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Heavy-to-light baryonic form factors at large recoil

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Abstract: We analyze heavy-to-light baryonic form factors at large recoil and derive the scaling behavior of these form factors in the heavy quark limit. It is shown that only one universal form factor is needed to parameterize $\Lambda_b \to p$ and $\Lambda_b \to \Lambda$ matrix elements in the large recoil limit of light baryons, while hadronic matrix elements of $\Lambda_b \to \Sigma$ transition vanish in the large energy limit of Σ baryon due to the space-time parity symmetry. The scaling law of the soft form factor $\eta(P' \cdot v)$, P' and v being the momentum of nucleon and the velocity of Λ_b baryon, responsible for $\Lambda_b \to p$ transitions is also derived using the nucleon distribution amplitudes in leading conformal spin. In particular, we verify that this scaling behavior is in full agreement with that from light-cone sum rule approach in the heavy-quark limit. With these form factors, we further investigate the Λ baryon polarization asymmetry α in $\Lambda_b \to \Lambda\gamma$ and the forward-backward asymmetry A_{FB} in $\Lambda_b \to \Lambda l^+ l^-$. Both two observables ($\alpha$ and A_{FB}) are independent of hadronic form factors in leading power of $1/m_b$ and in leading order of α_s. We also extend the analysis of hadronic matrix elements for $\Omega_b \to \Omega$ transitions to rare $\Omega_b \to \Omega\,\gamma$ and $\Omega_b \to \Omega\, l^+ l^-$ decays and find that radiative $\Omega_b \to \Omega\,\gamma$ decay is probably the most promising FCNC $b \to s$ radiative baryonic decay channel. In addition, it is interesting to notice that the zero-point of forward-backward asymmetry of $\Omega_b \to \Omega\, l^+ l^-$ is the same as the one for $\Lambda_b \to \Lambda l^+ l^-$ to leading order accuracy provided that the form factors $\bar{\zeta}_i$ $(i = 3,\, 4,\, 5\,)$ are numerically as small as indicated from the quark model.

Keywords: Heavy quark physics, Rare Decays, QCD.

Contents

1. Introduction

Heavy baryons have attracted renewed attention due to the expectation of the future data on processes involving these states. In particular, LHCb will open a window on these particles by producing a sizable number of bottom and also charmed baryons.

From the theoretical side these states have been investigated already in the early days of Heavy Quark Effective Theory (HQET) [1, 2, 3]. In fact, from the point of view of HQET the Λ_b baryon is the simplest state, since the light degrees of freedom are in a spin and isospin singlet state, hence the Λ_b spin is equal to the b quark spin.

However, HQET is applicable only in cases where the light degrees of freedom do not carry a large momentum in the rest frame of the heavy hadron. Thus the kinematics of processes which can be described in this way is strongly restricted. For semileptonic decays this means that the methods can be applied only for leptonic momentum transfer q^2 close to $q^2_{\rm max}$, where the recoil on the light degrees of freedom is small.

Various attempts have been made to formulate an effective theory for the situation where a weak process generates energetic light degrees of freedom in the rest frame of a decaying heavy hadron. For semileptonic processes this is the region close to $q^2 = 0$, where the light quark has an energy of the order of the mass of the heavy hadron. The first attempt, called Large Energy Effective Theory (LEET) [4, 5], had certain problems which were eventually cured by Soft Collinear Effective Theory (SCET) [6, 7], which is now considered to be the appropriate description of energetic light quarks and gluons.

Both LEET as well as SCET exhibit additional symmetries which are equivalent to the conformal spin symmetry of massless QCD. These symmetries have been used to restrict the number of form factors in the semileptonic decays of a B meson into a pion and a ρ meson to only three independent unknown functions [5]. SCET allows to compute corrections to these relations which hold in the infinite energy limit of the outgoing light meson [7, 8].

The purpose of the present paper is to apply the same methods to the case of baryonic transitions. It tuns out that (similar to the HQET application for soft light degrees of freedom) a significant reduction of the number of form factors is achieved also for the case of energetic light degrees of freedom. It turns out that in the infinite energy limit only a single form factor is needed to describe $\Lambda_b \to p$ and $\Lambda_b \to \Lambda$ transitions, while the $\Lambda_b \to \Sigma$ transition vanishes in this limit. The analysis is also extended to $\Omega_b \to \Omega$ transitions, which may also have interesting phenomenological applications.

In the next sections we shall briefly review the necessary ingredients of HQET and LEET/SCET. Section 3 is devoted to the derivation of the necessary tensor representations of heavy baryons and the light-cone projectors of light baryons consisting of energetic collinear quarks. The core relations for weak form factors are derived in section 4, while section 5 contains the phenomenological applications to Λ_b and Ω_b decays, where we focus on radiative and semileptonic FCNC decays, which may be an interesting target at LHCb. The concluding discussion is presented in section 6.

2. Brief Review of HQET and SCET

HQET is constructed using the limit $m_Q \to \infty$, where m_Q is the mass of the heavy quark. In this limit, the dynamics of heavy-light hadron system can be greatly simplified due to new symmetries which are absent in full QCD. The heavy quark in the heavy hadron acts as a static color source which binds the light degrees of freedom by the exchange of soft gluons. Since the color interaction of QCD is flavor blind, the light degrees of freedom are in the same state independent of the flavour of the heavy quark. Likewise, since the chromomagnetic moment of the heavy quark scales with $1/m_Q$, the spin of the heavy quark decouples in the infinite mass limit.

The momentum of the heavy quark p_Q bound in a heavy hadron moving with the four-velocity is decomposed according to

$$p_Q^\mu = m_Q v^\mu + k^\mu \,, \tag{2.1}$$

where the residual momentum is small, $k_\mu \sim \Lambda_{QCD}$ and reflects the off-shell fluctuations due to the soft interactions.

The heavy quark field $Q(x)$ can be decomposed using the velocoy vector v as

$$\begin{aligned} h_v(x) &= \frac{1+\not v}{2}\exp\{im_Q v\cdot x\}Q(x)\,, \\ H_v(x) &= \frac{1-\not v}{2}\exp\{im_Q v\cdot x\}Q(x)\,, \end{aligned} \tag{2.2}$$

which correspond to the large and small components of $Q(x)$.

The field $H(x)$ corresponds to massive fluctuations related to the scale $2m_Q$, while the field $h(x)$ corresponds to a massless excitation. Integrating out the the massive degree of freedom, we end up with the effective Lagrangian of HQET [9, 10, 11, 12, 13]

$$\mathcal{L}_{\rm HQET} = \bar{h}_v(iv\cdot D)h_v + O(1/m_Q)\,, \tag{2.3}$$

where $D_\mu = \partial_\mu - igA_\mu$ denotes the covariant derivative of QCD, involving the gluon field.

Note that the leading order interaction is independent on the heavy-quark mass and shows explicitly the heavy-flavor and spin symmetry mentioned above. Formally, the HQET Lagrangian is invariant under the transformation of $\mathrm{SU}(n_Q)_{flavor}\otimes\mathrm{SU}(2)_{spin}$ group, where n_Q denotes the number of heavy quarks [1].

Energetic light degrees of freedom are described in SCET, which was predated by an attempt to formulate an effective theory involving energetic partons (LEET) [4]. However, it was soon observed [14, 15] that LEET could not reproduce the the infrared physics of full QCD due to the absence of collinear gluon interaction with energetic quarks. A nonlocal counter term was introduced in [15] to cancel emerged non-local divergence, however large logarithms still remains in the matching coefficient from QCD to LEET. This development finally lead to the formulation of SCET. In Ref. [6], SCET involving both soft and collinear gluons coupling to energetic partons was formulated and infrared physics of QCD can be reproduced correctly. However, the inclusion of collinear models does not change the relations of soft form factors in the large energy limit.

Massless QCD as well as SCET has, to leading order, a conformal spin symmetry which - similar to the symmetries of HQET - lead to relations among form factors in the symmetry limit. In particular, applying this to heavy-to-light transition [5] one obtains relations among heavy-to-light mesonic form factors.

