λ_b , which describes the hadronic structure in rare parton configurations with a fixed number of Fock components at small transverse separation in the infinite momentum frame. The complete classification of three-quark distribution amplitudes of Λ_b baryon in the heavy quark limit has been carried out in Ref. [17], where the evolution equation for the leading twist distribution amplitude is also derived. It has been shown that the evolution equation for the “leading-twist” distribution amplitude contains one piece related to the Lange-Neubert kernel [18], which generates a radiative tail when either of the two momenta $\omega_{1,2}$ is large, and another piece related to the ERBL kernel [19], which redistributes the momenta within the spectator di-quark system.

The layout of this paper is as follows: In section II, we collect the distribution amplitudes of Λ_b baryon to the leading Fock state. Parameterizations of the various hadronic matrix element $\langle L(P) | \bar{q} \gamma_\mu b | \Lambda_b(P+q) \rangle$ (L denotes a light baryon) with Γ_i being all the possible Lorentz structures are presented in section III, where the applicability of OPE on the light cone for the correlation function is also briefly reviewed. The sum rules for the $\Lambda_b \rightarrow L$ transition form factors up to the twist-4 are then derived. It is shown that the relation of form factor in the heavy quark limit are well respected in the Λ_b baryon LCSR. Numerical analysis of LCSR for the transition form factors at large recoil region are displayed in section IV, where detailed comparisons of the results with that obtained in the conventional LCSR, three-point QCDSR and PQCD approaches are also discussed. In particular, the estimation of SU(3) symmetry breaking effect between the form factors $f_1^{\Lambda_b \rightarrow p}$ and $f_1^{\Lambda_b \rightarrow \Lambda}$ are included here. The last section is devoted to the conclusion.

II. DISTRIBUTION AMPLITUDES OF Λ_b BARYON

The complete set of three-quark distribution amplitudes of Λ_b baryon in the heavy quark limit can be constructed as [17]

$$\begin{aligned} \epsilon^{abc} \langle 0 | u^a(t_1 n) C \gamma_5 \not{n} d^b(t_2 n) h_v^c(0) | \Lambda(v) \rangle &= f_\Lambda^{(2)} \Psi_2(t_1, t_2) \Lambda(v), \\ \epsilon^{abc} \langle 0 | u^a(t_1 n) C \gamma_5 d^b(t_2 n) h_v^c(0) | \Lambda(v) \rangle &= f_\Lambda^{(1)} \Psi_3^s(t_1, t_2) \Lambda(v), \end{aligned}$$

$$\begin{aligned}
\epsilon^{abc}\langle 0|u^a(t_1n)C\gamma_5i\sigma_{\bar{n}n}d^b(t_2n)h_v^c(0)|\Lambda(v)\rangle &= 2f_\Lambda^{(1)}\Psi_3^\sigma(t_1,t_2)\Lambda(v), \\
\epsilon^{abc}\langle 0|u^a(t_1n)C\gamma_5\not{n}d^b(t_2n)h_v^c(0)|\Lambda(v)\rangle &= f_\Lambda^{(2)}\Psi_4(t_1,t_2)\Lambda(v),
\end{aligned} \tag{1}$$

where the subscript 2, 3, 4 refers to the twist of the diquark operator, t_i are arbitrary real numbers, describing the locations of the valence quarks inside the Λ_b baryon on the light-cone. The light-like unit vectors n_μ and \bar{n}_μ satisfy $n^2 = \bar{n}^2 = 0$, $n \cdot \bar{n} = 2$, therefore the four-velocity of the Λ_b baryon $v_\mu = (n_\mu + \bar{n}_\mu)/2$. The gauge links between the fields in the above have been suppressed for brevity.

It is straightforward to rewrite the Eq. (1) in the following form

$$\begin{aligned}
&\epsilon^{abc}\langle 0|u_\alpha^a(t_1n)d_\beta^b(t_2n)h_{v\gamma}^c(0)|\Lambda(v)\rangle \\
&= \frac{1}{8}f_\Lambda^{(2)}\Psi_2(t_1,t_2)(\not{n}\gamma_5 C)_{\alpha\beta}\Lambda_\gamma(v) + \frac{1}{4}f_\Lambda^{(1)}\Psi_3^s(t_1,t_2)(\gamma_5 C)_{\alpha\beta}\Lambda_\gamma(v) \\
&- \frac{1}{8}f_\Lambda^{(1)}\Psi_3^\sigma(t_1,t_2)(i\sigma_{\bar{n}n}\gamma_5 C)_{\alpha\beta}\Lambda_\gamma(v) + \frac{1}{8}f_\Lambda^{(2)}\Psi_4(t_1,t_2)(\not{n}\gamma_5 C)_{\alpha\beta}\Lambda_\gamma(v).
\end{aligned} \tag{2}$$

Each distribution amplitude $\Psi_i(t_1, t_2)$ can be expressed by a Fourier integral

$$\Psi_i(t_1, t_2) = \int_0^{+\infty} d\omega_1 \int_0^1 d\omega_2 e^{-i\omega_1 t_1 u - i t_2 \omega_2} \psi_i(\omega_1, \omega_2) = \int_0^{+\infty} \omega d\omega \int_0^1 du e^{-i\omega(t_1 u + t_2 \bar{u})} \tilde{\psi}_i(\omega, u), \tag{3}$$

where $\tilde{\psi}(\omega, u) = \psi(u\omega, \bar{u}\omega)$, $\bar{u} = 1 - u$, ω_i ($i = 1, 2$) are the energies of the u - and d - quarks. $\omega = \omega_1 + \omega_2$ is the total energy carried by light quarks and the dimensionless parameter u describes the momentum fraction carried by the u quark in the diquark system. The normalization of $\tilde{\psi}_i$ are

$$\int_0^{+\infty} \omega d\omega \int_0^1 \tilde{\psi}_2(\omega, u) = \int_0^{+\infty} \omega d\omega \int_0^1 \tilde{\psi}_3^s(\omega, u) = \int_0^{+\infty} \omega d\omega \int_0^1 \tilde{\psi}_4(\omega, u) = 1. \tag{4}$$

