

Semileptonic $\Lambda_{b,c}$ to Nucleon Transitions in Full QCD at Light Cone

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The tree level semileptonic $\Lambda_b \rightarrow p\nu$ and $\Lambda_c \rightarrow n\nu$ transitions are investigated using the light cone QCD sum rules approach in full theory. The spin-1/2, Λ_Q baryon with $Q = b$ or c , is considered by the most general form of its interpolating current. The time ordering product of the initial and transition currents is expanded in terms of the nucleon distribution amplitudes with different twists. Considering two sets of independent input parameters entering to the nucleon wave functions, namely, QCD sum rules and Lattice QCD parameters, the related form factors and their heavy quark effective theory limits are calculated and compared with the existing predictions of other approaches. It is shown that our results satisfy the heavy quark symmetry relations for lattice input parameters and b case exactly and the maximum violation is for charm case and QCD sum rules input parameters. The obtained form factors are used to compute the transition rates both in full theory and heavy quark effective theory. A comparison of the results on decay rate of $\Lambda_b \rightarrow p\nu$ with those predicted by other phenomenological methods or the same method in heavy quark effective theory with different interpolating current and distribution amplitudes of the Λ_b is also presented.

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I. INTRODUCTION

Motivated by the recent experimental progresses on the spectroscopy of the heavy baryons containing heavy b or c quark [1, 2, 3, 4, 5, 6, 7, 8], theoretical studies on these baryons gain pace. Because of the heavy quark, these states are expected to be narrow, experimentally, hence their isolation and detection are easy comparing with light systems. Theoretically, investigation of the semileptonic decays of the heavy baryons, whose experimental testing may be in the future program of the large hadron collider (LHC), have attracted interests beside their mass and electromagnetic properties. For instance, the semileptonic $\Lambda_b \rightarrow \Lambda_c$ and $\Lambda_c \rightarrow \Lambda$ decays have been investigated in three points QCD sum rules and heavy quark effective theory (HQET) in [9]. The $\Lambda_b \rightarrow p\bar{\nu}$ transition has also been studied in the same frameworks in [10] and using SU(3) symmetry and HQET in [11]. Constituent quark model have also been used to study the $\Lambda_c \rightarrow n\bar{\nu}$ and $\Lambda_b \rightarrow p\bar{\nu}$ form factors [12] and semileptonic decays of some heavy baryons containing single heavy quark in different quark models [12, 13, 14] are some other works in this respect.

In our recent work [15], we analyzed the semileptonic decay of Σ_b , which has different interpolating current and structure than Λ_Q , to proton in light cone QCD sum rules. In present study, we calculate the form factors related to the semileptonic decays of the $\Lambda_b \rightarrow p\nu$ and $\Lambda_c \rightarrow n\nu$ also in light cone QCD in full theory and HQET limit. In full theory, these transitions are governed by six form factors, but heavy quark effective theory limit reduces them to two. The vacuum to nucleon matrix element of the time ordering product is expanded in terms of nucleon distribution amplitudes (DA's) near light cone, $x^2 \simeq 0$. The nucleon wave functions contain eight independent parameters, which we consider two sets, namely, calculated using the QCD sum rules [16] and lattice QCD [17, 18, 19] approaches. In the calculations, the most general current of Λ_Q generalizing the Ioffe current is used. The obtained form factors are used to compute the corresponding transition rates both their numerical values and in terms of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. Studying such type of transition provides a better understanding of the internal structure of Λ_Q , information about the DA's and input parameters as well as determination of the CKM matrix elements. Note that, using different interpolating field, the semileptonic decay of bottom case, $\Lambda_b \rightarrow p\nu$, has already been investigated in references [20, 21] in the same framework but HQET limit. In [20], the nucleon distribution amplitudes are used only with QCD sum rules input parameters, while the distribution amplitudes of Λ_b have been utilized to calculate the form factors in [21].

The layout of the paper is as follows: in section II, the details of the calculation of the form factors in light cone QCD sum rules method are presented where the nucleon distribution amplitudes and the most general form of the

interpolating currents for the Λ_Q baryon are used. The heavy quark limit of the form factors and the relations between the form factors in this limit is also discussed in this section. Section III comprises numerical analysis of the form factors and our predictions for the decay rate obtained in two different ways: first, using the DA's obtained from QCD sum rules and second, the DA's calculated in lattice QCD. A comparison of our results on form factors and transition rates with the existing predictions of other approaches is also presented in this section.

II. THEORETICAL FRAMEWORK

In this section, following [15], we calculate the form factors of the $\Lambda_b \rightarrow p$ and $\Lambda_c \rightarrow n$ transitions in the framework of the light cone QCD sum rules and full theory. At quark level, these decays are governed by the tree level $Q \rightarrow q$ transition, where Q represents b (c) quark and q stands for u (d) quark for $\Lambda_b(\Lambda_c)$. The effective Hamiltonian responsible for these transitions at the quark level has the form

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{qQ} \bar{q} \gamma_\mu (1 - \gamma_5) Q \bar{l} \gamma^\mu (1 - \gamma_5) \nu. \quad (1)$$

To calculate the amplitude, we need to sandwich the above equation between the initial and final states and compute the matrix element $\langle N | \bar{q} \gamma_\mu (1 - \gamma_5) Q | \Lambda_Q \rangle$, which is needed to study the $\Lambda_Q \rightarrow N l \nu$ decay. The starting point is to consider the following correlation function

$$\Pi_\mu(p, q) = i \int d^4 x e^{iqx} \langle N(p) | T \{ J_\mu^{tr}(x) \bar{J}^{\Lambda_Q}(0) \} | 0 \rangle, \quad (2)$$

where, J^{Λ_Q} is interpolating currents of $\Lambda_{b(c)}$ baryon, $J_\mu^{tr} = \bar{q} \gamma_\mu (1 - \gamma_5) Q$ is transition current and $\langle N(p) |$ represents the nucleon state, where p denotes the proton (neutron) momentum and $q = (p + q) - p$ is the transferred momentum.

One further step of the calculation is the saturation of the correlation function by a tower of hadronic states having the same quantum numbers as the interpolating currents. The obtained result from this procedure is called the phenomenological or physical side of the correlation function. From the general philosophy of the QCD sum rules approach, this correlator is also calculated using the operator product expansion (OPE) in deep Euclidean region. This part is called the theoretical or QCD side. Match of these two different representations of the same correlation function gives sum rules for form factors. To suppress the contribution of the higher states and continuum, the Borel transformation is applied to both sides of the sum rules for physical quantities.

Let first calculate the phenomenological part. After the insertion of the complete set of the initial hadronic state and performing the integral over x , we obtain the physical side as:

$$\Pi_\mu(p, q) = \sum_s \frac{\langle N(p) | J_\mu^{tr}(x) | \Lambda_Q(p + q, s) \rangle \langle \Lambda_Q(p + q, s) | \bar{J}^{\Lambda_Q}(0) | 0 \rangle}{m_{\Lambda_Q}^2 - (p + q)^2} + \dots, \quad (3)$$

where, the ... represents the contribution of the higher states and continuum. The matrix element $\langle \Lambda_Q(p + q, s) | \bar{J}^{\Lambda_Q}(0) | 0 \rangle$ in (3) is given by:

$$\langle \Lambda_Q(p + q, s) | \bar{J}^{\Lambda_Q}(0) | 0 \rangle = \lambda_{\Lambda_Q} \bar{u}_{\Lambda_Q}(p + q, s), \quad (4)$$

where λ_{Λ_Q} is residue of Λ_Q baryon. The transition matrix element, $\langle N(p) | J_\mu^{tr} | \Lambda_Q(p + q, s) \rangle$ can be written as

$$\begin{aligned} \langle N(p) | J_\mu^{tr}(x) | \Lambda_Q(p + q) \rangle &= \bar{N}(p) \left[\gamma_\mu f_1(Q^2) + i \sigma_{\mu\nu} q^\nu f_2(Q^2) + q^\mu f_3(Q^2) + \gamma_\mu \gamma_5 g_1(Q^2) + i \sigma_{\mu\nu} \gamma_5 q^\nu g_2(Q^2) \right. \\ &\quad \left. + q^\mu \gamma_5 g_3(Q^2) \right] u_{\Lambda_Q}(p + q), \end{aligned} \quad (5)$$

where $Q^2 = -q^2$. The f_i and g_i are transition form factors in full theory and $N(p)$ and $u_{\Lambda_Q}(p + q)$ are the spinors of nucleon and Λ_Q , respectively. Using Eqs. (3), (4) and (5) and summing over spins of the Λ_Q baryon, i.e.,

$$\sum_s u_{\Lambda_Q}(p + q, s) \bar{u}_{\Lambda_Q}(p + q, s) = \not{p} + \not{q} + m_{\Lambda_Q}, \quad (6)$$

we attain the following expression

$$\begin{aligned} \Pi_\mu(p, q) = & \frac{\lambda_{\Lambda_Q}}{m_{\Lambda_Q}^2 - (p+q)^2} \bar{N}(p) \left[\gamma_\mu f_1(Q^2) + i\sigma_{\mu\nu} q^\nu f_2(Q^2 + q^\mu f_3(Q^2) + \gamma_\mu \gamma_5 g_1(Q^2) + i\sigma_{\mu\nu} \gamma_5 q^\nu g_2(Q^2) \right. \\ & \left. + q^\mu \gamma_5 g_3(Q^2) \right] (\not{p} + \not{q} + m_{\Lambda_Q}) + \dots \end{aligned} \quad (7)$$