Following closely to the derivation of HQET, the momentum of the energetic[2] parton can be split as

$$p_q^\mu = En^\mu + \tilde{k}^\mu, \qquad |\tilde{k}| \ll E \tag{2.4}$$

where E is the energy of light hadron and $n = p/E$ is the light-like vector in the direction of the outgoing light decay products. Making use of the velocity v of the decaying heavy hadron one may define a second light-like vector such that

$$v = \frac{1}{\sqrt{2}}(n+\bar{n})\,, \quad n^2 = 0 = \bar{n}^2\,, \quad n\bar{n} = 1\,.$$

[1] In fact, the symmetry is even larger, it is actually $\mathrm{SU}(2n_Q)$.

[2] This of course implies the definition of a reference frame in which the parton is energetic. For our purposes this frame is defined to be the rest frame of the decaying heavy hadron.

The quark field of full QCD $q(x)$ can be decomposed according to

$$q_n(x) = \frac{\not n\ \not{\bar n}}{2}\exp\{iEn\cdot x\}q(x)\,,$$
$$q_{\bar n}(x) = \frac{\not{\bar n}\ \not n}{2}\exp\{iEn\cdot x\}q(x)\,, \tag{2.5}$$

where the field $q_{\bar n}(x)$ is related to the large energy scale $2E$, while the field $q_n(x)$ contains the small energy scales. Similar to the case of HQET we may integrate out the field $q_{\bar n}(x)$ and obtain the leading order SCET Lagrangian

$$\mathcal{L}_{\rm SCET} = \bar q_n \left[in\cdot D + i\ \not D_c^{\perp}\frac{1}{2\,i\,\bar n\cdot D_c} i\ \not D_c^{\perp}\right] i\ \not{\bar n}\, q_n + O(1/E)\,, \tag{2.6}$$

where the soft gluon field has been separated out in $D_c^\mu = \partial^\mu - igA_c^\mu$ with A_c being the collinear component of the gluon field. It is worthwhile to point out that the projector prosperities of the collinear fields remain in this leading order Lagrangian [7], resulting in the symmetry relations between form factors observed in [5].

3. Tensor Representations and Light-cone projectors of baryons

In the following we discuss the transition matrix elements of heavy-to-light transitions for baryons at large recoil, i.e. the outgoing light baryon carries a large energy in the rest frame of the decaying heavy baryon. Thus there are various degrees of freedom involved: the heavy quark in the initial state accompanied by light degrees of freedom which are soft, with momenta of order $\Lambda_{\rm QCD}$; furthermore, the final state baryon with a large energy is assumed to consist of three collinear quarks. Since we shall exploit the collinear spin symmetry, we will make the spin indices of these collinear quarks explicitly by using the light-cone projectors of the light, energetic baryon states. Weak transition matrix elements can then be considered by a generalization of the well known trace formulae used in the case of the mesonic transitions [16].

3.1 Tensor representations for heavy baryons

Following Refs. [1, 2], the nomenclature of low-lying heavy baryons is

$$\begin{aligned}
\Lambda_Q &= [(qq')_0\, Q]_{1/2}\,, & \Xi_Q &= [(qs)_0\, Q]_{1/2}\,,\\
\Sigma_Q &= [(qq')_1\, Q]_{1/2}\,, & \Xi'_Q &= [(qs)_1\, Q]_{1/2}\,,\\
\Omega_Q &= [(ss)_1\, Q]_{1/2}\,, & \Sigma^*_Q &= [(qq')_1\, Q]_{3/2}\,,\\
\Xi^*_Q &= [(qs)_1\, Q]_{3/2}\,, & \Omega^*_Q &= [(ss)_1\, Q]_{3/2}\,.
\end{aligned} \tag{3.1}$$

The heavy baryon states Λ_Q and Ξ_Q are exactly analogous and identical in the SU(3) flavor symmetry limit. The other baryons shown in the above correspond to the heavy-baryon sextet categorized by the spin-parity of light-quark system, (Σ_Q, Σ^*_Q), (Ξ'_Q, Ξ^*_Q) and (Ω_Q, Ω^*_Q) degenerate in the heavy quark limit. For this reason, we will concentrate on the tensor representations of Λ_Q and Ω_Q baryons below.

The representation of Λ_b baryon is trivial since the soft light degrees of freedom are in a spinless state and the bottom quark carries all of the angular momentum of the baryon in the heavy quark limit. Thus we have

$$|\Lambda_b\rangle \mapsto \Lambda_b(v) \equiv b(v)\,, \tag{3.2}$$

where $\Lambda_b(v)$ ($b(v)$) on the right hand just denotes the Dirac spinor of Λ_b baryon (b-quark) with velocity v.

The representation of $\Omega_Q^{(*)}$ baryons involve some nontrivial Lorenz structures, since the light degrees of freedom now carry spin. Two equivalent representations (pseudovector and antisymmetric tensor) can be introduced to describe these states [2]. Here, we will stick to the pseudovector representation

$$|\Omega_Q^{(*)}\rangle \mapsto R^\mu(v) = A^\mu b(v)\,, \tag{3.3}$$

following the Ref. [17], where $R^\mu(v)$ satisfies the constraint of transversality.

The pseudovector-spinor object R^μ can be decomposed as

$$R^\mu_{3/2} = [g_{\mu\nu} - \frac{1}{3}(\gamma^\mu + v^\mu)\gamma_\nu]R^\nu \tag{3.4}$$

for a spin-$\frac{3}{2}$ baryon and

$$R^\mu_{1/2} = \frac{1}{3}(\gamma^\mu + v^\mu)\gamma_\nu R^\nu = \frac{1}{\sqrt{3}}(\gamma^\mu + v^\mu)\gamma_5 h_v \tag{3.5}$$

for a spin-$\frac{1}{2}$ baryon. $R^\mu_{3/2}$ satisfies the properties of Rartia-Schwinger vector spinor

$$\not{v} R^\mu_{3/2} = R^\mu_{3/2}\,, \qquad v_\mu R^\mu_{3/2} = 0\,, \qquad \gamma_\mu R^\mu_{3/2} = 0\,; \tag{3.6}$$

and $h_v = \frac{1}{\sqrt{3}}\gamma_5\gamma_\mu R^\mu_{1/2}$ is just the Dirac spinor of $\Omega_b(\frac{1}{2}^+)$ state moving with the velocity v_μ.

3.2 Light-cone projectors for light baryons

We assume that the quarks in the nucleon are collinear quarks, and hence we have collinear spin symmetries for these quarks. To the leading twist accuracy, the nucleon DA's at $z^2 \to 0$ are defined according to [18]:

$$\begin{aligned}\langle 0|\epsilon^{ijk} u^i_\alpha(a_1 z) u^j_\beta(a_2 z) d^k_\gamma(a_3 z)|N(P)\rangle &= \mathcal{V}_N\,(\not{P}C)_{\alpha\beta}\,(\gamma_5 N)_\gamma + \mathcal{A}_N\,(\not{P}\gamma_5 C)_{\alpha\beta}\,(N)_\gamma \\ &+ \mathcal{T}_N\,(P^\nu i\sigma_{\mu\nu}C)_{\alpha\beta}\,(\gamma^\mu\gamma_5 N)_\gamma\,. \end{aligned} \tag{3.7}$$

where α, β, γ are Dirac indices and z is a light-ray vector $z^2 = 0$. The calligraphic notations $\mathcal{F} = \{\mathcal{V}_N, \mathcal{A}_N, \mathcal{T}_N\}$ denote the integrals over the twist-3 nucleon DA's:

$$\mathcal{F} = \int \mathcal{D}x\, \exp\Big[-i\sum_{i=1}^{3} a_i x_i P\cdot z\Big] F(x_i, \mu)\,, \tag{3.8}$$

represented by the same non-calligraphic letters $F = \{V_N, A_N, T_N\}$. Here, $x_i = \{x_1, x_2, x_3\}$ with $0 \leq x_i \leq 1$ are the longitudinal momentum fractions of the quarks in the nucleon, μ is the normalization scale and the integral measures read

$$\int \mathcal{D}x \equiv \int dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3)\,. \tag{3.9}$$

This definition is equivalent to the following structure of the nucleon state [19, 20]

$$\begin{aligned}|N^\uparrow(P)\rangle = f_N \int \frac{\mathcal{D}x}{4\sqrt{24 x_1 x_2 x_3}} \Big\{ & V_N(x_i)\big|\big(u^\uparrow(x_1)u^\downarrow(x_2) + u^\downarrow(x_1)u^\uparrow(x_2)\big)d^\uparrow(x_3)\big\rangle \\ & -A_N(x_i)\big|\big(u^\uparrow(x_1)u^\downarrow(x_2) - u^\downarrow(x_1)u^\uparrow(x_2)\big)d^\downarrow(x_3)\big\rangle \\ & -2T_N(x_i)\big|u^\uparrow(x_1)u^\uparrow(x_2)d^\downarrow(x_3)\big\rangle \Big\}.\end{aligned} \tag{3.10}$$

The spin symmetry of $[u\,u]$ diquark implies that the distribution amplitudes V_N, A_N and T_N satisfy the following relations

$$\begin{aligned} V_N(x_1, x_2, x_3) &= V_N(x_2, x_1, x_3), \qquad A_N(x_1, x_2, x_3) = -A_N(x_2, x_1, x_3), \\ T_N(x_1, x_2, x_3) &= T_N(x_2, x_1, x_3), \end{aligned} \tag{3.11}$$

hence the distribution amplitude $A_1(x_i)$ vanishes in the leading conformal spin approximation, and we will drop out this term in the following analysis.