The explicit forms of the distribution amplitudes for the Λ_b baryon are proposed as

$$\begin{aligned}
\tilde{\psi}_2(\omega, u) &= \omega^2 u(1-u) \left[\frac{1}{\epsilon_0^4} e^{-\omega/\epsilon_0} + a_2 C_2^{3/2} (2u-1) \frac{1}{\epsilon_1^4} e^{-\omega/\epsilon_1} \right], \\
\tilde{\psi}_3^s(\omega, u) &= \frac{\omega}{2\epsilon_3^3} e^{-\omega/\epsilon_3}, \\
\tilde{\psi}_3^\sigma(\omega, u) &= \frac{\omega}{2\epsilon_3^3} (2u-1) e^{-\omega/\epsilon_3}, \\
\tilde{\psi}_4(\omega, u) &= 5\mathcal{N}^{-1} \int_{\omega/2}^{s_0^{\Lambda_b}} ds e^{-s/\tau} (s - \omega/2)^3,
\end{aligned} \tag{5}$$

where τ is taken to be in the interval $0.4 < \tau < 0.8 \text{ GeV}$, $s_0^{\Lambda_b} = 1.2 \text{ GeV}$ is the continuum threshold for the Λ_b channel in the heavy-quark effective theory (HQET) and the coefficient

$$\mathcal{N} = \int_0^{s_0^{\Lambda_b}} s^5 e^{-s/\tau}. \tag{6}$$

III. LIGHT-CONE SUM RULES FOR THE TRANSITION FORM FACTORS

A. Parameterizations of transition form factors

Generally, the hadronic matrix elements responsible for Λ_b decays to a light baryon L can be parameterized in terms of a series of form factors

$$\langle L(P) | \bar{q} \gamma_\mu b | \Lambda_b(P+q) \rangle = \bar{L}(P) (f_1 \gamma_\mu + f_2 i \sigma_{\mu\nu} q^\nu + f_3 q_\mu) \Lambda_b(P+q), \quad (7)$$

$$\langle L(P) | \bar{q} \gamma_\mu \gamma_5 b | \Lambda_b(P+q) \rangle = \bar{L}(P) (F_1 \gamma_\mu + F_2 i \sigma_{\mu\nu} q^\nu + F_3 q_\mu) \gamma_5 \Lambda_b(P+q), \quad (8)$$

$$\langle L(P) | \bar{q} i \sigma_{\mu\nu} q^\nu b | \Lambda_b(P+q) \rangle = \bar{L}(P) (g_1 \gamma_\mu + g_2 i \sigma_{\mu\nu} q^\nu + g_3 q_\mu) \Lambda_b(P+q), \quad (9)$$

$$\langle L(P) | \bar{q} i \sigma_{\mu\nu} q^\nu \gamma_5 b | \Lambda_b(P+q) \rangle = \bar{L}(P) (G_1 \gamma_\mu + G_2 i \sigma_{\mu\nu} q^\nu + G_3 q_\mu) \gamma_5 \Lambda_b(P+q), \quad (10)$$

where all the form factors f_i , F_i , g_i and G_i are functions of the momentum transfer q^2 .

For the completeness, we also present the parameterizations of matrix elements involving the scalar $\bar{q}b$ and pseudo-scalar $\bar{q}\gamma_5 b$ currents, which can be obtained from Eqs. (9,10) by contracting both sides to the four-momentum q^μ

$$\langle L(P) | \bar{q} b | \Lambda_b(P+q) \rangle = \frac{1}{m_b + m_q} \bar{L}(P) [g_1(m_{\Lambda_b} - m_L) + g_3 q^2] \Lambda_b(P+q), \quad (11)$$

$$\langle L(P) | \bar{q} \gamma_5 b | \Lambda_b(P+q) \rangle = \frac{1}{m_b - m_q} \bar{L}(P) [G_1(m_{\Lambda_b} + m_L) - G_3 q^2] \gamma_5 \Lambda_b(P+q). \quad (12)$$

In the heavy quark limit, the form factors f_i , F_i , g_i and G_i can be expressed by two independent functions ξ_1 and ξ_2

$$\langle L(P) | \bar{q} \Gamma b | \Lambda_b(P+q) \rangle = \bar{L}(P) [\xi_1(q^2) + \not{q} \xi_2(q^2)] \Gamma \Lambda_b(P+q) \quad (13)$$

with Γ being an arbitrary Lorentz structure. Comparing Eqs. (7)-(10) and Eq. (13), one can easily get

$$\begin{aligned} f_1 &= F_1 = g_2 = G_2 = \xi_1 + \frac{m_L}{m_{\Lambda_b}} \xi_2, \\ g_1 &= G_1 = \frac{q^2}{m_{\Lambda_b}} \xi_2, \\ f_2 &= F_2 = f_3 = F_3 = \frac{\xi_2}{m_{\Lambda_b}}, \\ g_3 &= \frac{m_L - m_{\Lambda_b}}{m_{\Lambda_b}} \xi_2, \\ G_3 &= \frac{m_L + m_{\Lambda_b}}{m_{\Lambda_b}} \xi_2, \end{aligned} \quad (14)$$

where m_L denotes the mass of light baryon. It is known that Eq. (13) is successful at the small-recoil region (with large q^2) in the heavy quark limit.

B. Correlation functions

Following Ref. [13], the correlation function of two quark currents relevant to the $\Lambda_b \rightarrow L$ transition is taken between the vacuum and the on-shell Λ_b baryon state

$$z_\mu T^\mu(P, q) = iz_\mu \int d^4x e^{ip \cdot x} \langle 0 | T j_1(x), j_2^\mu(0) | \Lambda_b(P + q) \rangle, \quad (15)$$

where $j_1(x)$ is the interpolating current for a light baryon and $j_2(0)$ denotes the weak transition current. The introduction of z_ν , satisfying $z^2 = 0$ and $z \cdot q = 0$, is to remove the contributions $\sim z^\nu$ that give subdominant contributions on the light-cone. The manifest forms of currents j_i ($i = 1, 2$) for $\Lambda_b \rightarrow p, \Lambda$ transitions and their form factors are grouped in Table I. The correlation function can be systematically expanded in terms of the heavy quark mass in HQET. The momentum of Λ_b baryon can be redefined as $P + q = m_b v + k$, where k is the residual momentum and the relativistic normalization of the state is $|\Lambda_b(P + q)\rangle = |\Lambda_b(v)\rangle$, up to $1/m_b$ corrections. Moreover, it is also convenient to rescale the b -quark field by introducing an effective field $h_v(x) = b(x)e^{im_b v x} + O(1/m_b)$. In the first approximation, $m_{\Lambda_b} = m_b + \bar{\Lambda}$ implying that $k_0 \sim \bar{\Lambda}$. Accordingly, the four-momentum transfer q is redefined by separating the “static” part of it: $q = m_b v + \tilde{q}$, hence we have $p + \tilde{q} = k$. Now, it is straight to translate the correlation function (15) to HQET

$$z_\mu T^\mu(P, q) = z_\mu \tilde{T}^\mu(P, \tilde{q}) + O(1/m_b) \quad (16)$$

where the effective correlation function

$$z_\mu \tilde{T}^\mu(P, \tilde{q}) = iz_\mu \int d^4x e^{ip \cdot x} \langle 0 | T j_1(x), \tilde{j}_2^\mu(0) | \Lambda_b(v) \rangle \quad (17)$$

does not depend on the b quark mass. $\tilde{j}_2^\mu(0)$ can be obtained from $j_2^\mu(0)$ by replacing b quark field with the effective field h_v . It can be found that the correlation function (17) is dominated by the light-cone region $x^2 \leq 1/P^2$, if both the four-momentum are spacelike $P^2 < 0$, \tilde{q}^2 and sufficiently large

$$|P^2|, |\tilde{q}^2| \gg \Lambda_{QCD}^2, \bar{\Lambda}, \quad (18)$$

apart from the requirement that the ratio $\xi = 2p \cdot k/P^2$ should be at least of $O(1)$. In the language of effective theory, the initial external momentum q can be written as

$$q^2 \approx m_b^2 + 2m_b \tilde{q}_0 \sim m_b^2 - m_b P^2 \xi / \bar{\Lambda}. \quad (19)$$

As a result, the operator product expansion (OPE) on the light-cone works well in the kinematical region

$$0 \leq q^2 < m_b^2 - m_b P^2 / \bar{\Lambda}. \quad (20)$$

TABLE I: The two currents involved in the correlation function (15) and the corresponding heavy-to-light form factors.