Using

$$\bar{N} \sigma_{\mu\nu} q^\nu u_{\Lambda_Q} = i \bar{N} [(m_N + m_{\Lambda_Q}) \gamma_\mu - (2p + q)_\mu] u_{\Lambda_Q}, \quad (8)$$

in Eq. (7), the following final expression for the physical side of the correlation function is obtained:

$$\begin{aligned} \Pi_\lambda(p, q) = & \frac{\lambda_{\Lambda_Q}}{m_{\Lambda_Q}^2 - (p+q)^2} \bar{N}(p) \left[2f_1(Q^2) p_\mu + \left\{ -f_1(Q^2)(m_N - m_{\Lambda_Q}) + f_2(Q^2)(m_N^2 - m_{\Lambda_Q}^2) \right\} \gamma_\mu \right. \\ & + \left\{ f_1(Q^2) - f_2(Q^2)(m_N + m_{\Lambda_Q}) \right\} \gamma_\mu \not{q} + 2f_2(Q^2) p_\mu \not{q} + \left\{ f_2(Q^2) + f_3(Q^2) \right\} (m_N + m_{\Lambda_Q}) q_\mu \\ & + \left\{ f_2(Q^2) + f_3(Q^2) \right\} q_\mu \not{q} - 2g_1(Q^2) p_\mu \gamma_5 + \left\{ g_1(Q^2)(m_N + m_{\Lambda_Q}) - g_2(Q^2)(m_N^2 - m_{\Lambda_Q}^2) \right\} \gamma_\mu \gamma_5 - \\ & \left\{ g_1(Q^2) - g_2(Q^2)(m_N - m_{\Lambda_Q}) \right\} \gamma_\mu \not{q} \gamma_5 - 2g_2(Q^2) p_\mu \not{q} \gamma_5 - \left\{ g_2(Q^2) + g_3(Q^2) \right\} (m_N - m_{\Lambda_Q}) q_\mu \gamma_5 \\ & \left. - \left\{ g_2(Q^2) + g_3(Q^2) \right\} q_\mu \not{q} \gamma_5 \right] + \dots \end{aligned} \quad (9)$$

In order to calculate the form factors f_1, f_2, f_3, g_1, g_2 and g_3 , we will choose the independent structures $p_\mu, p_\mu \not{q}, q_\mu \not{q}, p_\mu \gamma_5, p_\mu \not{q} \gamma_5$, and $q_\mu \not{q} \gamma_5$ from Eq. (7), respectively.

For the theoretical side, to evaluate the correlation function in deep Euclidean region where $(p+q)^2 \ll 0$, the explicit expression of the interpolating field of the Λ_Q baryon is needed. Considering the quantum numbers, the most general form of interpolating current which can create the Λ_Q from the vacuum is given as

$$\begin{aligned} J^{\Lambda_Q}(x) = & \frac{1}{\sqrt{6}} \epsilon_{abc} \left\{ 2(q_1^{aT} C q_2^b) \gamma_5 Q^c + \beta (q_1^{aT} C \gamma_5 q_2^b) Q^c + (q_1^{aT} C Q^b) \gamma_5 q_2^c \right. \\ & \left. + \beta (q_1^{aT} C \gamma_5 Q^b) q_2^c + (Q^{aT} C q_2^b) \gamma_5 q_1^c + \beta (Q^{aT} C \gamma_5 q_2^b) q_1^c \right\}, \end{aligned} \quad (10)$$

where q_1 and q_2 are the u and d quarks, respectively, a, b, c are the color indices and C is the charge conjugation operator and β is an arbitrary parameter with $\beta = -1$ corresponding to the Ioffe current. Using the transition current, $J_\mu^{tr} = \bar{q} \gamma_\mu (1 - \gamma_5) Q$ and J^{Λ_Q} and contracting out all quark pairs by the help of the Wick's theorem, we achieve

$$\begin{aligned} \Pi_\mu = & \frac{-i}{\sqrt{6}} \epsilon^{abc} \int d^4x e^{iqx} \left\{ \left[2(C)_{\eta\phi} (\gamma_5)_{\rho\beta} + (C)_{\eta\beta} (I)_{\rho\delta} (\gamma_5)_{\delta\phi} + (C)_{\beta\phi} (\gamma_5)_{\eta\rho} \right] + \beta \left[2(C\gamma_5)_{\eta\phi} (I)_{\rho\beta} \right. \right. \\ & \left. \left. + (C\gamma_5)_{\eta\beta} (I)_{\rho\phi} + (C\gamma_5)_{\beta\phi} (I)_{\eta\rho} \right] \right\} \left[(1 + \gamma_5) \gamma_\mu \right]_{\sigma\theta} S_Q(-x)_{\beta\sigma} \langle N(p) | \bar{u}_\eta^a(0) \bar{u}_\theta^b(x) \bar{d}_\phi^c(0) | 0 \rangle, \end{aligned} \quad (11)$$

where, $S_Q(x)$ is the heavy quark propagator which is given by [22]:

$$S_Q(x) = S_Q^{free}(x) - i g_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \int_0^1 dv \left[\frac{k + m_Q}{(m_Q^2 - k^2)^2} G^{\mu\nu}(vx) \sigma_{\mu\nu} + \frac{1}{m_Q^2 - k^2} vx_\mu G^{\mu\nu} \gamma_\nu \right]. \quad (12)$$

where

$$S_Q^{free} = \frac{m_Q^2}{4\pi^2} \frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} - i \frac{m_Q^2 \not{x}}{4\pi^2 x^2} K_2(m_Q \sqrt{-x^2}), \quad (13)$$

and K_i are the Bessel functions. Here, we neglect the terms proportional to the gluon field strength tensor since they can give contribution to four and five particle distribution functions and expected to be small [23, 24, 25].

The matrix element $\langle N(p) | \epsilon^{abc} \bar{u}_\eta^a(0) \bar{u}_\theta^b(x) \bar{d}_\phi^c(0) | 0 \rangle$ appearing in Eq. (11), which is the nucleon wave function, is represented as [16, 23, 24, 25, 26]:

$$\begin{aligned}
& 4 \langle 0 | \epsilon^{abc} u_\alpha^a(a_1 x) u_\beta^b(a_2 x) d_\gamma^c(a_3 x) | N(p) \rangle \\
&= \mathcal{S}_1 m_N C_{\alpha\beta}(\gamma_5 N)_\gamma + \mathcal{S}_2 m_N^2 C_{\alpha\beta}(\not{x} \gamma_5 N)_\gamma \\
&+ \mathcal{P}_1 m_N (\gamma_5 C)_{\alpha\beta} N_\gamma + \mathcal{P}_2 m_N^2 (\gamma_5 C)_{\alpha\beta} (\not{x} N)_\gamma + (\mathcal{V}_1 + \frac{x^2 m_N^2}{4} \mathcal{V}_1^M) (\not{p} C)_{\alpha\beta} (\gamma_5 N)_\gamma \\
&+ \mathcal{V}_2 m_N (\not{p} C)_{\alpha\beta} (\not{x} \gamma_5 N)_\gamma + \mathcal{V}_3 m_N (\gamma_\mu C)_{\alpha\beta} (\gamma^\mu \gamma_5 N)_\gamma + \mathcal{V}_4 m_N^2 (\not{x} C)_{\alpha\beta} (\gamma_5 N)_\gamma \\
&+ \mathcal{V}_5 m_N^2 (\gamma_\mu C)_{\alpha\beta} (i \sigma^{\mu\nu} x_\nu \gamma_5 N)_\gamma + \mathcal{V}_6 m_N^3 (\not{x} C)_{\alpha\beta} (\not{x} \gamma_5 N)_\gamma + (\mathcal{A}_1 \\
&+ \frac{x^2 m_N^2}{4} \mathcal{A}_1^M) (\not{p} \gamma_5 C)_{\alpha\beta} N_\gamma + \mathcal{A}_2 m_N (\not{p} \gamma_5 C)_{\alpha\beta} (\not{x} N)_\gamma + \mathcal{A}_3 m_N (\gamma_\mu \gamma_5 C)_{\alpha\beta} (\gamma^\mu N)_\gamma \\
&+ \mathcal{A}_4 m_N^2 (\not{x} \gamma_5 C)_{\alpha\beta} N_\gamma + \mathcal{A}_5 m_N^2 (\gamma_\mu \gamma_5 C)_{\alpha\beta} (i \sigma^{\mu\nu} x_\nu N)_\gamma + \mathcal{A}_6 m_N^3 (\not{x} \gamma_5 C)_{\alpha\beta} (\not{x} N)_\gamma \\
&+ (\mathcal{T}_1 + \frac{x^2 m_N^2}{4} \mathcal{T}_1^M) (p^\nu i \sigma_{\mu\nu} C)_{\alpha\beta} (\gamma^\mu \gamma_5 N)_\gamma + \mathcal{T}_2 m_N (x^\mu p^\nu i \sigma_{\mu\nu} C)_{\alpha\beta} (\gamma_5 N)_\gamma \\
&+ \mathcal{T}_3 m_N (\sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\nu} \gamma_5 N)_\gamma + \mathcal{T}_4 m_N (p^\nu \sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\rho} x_\rho \gamma_5 N)_\gamma \\
&+ \mathcal{T}_5 m_N^2 (x^\nu i \sigma_{\mu\nu} C)_{\alpha\beta} (\gamma^\mu \gamma_5 N)_\gamma + \mathcal{T}_6 m_N^2 (x^\mu p^\nu i \sigma_{\mu\nu} C)_{\alpha\beta} (\not{x} \gamma_5 N)_\gamma \\
&+ \mathcal{T}_7 m_N^2 (\sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\nu} \not{x} \gamma_5 N)_\gamma + \mathcal{T}_8 m_N^3 (x^\nu \sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\rho} x_\rho \gamma_5 N)_\gamma, \tag{14}
\end{aligned}$$

where, the calligraphic objects which have not definite twists are functions of the scalar product px and the parameters a_i , $i = 1, 2, 3$ and they are presented in terms of the nucleon distribution amplitudes (DA's) with definite and increasing twists. The scalar, pseudo-scalar, vector, axial vector and tensor DA's are explicitly shown in Tables I, II, III, IV and V, respectively.

$\mathcal{S}_1 = S_1$
$2px\mathcal{S}_2 = S_1 - S_2$

TABLE I: Relations between the calligraphic functions and nucleon scalar DA's.

$\mathcal{P}_1 = P_1$
$2px\mathcal{P}_2 = P_1 - P_2$

TABLE II: Relations between the calligraphic functions and nucleon pseudo-scalar DA's.