It is straightforward to derive the tensor representation of the nucleon state in the collinear limit, namely the nucleon light-cone projectors

$$\begin{aligned} \mathcal{M}_N^V &= (\not{n} C)_{\alpha\beta}\, [\gamma_5 N]_\gamma\,, \\ \mathcal{M}_N^T &= (n^\nu i\sigma_{\mu\nu} C)_{\alpha\beta}\, [\gamma^\mu \gamma_5 N]_\gamma\,, \end{aligned} \tag{3.12}$$

corresponding to the first and third configurations in Eq. (3.10), where the momentum of nucleon P_μ is chosen along the light-ray n_μ direction and ξ_n is the Dirac spinor of a fermion moving on the collinear direction n_μ ($\not{n}\xi_n = 0$). Likewise, the light-cone projectors for the nucleon in the final state can be written as

$$\begin{aligned} \overline{\mathcal{M}}_N^V &= -(C\,\not{n})_{\alpha\beta}\, [\bar{N}\gamma_5]_\gamma\,, \\ \overline{\mathcal{M}}_N^T &= -(n^\nu iC\sigma_{\mu\nu})_{\alpha\beta}\, [\bar{N}\gamma^\mu\gamma_5]_\gamma\,. \end{aligned} \tag{3.13}$$

Similarly, one can define the leading-twist distribution amplitudes of Λ baryon [21]

$$\begin{aligned} \langle 0|\epsilon^{ijk} u^i_\alpha(a_1 z) d^j_\beta(a_2 z) s^k_\gamma(a_3 z)|N(P)\rangle &= \mathcal{V}_\Lambda\, (\not{P}C)_{\alpha\beta}\, (\gamma_5\Lambda)_\gamma + \mathcal{A}_\Lambda\, (\not{P}\gamma_5 C)_{\alpha\beta}\, (\Lambda)_\gamma \\ &\quad + \mathcal{T}_\Lambda\, (P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta}\, (\gamma^\mu\gamma_5\Lambda)_\gamma\,, \end{aligned} \tag{3.14}$$

which is equivalent to the following structure of the Λ baryon state

$$\begin{aligned}|\Lambda^\uparrow(P)\rangle = \int \frac{\mathcal{D}x}{4\sqrt{24 x_1 x_2 x_3}} \Big\{ & f_\Lambda^V V_\Lambda(x_i)\big|\big(u^\uparrow(x_1)d^\downarrow(x_2) + u^\downarrow(x_1)d^\uparrow(x_2)\big)s^\uparrow(x_3)\big\rangle \\ & -f_\Lambda^V A_\Lambda(x_i)\big|\big(u^\uparrow(x_1)d^\downarrow(x_2) - u^\downarrow(x_1)d^\uparrow(x_2)\big)s^\downarrow(x_3)\big\rangle \\ & -2f_\Lambda^T T_\Lambda(x_i)\big|u^\uparrow(x_1)d^\uparrow(x_2)s^\downarrow(x_3)\big\rangle \Big\}.\end{aligned} \tag{3.15}$$

The isospin symmetry of $[u\,d]$ diquark implies that distribution amplitudes V_Λ, A_Λ and T_Λ respect the following relations

$$
\begin{aligned}
V_\Lambda(x_1,x_2,x_3) &= -V_\Lambda(x_2,x_1,x_3), \qquad A_\Lambda(x_1,x_2,x_3) = A_\Lambda(x_2,x_1,x_3), \\
T_\Lambda(x_1,x_2,x_3) &= -T_\Lambda(x_2,x_1,x_3),
\end{aligned} \tag{3.16}
$$

therefore $V_\Lambda(x_1,x_2,x_3)$ and $T_\Lambda(x_1,x_2,x_3)$ vanish to the leading conformal spin accuracy, and only the second term in Eq. (3.15) will be considered below. The light-cone projector of Λ baryon can be written as

$$
\mathcal{M}^A_\Lambda = (\not{n}\gamma_5 C)_{\alpha\beta}\ (\Lambda)_\gamma\,, \qquad \overline{\mathcal{M}}^A_\Lambda = (C\ \not{n}\gamma_5)_{\alpha\beta}\ (\bar{\Lambda})_\gamma\,, \tag{3.17}
$$

for the initial and final states, respectively.

In addition, the light-cone projectors of Σ and Ξ baryons are the same as the ones for the nucleon state. Below, we explicitly present the projectors of Σ baryon

$$
\begin{aligned}
\mathcal{M}^V_\Sigma &= (\not{n}C)_{\alpha\beta}\ [\gamma_5\Sigma]_\gamma\,, \qquad \mathcal{M}^T_\Sigma = (n^\nu i\sigma_{\mu\nu}C)_{\alpha\beta}\ [\gamma^\mu\gamma_5\Sigma]_\gamma\,, \\
\overline{\mathcal{M}}^V_\Sigma &= -(C\ \not{n})_{\alpha\beta}\ [\bar{\Sigma}\gamma_5]_\gamma\,, \qquad \overline{\mathcal{M}}^T_\Sigma = -(n^\nu iC\sigma_{\mu\nu})_{\alpha\beta}\ [\bar{\Sigma}\gamma^\mu\gamma_5]_\gamma\,,
\end{aligned} \tag{3.18}
$$

for the initial and final states.

3.3 Light-cone projectors of baryon decuplet

Following Refs. [19, 22], the distribution amplitudes of spin-$\frac{3}{2}$ Ω baryon are defined as

$$
\begin{aligned}
&\langle 0|\epsilon^{ijk} s^i_\alpha(a_1 z) s^j_\beta(a_2 z) s^k_\gamma(a_3 z)|\Omega(P)\rangle \\
&= \frac{\lambda^{1/2}_\Omega}{4}\left[\mathcal{V}_\Omega\,(\gamma_\mu C)_{\alpha\beta}\,(\Omega^\mu)_\gamma + \mathcal{A}_\Omega\,(\gamma_\mu\gamma_5 C)_{\alpha\beta}\,(\gamma_5\Omega^\mu)_\gamma - \frac{\mathcal{T}_\Omega}{2}\,(i\sigma_{\mu\nu}C)_{\alpha\beta}\,(\gamma^\mu\Omega^\nu)_\gamma\right] \\
&\quad -\frac{1}{4} f^{3/2}_\Omega \Phi_\Omega (i\sigma_{\mu\nu}C)_{\alpha\beta}(P_\mu\Omega^\nu - \frac{1}{2}m_\Omega\gamma_\mu\Omega^\nu)_\gamma\,,
\end{aligned} \tag{3.19}
$$

where $f^{3/2}_\Omega = \sqrt{\frac{2}{3}}\lambda^{1/2}_\Omega/m_\Omega$ and Ω^μ_γ is the Ω resonance spin-$\frac{3}{2}$ vector

$$
(\not{P} - m_\Omega)\Omega^\mu = 0\,, \qquad \bar{\Omega}^\mu\Omega_\mu = -2m_\Omega\,, \qquad \gamma_\mu\Omega^\mu = P_\mu\Omega^\mu = 0\,. \tag{3.20}
$$