Transition	j_1	j_2^μ	form factors
$\Lambda_b \rightarrow p$	$\epsilon^{ijk}[u^i(x)C \not{\epsilon} u^j(x)] \gamma_5 \not{\epsilon} d^k(x)$	$\bar{u}(0)\gamma^\mu(1 - \gamma_5)b(0)$	$f_i^{\Lambda_b \rightarrow p}, F_i^{\Lambda_b \rightarrow p}$
$\Lambda_b \rightarrow \Lambda$	$\epsilon^{ijk}[u^i(x)C\gamma_5 \not{\epsilon} d^j(x)] \not{\epsilon} s^k(x)$	$\bar{s}(0)\gamma_\mu(1 - \gamma_5)b(0)$	$f_i^{\Lambda_b \rightarrow \Lambda}, F_i^{\Lambda_b \rightarrow \Lambda}$
$\Lambda_b \rightarrow \Lambda$	$\epsilon^{ijk}[u^i(x)C\gamma_5 \not{\epsilon} d^j(x)] \not{\epsilon} s^k(x)$	$\bar{s}(0)i\sigma^{\mu\nu}q_\nu(1 - \gamma_5)b(0)$	$g_i^{\Lambda_b \rightarrow \Lambda}, G_i^{\Lambda_b \rightarrow \Lambda}$

C. Sum rules for baryonic transition form factors

The sum rules of form factors can be derived from the standard procedure, that is, matching the correlation function computed in the hadron and quark representations with the help of the dispersion relation under the assumption of quark-hadron duality.

1. $\Lambda_b \rightarrow p$ transition form factors

Inserting the complete set of states between the currents in Eq. (15) with the same quantum numbers as proton, we arrive at the hadronic representation of the correlator

$$z_\mu T^\mu(P, q) = -2f_N \frac{(z \cdot p)^2}{P^2 - m_N^2} [f_1^{\Lambda_b \rightarrow p} \not{\epsilon} - f_2^{\Lambda_b \rightarrow p} \not{\epsilon} \not{q} - F_1^{\Lambda_b \rightarrow p} \not{\epsilon} \gamma_5 + F_2^{\Lambda_b \rightarrow p} \not{\epsilon} \not{q} \gamma_5] \Lambda_b(P + q) + \dots (21)$$

where the ellipsis stands for the contribution from the higher resonance states of the proton channel and

$$\langle 0 | \epsilon^{ijk} [u^i(0)C \not{\epsilon} u^j(0)] \gamma_5 \not{\epsilon} d^k(0) | p(P) \rangle = f_N (z \cdot P) \not{\epsilon} p(P) \quad (22)$$

have been employed in the above derivations. It needs to be pointed out that the choice of the interpolating current for the baryon is not unique generally. There is no general recipe to discriminate various choices for the interpolating field of baryon. A practical criterion is that the coupling between the interpolating current and the given state should be strong enough.

On the theoretical side, the correlation function (15) can be calculated in the perturbation theory using the light-cone OPE. To the leading order of α_s , the correlation function may be computed by contracting the u quark field in (15) and inserting the free u quark propagator

$$\begin{aligned} z_\mu T^\mu(P, q) = & -2(C \not{\epsilon})_{\alpha\beta} (\gamma_5 \not{\epsilon})_{\gamma'\gamma} [\not{\epsilon}(1 - \gamma_5)]_{\rho\tau} \int d^4x \int \frac{d^4k}{(2\pi)^4} e^{i(p-k)\cdot x} \frac{1}{k^2 - m_u^2} (\not{k} + m_u)_{\alpha\rho} \\ & \times \langle 0 | \epsilon^{ijk} u_\beta^i(x) d_\gamma^j(x) b_\tau^k(0) | \Lambda_b(P + q) \rangle \end{aligned} \quad (23)$$

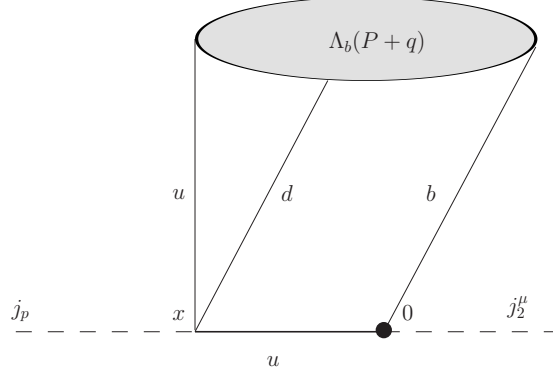


FIG. 1: The tree-level contribution to the correlation function (15), where the black solid dot represent the weak transition vertex.

as shown in Fig. 1. The full quark propagator also receives corrections from the background field [20, 21], which can be written as

$$\begin{aligned} \langle 0 | T \{ q_i(x) \bar{q}_j(0) \} | 0 \rangle &= \delta_{ij} \int \frac{d^4 k}{(2\pi)^4} e^{-ikx} \frac{i}{\not{k} - m_q} - ig \int \frac{d^4 k}{(2\pi)^4} e^{-ikx} \int_0^1 dv \left[\frac{1}{2} \frac{\not{k} + m_q}{(m_q^2 - k^2)^2} G_{ij}^{\mu\nu}(vx) \sigma_{\mu\nu} \right. \\ &\quad \left. + \frac{1}{m_q^2 - k^2} vx_\mu G^{\mu\nu}(vx) \gamma_\nu \right], \end{aligned} \quad (24)$$

where the first term is the free-quark propagator and $G_{ij}^{\mu\nu} = G_{\mu\nu}^a T_{ij}^a$ with $\text{Tr}[T^a T^b] = \frac{1}{2} \delta^{ab}$. Inserting the second term proportional to the gluon field strength into the correlation function can result in the distribution amplitudes corresponding to the higher Fock states of Λ_b baryon. It is expected that such corrections associating with the distribution amplitudes of higher Fock states do not play any significant role in the sum rules for transition form factors [22], therefore can be neglected safely in the presented work.