$\mathcal{V}_1 = V_1$
$2px\mathcal{V}_2 = V_1 - V_2 - V_3$
$2\mathcal{V}_3 = V_3$
$4px\mathcal{V}_4 = -2V_1 + V_3 + V_4 + 2V_5$
$4px\mathcal{V}_5 = V_4 - V_3$
$4(px)^2\mathcal{V}_6 = -V_1 + V_2 + V_3 + V_4 + V_5 - V_6$

TABLE III: Relations between the calligraphic functions and nucleon vector DA's.

The distribution amplitudes $F(a_i px) = S_i, P_i, V_i, A_i, T_i$ can be written as:

$$F(a_i px) = \int dx_1 dx_2 dx_3 \delta(x_1 + x_2 + x_3 - 1) e^{-ipx \Sigma_i x_i a_i} F(x_i). \tag{15}$$

where, x_i with $i = 1, 2, 3$ corresponds to the longitudinal momentum fractions carried by the quarks.

$\mathcal{A}_1 = A_1$
$2px\mathcal{A}_2 = -A_1 + A_2 - A_3$
$2\mathcal{A}_3 = A_3$
$4px\mathcal{A}_4 = -2A_1 - A_3 - A_4 + 2A_5$
$4px\mathcal{A}_5 = A_3 - A_4$
$4(px)^2\mathcal{A}_6 = A_1 - A_2 + A_3 + A_4 - A_5 + A_6$

TABLE IV: Relations between the calligraphic functions and nucleon axial vector DA's.

$T_1 = T_1$
$2pxT_2 = T_1 + T_2 - 2T_3$
$2T_3 = T_7$
$2pxT_4 = T_1 - T_2 - 2T_7$
$2pxT_5 = -T_1 + T_5 + 2T_8$
$4(px)^2T_6 = 2T_2 - 2T_3 - 2T_4 + 2T_5 + 2T_7 + 2T_8$
$4pxT_7 = T_7 - T_8$
$4(px)^2T_8 = -T_1 + T_2 + T_5 - T_6 + 2T_7 + 2T_8$

TABLE V: Relations between the calligraphic functions and nucleon tensor DA's.

In order to obtain the QCD or theoretical representation of the correlation function, the heavy quark propagator and nucleon distribution amplitudes are used in Eq. (11). Performing integral over x , equating the corresponding structures from both representations of the correlation function through the dispersion relations and applying Borel transformation with respect to $(p+q)^2$ to suppress the contribution of the higher states and continuum, one can obtain sum rules for the form factors f_1, f_2, f_3, g_1, g_2 and g_3 .

By means of the heavy quark effective theory (HQET), the number of independent form factors is reduced to two, F_1 and F_2 . Hence, the transition matrix element can be parameterized in terms of these two form factors as [27, 28]:

$$\langle N(p) | \bar{u}\Gamma b | \Lambda_Q(p+q) \rangle = \bar{N}(p)[F_1(Q^2) + \not{p}F_2(Q^2)]\Gamma u_{\Lambda_Q}(p+q), \quad (16)$$

where, Γ is any Dirac matrices and $\not{p} = \frac{p\!\!\!/}{m_{\Lambda_Q}}$. One can immediately obtain the following relations among the form factors in HQET limit comparing the Eq. (16) with the general definition of the form factors in Eq. (5) (see also [29, 30])

$$\begin{aligned} g_1 &= f_1 = F_1 + \frac{m_N}{m_{\Sigma_b}} F_2 \\ g_2 &= f_2 = g_3 = f_3 = \frac{F_2}{m_{\Sigma_b}} \end{aligned} \quad (17)$$

Considering the above relations, instead of giving the explicit expressions of the sum rules for the all form factors which are very lengthy, we will present only the expressions for f_1 and f_2 in the Appendix-A. However, we will give the extrapolation of all form factors in finite mass in terms of q^2 in the numerical analysis section.

In the following, some remarks about how the HQET limit of the form factors satisfy the above relations are in order. In HQET, all the ratios, $\frac{f_1}{g_1}, \frac{f_2}{g_2}, \frac{f_3}{g_3}, \frac{f_2}{g_3}, \frac{f_3}{g_2}, \frac{f_2}{f_3}$ and $\frac{g_2}{g_3}$ should be equal to one. The deviation of those ratios from unity are presented in Tables VI and VII for $\Lambda_b \rightarrow p\ell\nu$ and $\Lambda_c \rightarrow n\ell\nu$, respectively. The bottom case and Lattice QCD input parameters satisfies the HQET relations exactly, while the maximum violation of this symmetry is related to the charm case and QCD input parameters. When we consider all relations, we see that the violations for charm case is larger than that of the bottom one.

The explicit expressions of the sum rules for form factors reveals that to get the numerical values of the form factors, the expression for residue λ_{Λ_Q} is needed. This residue has been calculated in [31] using two-point QCD sum rules method:

$$-\lambda_{\Lambda_Q}^2 e^{-m_{\Lambda_Q}^2/M_B^2} = \int_{m_Q^2}^{s_0} e^{\frac{-s}{M_B^2}} \rho(s) ds + e^{\frac{-m_Q^2}{M_B^2}} \Gamma, \quad (18)$$

	HQET	
	QCD sum rules input parameters	Lattice QCD input parameters
$\frac{f_1}{g_1}$	0	0
$\frac{f_2}{g_2}$	20 %	0
$\frac{f_3}{g_3}$	20 %	0
$\frac{f_2}{g_3}$	20 %	0
$\frac{f_3}{g_2}$	20 %	0
$\frac{f_2}{f_3}$	0	0
$\frac{g_2}{g_3}$	0	0

TABLE VI: Deviation of the ratio of the form factors from unity (violation of HQET symmetry relations) for $\Lambda_b \rightarrow p \ell \nu$.

	HQET	
	QCD sum rules input parameters	Lattice QCD input parameters
$\frac{f_1}{g_1}$	0	0
$\frac{f_2}{g_2}$	45 %	40 %
$\frac{f_3}{g_3}$	45 %	40 %
$\frac{f_2}{g_3}$	45 %	35 %
$\frac{f_3}{g_2}$	40 %	35 %
$\frac{f_2}{f_3}$	0	0
$\frac{g_2}{g_3}$	0	0

TABLE VII: Deviation of the ratio of the form factors from unity (violation of HQET symmetry relations) for $\Lambda_c \rightarrow n \ell \nu$.

with

$$\begin{aligned}
\rho(s) = & (< \bar{d}d > + < \bar{u}u >) \frac{(\beta - 1)}{192\pi^2} \left\{ \frac{m_0^2}{4m_Q} [6(1 + \beta)\psi_{00} - (7 + 11\beta)\psi_{02} \right. \\
& - 6(1 + \beta)\psi_{11}] + (1 + 5\beta)m_Q(2\psi_{10} - \psi_{11} - \psi_{12} + 2\psi_{21}) \Big\} \\
& + \frac{m_Q^4}{2048\pi^4} [5 + \beta(2 + 5\beta)][12\psi_{10} - 6\psi_{20} + 2\psi_{30} - 4\psi_{41} + \psi_{42} - 12\ln(\frac{s}{m_Q^2})],
\end{aligned} \tag{19}$$

$$\Gamma = \frac{(\beta - 1)}{72} < \bar{d}d > < \bar{u}u > \left[\frac{m_Q^2 m_0^2}{2M_B^4} (13 + 11\beta) + \frac{m_0^2}{4M_B^2} (25 + 23\beta) - (13 + 11\beta) \right]. \tag{20}$$

where, s_0 is continuum threshold, M_B^2 is the Borel mass parameter and $\psi_{nm} = \frac{(s - m_Q^2)^n}{s^m (m_Q^2)^{n-m}}$ are some dimensionless functions.

	QCD sum rules [16]	Lattice QCD [17, 18, 19]
f_N	$(5.0 \pm 0.5) \times 10^{-3} \text{ GeV}^2$	$(3.234 \pm 0.063 \pm 0.086) \times 10^{-3} \text{ GeV}^2$
λ_1	$-(2.7 \pm 0.9) \times 10^{-2} \text{ GeV}^2$	$(-3.557 \pm 0.065 \pm 0.136) \times 10^{-2} \text{ GeV}^2$
λ_2	$(5.4 \pm 1.9) \times 10^{-2} \text{ GeV}^2$	$(7.002 \pm 0.128 \pm 0.268) \times 10^{-2} \text{ GeV}^2$
V_1^d	0.23 ± 0.03	$0.3015 \pm 0.0032 \pm 0.0106$
A_1^u	0.38 ± 0.15	$0.1013 \pm 0.0081 \pm 0.0298$
f_1^d	0.40 ± 0.05	—
f_1^u	0.07 ± 0.05	—
f_2^d	0.22 ± 0.05	—

TABLE VIII: The values of independent parameters entering to the nucleon DA's. The first errors in lattice values are statistical and the second errors represent the uncertainty due to the Chiral extrapolation and renormalization.

III. NUMERICAL RESULTS

The numerical analysis of the form factors and total decay rate for $\Lambda_{b(c)} \rightarrow p(n)\ell\nu$ transition are presented in this section. Some input parameters used in the analysis of the sum rules for the form factors are $\langle\bar{u}u\rangle(1 \text{ GeV}) = \langle\bar{d}d\rangle(1 \text{ GeV}) = -(0.243)^3 \text{ GeV}^3$, $m_n = 0.939 \text{ GeV}$, $m_p = 0.938 \text{ GeV}$, $m_b = 4.7 \text{ GeV}$, $m_c = 1.23 \text{ GeV}$, $m_{\Lambda_b} = 5.620 \text{ GeV}$, $m_{\Lambda_c} = 2.286 \text{ GeV}$ and $m_0^2(1 \text{ GeV}) = (0.8 \pm 0.2) \text{ GeV}^2$ [32]. The main inputs which are the nucleon DA's can be found in [16]. These DA's contain eight independent parameters f_N , λ_1 , λ_2 , V_1^d , A_1^u , f_1^d , f_1^u and f_2^d . These parameters have been calculated in the light cone QCD sum rules [16] and also most of these parameters have been computed in the framework of the lattice QCD [17, 18, 19]. For those parameters which have not calculated in lattice, the data from QCD input parameters will be used. These parameters are given in Table VIII.