The following symmetry relations among the distribution amplitudes

$$
\begin{aligned}
V_\Omega(x_1,x_2,x_3) &= V_\Omega(x_2,x_1,x_3)\,, \qquad A_\Omega(x_1,x_2,x_3) = -A_\Omega(x_2,x_1,x_3)\,, \\
T_\Omega(x_1,x_2,x_3) &= T_\Omega(x_2,x_1,x_3)\,, \qquad \Phi_\Omega(x_1,x_2,x_3) = \Phi_\Omega(x_2,x_1,x_3)\,,
\end{aligned} \tag{3.21}
$$

can be identified. The light-cone projectors of Ω-baryon can be written as

$$
\begin{aligned}
\mathcal{M}^V_\Omega &= (\gamma_\mu C)_{\alpha\beta}\ [\Omega^\mu]_\gamma\,, \qquad M^T_\Omega = (i\sigma_{\mu\nu}C)_{\alpha\beta}\ [\gamma^\mu\Omega^\nu]_\gamma\,, \\
\mathcal{M}^\Phi_\Omega &= (i\sigma_{\mu\nu}C)_{\alpha\beta}\ [n^\mu\Omega^\nu]_\gamma\,,
\end{aligned} \tag{3.22}
$$

for the initial state and

$$
\begin{aligned}
\overline{\mathcal{M}}^V_\Omega &= (C\gamma_\mu)_{\alpha\beta}\ [\bar{\Omega}^\mu]_\gamma\,, \qquad \overline{\mathcal{M}}^T_\Omega = (Ci\sigma_{\mu\nu})_{\alpha\beta}\ [\bar{\Omega}^\nu\gamma^\mu]_\gamma\,, \\
\mathcal{M}^\Phi_\Omega &= (Ci\sigma_{\mu\nu})_{\alpha\beta}\ [\bar{\Omega}^\nu n^\mu]_\gamma\,,
\end{aligned} \tag{3.23}
$$

for the final state.

4. Weak form factors for bottom baryon decays

4.1 $\Lambda_b \to p$, $\Lambda_b \to \Lambda$ and $\Lambda_b \to \Sigma$ form factors

Neglecting the hard interactions, $\Lambda_b \to p$ form factors at large recoil can can be written down in terms of the tensor representations of the participating baryons. Heavy quark and collinear spin symmetries imply

$$\begin{aligned}\langle N(P')|\bar{u}\Gamma b|\Lambda_b(v)\rangle &= \sum_{l=V,T} [\overline{\mathcal{M}}^l_N]_{\alpha\beta,\gamma}\,[\mathcal{J}_l C^T]_{\gamma\beta}\,\delta_{\alpha\tau}\,[\Gamma\,\Lambda_b(v)]_\tau \\ &= \bar{N}(P')\gamma_5\mathcal{J}_1\, \not{n}\Gamma\Lambda_b(v) + \bar{N}(P')\gamma^\mu\gamma_5\mathcal{J}_2 i n^\nu \sigma_{\mu\nu}\Gamma\Lambda_b(v)\,, \end{aligned} \tag{4.1}$$

where the non-perturbative dynamics is embedded in the coefficient functions $\mathcal{J}_i$ $(i = 1, 2)$. The most general structures of $\mathcal{J}_i$ can be written as

$$\mathcal{J}_i = (a_i + b_i\, \not{n} + c_i\, \bar{\not{n}} + d_i\, \not{n}\, \bar{\not{n}})\gamma_5\,, \tag{4.2}$$

where the coefficients a_i, b_i, c_i and d_i are functions of $P' \cdot v$. Using the equation of motion $\not{n}\xi_n = 0$, the matrix element (4.1) can be reduced to

$$\langle P(P')|\bar{u}\Gamma b|\Lambda_b(v)\rangle = \bar{N}(P')(\eta_1 + \eta_2\, \not{v})\Gamma\Lambda_b(v)\,, \tag{4.3}$$

which is the same as that obtained at small recoil, involving only soft light degrees of freedom. However, we may further simplify it by considering the matrix element with $\Gamma = \not{n}$, for which we have

$$0 = \langle P(P')|\bar{u}\, \not{n}\, b|\Lambda_b(v)\rangle = \bar{N}(P')(\eta_1 + \eta_2\, \not{v})\, \not{n}\Lambda_b(v) = \sqrt{2}\,\eta_2\,\bar{N}(P')\Lambda_b(v)\,, \tag{4.4}$$

indicating that the soft form factor η_2 vanishes at large recoil to the leading-power accuracy. Then, the $\Lambda_b \to p$ transition matrix element is written as

$$\langle P(P')|\bar{u}\Gamma b|\Lambda_b(v)\rangle = \eta(P' \cdot v)\bar{N}(P')\Gamma\Lambda_b(v)\,, \tag{4.5}$$

where only one universal form factor $\eta(P' \cdot v)$ appears to leading order. In full QCD, the weak form factors of $\Lambda_b \to p$ induced by $V - A$ current are defined as

$$\begin{aligned}\langle N(P')|\bar{c}\,\gamma_\mu\, u|\Lambda_b(P)\rangle &= \bar{N}(P')\left\{ f_1(q^2)\,\gamma_\mu + i\frac{f_2(q^2)}{m_{\Lambda_b}}\,\sigma_{\mu\nu}q^\nu + \frac{f_3(q^2)}{m_{\Lambda_b}}\,q_\mu \right\}\Lambda_b(P)\,, \\ \langle N(P')|\bar{c}\,\gamma_\mu\gamma_5\, u|\Lambda_b(P)\rangle &= \bar{N}(P')\left\{ g_1(q^2)\,\gamma_\mu + i\frac{g_2(q^2)}{m_{\Lambda_b}}\,\sigma_{\mu\nu}q^\nu + \frac{g_3(q^2)}{m_{\Lambda_b}}\,q_\mu \right\}\gamma_5\Lambda_b(P)\,,\end{aligned} \tag{4.6}$$

with $q = P - P'$. Following the Ref. [23], we rescale the hadronic states and spinors in the following way

$$\begin{aligned}|H_Q(P)\rangle &= \sqrt{m_Q}\,[\,|H(v)\rangle + O(1/m_Q)\,]\,, \\ u_Q(P,\lambda) &= \sqrt{m_Q}\,u(v,\lambda)\,,\end{aligned} \tag{4.7}$$

so that the heavy-quark mass dependence is removed from the states and spinors labelled by the subscript Q. Then, one can immediately find that the QCD matrix element is equal

form factors	$\eta^{(\mathcal{A})}_{\Lambda_b}$	$\eta^{(\mathcal{P})}_{\Lambda_b}$
$f_1(0)$	$0.14^{+0.03}_{-0.03}$	$0.12^{+0.03}_{-0.04}$
$f_2(0)$	$-0.054^{+0.016}_{-0.013}$	$-0.047^{+0.015}_{-0.013}$
$g_1(0)$	$0.14^{+0.03}_{-0.03}$	$0.12^{+0.03}_{-0.03}$
$g_2(0)$	$-0.028^{+0.012}_{-0.009}$	$-0.016^{+0.007}_{-0.005}$

Table 1: Numerical results of $\Lambda_b \to p$ transition form factors at zero momentum transfer calculated in LCSR with different interpolating currents of Λ_b baryon from [24].

to the one in the effective theory up to a $O(1/m_Q)$ correction. Comparing two definitions (4.5) and (4.6), one can find the following relations among the form factors

$$f_1(q^2) = g_1(q^2) = \eta(P' \cdot v)\,,$$
$$f_2(q^2) = g_2(q^2) = f_3(q^2) = g_3(q^2) = 0\,. \tag{4.8}$$

Numerical results of $\Lambda_b \to p$ form factors from light-cone sum rules (LCSR) [24] have been collected in Table 1 implying that the relations derived from heavy-quark and large-energy symmetries are well respected [3]. It needs to stress that two form factors are needed to parameterize $\Lambda_b \to p$ transition matrix elements if only heavy-quark spin symmetry is employed. Large-energy symmetry can further simplify the long-distance physics of $\Lambda_b \to p$ transitions in the large-recoil region.