Substituting Eq. (2) into Eq. (23) and performing the integral in the coordinate space, we can achieve the correlation function in the momentum representation at the quark level as follows:

$$\begin{aligned} z_\mu T^\mu(P, q) &= 2f_\Lambda^{(2)}(z \cdot P)^2 \not{z}(1 - \gamma_5) \Lambda(v) \int_0^\infty d\omega \int_0^1 du \left\{ \frac{m_u^2 - q^2}{m_{\Lambda_b}^2} \frac{1}{[(P - \omega v)^2 - m_u^2]^2} \bar{\psi}_2(\omega, u) \right. \\ &\quad \left. + \frac{\bar{\sigma}^2}{[(P - \omega v)^2 - m_u^2]^2} \bar{\psi}_4(\omega, u) \right\} + 2f_\Lambda^{(1)}(z \cdot P)^2 \not{z} \not{A}(1 - \gamma_5) \int_0^\infty d\omega \int_0^1 du \\ &\quad \frac{1}{m_{\Lambda_b}} \frac{\bar{\sigma}}{[(P - \omega v)^2 - m_u^2]^2} \bar{\psi}_3^\sigma(\omega, u) + \dots, \end{aligned} \quad (25)$$

where the subleading terms in the infinite-momentum frame kinematics denoted by the ellipsis are suppressed. The functions $\bar{\psi}_i(\omega, u)$ are defined as

$$\bar{\psi}_i(\omega, u) = \int_0^\omega d\tau \tau \tilde{\psi}_i(\tau, u), \quad (26)$$

originating from the partial integral in the variable ω in order to eliminate the factor $1/v \cdot x$ due to the insertion of distribution amplitudes in Eq. (2)¹.

For the convenience of matching the correlation in QCD representation and hadronic level, Eq. (25) is usually written in a form of dispersion integral as

$$z^\mu T_\mu = (z \cdot P)^2 \int_0^\infty \frac{\rho_V(s, q^2) \not{z}(1 - \gamma_5) + \rho_T(s, q^2) \not{z} \not{q}(1 - \gamma_5)}{s - P^2} \Lambda(v) + \dots \quad (27)$$

With the assumption of quark-hadron duality, the higher states in the proton channel can be given by the same dispersion integral only with the lower bound replaced by the effective threshold parameter s_0 . Besides, the Borel transformation is commonly introduced in the standard procedure of sum rules approach for the sake of compensating the deficiency due to the approximation of quark-hadron duality.

Mathematically, the subtraction of higher states can be realized by making use of the following replacements

$$\begin{aligned} \int_0^\infty d\sigma \frac{\rho(\sigma, u)}{(P - m_{\Lambda_b} \sigma v)^2 - m_q^2} &\rightarrow - \int_0^{\sigma_0} \frac{d\sigma}{\bar{\sigma}} \frac{\rho(\sigma, u)}{s - P^2}, \\ \int_0^\infty d\sigma \frac{\rho(\sigma, u)}{(P - m_{\Lambda_b} \sigma v)^2 - m_q^2}^2 &\rightarrow \int_0^{\sigma_0} \frac{d\sigma}{\bar{\sigma}^2} \frac{\rho(\sigma, u)}{(s - P^2)^2} + \frac{\rho(\sigma_0, u)}{\bar{\sigma}_0^2} \frac{1}{s_0 - P^2} \eta(\sigma_0), \end{aligned} \quad (28)$$

where the involved parameters are defined as

$$\sigma = \frac{\omega}{m_{\Lambda_b}}, \quad \bar{\sigma} = 1 - \sigma, \quad s = \sigma m_{\Lambda_b}^2 + \frac{m_q^2 - \sigma q^2}{\bar{\sigma}}, \quad \eta(\sigma_0) = \left(1 + \frac{m_q^2 - q^2}{\bar{\sigma}^2 m_{\Lambda_b}^2}\right)^{-1}, \quad (29)$$

and σ_0 is the positive solution of the corresponding quadratic equation for $s = s_0$:

$$\sigma_0 = \frac{(s_0 + m_{\Lambda_b}^2 - q^2) - \sqrt{(s_0 + m_{\Lambda_b}^2 - q^2)^2 - 4m_{\Lambda_b}^2(s_0 - m_q^2)}}{2M^2}. \quad (30)$$

Performing the Borel transformation, we can finally obtain the sum rules for the transition form factors

$$\begin{aligned} f_N f_1^{\Lambda_b \rightarrow p}(q^2) e^{-\frac{m_N^2}{M^2}} &= f_\Lambda^{(2)} \int_0^1 du \left\{ \frac{m_{\Lambda_b}}{M^2} \int_0^{\sigma_0} \frac{d\sigma}{\bar{\sigma}^2} \left[\frac{m_u^2 - q^2}{m_{\Lambda_b}^2} \bar{\psi}_2(\omega, u) + \bar{\sigma}^2 \bar{\psi}_4(\omega, u) \right] e^{-s/M^2} \right. \\ &\quad \left. + \frac{1}{m_{\Lambda_b} \bar{\sigma}_0^2} \left[\frac{m_u^2 - q^2}{m_{\Lambda_b}^2} \bar{\psi}_2(\omega_0, u) + \bar{\sigma}_0^2 \bar{\psi}_4(\omega_0, u) \right] \eta(\sigma_0) e^{-s_0^p/M^2} \right\}, \\ f_N f_2^{\Lambda_b \rightarrow p}(q^2) e^{-\frac{m_N^2}{M^2}} &= -f_\Lambda^{(1)} \int_0^1 du \left\{ \frac{1}{M^2} \int_0^{\sigma_0} \frac{d\sigma}{\bar{\sigma}} \bar{\psi}_3^\sigma(\omega, u) e^{-s/M^2} + \frac{1}{m_{\Lambda_b}^2} \frac{1}{\bar{\sigma}_0} \bar{\psi}_3^\sigma(\omega_0, u) \eta(\sigma_0) e^{-s_0^p/M^2} \right\}, \end{aligned} \quad (31)$$

¹ The two light-like vector n_μ and \bar{n}_μ can be expressed by the four-velocity vector v_μ and the coordinate vector x_μ :

$$n_\mu = \frac{1}{v \cdot x} x_\mu, \quad \bar{n}_\mu = 2v_\mu - \frac{1}{v \cdot x} x_\mu.$$

with $\omega_0 = m_{\Lambda_b}\sigma_0$ and s_0^p being the threshold value of proton channel. It may be easily observed that the sum rules for the form factors $F_1^{\Lambda_b \rightarrow p}(q^2)$ and $F_2^{\Lambda_b \rightarrow p}(q^2)$ are the same as those for $f_1^{\Lambda_b \rightarrow p}(q^2)$ and $f_2^{\Lambda_b \rightarrow p}(q^2)$ respectively

$$F_1^{\Lambda_b \rightarrow p}(q^2) = f_1^{\Lambda_b \rightarrow p}(q^2), \quad F_2^{\Lambda_b \rightarrow p}(q^2) = f_2^{\Lambda_b \rightarrow p}(q^2). \quad (32)$$