Three auxiliary parameters are encountered to the expression of the sum rules for form factors, continuum threshold s_0 , Borel mass parameter M_B^2 and general parameter β entering to the most general form of the interpolating current for Λ_Q baryon. A working region should be determined for these auxiliary and mathematical parameters such that the form factors as physical quantities should be independent of them. The continuum threshold, s_0 is not completely arbitrary and it is related to the energy of the excited states. From our results, we observed that the form factors are weakly dependent on s_0 in the interval, $(m_{\Lambda_Q} + 0.5)^2 \leq s_0 \leq (m_{\Lambda_Q} + 0.7)^2$. To determine the working region for β , we look at the variation of the form factors with respect to $\cos\theta$ in the interval $-1 \leq \cos\theta \leq 1$ which corresponds to $-\infty \leq \beta \leq \infty$, where $\beta = \tan\theta$. As a result, we attain a region at which the dependency is weak. The working region for β is obtained to be $-0.75 \leq \cos\theta \leq 0.25$ for Λ_b and $-0.25 \leq \cos\theta \leq 0.25$ for Λ_c . The Ioffe current which corresponds to $\cos\theta = -0.71$ is inside the working region for Λ_b but out of the region for Λ_c .

For further analysis, the upper and lower limits of M_B^2 should be determined. To do that, we apply two conditions: The first one, which gives the upper limit, is that the series of the light cone expansion with increasing twist should be convergent, and the second one, which determine the lower limit, is that the contribution of higher states and continuum to the correlation function should be enough small i.e., the contribution of the highest term with power $1/M_B^2$ is less than, say, 20–25% of the highest power of M_B^2 . In the present work, both conditions are satisfied in the region $15 \text{ GeV}^2 \leq M_B^2 \leq 30 \text{ GeV}^2$ for Λ_b and $4 \text{ GeV}^2 \leq M_B^2 \leq 12 \text{ GeV}^2$ for Λ_c , which we will use in numerical analysis. Taking into account the above requirements, we obtained that the form factors obey the following extrapolations in terms of q^2 :

$$f_i(q^2)[g_i(q^2)] = \frac{a}{(1 - \frac{q^2}{m_{fit}^2})} + \frac{b}{(1 - \frac{q^2}{m_{fit}^2})^2}, \quad (21)$$

The values of the parameters a , b and m_{fit} for form factors and their HQET limit are given in Tables IX, X, XI and XII related to the QCD sum rules and lattice QCD input parameters. Because of the working near the light cone, $x^2 \simeq 0$ and concerning the considered correlation function, the results are not reliable at low q^2 , hence to make the extension of our predictions to full physical region, we need to the above parameterization. From those Tables, we see that the pole of the form factors exist outside the physical region and the form factors are analytic in the whole physical interval. The values of form factors at $q^2 = 0$ obtained from fit functions are shown in Tables XIII and XIV for $\Lambda_b \rightarrow p\ell\nu$ and $\Lambda_c \rightarrow n\ell\nu$, respectively. A comparison of the existing predictions from other approaches is also presented for bottom case. The Table XIII depicts a good consistency on our result for $f_1(0)$ HQET limit obtained from lattice QCD input parameters with the prediction of [21], however the $f_1(0)$ HQET limit obtained from

QCD sum rules parameters is almost four times larger than that of [21] prediction. On the other hand, the similar comparison of our result on form factor $f_2(0)$ at HQET and prediction of [21] shows that the value presented in [21] is almost two times greater than our result obtained from Lattice QCD input parameters and 1.5 times smaller than our result obtained from QCD input parameters.

	QCD sum rules			Lattice QCD		
	a	b	m_{fit}	a	b	m_{fit}
f_1	0.025	0.052	4.91	0.048	0.016	4.89
f_2	0.007	-0.050	4.92	-0.003	-0.006	4.92
f_3	0.052	-0.13	4.99	0.028	-0.063	4.96
g_1	-0.059	0.13	5.29	-0.17	0.32	5.32
g_2	0.011	-0.050	5.20	0.019	-0.040	5.40
g_3	-0.009	-0.017	4.90	-0.015	0.012	4.98

TABLE IX: Parameters appearing in fit function of the original form factors for $\Lambda_b \rightarrow p\ell\nu$.

	QCD sum rules			Lattice QCD		
	a	b	m_{fit}	a	b	m_{fit}
f_1	-0.034	0.20	1.59	-0.14	0.64	1.55
f_2	-0.015	-0.77	1.57	0.018	-0.32	1.60
f_3	-0.062	-1.23	1.48	0.12	-1.09	1.56
g_1	-0.015	0.54	1.53	-0.20	0.71	1.59
g_2	-0.11	-0.20	1.52	-0.034	-0.14	1.65
g_3	-0.088	0.085	1.48	0.009	-0.41	1.50

TABLE X: Parameters appearing in fit function of the original form factors for $\Lambda_c \rightarrow n\ell\nu$.

	QCD sum rules			Lattice QCD		
	a	b	m_{fit}	a	b	m_{fit}
f_1	0.041	0.040	4.82	0.0042	0.016	4.92
f_2	0.033	-0.097	4.83	0.013	-0.030	5.92
f_3	0.060	-0.14	4.90	0.016	-0.040	4.94
g_1	-0.0012	0.096	5.10	-0.0022	0.029	5.30
g_2	-0.0094	-0.018	5.36	0.0017	-0.0043	5.36
g_3	-0.040	0.025	4.95	-0.018	0.015	4.98

TABLE XI: Parameters appearing in the fit function of the form factors at HQET limit for $\Lambda_b \rightarrow p\ell\nu$.

In the next step, we calculate the total decay rate of $\Lambda_Q \rightarrow N\ell\nu$ transition in the whole physical region, i.e., $m_l^2 \leq q^2 \leq (m_{\Lambda_Q} - m_N)^2$. The decay width for such transition is given by the following expression [33, 34]

$$\Gamma(\Lambda_Q \rightarrow N\ell\nu) = \frac{G_F^2}{384\pi^3 m_{\Lambda_Q}^3} |V_{qQ}|^2 \int_{m_l^2}^{\Delta^2} dq^2 (1 - m_l^2/q^2)^2 \sqrt{(\Sigma^2 - q^2)(\Delta^2 - q^2)} N(q^2) \quad (22)$$

	QCD sum rules			Lattice QCD		
	a	b	m_{fit}	a	b	m_{fit}
f_1	-0.066	1.14	1.51	-0.039	0.37	1.55
f_2	0.046	-1.14	1.53	0.047	-0.63	1.48
f_3	0.071	-1.33	1.50	0.039	-0.52	1.53
g_1	-0.10	1.21	1.57	-0.039	0.39	1.55
g_2	-0.070	-0.11	1.56	-0.027	-0.046	1.60
g_3	-0.076	-0.91	1.54	-0.034	-0.032	1.54

TABLE XII: Parameters appearing in the fit function of the form factors at HQET limit for $\Lambda_c \rightarrow n\ell\nu$.

	Original		HQET		
	QCD sum rules	Lattice QCD	QCD sum rules	Lattice QCD	[21]
$f_1(0)$	0.077	0.064	0.081	0.021	$0.023^{+0.006}_{-0.005}$
$f_2(0)$	-0.044	-0.013	-0.064	-0.018	$-0.039^{+0.006}_{-0.009}$
$f_3(0)$	-0.079	-0.036	—	—	
$g_1(0)$	0.073	0.15	—	—	
$g_2(0)$	-0.039	-0.021	—	—	
$g_3(0)$	-0.026	-0.0035	—	—	

TABLE XIII: The values of the form factors at $q^2 = 0$ for $\Lambda_b \rightarrow p\ell\nu$.

where

$$\begin{aligned}
N(q^2) = & F_1^2(q^2)(\Delta^2(4q^2 - m_l^2) + 2\Sigma^2\Delta^2(1 + 2m_l^2/q^2) - (\Sigma^2 + 2q^2)(2q^2 + m_l^2)) \\
& + F_2^2(q^2)(\Delta^2 - q^2)(2\Sigma^2 + q^2)(2q^2 + m_l^2)/m_{\Sigma_b}^2 + 3F_3^2(q^2)m_l^2(\Sigma^2 - q^2)q^2/m_{\Sigma_b}^2 \\
& + 6F_1(q^2)F_2(q^2)(\Delta^2 - q^2)(2q^2 + m_l^2)\Sigma/m_{\Sigma_b} - 6F_1(q^2)F_3(q^2)m_l^2(\Sigma^2 - q^2)\Delta/m_{\Sigma_b} \\
& + G_1^2(q^2)(\Sigma^2(4q^2 - m_l^2) + 2\Sigma^2\Delta^2(1 + 2m_l^2/q^2) - (\Delta^2 + 2q^2)(2q^2 + m_l^2)) \\
& + G_2^2(q^2)(\Sigma^2 - q^2)(2\Delta^2 + q^2)(2q^2 + m_l^2)/m_{\Sigma_b}^2 + 3G_3^2(q^2)m_l^2(\Delta^2 - q^2)q^2/m_{\Sigma_b}^2 \\
& - 6G_1(q^2)G_2(q^2)(\Sigma^2 - q^2)(2q^2 + m_l^2)\Delta/m_{\Sigma_b} + 6G_1(q^2)G_3(q^2)m_l^2(\Delta^2 - q^2)\Sigma/m_{\Sigma_b}.
\end{aligned} \tag{23}$$

Here, $F_1(q^2) = f_1(q^2)$, $F_2(q^2) = m_{\Lambda_Q} f_2(q^2)$, $F_3(q^2) = m_{\Lambda_Q} f_3(q^2)$, $G_1(q^2) = g_1(q^2)$, $G_2(q^2) = m_{\Lambda_Q} g_2(q^2)$, $G_3(q^2) = m_{\Lambda_Q} g_3(q^2)$, $\Sigma = m_{\Lambda_Q} + m_N$ and $\Delta = m_{\Lambda_Q} - m_N$. $G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi coupling constant, and m_l is the leptonic (electron, muon or tau) mass. For the corresponding CKM matrix element $V_{ub} = (4.31 \pm 0.30) 10^{-3}$ and $V_{cd} = (0.230 \pm 0.011)$ are used [35].