As mentioned above, the relations shown in Eq. (4.8) are only valid for soft form factors at leading-order of $\Lambda_{\rm QCD}/m_Q$ and α_s, and they can be violated taking into account the hard interactions. In other words, these nontrivial relations hold only for the Feynman-mechanism contribution to the form factors, which are suppressed by the nucleon distribution amplitudes in the end-point region. In view of this observation, it is also possible to derive the scaling behavior of the soft form factor $\eta(P' \cdot v)$ in the heavy quark limit making use of the asymptotic behavior of nucleon distribution amplitudes. To the lowest conformal spin, the distribution amplitudes of nucleon are given by [18]

$$V_N(x_1, x_2, x_3) = T_N(x_1, x_2, x_3) = \phi^{asy}(x_1, x_2, x_3) \equiv 120 x_1 x_2 x_3\,. \tag{4.9}$$

[3] We stress that LCSR predictions of the $\Lambda_b \to p$ form factors $f_1(0)$ and $g_1(0)$ presented in [24] are in good agreement with that extracted from CDF measurement of non-leptonic decay $\Lambda_b \to p\pi$ [25] in the factorization limit. A rather small value of the form factor $f_1(0) = (2.3^{+0.6}_{-0.5}) \times 10^{-2}$ [26] from LCSR with Λ_b distribution amplitudes is probably due to the fact that the sum rules are constructed from the correlation function involving the nucleon interpolating current $\eta = (u^{\rm T} C \not{z})\gamma_5 \not{z} d$ which couples to both Δ-resonances and negative-parity baryons. For more detailed discussions on this issue, we refer the reader to [24].

Integrating over the end-point region for the momentum fraction of the recoiled quark $x_1 \sim 1 - \Lambda/E$, one can obtain the scaling behavior

$$\eta(P' \cdot v) \sim \int_{1-\frac{\Lambda}{E}}^{1} dx_1 \int_0^{1-x_1} dx_2\, \phi^{asy}(x_1, x_2, 1 - x_1 - x_2) \sim \frac{\Lambda^3}{E^3}\,. \tag{4.10}$$

Now, we will show that the scaling law of $\eta(P' \cdot v)$ derived above is compatibles with that from the LCSR. Very recently, $\Lambda_b \to p$ transition form factors were revisited in Ref. [24] from the improved sum rules approach, where the contribution of ground state negative-parity bottom baryon has been separated out from the Λ_b contribution without absorbing it into the continuum in the hadronic dispersion relation. An advantage of this approach is that the physical form factors are insensitive to the specific choice of the interpolating current of the heavy baryon as observed from Table 1.

To work out $\Lambda_b \to p$ form factors at $q^2 = 0$ in the heavy quark limit $m_b \to \infty$, we rescale Borel mass, threshold parameter and the decay constant of heavy baryon $\lambda^{(i)}_{\Lambda_b}$ following the Ref. [27]

$$\begin{aligned} M^2 &= 2 m_b \tau\,, & s_0 &= m_b^2 + 2 m_b \omega_0\,, \\ \lambda^{(i)}_{\Lambda_b} &= \frac{\tilde{f}_{\Lambda_b}}{m_b}\,, & m_{\Lambda_b} - m_b &= \bar{\Lambda}\,. \end{aligned} \tag{4.11}$$

Here, τ and ω_0 correspond to the nonrelativistic Borel mass and threshold parameter. The decay constants $\lambda^{(i)}_{\Lambda_b}$ and $\tilde{f}_{\Lambda_b}$ are given by

$$\langle 0 | \eta^{(i)}_{\Lambda_b} | \Lambda_b(P) \rangle = m_{\Lambda_b} \lambda^{(i)}_{\Lambda_b}\, \Lambda_b(P)\,, \qquad \langle 0 | (u\, C\, \Gamma\, d)\, b_v | \Lambda_b(v) \rangle = \tilde{f}_{\Lambda_b} \Lambda_b(v)\,. \tag{4.12}$$

It is clear that Λ_b decay constants defined by various currents degenerate in the heavy-quark limit. We then derive the sum rules of the form factor $f_1(0)$ in the heavy quark limit

$$f_1(0) = \frac{12}{m_b^3 \tilde{f}_{\Lambda_b}} \begin{cases} (\lambda_2 - 2\lambda_1)\, m_N\, e^{\bar{\Lambda}/\tau} \int_0^{\omega_0} d\omega\, \omega^2 e^{-\omega/\tau}\,, \\[2ex] 20 f_N\, e^{\bar{\Lambda}/\tau} \int_0^{\omega_0} d\omega\, \omega^3 e^{-\omega/\tau}\,, \end{cases} \tag{4.13}$$

where the upper (lower) sum rule is constructed from the correlation function with pseudoscalar (axial-vector) Λ_b current and the nucleon decay constants f_N, λ_1 and λ_2 are defined as [18]

$$\begin{aligned} \langle 0 | \epsilon^{ijk} \left[u^i(0) C\, \not{n} u^j(0) \right] \gamma_5\, \not{n}\, d^k_\gamma(0) | N(P) \rangle &= f_N (\bar{n} \cdot P)\, \not{n} N(P)\,, \\ \langle 0 | \epsilon^{ijk} \left[u^i(0) C \gamma_\mu u^j(0) \right] \gamma_5 \gamma^\mu\, d^k_\gamma(0) | N(P) \rangle &= \lambda_1 m_N N(P)\,, \\ \langle 0 | \epsilon^{ijk} \left[u^i(0) C \sigma_{\mu\nu} u^j(0) \right] \gamma_5 \sigma^{\mu\nu}\, d^k_\gamma(0) | N(P) \rangle &= \lambda_2 m_N N(P)\,. \end{aligned}$$

Comparing Eq. (4.10) with Eq. (4.13), one can observe that LCSR and HQET/SCET formalism predict consistent scaling behavior of the soft baryonic form factor $\eta(P' \cdot v) \sim (\Lambda_{\rm QCD}/m_Q)^3$, which is different from the scaling of heavy-to-light mesonic soft form factor $\xi(P' \cdot v) \sim (\Lambda_{\rm QCD}/m_Q)^{3/2}$ following from the symmetry argument [5]. Assuming the scaling of the inner sum rule parameters $\omega_0 \sim \tau \sim m_N$, we can extract the relation of nucleon decay constants f_N, λ_1 and λ_2

$$\frac{\lambda_2 - 2\lambda_1}{f_N} = 14.2\,, \tag{4.14}$$

from the matching of two sum rules presented in Eq. (4.13), which is consistent with the prediction from two-point QCD sum rules [18].

Applying the same technique to $\Lambda_b \to \Lambda$ form factors, we obtain

$$\langle \Lambda(P')|\bar{s}\Gamma b|\Lambda_b(v))\rangle = \bar{\Lambda}(P')\Gamma\Lambda_b(v)\,\mathrm{Tr}[\not{n}\gamma_5\bar{\mathcal{J}}] = \bar{\eta}(P'\cdot v)\bar{\Lambda}(P')\Gamma\Lambda_b(v)\,, \tag{4.15}$$

indicating that ten $\Lambda_b \to \Lambda$ form factors can be reduced to one universal form factor in the large recoil region in the heavy-quark limit. This can be also easily understood from the fact that the spin of the strange quark is the same as that of the Λ baryon in the quark model and the effective Lagrangian describing the interaction of energetic quarks with soft gluon does not involve nontrivial Dirac dynamics. Similar observation was also made in [28] with slightly different arguments.

For $\Lambda_b \to \Sigma$ transition, the hadronic matrix element in the heavy quark limit can be simplified as

$$\begin{aligned}\langle \Sigma(P')|\bar{s}\Gamma b|\Lambda_b(v))\rangle &= \bar{\Sigma}(P')\gamma_5\Gamma\Lambda_b(v)\,\mathrm{Tr}[\not{n}\tilde{\mathcal{J}}_1] + \bar{\Sigma}(P')\gamma^\mu\gamma_5\Gamma\Lambda_b(v)\,\mathrm{Tr}[n^\nu i\sigma_{\mu\nu}\tilde{\mathcal{J}}_2] \\ &= 0\,,\end{aligned} \tag{4.16}$$

as a consequence of the space-time parity symmetry, stating that all the ten $\Lambda_b \to \Sigma$ transition form factors should vanish in the large recoil limit.