2. $\Lambda_b \rightarrow \Lambda$ transition form factors

Following the same procedure, we can achieve the hadronic representation of the correlator responsible for the tensor $\Lambda_b \rightarrow \Lambda$ transition

$$z_\mu T^\mu(P, q) = -2f_\Lambda \frac{(z \cdot p)^2}{P^2 - m_\Lambda^2} [g_1^{\Lambda_b \rightarrow \Lambda} \not{z} - g_2^{\Lambda_b \rightarrow \Lambda} \not{q} + G_1^{\Lambda_b \rightarrow \Lambda} \not{z}\gamma_5 - G_2^{\Lambda_b \rightarrow \Lambda} \not{q}\gamma_5] \Lambda_b(P + q) + \dots (33)$$

where the ellipsis stands for the contribution from the higher resonance states of Λ baryon channel. The coupling between the selected current and Λ baryon is given by

$$\langle 0 | \epsilon^{ijk} [u^i(0) C \gamma_5 \not{z} d^j(0)] \not{z} s^k(0) | \Lambda(P) \rangle = f_\Lambda (z \cdot P) \not{z} \Lambda(P). \quad (34)$$

Similarly, one can also obtain the QCD representation of the correlation function in terms of the light-cone OPE

$$z_\mu T^\mu(P, q) = 2f_\Lambda^{(2)} (z \cdot P)^2 \not{z} \not{q} (1 + \gamma_5) \Lambda(v) \\ \times \int_0^\infty d\omega \int_0^1 du \frac{1}{[(P - \omega v)^2 - m_s^2]^2} \left[\frac{q^2 - m_s^2}{m_{\Lambda_b}^2} \bar{\psi}_2(\omega, u) - \bar{\sigma}^2 \bar{\psi}_4(\omega, u) \right] + \dots, \quad (35)$$

where the subleading terms in the infinite-momentum frame kinematics represented by the ellipsis are neglected. It is obvious that the form factors $g_1^{\Lambda_b \rightarrow \Lambda}(q^2)$ and $G_1^{\Lambda_b \rightarrow \Lambda}(q^2)$ do not contribute to the correlation function associated with the $\Lambda_b \rightarrow \Lambda$ transition at the leading power.

Matching the correlation function in the above two representations and performing the Borel transformation with the variable P^2 , one can arrive at the sum rules for the transition form factors

$$f_\Lambda g_2^{\Lambda_b \rightarrow \Lambda}(q^2) e^{-\frac{m_\Lambda^2}{M^2}} = f_\Lambda^{(2)} \int_0^1 du \left\{ \frac{m_{\Lambda_b}}{M^2} \int_0^{\sigma_0} d\sigma \left[\frac{m_s^2 - q^2}{m_{\Lambda_b}^2} \bar{\psi}_2(\omega, u) + \bar{\sigma}^2 \bar{\psi}_4(\omega, u) \right] e^{-s/M^2} \right. \\ \left. + \frac{1}{m_{\Lambda_b} \bar{\sigma}_0^2} \left[\frac{m_s^2 - q^2}{m_{\Lambda_b}^2} \bar{\psi}_2(\omega_0, u) + \bar{\sigma}_0^2 \bar{\psi}_4(\omega_0, u) \right] \eta(\sigma_0) e^{-s_0^\Lambda/M^2} \right\}, \quad (36)$$

where s_0^Λ is the duality-threshold parameter of Λ channel. In addition, the sum rules for the form factors satisfy

$$f_1^{\Lambda_b \rightarrow \Lambda}(q^2) = F_1^{\Lambda_b \rightarrow \Lambda}(q^2) = G_2^{\Lambda_b \rightarrow \Lambda}(q^2) = g_2^{\Lambda_b \rightarrow \Lambda}(q^2). \quad (37)$$

IV. NUMERICAL ANALYSIS OF SUM RULES FOR FORM FACTORS

Now, we are going to calculate the form factors $f_1^{\Lambda_b \rightarrow p}(q^2)$, $f_2^{\Lambda_b \rightarrow p}(q^2)$ responsible for the $\Lambda_b \rightarrow p$ decay and $g_2^{\Lambda_b \rightarrow \Lambda}(q^2)$ relevant to the $\Lambda_b \rightarrow \Lambda$ transition numerically. Firstly, we collect the input parameters used in this paper [17, 23, 24, 25, 26]

$$\begin{aligned}
m_s(1\text{GeV}) &= 142\text{MeV}, & m_u(1\text{GeV}) &= 2.8\text{MeV}, \\
m_{\Lambda_b} &= 5.62\text{GeV}, & m_p &= 0.938\text{GeV}, \\
m_\Lambda &= 1.12\text{GeV}, & f_\Lambda^{(1)} = f_\Lambda^{(2)} &= 0.030 \pm 0.005\text{GeV}^3 \\
f_N &= (5.0 \pm 0.5) \times 10^{-3}\text{GeV}^2, & f_\Lambda &= (6.0 \pm 0.3) \times 10^{-3}\text{GeV}^2 \\
s_0^p &= (2.25 \pm 0.10)\text{GeV}^2, & s_0^\Lambda &= (2.55 \pm 0.10)\text{GeV}^2.
\end{aligned} \tag{38}$$

The normalization constants of the light-cone distribution amplitudes for the proton, Λ baryon, and Λ_b baryon, namely, f_N , f_Λ , $f_\Lambda^{(1)}$ and $f_\Lambda^{(2)}$ are all evaluated at the renormalization scale $\mu = 1\text{GeV}$. As for the choice of the threshold parameters s_0^p and s_0^Λ , one should determine it by demanding the sum rules to be relatively stable in allowed regions for Borel mass M_B^2 , the value of which should be around the mass square of the corresponding first excited states. As for the heavy-light systems, the standard value of the threshold in the X channel would be $s_0^X = (m_X + \Delta_X)^2$, where Δ_X is about 0.5 GeV [27, 28, 29, 30, 31, 32].

With all the parameters listed above, we can proceed to compute the numerical values of the form factors. The form factors should not depend on the the Borel mass M^2 in a complete theory. However, as we truncate the operator product expansion up to next-to-leading conformal spin for the Λ_b bayon in the leading Fock configuration and keep the perturbative expansion in α_s to leading order, a manifest dependence of the form factors on the Borel parameter M^2 would emerge in practice. Therefore, one should look for a working “window”, where the results only mildly vary with respect to the Borel mass, so that the truncation is acceptable.