Our final results for total decay rates are given in Table XV. As it can be seen from this Table, our results for e and μ and Λ_b cases are consistent for two sets of input parameters when the original form factors are used especially, when we consider the uncertainties. However, QCD input parameters result is 1.5 times greater than that of the lattice input parameter for the decay rates of τ and bottom case. If we consider Λ_c , QCD sum rules input parameters gives the result 2 times greater than the Lattice QCD input parameters. On the other hand, when we consider the uncertainties, results obtained using both sets of input parameters and original form factors coincide for all leptons. At HQET limit and QCD sum rules input parameters, our predictions for the decay rates are in the same order of magnitude with the original form factors and two sets for all leptons and both charm and bottom cases. In contrast,

	Original		HQET	
	QCD sum rules	Lattice QCD	QCD sum rules	Lattice QCD
$f_1(0)$	0.17	0.50	1.078	0.33
$f_2(0)$	-0.78	-0.31	-1.09	-0.58
$f_3(0)$	1.29	0.98	-	-
$g_1(0)$	0.52	0.51	-	-
$g_2(0)$	-0.31	-0.18	-	-
$g_3(0)$	-0.0032	-0.31	-	-

TABLE XIV: The values of the form factors at $q^2 = 0$ for $\Lambda_c \rightarrow n\ell\nu$.

	$\Lambda_b \rightarrow p\mu\nu_\mu$	$\Lambda_b \rightarrow p e \nu_e$	$\Lambda_b \rightarrow p\tau\nu_\tau$	$\Lambda_c \rightarrow n\mu\nu_\mu$	$\Lambda_c \rightarrow n e \nu_e$	$\Lambda_c \rightarrow n\tau\nu_\tau$
For QCD sum rules inputs	$(3.07 \pm 1.05) \times 10^{-15}$	$(3.065 \pm 1.05) \times 10^{-15}$	$(3.82 \pm 1.35) \times 10^{-15}$	$(2.89 \pm 0.95) \times 10^{-13}$	$(2.86 \pm 0.95) \times 10^{-13}$	-
For lattice QCD inputs	$(2.87 \pm 0.95) \times 10^{-15}$	$(2.87 \pm 0.95) \times 10^{-15}$	$(2.55 \pm 0.85) \times 10^{-15}$	$(1.35 \pm 0.45) \times 10^{-13}$	$(1.33 \pm 0.43) \times 10^{-13}$	-
HQET limit for QCD sum rules inputs	$(5.84 \pm 1.81) \times 10^{-15}$	$(5.83 \pm 1.81) \times 10^{-15}$	$(7.90 \pm 2.45) \times 10^{-15}$	$(5.08 \pm 1.65) \times 10^{-13}$	$(5.01 \pm 1.60) \times 10^{-13}$	-
HQET limit for lattice QCD inputs	$(4.70 \pm 1.60) \times 10^{-17}$	$(4.60 \pm 1.55) \times 10^{-17}$	$(2.36 \pm 0.85) \times 10^{-16}$	$(8.75 \pm 2.85) \times 10^{-14}$	$(8.74 \pm 2.83) \times 10^{-14}$	-

TABLE XV: Values of the $\Gamma(\Lambda_Q \rightarrow N\ell\nu)$ in GeV for different leptons and two sets of input parameters obtained from QCD sum rules and lattice QCD and also their HQET limit.

the results at HQET limit and lattice parameters are two orders of magnitude less than HQET limit and sum rules inputs as well as original form factors for bottom and e and μ cases. For τ and bottom, and e and μ and charm cases, this difference is approximately one order of magnitude. We also compare our results on decay rates in units of $|V_{qQ}|^2 s^{-1}$ with the predictions of references [9, 10, 11, 12, 20, 21] in Table XVI. From this Table, it is clear that our results for bottom case, lattice parameters and HQET limit are in the same order of magnitude with the predictions of [11, 12] and HQET-[20]. For all other cases the difference between our results with the existing predictions of the other approaches presented in Table XVI is one-two order of magnitudes. In Table XVI, HOSR refers to harmonic oscillator semi relativistic and HONR stands for harmonic oscillator non relativistic constituent quark models.

To summarize, using the most general form of the interpolating currents of Λ_Q and nucleon DA's with two sets of input parameters, namely QCD sum rules and lattice QCD inputs, the transition form factors of the semileptonic $\Lambda_Q \rightarrow N\ell\nu$ have been calculated in the framework of the light cone QCD sum rules in full theory and HQET. The lattice input parameters satisfy the HQET relations exactly for bottom case, while the maximum violation is for charm case and QCD input parameters. The results of the form factors at HQET and $q^2 = 0$ have been compared with the existing predictions of the other approaches. These transition form factors have been used to estimate the corresponding tree level semileptonic decay rates both in full theory and HQET limit. A comparison of the obtained results and the existing predictions of the other approaches which all are at HQET limit, was also presented. The best consistency between our results and those predictions is related to the bottom case and lattice QCD input parameters at HQET. Our results can be checked in experiments hold in future such as LHC. Comparison between the experimental data and our results could give essential information about the nature of the Λ_Q , Nucleon distribution amplitudes as well as determination of the CKM matrix elements, V_{ub} and V_{cd} .

	$\Lambda_b \longrightarrow p\mu\nu\mu$	$\Lambda_b \longrightarrow p e \nu e$	$\Lambda_b \longrightarrow p \tau \nu \tau$	$\Lambda_c \longrightarrow n\mu\nu\mu$	$\Lambda_c \longrightarrow n e \nu e$	$\Lambda_c \longrightarrow n \tau \nu \tau$
For QCD sum rules	$(2.5 \pm 0.85) \times 10^{14}$	$(2.5 \pm 0.85) \times 10^{14}$	$(3.12 \pm 1.05) \times 10^{14}$	$(8.3 \pm 2.85) \times 10^{12}$	$(8.21 \pm 2.80) \times 10^{12}$	
For lattice QCD	$(2.35 \pm 0.85) \times 10^{14}$	$(2.35 \pm 0.85) \times 10^{14}$	$(2.08 \pm 0.70) \times 10^{14}$	$(3.88 \pm 1.25) \times 10^{12}$	$(3.82 \pm 1.20) \times 10^{12}$	
HQET limit for QCD sum rules	$(4.78 \pm 1.75) \times 10^{14}$	$(4.77 \pm 1.75) \times 10^{14}$	$(6.46 \pm 2.15) \times 10^{14}$	$(1.46 \pm 0.55) \times 10^{13}$	$(1.44 \pm 0.55) \times 10^{13}$	
HQET limit for lattice QCD	$(3.84 \pm 1.25) \times 10^{12}$	$(3.76 \pm 1.20) \times 10^{12}$	$(1.93 \pm 0.70) \times 10^{12}$	$(2.51 \pm 0.85) \times 10^{12}$	$(2.51 \pm 0.85) \times 10^{12}$	
[10]	2.05×10^{13}					
[9]	2.58×10^{13}					
[11]	6.48×10^{12}					
QCD sum rules[20]	3.65×10^{13}					
HQET[20]	5.62×10^{12}					
[12]	4.55×10^{12} (HONR)		4.01×10^{12} (HONR)	1.02×10^{10} (HONR)		
	7.55×10^{12} (HOSR)		6.55×10^{12} (HOSR)	1.35×10^{10} (HOSR)		

TABLE XVI: Values of the total decay rate (in $|V_{qQ}|^2 s^{-1}$) of the $\Lambda_Q \longrightarrow N\ell\nu$ transition for different leptons and two sets of input parameters obtained from QCD sum rules and lattice QCD and also their HQET limit compared to the [9, 10, 11, 12, 20, 21].

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Appendix A

In this Appendix, the explicit expressions for the form factors f_1 and f_2 are given:

$$\begin{aligned}
f_1(Q^2) = & \frac{1}{2\sqrt{\lambda_{\Lambda_b}}} e^{m_{\Lambda_b}^2/M_B^2} \left\{ \int_{t_0}^1 dx_2 \int_0^{1-x_2} dx_1 e^{-s(x_2, Q^2)/M_B^2} \frac{1}{2\sqrt{6}x_2} \left[m_b(-1+\beta)\mathcal{H}_{+11,+17_2,+5}(x_i) \right. \right. \\
& - m_N x_2 \left(\mathcal{H}_{-11,-13_3,+17_{10},-19_{18},+3_2,+5_3,-7_5}(x_i) + \beta\mathcal{H}_{+11,+13_3,+17_8,-19_{28},+3_4,-5_3,+7_5}(x_i) + 2\mathcal{H}_1(2+\beta) \right) \Big] \\
& + \int_{t_0}^1 dx_2 \int_0^{1-x_2} dx_1 \int_{t_0}^{x_2} dt_1 e^{-s(t_1, Q^2)/M_B^2} \left[\frac{1}{12\sqrt{6}M_B^4 t_1^2} \left\{ m_N^5 x_2 \left(\mathcal{H}_{+10_3,+16,-22_8,+24_{10}}(x_i) \right. \right. \right. \\
& - \beta\mathcal{H}_{+10_3,+16,+22_4,-24_8}(x_i) \Big) + m_N^4 m_b \mathcal{H}_{22}(x_i)(1-\beta)(2+3x_2) - m_N^2 m_b x_2 \mathcal{H}_{22}(x_i)(1-\beta)(Q^2 + s(t_1, Q^2)) \Big\} \\
& + \frac{1}{12\sqrt{6}M_B^4 t_1} \left\{ m_N^5 \left(\mathcal{H}_{+10_3,+16}(x_i)(-1+\beta)(1+x_2) - 2\mathcal{H}_{24}(x_i)(5+4\beta)(1+x_2) + 4\mathcal{H}_{22}(x_i)(2+\beta)(1+2x_2) \right) \right. \\
& + m_N^4 m_b(-1+\beta) \left(x_2 \mathcal{H}_{-10,+16,-24_2}(x_i) + \mathcal{H}_{22}(x_i)(3+x_2) \right) - m_N^3 x_2 (Q^2 + s(t_1, Q^2)) \left(\mathcal{H}_{+22_8,-24_{10}}(x_i) \right. \\
& + \beta\mathcal{H}_{+22_4,-24_8}(x_i) + (-1+\beta)\mathcal{H}_{+10_3,+16}(x_i) \Big) - m_N^2 m_b \mathcal{H}_{22}(x_i)(-1+\beta) \left(Q^2 + s(t_1, Q^2) + 3Q^2 x_2 + s(t_1, Q^2)x_2 \right) \Big\} \\
& + \frac{1}{24\sqrt{6}M_B^4 t_1} \left\{ m_N^3 \left(\mathcal{H}_{-12_{10},+18,-20_{23},-6_6}(x_i) + \beta\mathcal{H}_{+12_{10},-18_7,-20_{85},+6_6}(x_i) + x_2 \mathcal{H}_{+10_6,+16_2,+24_{20}}(x_i) \right. \right. \\
& + \beta x_2 \mathcal{H}_{-10_6,-16_2,+24_{16}}(x_i) \Big) - m_N^2 m_b(-1+\beta) \left(\mathcal{H}_{+12_2,+14,+15,-20_4,+21_4,-6_2,+8,+9}(x_i) - 4x_2 \mathcal{H}_{22}(x_i) \right) \\
& + m_N(Q^2 + s(t_1, Q^2)) \left(\mathcal{H}_{-18_3,-20_3,-6_2}(x_i) + \beta\mathcal{H}_{+18,+20_{19},+6_2}(x_i) - 2\mathcal{H}_{12}(x_i)(-1+\beta) \right) \Big\} \\
& + \frac{m_N^4 m_b}{\sqrt{6}M_B^4 t_1^3} x_2 \mathcal{H}_{22}(x_i)(-1+\beta) - \frac{m_N^3}{2\sqrt{6}M_B^2 t_1^2} \left(\mathcal{H}_{+18,+20,+6}(x_i) - \beta\mathcal{H}_{+18,+20_{11},+6}(x_i) + \mathcal{H}_{12}(x_i)(-1+\beta) \right) \\
& + \frac{1}{12\sqrt{6}M_B^4} \left\{ m_N^5 \left(\mathcal{H}_{+16,+10_3,+24_{10}}(x_i) + \beta\mathcal{H}_{-10_3,-16,+24_8}(x_i) + 2\mathcal{H}_{22}(x_i)(2+\beta)(-4+t_1-x_2) \right) \right. \\
& + m_N^4 m_b(-1+\beta)\mathcal{H}_{+10,-16,-22,+24_2}(x_i) + m_N^3 \left[(-1+\beta) \left(Q^2 + s(t_1, Q^2) - Q^2 t_1 + Q^2 x_2 \right) \mathcal{H}_{+10_3,+16}(x_i) \right. \\
& - 2\mathcal{H}_{22}(x_i)(2+\beta) \left(Q^2(-2+3t_1-3x_2) + s(t_1, Q^2)(-2+t_1-x_2) \right) - 2\mathcal{H}_{24}(x_i)(5+4\beta) \left(Q^2 + s(t_1, Q^2) \right. \\
& \left. \left. - Q^2 t_1 + Q^2 x_2 \right) \right] + m_N^2 m_b \mathcal{H}_{22}(x_i)(-1+\beta) \left(3Q^2 + s(t_1, Q^2) \right) \Big\} + \frac{1}{24\sqrt{6}M_B^2} \left\{ -4m_N^2 m_b \mathcal{H}_{22}(x_i)(-1+\beta) \right. \\
& + m_N^3 \left(\mathcal{H}_{+12_8,-14,+15_5,-16_2,+18_5,-2_8,+20_{21}+21_{20},+23_{36},-24_{20},+4_4,+6_8,+8_3,-9_7}(x_i) + 8\mathcal{H}_{22}(x_i)((2+\beta)t_1 + (2+\beta)x_2) \right. \\
& + 6\mathcal{H}_{10}(x_i)(-1+\beta) + \beta\mathcal{H}_{-12_8,+14,-15_5,+16_2,+18_7,-2_4,+20_{61}+21_{16},+23_{56},-24_{16},+4_8,-6_8,-8_3,+9_7}(x_i) \Big) \\
& + m_N \left[Q^2 \mathcal{H}_{-12_6,+18,-20_{11},-6_2}(x_i) - s(t_1, Q^2) \mathcal{H}_{+12_4,+18,+20_{13},+6_4}(x_i) + \beta \left(Q^2 \mathcal{H}_{+12_6,-18_5,-20_{55},+6_2}(x_i) \right. \right. \\
& \left. \left. + s(t_1, Q^2) \mathcal{H}_{+12_4,-18_3,-20_{33},+6_4}(x_i) \right) \right] \Big\} - \frac{m_N}{4\sqrt{6}t_1} \left(\mathcal{H}_{+12_2,+18_3,+20_{15},+6_6}(x_i) + \beta\mathcal{H}_{-12_2,+18,+20_{11},-6_6}(x_i) \right) \\
& + \frac{m_N}{4\sqrt{6}} \left(\mathcal{H}_{+12_4,+18,+20_{13},+6_4}(x_i) + \beta\mathcal{H}_{-12_4,+18_3,+20_{33},-6_4}(x_i) \right) \Big] \\
& + \int_{t_0}^1 dx_2 \int_0^{1-x_2} dx_1 e^{-s_0/M_B^2} \left[\frac{1}{(Q^2 + m_N^2 t_0^2)^3 24\sqrt{6}} \left\{ -\frac{2m_N^9 t_0^4}{M_B^2} (t_0 - x_2) \left[(-1+\beta)(-1+t_0)\mathcal{H}_{+10_3,+16}(x_i) \right. \right. \right. \\
& - 2\mathcal{H}_{24}(x_i)(5+4\beta)(-1+t_0) - 2\mathcal{H}_{22}(x_i)(2+\beta) \left(2-4t_0+t_0^2 \right) \Big] - \frac{2m_N^8 m_b t_0^3}{M_B^2} (-1+\beta)(t_0 - x_2) \Big\} \right]
\end{aligned}$$