4.2 $\Omega_b \to \Xi$ and $\Omega_b \to \Omega$ form factors

Repeating the same procedure for $\Omega_b \to \Xi$ transition, we can obtain

$$\begin{aligned}\langle \Xi(P')|\bar{u}\Gamma b|\Omega_b(v)\rangle &= \bar{\Xi}(P')\gamma_5\Gamma R^\rho_{1/2}(v)\,\mathrm{Tr}[\not{n}(\mathcal{K}_1)_\rho] \\ &\quad + \bar{\Xi}(P')\gamma^\mu\gamma_5\Gamma R^\rho_{1/2}(v)\,\mathrm{Tr}[in^\nu\sigma_{\mu\nu}(\mathcal{K}_2)_\rho]\,,\end{aligned} \tag{4.17}$$

where again the nonperturbative functions $\mathcal{K}_i$ $(i = 1, 2)$ involve the matrices $\not{n}$ and $\not{v}$, however they are independent of the Dirac structure Γ of the transition current. Using the equation of motion $\not{n}\xi_n = 0$ and performing the replacement $R^\mu_{1/2} \to \frac{1}{\sqrt{3}}(\gamma^\mu + v^\mu)\gamma_5 h_v$, the above hadronic matrix element can be simplified as

$$\begin{aligned}\langle \Xi(P')|\bar{u}\Gamma b|\Omega_b(v)\rangle &= \zeta_1(P'\cdot v)\,\bar{\Xi}(P')\gamma_5\Gamma\frac{n_\mu}{\sqrt{2}}(\gamma^\mu + v^\mu)\gamma_5\Omega_b(v) \\ &\quad + \zeta_2(P'\cdot v)\,\bar{\Xi}(P')\gamma_5\,\not{\bar{n}}\Gamma\frac{n_\mu}{\sqrt{2}}(\gamma^\mu + v^\mu)\gamma_5\Omega_b(v) \\ &\quad + \zeta_3(P'\cdot v)\,\bar{\Xi}(P')\gamma_5\gamma^\mu\Gamma(\gamma^\mu + v^\mu)\gamma_5\Omega_b(v)\,,\end{aligned} \tag{4.18}$$

in the large recoil limit.

Similarly, one can derive the $\Omega_b \to \Omega$ transition matrix element in the heavy quark limit as

$$\begin{aligned}\langle\Omega(P')|\bar{s}\Gamma b|\Omega_b(v)\rangle &= \bar{\Omega}^\mu(P')\Gamma R^\rho_{1/2}(v)\,\mathrm{Tr}[\gamma_\mu(\bar{\mathcal{K}}_1)_\rho] + \bar{\Omega}^\nu(P')\gamma^\mu R^\rho_{1/2}(v)\,\mathrm{Tr}[i\sigma_{\mu\nu}(\bar{\mathcal{K}}_2)_\rho] \\ &\quad +\bar{\Omega}^\nu(P')n^\mu\Gamma R^\rho_{1/2}(v)\,\mathrm{Tr}[i\sigma_{\mu\nu}(\bar{\mathcal{K}}_3)_\rho]\,. \end{aligned} \tag{4.19}$$

Making use of the equation of motion, we obtain

$$\begin{aligned}\langle\Omega(P')|\bar{s}\Gamma b|\Omega_b(v)\rangle &= \bar{\zeta}_1(P'\cdot v)\,\bar{\Omega}^\mu(P')\,\bar{n}_\mu\Gamma n_\rho\,(\gamma^\rho+v^\rho)\gamma_5\Omega_b(v) \\ &\quad +\bar{\zeta}_2(P'\cdot v)\,\bar{\Omega}^\mu(P')\,g_{\mu\rho}\,\Gamma\,(\gamma^\rho+v^\rho)\gamma_5\Omega_b(v) \\ &\quad +\bar{\zeta}_3(P'\cdot v)\,\bar{\Omega}^\mu(P')\,\frac{\bar{n}_\mu}{\sqrt{2}}\,\not{\bar{n}}\Gamma n_\rho\,(\gamma^\rho+v^\rho)\gamma_5\Omega_b(v) \\ &\quad +\bar{\zeta}_4(P'\cdot v)\,\bar{\Omega}^\mu(P')\,\frac{\bar{n}_\mu}{\sqrt{2}}\gamma_\rho\Gamma\,(\gamma^\rho+v^\rho)\gamma_5\Omega_b(v) \\ &\quad +\bar{\zeta}_5(P'\cdot v)\,\bar{\Omega}^\mu(P')\,g_{\mu\rho}\frac{\not{\bar{n}}}{\sqrt{2}}\,\Gamma\,(\gamma^\rho+v^\rho)\gamma_5\Omega_b(v)\,, \end{aligned} \tag{4.20}$$

where five soft form factors are necessary to parameterize the nonperturbative dynamics, and the normalization factor "$1/\sqrt{2}$" is introduced for later convenience. Without employing the large-energy symmetry, an additional soft form factor associating with the spin structure

$$\bar{\Omega}^\mu(P')\,\bar{n}_\mu\,\gamma_\rho\;\not{v}\,\Gamma\,(\gamma^\rho+v^\rho)\gamma_5\Omega_b(v) \tag{4.21}$$

should be included as shown in [29].

5. Applications to FCNC Λ_b and Ω_b decays

5.1 Rare decays of $\Lambda_b \to \Lambda\,\gamma$ and $\Lambda_b \to \Lambda\, l^+ l^-$

5.1.1 Radiative decay $\Lambda_b \to \Lambda\,\gamma$

The underlying flavour-changing $b \to s$ transition in the Standard Model (SM) is described by the effective Hamiltonian:

$$H_{eff} = -\frac{4G_F}{\sqrt{2}}V_{tb}V^*_{ts}\sum_{i=1}^{10}C_i(\mu)O_i(\mu)\,, \tag{5.1}$$

a linear combination of the effective operators O_i weighted by their Wilson coefficients C_i. The leading contribution of $\Lambda_b \to \Lambda\gamma$ is generated by the electromagnetic penguin operator

$$O_{7\gamma} = -\frac{e}{16\pi^2}\bar{s}\sigma_{\mu\nu}(m_s L + m_b R)bF^{\mu\nu}\,, \tag{5.2}$$

with the notation $L(R) = \frac{1-(+)\gamma_5}{2}$. Allowing for couplings beyond the SM, we introduce a more general dipole transition operator

$$\tilde{O}_{7\gamma} = -\frac{e}{32\pi^2}m_b\,\bar{s}\sigma_{\mu\nu}(g_V - g_A\gamma_5)bF^{\mu\nu}\,. \tag{5.3}$$

Considering the Λ-baryon polarization asymmetry in $\Lambda_b \to \Lambda + \gamma$, we firstly define the four-spin vector s^μ of Λ baryon in its rest frame

$$(s^\mu)_{r.s.} = (0,\ \hat{\xi}), \tag{5.4}$$

which can be boosted into the rest frame of Λ_b baryon

$$s^\mu = (\frac{\mathbf{P}_\Lambda \cdot \hat{\xi}}{m_\Lambda},\ \hat{\xi} + \frac{s_0}{E_\Lambda + m_\Lambda}\mathbf{P}_\Lambda), \tag{5.5}$$

with $\mathbf{P}_\Lambda$ and E_Λ being the three-momentum and energy of Λ baryon. It is straightforward to derive that

$$v \cdot s = \frac{1 - x_\Lambda^2}{1 + x_\Lambda^2}\hat{\mathbf{p}} \cdot \mathbf{s} = \frac{1 - x_\Lambda^2}{2x_\Lambda}\hat{\mathbf{p}} \cdot \hat{\xi}, \tag{5.6}$$

where $x_\Lambda = m_\Lambda / m_{\Lambda_b}$ and $\hat{\mathbf{p}}$ is a unite vector along the direction of Λ-baryon momentum.