Firstly, we focus on the form factors at the zero-momentum transfer. As shown in Fig. 2, the form factor $f_1^{\Lambda_b \rightarrow p}$ is rather stable with the selected Borel mass $M^2 \in [1.5, 2.5]\text{GeV}^2$, which is consistent with that determined from the two-point sum rules for the nucleon form factors [33]. In principle, the Borel parameter M^2 should not be too large in order to insure that the contributions from the higher states are exponentially damped as can be observed from Eq. (31) and the global quark-hadron duality is satisfactory. On the other hand, the Borel mass could not be too small for the validity of OPE near the light-cone for the correlation function in deep Euclidean region, since the contributions of higher twist distribution amplitudes amount the higher order of $1/M^2$ to the leading contributions. The value of $f_1^{\Lambda_b \rightarrow p}(q^2 = 0)$ is $0.023_{-0.005}^{+0.006}$, where the uncertainties from

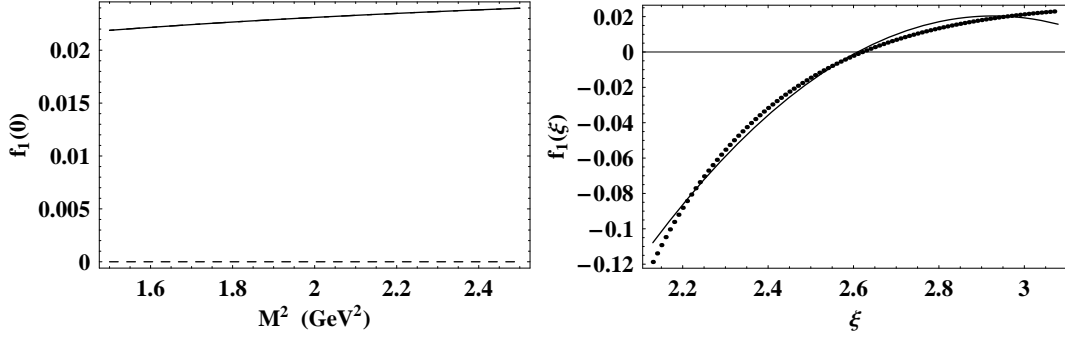


FIG. 2: Left panel: The dependence of form factor $f_1^{\Lambda_b \rightarrow p}(q^2 = 0)$ on the Borel mass M^2 (solid line) and the contribution from the twist-2 distribution amplitude in the whole sum rules (dashed line); Right panel: The dependence of form factor $f_1^{\Lambda_b \rightarrow p}$ on the momentum transfer q^2 within the kinematical region, where the light-cone OPE for the correlation function works well. The dashed line represents the result of form factor $f_1^{\Lambda_b \rightarrow p}$ predicted by the Λ_b -baryon LCSR, while the solid line describes the form factor $f_1^{\Lambda_b \rightarrow p}$ determined by Eq. (40).

the variations of Borel parameters, the fluctuation of the threshold value and the uncertainties from the normalization constants of the hadronic distribution amplitudes are combined together. Following the same procedure, we can continue to estimate the numerical results for the form factor $f_2^{\Lambda_b \rightarrow p}(q^2 = 0)$ at the zero-momentum transfer within the chosen Borel window as displayed in Fig. 3. It may be observed that $f_2^{\Lambda_b \rightarrow p}(q^2 = 0) = -0.039_{-0.009}^{+0.009} \text{ GeV}^{-1}$ with the given Borel window $M^2 \in [1.5, 2.5] \text{ GeV}^2$. As for the $\Lambda_b \rightarrow \Lambda$ transition, the Borel platform for the form factor $g_2^{\Lambda_b \rightarrow \Lambda}$ at the zero-momentum transfer is determined as $M^2 \in [2.0, 3.0] \text{ GeV}^2$ with the threshold parameter $s_0^\Lambda = 2.55 \text{ GeV}^2$. It can be observed from Fig. 4 that $g_2^{\Lambda_b \rightarrow \Lambda}(q^2 = 0) = 0.018_{-0.003}^{+0.003}$. To illustrate the SU(3)-breaking effects predicted in the Λ_b -baryon LCSR, it is helpful to define the following ratio

$$R \equiv \frac{f_1^{\Lambda_b \rightarrow p}(q^2 = 0)}{f_1^{\Lambda_b \rightarrow \Lambda}(q^2 = 0)} = 1.28_{-0.08}^{+0.14}. \quad (39)$$

In particular, this ratio is less sensitive to the variations of hadronic parameters involved in the Λ_b baryon distribution amplitudes than the individual form factors. It can be easily observed that SU(3) violating effects are attributed to the differences between the masses of s and u quarks and the discrepancy in the duality-threshold parameters for proton and Λ baryon.

In Table II, the numbers of the various transition form factors predicted in the Λ_b -baryon LCSR, light-baryon LCSR [10, 11], three-point QCDSR [7, 8] as well as PQCD approach [6, 34] are grouped

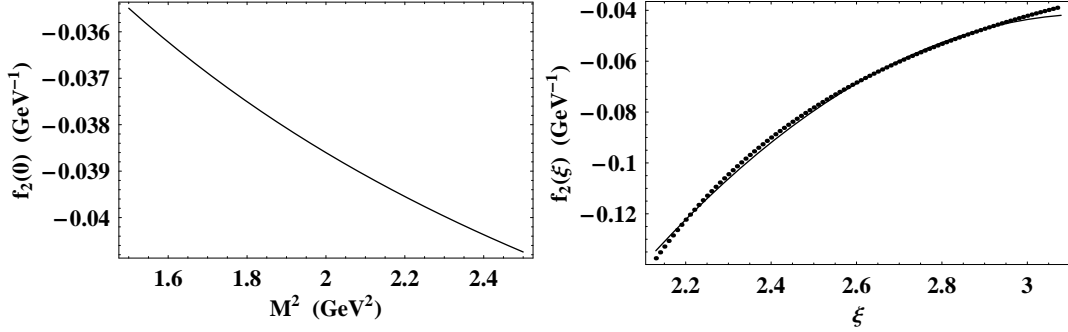


FIG. 3: Left panel: The dependence of form factor $f_2^{\Lambda_b \rightarrow p}(q^2 = 0)$ on the Borel mass M^2 ; Right panel: The dependence of form factor $f_2^{\Lambda_b \rightarrow p}$ on the momentum transfer q^2 within the kinematical region, where the light-cone OPE for the correlation function is valid. The dashed line represents the result of form factor $f_2^{\Lambda_b \rightarrow p}$ predicted by the Λ_b -baryon LCSR, while the solid line describes the form factor $f_2^{\Lambda_b \rightarrow p}$ determined by Eq. (40).