$$\begin{aligned}
& \left(\mathcal{H}_{22}(x_i)(2 - 3t_0) + t_0^2 \mathcal{H}_{-10,+16,+22,-24_2}(x_i) \right) + m_N^7 \left[\frac{2t_0^7}{M_B^2} \left\{ Q^2 \left(\mathcal{H}_{+10_3,+16,+24_{10}}(x_i) - \beta \mathcal{H}_{+10_3,+16,-24_8}(x_i) \right) \right. \right. \\
& + 2\mathcal{H}_{22}(x_i)(2 + \beta) \left(2M_B^2 - 3Q^2 - s(s_0, Q^2) \right) \left. \right\} + t_0^6 \left\{ (-1 + \beta) \mathcal{H}_{+10_6,-12_8,+14,-15_5,+16_2,-6_8,-8_3,+9_7}(x_i) \right. \\
& + \mathcal{H}_{+18_5,-2_8,+20_{21},+21_{20},+22_{24},+23_{36},-24_{20},+4_4}(x_i) + \beta \mathcal{H}_{+18_7,-2_4,+20_{61},+21_{16},+22_{12},+23_{56},-24_{16},+4_8}(x_i) \\
& + \frac{1}{M_B^2} \left[Q^2 \left(\mathcal{H}_{-10_6,-16_2,+22_{16},-24_{20}}(x_i) + \beta \mathcal{H}_{+10_6,+16_2,+22_8,-24_{16}}(x_i) + x_2 \mathcal{H}_{-10_6,-16_2,+22_{24},-24_{20}}(x_i) \right. \right. \\
& + \beta x_2 \mathcal{H}_{+10_6,+16_2,+22_{12},-24_{16}}(x_i) \left. \right] + s(s_0, Q^2) \left(\mathcal{H}_{-10_6,-16_2,+22_{16},-24_{20}}(x_i) + \beta \mathcal{H}_{+10_6,+16_2,+22_8,-24_{16}}(x_i) \right. \\
& + 4x_2 \mathcal{H}_{22}(x_i)(2 + \beta) \left. \right) \left. \right\} + t_0^5 \left\{ \mathcal{H}_{+10_{18},-12_{10},+16_6,+18,-20_{23},-22_{96},+24_{60},+6_6}(x_i) + x_2 \mathcal{H}_{+10_6,+16_2,-22_{24},+24_{20}}(x_i) \right. \\
& + \beta \mathcal{H}_{-10_{18},+12_{10},-16_6,-18_7,-20_{85},-22_{48},+24_{48},+6_6}(x_i) + \beta x_2 \mathcal{H}_{+10_6,-16_2,-22_{12},+24_{16}}(x_i) + \frac{1}{M_B^2} \left[8Q^2 \mathcal{H}_{22}(x_i)(2 + \beta) \right. \\
& + Q^2 x_2 \left(\mathcal{H}_{+10_6,+16_2,-22_{16},+24_{20}}(x_i) + \beta \mathcal{H}_{-10_6,-16_2,-22_8,+24_{16}}(x_i) \right) + x_2 s(s_0, Q^2) \left(\mathcal{H}_{+10_6,+16_2,-22_{16},+24_{20}}(x_i) \right. \\
& \left. \left. - \beta \mathcal{H}_{+10_6,+16_2,+22_8,+24_{16}}(x_i) \right) \right] \left. \right\} + \frac{2t_0^4}{M_B^2} \left\{ M_B^2 \mathcal{H}_{-16_3,-18,-20,+22_{24},-24_{30},-6}(x_i) + Q^2 \beta \mathcal{H}_{-16_2,-22_{16},+24_{16}}(x_i) \right. \\
& + M_B^2 \beta \mathcal{H}_{+16_3,+18,+20_{11},+22_{12},-24_{24},+6}(x_i) + Q^2 \mathcal{H}_{+16_2,-22_{32},+24_{20}}(x_i) + M_B^2 \mathcal{H}_{12}(x_i)(1 - \beta) + x_2 \left(-3M_B^2 \mathcal{H}_{16}(x_i) \right. \\
& \left. \left. (1 - \beta) + 4\mathcal{H}_{22}(x_i)(2 + \beta)(6^2 - Q^2) - 6M_B^2 \mathcal{H}_{24}(x_i)(5 + 4\beta) \right) - 3\mathcal{H}_{10}(x_i)(1 - \beta)(-2Q^2 + 3M_B^2(1 + x_2)) \right\} \\
& + \frac{2t_0^3}{M_B^2} \left\{ \left(\mathcal{H}_{+16,-10_3}(x_i)(1 - \beta) + 2\mathcal{H}_{24}(x_i)(5 + 4\beta) \right) (3M_B^2 x_2 - 2Q^2(1 + x_2)) - 4\mathcal{H}_{22}(x_i)(2 + \beta) \right. \\
& \left. (3M_B^2 x_2 - 2Q^2(1 + 2x_2)) \right\} - \frac{4Q^2 t_0^2 x_2}{M_B^2} \left(\mathcal{H}_{+22_8,-24_{10}}(x_i) - \mathcal{H}_{+10_3,+16}(x_i)(1 - \beta) + \beta \mathcal{H}_{+22_4,-24_8}(x_i) \right) \left. \right] \\
& + \frac{m_N^6 m_b(1 - \beta)t_0}{M_B^2} \left\{ -2\mathcal{H}_{22}(x_i)(t_0 - x_2) \left[-M_B^2 t_0 \left(6 + t_0(3 + t_0)(-3 + 2t_0) \right) + Q^2 \left(-4 + t_0(6 + t_0(-1 + t_0) \right. \right. \right. \\
& \left. \left. (2 + 3t_0) \right) \right) + (-1 + t_0)t_0^3 s(s_0, Q^2) \right] - 4t_0^3 Q^2 \mathcal{H}_{+10,-16,+24_2}(x_i) + 2x_2 t_0^2 \mathcal{H}_{+10,-16,+24_2}(x_i)(2Q^2 + 3M_B^2 t_0) \\
& + t_0^4 M_B^2 \mathcal{H}_{-10_6,+12_2,+14,+15,+16_6,-20_4,+21_4,-24_{12},-6_2,+8,+9}(x_i) \left. \right\} + \frac{m_N^5}{M_B^2} \left\{ Q^2 t_0 \left(\mathcal{H}_{-16_2,+22_{16},-24_{20}}(x_i) \right. \right. \\
& + \beta \mathcal{H}_{+16_2,+22_8,-24_{16}}(x_i) \left. \right) - M_B^2 Q^2 t_0^2 \mathcal{H}_{+12_4,-16_2,-18_4,-20_4,+22_{16},-24_{20},-6_4}(x_i) + 8Q^2 t_0^3 \mathcal{H}_{22}(x_i) \\
& + M_B^2 Q^2 \beta t_0^2 \mathcal{H}_{-12_4,+16_2,+18_4,+20_{44},+22_8,-24_{16},+6_4}(x_i) + Q^4 \beta t_0^4 \mathcal{H}_{+16_4,+22_{16},-24_{32}}(x_i) \\
& + Q^2 \beta t_0^2 \mathcal{H}_{-16_2,-22_{16},+24_{16}}(x_i) + M_B^2 Q^2 t_0^3 \mathcal{H}_{-12_{20},+16_2,+18_2,-20_{46},-22_{32},+24_{20},-6_{12}}(x_i) \\
& + M_B^2 Q^2 t_0^4 \mathcal{H}_{-14_2,+15_{10},-16_4,+18_{10},-2_{16},+20_{42},+21_{40},+22_8,+23_{72},-24_{40},+4_8,+6_{16},+8_6,-9_{14}}(x_i) + Q^2 \beta t_0^3 \mathcal{H}_{22_4}(x_i) \\
& + M_B^2 Q^2 \beta t_0^3 \mathcal{H}_{+12_{20},-16_2,-18_{14},-20_{170},-22_{16},+24_{16},+6_{12}}(x_i) + Q^4 t_0^4 \mathcal{H}_{-16_4,+22_{32},-24_{40}}(x_i) \\
& + M_B^2 Q^2 \beta t_0^4 \mathcal{H}_{-12_{16},+14_2,-15_{10},+16_4,+18_{14},-2_8,+20_{122},+21_{32},+22_4,+23_{112},-24_{32},+4_{16},-6_{16},-8_6,+9_{14}}(x_i) \\
& + M_B^2 Q^2 t_0^5 \mathcal{H}_{+12_2,-16_6,-18_3,-20_3,+22_{80},-24_{60},-6_2}(x_i) + Q^4 t_0^5 \mathcal{H}_{+16_4,-22_{48},+24_{40}}(x_i) + Q^4 \beta t_0^5 \mathcal{H}_{-16_4,-22_{24},+24_{32}}(x_i) \\
& + M_B^2 Q^2 \beta t_0^5 \mathcal{H}_{-12_2,+16_6,+18,+20_{19},+22_{40},-24_{48},+6_2}(x_i) + M_B^2 Q^2 t_0^6 \mathcal{H}_{-12_6,+16_6,+18,-20_{11},-22_{72},+24_{60},-6_2}(x_i) \\
& + M_B^2 Q^2 \beta t_0^6 \mathcal{H}_{+12_6,-16_6,-18_5,-20_{55},-22_{36},+24_{48},+6_2}(x_i) + s(s_0, Q^2) \left\{ Q^2 t_0^4 \mathcal{H}_{-16_4,+22_{32},-24_{40}}(x_i) \right.
\end{aligned}$$

$$\begin{aligned}
& + Q^2 t_0^4 \beta \mathcal{H}_{+16_4, +22_{16}, -24_{32}}(x_i) + M_B^2 t_0^5 \mathcal{H}_{+12_2, -16_6, -18_3, -20_3, +22_{48}, -24_{60}, -6_2}(x_i) - 8Q^2 t_0^5 \mathcal{H}_{22}(x_i)(2 + \beta) \\
& + M_B^2 t_0^5 \left(\beta \mathcal{H}_{-12_2, +16_6, +18, +20_{19}, +22_{24}, -24_{48}, +6_2}(x_i) + t_0 \mathcal{H}_{-12_4, -18, -20_{13}, -22_{24}, -6_4}(x_i) \right. \\
& \left. + t_0 \beta \mathcal{H}_{+12_4, -18_3, -20_{33}, -22_{12}, +6_4}(x_i) \right) \Bigg\} + \left\{ Q^4(-1 + t_0)(1 + 2t_0^3) + Q^2 M_B^2 t_0(-1 + t_0 - 2t_0^2 - 3t_0^3 + 3t_0^4) \right. \\
& \left. - t_0^3(2Q^2 + 3M_B^2 t_0)s(s_0, Q^2) \right\} \left[(1 - \beta) \left(6\mathcal{H}_{10}(x_i)(t_0 - x_2) + 2x_2 \mathcal{H}_{16}(x_i) \right) - 4x_2 \mathcal{H}_{24}(x_i)(5 + 4\beta) \right] \\
& + 4\mathcal{H}_{22}(x_i)(2 + \beta) \left[Q^2 \left(M_B^2(-1 + t_0)(2t_0 - 2t_0^2 - t_0^3 + 9t_0^4) + Q^2(-2 + 4t_0 - t_0^2 - 4t_0^3 + 6t_0^4) \right) \right. \\
& \left. + t_0^3(-2 + t_0)(2Q^2 + 3M_B^2 t_0)s(s_0, Q^2) \right] \Bigg\} + \frac{2m_N^4 m_b}{M_B^2 t_0}(-1 + \beta) \left\{ \mathcal{H}_{22}(x_i)(t_0 - x_2) \left(Q^4(-2 + 3t_0 - t_0^2 - 2t_0^3 + 6t_0^4) \right. \right. \\
& \left. \left. + M_B^2 Q^2(-2t_0 + 3t_0^2 - t_0^3 - 7t_0^4 + 9t_0^5) + t_0^3(-1 + t_0)(2Q^2 + 3t_0 M_B^2)s(s_0, Q^2) \right) + Q^2 t_0^2 \left[-Q^2 t_0 \mathcal{H}_{+16, -24_2}(x_i) \right. \right. \\
& \left. \left. - t_0^2 M_B^2 \mathcal{H}_{+12_2, +14, +15, +15, -20_4, +21_4 - 24_2, -6_2, +8, +9}(x_i) + t_0 \mathcal{H}_{10}(x_i)(Q^2 + t_0 M_B^2) - x_2(Q^2 + t_0 M_B^2) \mathcal{H}_{+10, -16, +24_2}(x_i) \right] \right\} \\
& + \frac{m_N^3 Q^2}{M_B^2} \left\{ -M_B^2 Q^2(-1 + \beta) \mathcal{H}_{+12_2, -18_2, -6_2}(x_i) - M_B^2 Q^2 \mathcal{H}_{20}(x_i)(2 - 22\beta) + M_B^2 Q^2 t_0(-1 + \beta) \mathcal{H}_{+12_{10}, -18, +6_6}(x_i) \right. \\
& - M_B^2 Q^2 t_0 \mathcal{H}_{20}(x_i)(23 + 85\beta) + Q^4 t_0^2 \beta \mathcal{H}_{+10_6, +16_2, +22_8, -24_{16}}(x_i) + Q^4 t_0^2 \mathcal{H}_{-10_6, -16_2, +22_{16}, -24_{20}}(x_i) \\
& + M_B^2 Q^2 t_0^2 \mathcal{H}_{-10_6, +12_8, -14, +15_5, -16_2, +18_5, -2_8, +20_{21}, +21_{20}, +23_{36}, -24_{20}, +4_4, +6_8, +8_3, -9_7}(x_i) \\
& + M_B^2 Q^2 t_0^2 \beta \mathcal{H}_{+10_6, -12_8, +14, -15_5, +16_2, +18_7, -4_2, +20_{61}, +21_{16}, +23_{56}, -24_{16}, +4_8, -6_8, -8_3, +9_7}(x_i) \\
& + M_B^2 Q^2 t_0^3 \mathcal{H}_{-10_6, +12_4, -16_2, -18_6, -20_6, +22_{32}, -24_{20}, -6_4}(x_i) + Q^4 t_0^3 \mathcal{H}_{+10_6, +16_2, -22_{24}, +24_{20}}(x_i) \\
& + M_B^2 Q^2 t_0^3 \beta \mathcal{H}_{+10_6, +12_4, +16_2, +18_2, +20_{38}, +22_{16}, -24_{16}, +6_4}(x_i) + Q^4 t_0^3 \beta \mathcal{H}_{-10_6, -16_2, -22_{12}, +24_{16}}(x_i) \\
& + M_B^2 Q^2 t_0^4 \mathcal{H}_{+10_6, -12_{12}, +16_2, +18_2, -20_{22}, -22_{24}, +24_{20}, -6_4}(x_i) + M_B^2 Q^2 t_0^4 \beta \mathcal{H}_{-10_6, +12_{12}, -16_2, -18_{10}, -20_{110}, -22_{12}, +24_{16}, +6_4}(x_i) \\
& + s(s_0, Q^2) \left[Q^2 t_0^2 \mathcal{H}_{-10_6, -16_2, +22_{16}, -24_{20}}(x_i) + Q^2 t_0^2 \beta \mathcal{H}_{+10_6, +16_2, +22_8, -24_{16}}(x_i) - 4Q^2 t_0^3 \mathcal{H}_{22}(x_i)(2 + \beta) \right. \\
& + M_B^2 t_0^3 \mathcal{H}_{-10_6, +12_4, -16_2, -18_6, -20_6, +22_{16}, -24_{20}, -6_4}(x_i) + M_B^2 t_0^3 \beta \mathcal{H}_{+10_6, -12_4, +16_2, +18_2, +20_{38}, +22_8, -24_{16}, +6_4}(x_i) \\
& + M_B^2 t_0^4 \mathcal{H}_{-12_8, -18_2, -20_{26}, -22_8, +6_8}(x_i) + M_B^2 t_0^4 \beta \mathcal{H}_{+12_8, -18_6, -20_{66}, -22_4, +6_8}(x_i) \Bigg] + 2t_0 x_2 \left[2Q^2 \mathcal{H}_{22}(x_i)(2 + \beta) \right. \\
& \left. \left(M_B^2 t_0(-4 + 3t_0) + Q^2(-2 + 3t_0) \right) - 2Q^2 \mathcal{H}_{24}(x_i)(5 + 4\beta) \left(Q^2(-1 + t_0) + M_B^2(-1 - t_0 + t_0^2) \right) \right. \\
& \left. + 2 \left(\mathcal{H}_{24}(x_i)(5 + 4\beta) + \mathcal{H}_{22}(x_i)(2 + \beta)(-2 + t_0) \right) (Q^2 + M_B^2 t_0)s(s_0, Q^2) + \mathcal{H}_{+10_3, 16}(x_i)(-1 + \beta) \right. \\
& \left. \left(Q^4(-1 + t_0) + Q^2 M_B^2(-1 - t_0 + t_0^2) - (Q^2 + M_B^2 t_0)s(s_0, Q^2) \right) \right] \Bigg\} - \frac{m_N^2 m_b Q^2}{M_B^2}(-1 + \beta) \left\{ Q^2 t_0 M_B^2 \right. \\
& \mathcal{H}_{+12_2, +14, +15, -20_4, +21_4, -6_2, +8, +9}(x_i) + 2Q^2 t_0 \mathcal{H}_{22}(x_i) \left(Q^2(2 - 3t_0) + 6M_B^2 t_0(1 - t_0) \right) - 2\mathcal{H}_{22}(x_i) \left[(-1 + t_0)t_0 \right. \\
& \left. (Q^2 + M_B^2 t_0)s(s_0, Q^2) - x_2 Q^4(-1 + 3t_0) + x_2(-1 + t_0) \left(M_B^2 t_0 s(s_0, Q^2) + Q^2(3M_B^2 t_0 + s(s_0, Q^2)) \right) \right] \Bigg\} \\
& + m_N Q^4 t_0 \left\{ Q^2 \left(2\mathcal{H}_6(x_i)(-1 + \beta)(1 + t_0) + \mathcal{H}_{18}(x_i)(-3 + \beta + t_0 - 5t_0\beta) - \mathcal{H}_{20}(x_i)(3 - 19\beta + 11t_0 + 55t_0\beta) \right) \right.
\end{aligned}$$