Following Refs. [30, 31], the polarized decay width of $\Lambda_b \to \Lambda + \gamma$ has a form

$$\Gamma(\Lambda_b \to \Lambda\gamma) = \frac{1}{2}\Gamma_0[1 + \alpha\ \hat{\mathbf{p}} \cdot \mathbf{s}] = \frac{1}{2}\Gamma_0[1 + \alpha'\ \hat{\mathbf{p}} \cdot \hat{\xi}], \tag{5.7}$$

where Γ_0 is the total decay with of $\Lambda_b \to \Lambda + \gamma$

$$\Gamma_0 = \frac{G_F^2 \alpha_{em}}{64\pi^4}|V_{tb}V_{ts}^*|^2 m_b^2 m_{\Lambda_b}^3 (1 - x_\Lambda^2)^3 \big|\bar{\eta}(P' \cdot v)\big|^2 (|g_V|^2 + |g_A|^2)|\tilde{C}_{7\gamma}^{eff}|^2 \tag{5.8}$$

and the polarization asymmetry reads

$$\alpha = \frac{2x_\Lambda}{1 + x_\Lambda^2}\alpha' = \frac{2x_\Lambda}{1 + x_\Lambda^2}\frac{2g_V g_A}{g_V^2 + g_A^2} + O(\frac{\Lambda_{\rm QCD}}{m_b}) + O\big(\alpha_s\big)\,. \tag{5.9}$$

Taking the soft form factor $\bar{\eta}(q^2 = 0) = 0.15^{+0.02}_{-0.02}$ from QCD LCSR [32] and neglecting the long-distance contribution,[4] we predict the branching ratio $\mathrm{BR}(\Lambda_b \to \Lambda + \gamma) = \big(7.7^{+2.2}_{-1.9}\big) \times 10^{-6}$ in the SM, which is compatible with that estimated in [30] based on the heavy-to-light baryonic form factors from the pole model. We also mention in passing that Λ-baryon polarization asymmetry, to the leading order, is only determined by the short distance coefficients of partonic transition as already observed in [31, 33].

5.1.2 Semileptonic decay $\Lambda_b \to \Lambda\, l^+ l^-$

The dominant contributions to $\Lambda_b \to \Lambda\, l^+ l^-$ are generated by the operators $O_{7\gamma}$ and $O_{9,10}$

$$O_9 = \frac{\alpha_{em}}{4\pi}\,(\bar{s}\gamma_\rho L b)\,\big(\bar{l}\gamma^\rho l\big)\ , \qquad O_{10} = \frac{\alpha_{em}}{4\pi}\,(\bar{s}\gamma_\rho L b)\,\big(\bar{l}\gamma^\rho \gamma_5 l\big)\ . \tag{5.10}$$

[4] A preliminary study on some possible long-distance contributions, for instance charm-quark loop, internal W-exchange and light-quark loop, was already performed in [30], where long-distance contribution was found to be suppressed either by the large virtuality of the charm propagators or by the CKM matrix elements.

The free quark decay amplitude for $b \to s l^+ l^-$ process reads

$$\mathcal{A}(b \to s l^+ l^-) = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha_{em}}{\pi} \Big\{ -\frac{2i}{q^2} C_7^{eff}(\mu) \bar{s} \sigma_{\mu\nu} q^\nu (m_b R + m_s L) b \, \bar{l} \gamma^\mu l + C_9^{eff}(\mu) \bar{s} \gamma_\mu L b \, \bar{l} \gamma^\mu l + C_{10} \bar{s} \gamma_\mu L b \, \bar{l} \gamma^\mu \gamma_5 l \Big\}. \tag{5.11}$$

In the leading-order, the hadronic decay amplitude $\mathcal{A}_{\Lambda_b \to \Lambda l^+ l^-}$ can be derived by sandwiching the free quark amplitude (5.11) between the initial and final states. Defining the differential forward-backward asymmetry for the semileptonic decay

$$\frac{dA_{FB}(q^2)}{dq^2} = \int_0^1 dz \frac{d^2\Gamma(q^2, z)}{dq^2 dz} - \int_{-1}^0 dz \frac{d^2\Gamma(q^2, z)}{dq^2 dz} \,, \tag{5.12}$$

we obtain the following expression for $\Lambda_b \to \Lambda l^+ l^-$ transition in the SM

$$\frac{dA_{FB}(\Lambda_b \to \Lambda + l^+ l^-)}{dq^2} = \frac{G_F^2 \alpha_{em}^2}{256 m_{\Lambda_b}^3 \pi^5} |V_{tb} V_{ts}^*|^2 \lambda(m_{\Lambda_b^2}, m_\Lambda^2, q^2)(1 - \frac{4 m_l^2}{q^2}) R_{FB}(q^2), \tag{5.13}$$

with $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$ and

$$R_{FB}(q^2) = \big[2 m_b \, m_{\Lambda_b} \mathrm{Re}(C_{7\gamma}^{eff} C_{10}^*) + q^2 \mathrm{Re}(C_9^{eff} C_{10}^*)\big] \big|\bar{\eta}(P' \cdot v)\big|^2 + O\big(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\big) + O\big(\alpha_s\big). \tag{5.14}$$

It is obvious that the differential forward-backward asymmetry in $\Lambda_b \to \Lambda l^+ l^-$ decay only depends on the Wilson coefficients $\mathrm{Re}(C_7^{eff} C_{10}^*)$ and $\mathrm{Re}(C_9^{eff} C_{10}^*)$. The zero-position t_0 of forward-backward asymmetry is given by

$$t_0(\Lambda_b \to \Lambda + l^+ l^-) = -2 \, m_b \, m_{\Lambda_b} \frac{\mathrm{Re}(C_{7\gamma}^{eff} C_{10}^*)}{\mathrm{Re}(C_9^{eff} C_{10}^*)} + O\big(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\big) + O\big(\alpha_s\big) \,. \tag{5.15}$$

The expression of $t_0(\Lambda_b \to \Lambda + l^+ l^-)$ is exactly the same as that in the case of $B \to K^* l^+ l^-$[34], and it is free of hadronic uncertainties in the large recoil limit of Λ baryon. Substituting the values of Wilson coefficients at next-to-leading-logarithmic order as well as the pole-mass of the b quark m_b [35], we find $t_0(\Lambda_b \to \Lambda + l^+ l^-) = 3.8 \text{ GeV}^2$ which is consistent with those derived in [32] using $\Lambda_b \to \Lambda$ form factors from LCSR.

5.2 Rare decays of $\Omega_b \to \Omega \, \gamma$ and $\Omega_b \to \Omega \, l^+ l^-$

5.2.1 Radiative decay $\Omega_b \to \Omega \, \gamma$

Taking into account the left-hand coupling, the decay amplitude of $\Omega_b \to \Omega \, \gamma$ can be written as

$$\mathcal{A}(\Omega_b \to \Omega \, \gamma) = \frac{4 G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e}{32 \pi^2} m_b \langle \Omega | \tilde{C}_{7\gamma}^{eff} \, \tilde{O}_{7\gamma} | \Omega_b \rangle \,. \tag{5.16}$$

In terms of the hadronic matrix element given in Eq. (4.20), one can derive the decay width of the radiative decay $\Omega_b \to \Omega\,\gamma$ as

$$\Gamma(\Omega_b \to \Omega\,\gamma) = \frac{G_F^2 \alpha_{em}}{384\pi^4} |V_{tb}V_{ts}^*|^2 m_b^2 m_{\Omega_b}^3 \frac{(1-x_\Omega^2)^3}{x_\Omega^2} \big\{ \bar{\zeta}_1^2 + 2\bar{\zeta}_1 \big[(1+x_\Omega^2)\bar{\zeta}_2 - x_\Omega \bar{\zeta}_4 \big] \\ +(1+6x_\Omega^2+x_\Omega^4)\bar{\zeta}_2^2 + x_\Omega^2 \bar{\zeta}_2^4 - 2x_\Omega(1+3x_\Omega^2)\bar{\zeta}_2\bar{\zeta}_4 \big\} \\ \times(|g_V|^2+|g_A|^2)|\tilde{C}_{7\gamma}^{eff}|^2\,, \tag{5.17}$$

where $x_\Omega = m_\Omega/m_{\Omega_b}$ and the polarization sum of Rarita-Schwinger spin vectors

$$\Omega_\mu(P')\bar{\Omega}_\nu(P') = -(\not{P}'+m_\Omega)\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2}{3m_\Omega^2}P'_\mu P'_\nu + \frac{\gamma_\nu P'_\mu - \gamma_\mu P'_\nu}{3m_\Omega} \right]. \tag{5.18}$$

has been employed. In the massless limit of Ω baryon, the decay width is further simplified as

$$\Gamma(\Omega_b \to \Omega\,\gamma) = \frac{G_F^2 \alpha_{em}}{384\pi^4} |V_{tb}V_{ts}^*|^2 \frac{m_b^2 m_{\Omega_b}^3}{x_\Omega^2} \big(\bar{\zeta}_1^2 + \bar{\zeta}_2^2\big)(|g_V|^2+|g_A|^2)|\tilde{C}_{7\gamma}^{eff}|^2\,, \tag{5.19}$$

indicating that the radiative decay $\Omega_b \to \Omega\,\gamma$ is strongly enhanced by a factor $1/x_\Omega^2$ coming from the helicity-1/2 Ω contribution. Taking the ratio of decay width between $\Omega_b \to \Omega\,\gamma$ and $\Lambda_b \to \Lambda\,\gamma$ in the massless limit of light baryon, we obtain

$$\frac{\Gamma(\Omega_b \to \Omega\,\gamma)}{\Gamma(\Lambda_b \to \Lambda\,\gamma)} = \frac{\bar{\zeta}_1^2 + \bar{\zeta}_2^2}{6\,\bar{\eta}^2} \cdot \big(\frac{m_{\Omega_b}}{m_{\Lambda_b}}\big)^3 \cdot \big(\frac{m_{\Omega_b}}{m_\Omega}\big)^2\,, \tag{5.20}$$

showing that radiative decay $\Omega_b \to \Omega\,\gamma$ is probably the most promising FCNC $b \to s$ radiative baryonic decay channel and would be a golden channel to extract the helicity structures of weak effective Hamiltonian.

