$\Lambda_b \to p$ form factors

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1 Relativistic form factors

The authors of Ref. [1] define in their Eq. (73)

$$\langle N^+(p')|\overline{u}\,\gamma^\mu\, b|\Lambda_b(p)\rangle = \overline{u}_N(p')\left[\tilde{F}_1^V\,\gamma^\mu + \tilde{F}_2^V\,v^\mu + \tilde{F}_3^V\,v'^\mu\right]u_{\Lambda_b}(p), \tag{1}$$

$$\langle N^+(p')|\overline{u}\,\gamma^\mu\gamma_5\, b|\Lambda_b(p)\rangle = -\overline{u}_N(p')\left[\tilde{F}_1^A\,\gamma^\mu + \tilde{F}_2^A v^\mu + \tilde{F}_3^A v'^\mu\right]\gamma_5\,u_{\Lambda_b}(p), \tag{2}$$

where $v^\mu = p^\mu/m_{\Lambda_b}$, $v'^\mu = p'^\mu/m_N$.

2 HQET form factors

In Ref. [2] we use HQET for the b quark, and calculate the form factors F_1 and F_2, which are defined as

$$\langle N^+(p')|\,\bar{u}\Gamma b\,|\Lambda_b(p)\rangle = \overline{u}_N(p')\,[F_1 + \not{v}\,F_2]\,\Gamma\,u_{\Lambda_b}(p). \tag{3}$$

3 Relativistic form factors in terms of HQET form factors

We insert the vector current into Eq. (3):

$$\begin{aligned}
\langle N^+(p')|\,\bar{u}\gamma^\mu b\,|\Lambda_b(p)\rangle &= \overline{u}_N(p')\,[F_1 + \not{v}\,F_2]\,\gamma^\mu\,u_{\Lambda_b}(p)\\
&= \overline{u}_N(p')\,[F_1\gamma^\mu + F_2\,(-\gamma^\mu\not{v} + \gamma^\mu\not{v} + \not{v}\gamma^\mu)\,]\;u_{\Lambda_b}(p)\\
&= \overline{u}_N(p')\,\big[F_1\gamma^\mu + F_2\,\big(-\gamma^\mu\not{v} + v_\nu(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu)\big)\,\big]\;u_{\Lambda_b}(p)\\
&= \overline{u}_N(p')\,[F_1\gamma^\mu + F_2\,(-\gamma^\mu\not{v} + 2v^\mu)\,]\;u_{\Lambda_b}(p)\\
&= \overline{u}_N(p')\,[F_1\gamma^\mu + F_2\,(-\gamma^\mu + 2v^\mu)\,]\;u_{\Lambda_b}(p)\\
&= \overline{u}_N(p')\,[(F_1 - F_2)\gamma^\mu + 2\,F_2\,v^\mu]\;u_{\Lambda_b}(p).
\end{aligned} \tag{4}$$

Here we have used the Dirac equation $\not{v}u_{\Lambda_b} = u_{\Lambda_b}$. By comparing Eqs. (4) and (1), we see that, in HQET,

$$\tilde{F}_1^V = F_1 - F_2, \tag{5}$$

$$\tilde{F}_2^V = 2\,F_2, \tag{6}$$

$$\tilde{F}_3^V = 0. \tag{7}$$

Similarly, for the the axial vector current, one finds

$$\langle N^+(p')|\,\bar{u}\gamma^\mu\gamma_5 b\,|\Lambda_b(p)\rangle = \overline{u}_N(p')\,[(F_1 + F_2)\gamma^\mu + 2\,F_2\,v^\mu]\,\gamma_5\,u_{\Lambda_b}(p). \tag{8}$$

By comparing Eqs. (8) and (2), we see that, in HQET,

$$\tilde{F}_1^A = -(F_1 + F_2), \tag{9}$$

$$\tilde{F}_2^A = -2\,F_2, \tag{10}$$

$$\tilde{F}_3^A = 0. \tag{11}$$

The form factors F_1 and F_2 in Ref. [2] are written as functions of $E_N - m_N$, where E_N is the energy of the proton in the Λ_b rest frame. In terms of q^2, we have

$$E_N - m_N = \frac{m_{\Lambda_b}^2 + m_N^2 - q^2}{2m_{\Lambda_b}} - m_N. \tag{12}$$

References

[1] F. Hussain, D.-S. Liu, M. Kramer, J. G. Körner, and S. Tawfiq, Nucl. Phys. B **370**, 259 (1992).

[2] W. Detmold, C.-J. D. Lin, S. Meinel, and M. Wingate, arXiv:1306.0446 [hep-lat].