together. It is worthwhile to remind that the quark-hadron duality is employed differently in the Λ_b -baryon and light-baryon LCSR. Consequently, the difference between the predictions of two kinds of LCSR for the same transition form factors can be taken into account as a quantitative estimation of the systematic uncertainty. As shown in this table, the $\Lambda_b \rightarrow p$ transition form factors evaluated in Λ_b -baryon LCSR and light-baryon LCSR with full QCD are basically consistent with each other, which implies that the power corrections to the Λ_b -baryon LCSR are numerically small. The predictions of $\Lambda_b \rightarrow p$ transition form factors in terms of the light-baryon LCSR with HQET deviate distinctly from that obtained in the light-baryon LCSR with full QCD. Different versions of LCSR can be easily discriminated by measuring the semileptonic $\Lambda_b \rightarrow p$ decay at the ongoing and forthcoming colliders. The $\Lambda_b \rightarrow \Lambda$ transition form factor $g_2^{\Lambda_b \rightarrow \Lambda}(q^2 = 0)$ estimated in light-baryon LCSR is almost one order larger than that given by the Λ_b -baryon LCSR. Such distinct discrepancy can be attributed to the fact that only the asymptotic contributions of distribution amplitudes for the Λ baryon are included in the sum rules for the form factor $g_2^{\Lambda_b \rightarrow \Lambda}$, apart from the systematic uncertainties coming from the different quark-hadron duality assumptions as mentioned above.

It is also observed from Table II that the hard contributions to the form factor $f_1^{\Lambda_b \rightarrow p}$ involving two hard-gluons' exchange, estimated in the PQCD approach, is approximately one order smaller than those contributions dominated by the soft gluon exchange as estimated in Λ_b -baryon LCSR.

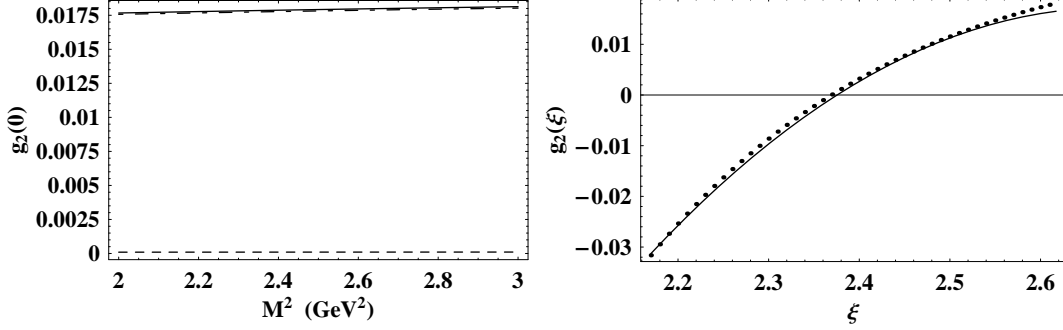


FIG. 4: Left panel: The dependence of form factor $g_2^{\Lambda_b \rightarrow \Lambda}(q^2 = 0)$ on the Borel mass M^2 (solid line) and the contribution from the twist-2 distribution amplitude in the whole sum rules (dashed line); Right panel: The dependence of form factor $g_2^{\Lambda_b \rightarrow \Lambda}$ on the momentum transfer q^2 within the kinematical region, where the light-cone OPE for the correlation function works well. The dashed line represents the result of form factor $g_2^{\Lambda_b \rightarrow \Lambda}$ predicted by the Λ_b -baryon LCSR, while the solid line describes the form factor $g_2^{\Lambda_b \rightarrow \Lambda}$ determined by Eq. (40).

In other words, the $\Lambda_b \rightarrow p$ transition form factors are dominated by the non-perturbative contributions, which may not be estimated reliably in the PQCD approach. As for the $\Lambda_b \rightarrow \Lambda$ transition, the value of form factor $f_1^{\Lambda_b \rightarrow \Lambda}$ ($=g_2^{\Lambda_b \rightarrow \Lambda}$) is approximately 5 times larger than that of $f_1^{\Lambda_b \rightarrow \Lambda}$ in PQCD approach, implying unexpectable SU(3) symmetry breaking effects. A potential reason responsible for such impenetrable observation is that the distribution amplitudes of Λ baryon employed in the analysis of Ref. [6] are motivated by quark model, which are not consistent with the QCD constraints.

In the next place, we can further investigate the q^2 dependence of the $\Lambda_b \rightarrow p, \Lambda$ form factors based on the sum rules given in (31) and (36). As already mentioned, the light-cone OPE is expected to be successful at the region (20), which indicates that the sum rules for transition form factors are reliable only for $0 < q^2 < 10 \text{ GeV}^2$. LCSR with the light-baryon distribution amplitudes are applicable at larger momentum transfer, up to $14 - 16 \text{ GeV}^2$ [11]. The dependence of form factors $f_1^{\Lambda_b \rightarrow p}$, $f_2^{\Lambda_b \rightarrow p}$ and $g_2^{\Lambda_b \rightarrow \Lambda}$ on the momentum transfer have been plotted in Fig. (2), (3) and (4) respectively.

Following Ref. [10], we fit the results of form factors given by the Λ_b -LCSR at $0 < q^2 < 10 \text{ GeV}^2$ to the following parameterization

$$\eta_i(\xi) = a_i + b_i \xi + c_i \xi, \quad (40)$$

TABLE II: $\Lambda_b \rightarrow p$ and $\Lambda_b \rightarrow \Lambda$ transition form factors computed in the LCSR with Λ_b distribution amplitudes, where the uncertainties from the Borel mass, threshold parameter and normalization constants of the hadronic distribution amplitudes are combined together. For comparison, we also cite the theoretical estimations of the form factors in the LCSR with light baryon distribution amplitudes, three-point QCD sum rules and PQCD approach.

form factor	Λ_b baryon LCSR	light-baryon LCSR	light-baryon LCSR	3-point QCDSR	PQCD
	HQET		full QCD		
$f_1^{\Lambda_b \rightarrow p}(q^2 = 0)$	$0.023^{+0.006}_{-0.005}$	-2.14×10^{-3} [10]	0.018 [10]	0.22 [7]	$2.2^{+0.8}_{-0.5} \times 10^{-3}$ [34]
$f_2^{\Lambda_b \rightarrow p}(q^2 = 0) (\text{GeV}^{-1})$	$-0.039^{+0.009}_{-0.009}$	-0.015 [10]	-0.028 [10]	0.71×10^{-2} [7]	—
$g_2^{\Lambda_b \rightarrow \Lambda}(q^2 = 0)$	$0.018^{+0.003}_{-0.003}$	—	$0.14^{+0.02}_{-0.01}$ [11]	0.45 [8]	$(1.2 - 1.6) \times 10^{-2}$ [6]

TABLE III: The parameters a_i , b_i and c_i in Eq. (40) determined by the Λ_b -baryon LCSR at the region $0 < q^2 < 10\text{GeV}^2$, where the uncertainties from the Borel mass, threshold parameter and normalization constants of the hadronic distribution amplitudes are combined together. The results obtained in the light-baryon LCSR [10] are also collected for comparison.