5.2.2 Semileptonic decay $\Omega_b \to \Omega\, l^+ l^-$

The decay amplitude $\mathcal{A}_{\Omega_b \to \Omega l^+ l^-}$ responsible for $\Omega_b \to \Omega\, l^+ l^-$ transition can be calculated following a similar way for $\mathcal{A}_{\Lambda_b \to \Lambda l^+ l^-}$ albeit with more involved spin structures. The differential forward-backward asymmetry in $\Omega_b \to \Omega\, l^+ l^-$ decay is calculated as

$$\frac{dA_{FB}(\Omega_b \to \Omega\, l^+ l^-)}{dq^2} = \frac{G_F^2 \alpha_{em}^2}{1536 m_{\Omega_b}^3 \pi^5} |V_{tb}V_{ts}^*|^2 \lambda^{1/2}(m_{\Omega_b^2}, m_\Omega^2, q^2) \sqrt{1-\frac{4m_l^2}{q^2}}\, \tilde{R}_{FB}(q^2), \tag{5.21}$$

with

$$\tilde{R}_{FB}(q^2) = -m_b\, m_{\Omega_b}^3 \left(1-\frac{q^2}{m_{\Omega_b}^2}\right)^3 \left(\frac{m_{\Omega_b}}{m_\Omega}\right)^2 \Big\{ 2\big[(\bar{\zeta}_1+\bar{\zeta}_2)^2 - \frac{q^2}{m_{\Omega_b}^2}(\bar{\zeta}_3+\bar{\zeta}_4+\bar{\zeta}_5)^2\big] \mathrm{Re}(C_{7\gamma}^{eff} C_{10}^*) \\ + \frac{q^2}{m_b\, m_{\Omega_b}} \big[(\bar{\zeta}_1+\bar{\zeta}_2)^2 - (\bar{\zeta}_3+\bar{\zeta}_4+\bar{\zeta}_5)^2\big] \mathrm{Re}(C_9^{eff} C_{10}^*) \Big\} \\ + O\big(\frac{\Lambda_{\rm QCD}}{m_b}\big) + O(\alpha_s). \tag{5.22}$$

An estimate from the quark model [36] indicates that the form factors $\bar{\zeta}_i$ $(i = 3,\, 4,\, 5\,)$ are negligible numerically. In this case, one can easily derive the zero-point of the forward-backward asymmetry

$$t_0(\Omega_b \to \Omega\, l^+ l^-) = -2\, m_b\, m_{\Omega_b}\, \frac{\mathrm{Re}(C_7^{eff} C_{10}^*)}{\mathrm{Re}(C_9^{eff} C_{10}^*)} + O\big(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\big) + O\big(\alpha_s\big)\,, \tag{5.23}$$

which is again free of hadronic uncertainties in leading power of $\Lambda_{\mathrm{QCD}}/m_b$ and in leading order of α_s. It is manifest that the expression of t_0 in $\Omega_b \to \Omega\, l^+ l^-$ is the same as the one for semileptonic $\Lambda_b \to \Lambda\, l^+ l^-$ decay shown in Eq. (5.15). Substituting the values of the Wilson coefficients yields $t_0(\Omega_b \to \Omega\, l^+ l^-) = 4.1\ \mathrm{GeV}^2$ numerically, where the uncertainties are not expected to be larger than $\Lambda_{\mathrm{QCD}}/m_b$ owing to the cancellation of nonperturbative effect in the forward-backward asymmetry. More dedicated work on $\Omega_b \to \Omega$ transition form factors from nonperturbative approaches based on QCD, such as Lattice QCD and QCD LCSR, is highly demanded to provide nontrivial tests of the predictions presented here.

6. Discussion

Weak decays of heavy baryons containing a bottom quark are among the topics of central interest in heavy flavor physics for many reasons. In contrast to the B-meson decays, an attractive peculiarity of these decays is that they allow the study of spin correlation providing an unique ground to extract the helicity structure of the flavor changing currents. Heavy-to-light baryon form factors embedding the long-distance hadronic dynamics are essential to describe semileptonic bottom baryon decays and also enter into the factorization formulae of nonleptonic bottom baryon decays. On account of a large amount of baryonic form factors in QCD, reduction of independent nonperturbative functions with the help of an effective theory can help to simplify complicated infrared dynamics in specific kinematical limit. One classical example is that in the small recoil limit two soft form factors are adequate to parameterize the hadronic matrix element responsible for $\Lambda_b \to \Lambda$ transition in HQET [2]. It was the aim of this work to explore the relations among heavy-to-light baryonic form factors in the opposite kinematical limit, the large recoil region.

We discuss the tensor representations for bottom baryons in the heavy-quark limit following [2, 17] and then work out the light-cone projectors for both baryon-octet and baryon-decuplet. With the tensor formalism introduced in [16], we show that only one form factor is essential to parameterize $\Lambda_b \to p$ and $\Lambda_b \to \Lambda$ matrix elements in the heavy quark limit and in the large energy limit of the light baryon; while $\Lambda_b \to \Sigma$ transition form factors should vanish in the same limit due to the violation of space-time parity symmetry. The scaling behavior of the $\Lambda_b \to p$ soft form factor is also derived with the nucleon distribution amplitudes in the leading conformal spin and the yielding scaling law $\xi(P' \cdot v) \sim (\Lambda_{\mathrm{QCD}}/m_Q)^3$ is exactly the same as that derived from QCD LCSR. We then observe that three form factors are needed to describe $\Omega_b \to \Xi$ decays and five form factors are required to parameterize $\Omega_b \to \Omega$ transition matrix element remembering that one additional form factor should be included in the small recoil region.

Applying the relations of form factors to FCNC transitions $\Lambda_b \to \Lambda\,\gamma$ and $\Lambda_b \to \Lambda\,l^+l^-$, we confirm that the polarization asymmetry in radiative $\Lambda_b \to \Lambda\,\gamma$ decay is free of hadronic uncertainties in the leading power of $1/m_b$ and in the leading order of α_s; the forward-backward asymmetry of $\Lambda_b \to \Lambda\,l^+l^-$ is the same as the one in $B \to K^*\,l^+l^-$ to the same accuracy. It is very interesting to notice that the radiative decay $\Omega_b \to \Omega\,\gamma$ is strongly enhanced by a factor of $m_{\Omega_b}^2/m_{\Omega}^2$ contributed from a helicity-1/2 Ω baryon and this channel would be among the most valuable probes on the chirality information of short-distance dipole transition. Forward-backward asymmetry of semileptonic $\Omega_b \to \Omega\,l^+l^-$ decay is generally dependent on the long-distance hadronic dynamics reflected in the transition form factors, however, this asymmetry will be only determined by the short-distance coefficients in an exact form of that for $\Lambda_b \to \Lambda\,l^+l^-$ channel provided that the form factors $\bar{\zeta}_i$ $(i = 3,\,4,\,5\,)$ are negligible as indicated from the quark model.

Note added: While completing this work we have been informed of related work by Thorsten Feldmann and Matthew W Y Yip [37], where they compute $\Lambda_b \to \Lambda$ transition form factors in the framework of SCET sum rules. These authors discuss similar issues and include also symmetry breaking effects due to the sub-leading currents in SCET for the relations of form factors discussed here.

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