form factor	a_i	b_i	c_i
$f_1^{\Lambda_b \rightarrow p}$	$-1.71^{+0.44}_{-0.42}$	$1.18^{+0.28}_{-0.30}$	$-0.20^{+0.05}_{-0.05}$
	-1.14 [10]	0.75 [10]	-0.12 [10]
$f_2^{\Lambda_b \rightarrow p} (\text{GeV}^{-1})$	$-0.92^{+0.18}_{-0.19}$	$0.56^{+0.11}_{-0.11}$	$-0.088^{+0.018}_{-0.018}$
	0.027 [10]	-0.040 [10]	0.0085 [10]
$g_2^{\Lambda_b \rightarrow \Lambda}$	$-1.33^{+0.26}_{-0.26}$	$1.01^{+0.20}_{-0.20}$	$-0.19^{+0.04}_{-0.03}$

where the label ξ_i denote the form factors $f_1^{\Lambda_b \rightarrow p}$, $f_2^{\Lambda_b \rightarrow p}$ and $g_2^{\Lambda_b \rightarrow \Lambda}$. The viable ξ is defined as $\xi = \frac{v \cdot P}{m_L}$, whose number in the whole kinematical region is $1 \leq \xi \leq \xi_{\max} \equiv \frac{m_{\Lambda_b}^2 + m_L^2}{m_{\Lambda_b} m_L}$. The values of parameters a_i , b_i and c_i are tabulated in Table III. It is clear from this table that two different LCSR can lead to the consistent numbers for the q^2 -dependence of the form factor $f_1^{\Lambda_b \rightarrow p}$, which have been displayed in Fig. 5. However, the form factor $f_2^{\Lambda_b \rightarrow p}$ estimated in the Λ_b -baryon LCSR deviates from that predicted in the light-baryon LCSR significantly, implying that the power corrections in these two LCSR differ from each other remarkably. To illustrate the discrepancy more quantitatively, we present the q^2 -dependent behavior of the form factor $f_2^{\Lambda_b \rightarrow p}$ in two LCSR manifestly. As shown in Fig. 5, the form factor $f_2^{\Lambda_b \rightarrow p}$ rises drastically with increasing

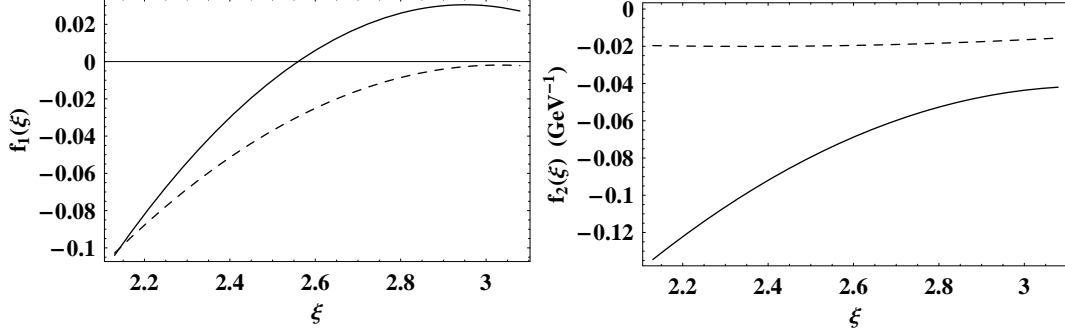


FIG. 5: The dependence of form factors $f_1^{\Lambda_b \rightarrow p}$ and $f_2^{\Lambda_b \rightarrow p}$ on the momentum transfer evaluated in two LCSR: the solid line denotes the results given by the Λ_b -baryon LCSR, while the dashed line describes the predictions from the light-baryon LCSR.

squared momentum transfer q^2 in the Λ_b -baryon LCSR, however, it almost does not change with the variation of q^2 in the light-baryon LCSR.

V. DISCUSSIONS AND CONCLUSIONS

Employing the distribution amplitudes of Λ_b baryon, we investigate the $\Lambda_b \rightarrow p$, Λ transition form factors in the framework of LCSR approach, which serves as a further development of B -meson LCSR suggested in Ref. [12, 13]. Such transition form factors play the role of a corner stone to explore the quark-flavor structure of the SM as well as determine its fundamental parameters such as the CKM matrix. Pinning down the uncertainties of transition form factors in many cases is an essential prescription to improve the accuracy of theoretical predictions.

Our results indicate that $\Lambda_b \rightarrow p$ transition form factors computed in the Λ_b -baryon LCSR are consistent with that given by the standard LCSR in full QCD within the error bars. Such agreement possibly illustrate that the power corrections to the Λ_b -baryon LCSR are not sizable. The prediction of form factor $g_2^{\Lambda_b \rightarrow \Lambda}(q^2 = 0)$ in light-baryon LCSR is about one order larger than that estimated in the Λ_b -baryon LCSR. It is not difficult to understand these results from two kinds of LCSR. In the Λ_b -baryon LCSR, the distribution amplitudes of Λ_b baryon are the universal nonperturbative inputs, which parameterize the long distance dynamics of $\Lambda_b \rightarrow p$, Λ form factors. However, it is the distribution amplitudes of light baryon that describe the nonperturbative contributions to the form factors in the standard LCSR. It is well known that hadronic distribution amplitudes

are of limited accuracy due to our lack good understanding of QCD at low energies. Reasonable prediction on the SU(3) symmetry breaking effects between the form factors $f_1^{\Lambda_b \rightarrow p}$ and $f_1^{\Lambda_b \rightarrow \Lambda}$ in the Λ_b -baryon LCSR can be ascribed to the same hadronic distribution amplitudes involved in the sum rules, which can reduce the theoretical uncertainties significantly. As for the light baryon LCSR, the distribution amplitudes of proton are considered up to the next-to-leading conformal spin accuracy in Ref. [10]; while only the asymptotic forms of Λ baryon distribution amplitudes are included in the sum rules for the form factor $g_2^{\Lambda_b \rightarrow \Lambda}$ in Ref. [11]. Hence, it is probable that the pre-asymptonic corrections to the Λ baryon are quite crucial to reconcile the existing discrepancy. As a matter of fact, large pre-asymptonic corrections to the distribution amplitudes of proton have been observed in the form factors responsible for $\Lambda_b \rightarrow p$ transition in Ref. [10].

Within the framework of Λ_b -baryon LCSR, we also study the dependence of form factors on the momentum transfer q^2 . It is shown that the Λ_b -baryon LCSR prediction of $f_1^{\Lambda_b \rightarrow p}$ is in accord with that estimated in the light-baryon LCSR. Moreover, radiative corrections to the $\Lambda_b \rightarrow p$, Λ transition form factors can be further carried out, once the renormalization-group evolutions for the Λ_b distribution amplitudes are available.

Acknowledgments

This work is partly supported by the National Science Foundation of China under Grant No.10735080 and 10625525. The authors would like to thank Vladimir Braun for helpful discussions.

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