$$\begin{aligned}
& - \left(-2\mathcal{H}_6(x_i)(-1+\beta)(1+2t_0) + \mathcal{H}_{20}(x_i)(3-19\beta+13t_0+33t_0\beta) + \mathcal{H}_{18}(x_i)(3+t_0-\beta+3t_0\beta) \right) s(s_0, Q^2) \\
& + 2\mathcal{H}_{12}(x_i)(-1+\beta) \left(Q^2(-1+3t_0) + (-1+2t_0)s(s_0, Q^2) \right) \Big\} \Big\} + \frac{m_N^2}{(Q^2 + m_N^2 t_0^2)^2 2\sqrt{6}} (t_0 - x_2) \left\{ -m_N t_0 \left[\mathcal{H}_{16}(x_i) \right. \right. \\
& (-1+\beta) \left(m_N^2(-1+t_0) + m_N m_b t_0 + Q^2 t_0(-1+t_0) - t_0 s(s_0, Q^2) \right) + \mathcal{H}_{10}(x_i)(-1+\beta) \left(3m_N^2(-1+t_0) - m_N m_b t_0 \right. \\
& + 3t_0(Q^2(-1+t_0) - s(s_0, Q^2)) \Big) - 2\mathcal{H}_{24}(x_i) \left(m_N^2(5+4\beta)(-1+t_0) + m_N^2 m_b(-1+\beta)t_0 + (5+4\beta)t_0 \left(Q^2(-1+t_0) \right. \right. \\
& \left. \left. - s(s_0, Q^2) \right) \right) \Big] + \mathcal{H}_{22}(x_i) \left[-m_N^2 m_b(-1+\beta)(-2+t_0)(-1+t_0) + 2m_N^3(2+\beta)(2t_0 - 4t_0^2 + t_0^3) - 2m_N(2+\beta) \right. \\
& \left. t_0^2 \left(Q^2(-2+3t_0) + (-2+t_0)s(s_0, Q^2) \right) + m_b(-1+\beta)t_0 \left(Q^2(-1+3t_0) + (-1+t_0)s(s_0, Q^2) \right) \right] \Big\} \\
& + \frac{m_N}{(Q^2 + m_N^2 t_0^2) 4\sqrt{6} M_B^2 t_0} \left\{ 2m_N \mathcal{H}_{22}(x_i)(t_0 - x_2) \left[-m_N^2 m_b(-1+\beta)(-2+t_0)(-1+t_0) + 2m_N^3(2+\beta)t_0 \right. \right. \\
& (2 - 4t_0 + t_0^2) - m_b(-1+\beta)t_0 \left(Q^2 + 2t_0 M_B^2 - 3Q^2 t_0 + s(s_0, Q^2) + t_0 s(s_0, Q^2) \right) + 2m_N(2+\beta)t_0^2 \left(Q^2(2 - 3t_0) \right. \\
& \left. \left. + 2t_0 M_B^2 + 2s(s_0, Q^2) - t_0 s(s_0, Q^2) \right) \right] + t_0 \left[-2m_N^2 M_B^2 \mathcal{H}_{18,+20,+6}(x_i) + m_N^2 M_B^2 \beta \mathcal{H}_{182,+2022,+62}(x_i) \right. \\
& - m_N^4 t_0 \mathcal{H}_{+106,+162,+2024}(x_i) + m_N^2 M_B^2 t_0 \mathcal{H}_{+18,-2023,-66}(x_i) + m_N m_b M_B^2 t_0 \mathcal{H}_{+14,+15,-204,+214,-62,+8,+9}(x_i) \\
& - M_B^2 Q^2 t_0 \mathcal{H}_{+183,-203,-62}(x_i) + m_N^4 \beta t_0 \mathcal{H}_{+106,+162,-2416}(x_i) - m_N^2 M_B^2 t_0 \beta \mathcal{H}_{-187,-2085,+66}(x_i) \\
& - m_N m_b M_B^2 t_0 \beta \mathcal{H}_{+14,+15,-204,+214,-62,+8,+9}(x_i) + M_B^2 Q^2 t_0 \beta \mathcal{H}_{+18,+2019,+62}(x_i) + m_N^2 t_0^2 (m_N^2 - Q^2) \\
& \mathcal{H}_{+106,+162,+2420}(x_i) - m_N^3 m_b t_0^2 \mathcal{H}_{+102,-162,+244}(x_i) + m_N^2 \beta t_0^2 (Q^2 - m_N^2) \mathcal{H}_{+106,+162,-2416}(x_i) \\
& - m_N^2 M_B^2 t_0^2 \mathcal{H}_{+106,+14,-155,+162,-185,+82,-2021,+2120,-2336,+2420,-44,-68,-83,+97}(x_i) + m_N^3 m_b \beta t_0^2 \mathcal{H}_{+102,-162,+244}(x_i) \\
& + M_B^2 Q^2 t_0^2 \mathcal{H}_{+18,-2011,-62}(x_i) + m_N^2 M_B^2 t_0^2 \beta \mathcal{H}_{+106,+14,-155,+162,+187,-24,+2061,+2116,+2356,-2416,+48,+68,-86,+97}(x_i) \\
& - M_B^2 Q^2 \beta t_0^2 \mathcal{H}_{+185,+2055,-62}(x_i) + m_N^2 Q^2 t_0^3 \mathcal{H}_{+106,+162,+2420}(x_i) - m_N^2 Q^2 \beta t_0^3 \mathcal{H}_{+106,+162,-2416}(x_i) \\
& + s(s_0, Q^2) \left[-M_B^2 t_0 \mathcal{H}_{+183,+203,+62}(x_i) + M_B^2 \beta t_0 \mathcal{H}_{+18,+2019,+62}(x_i) - m_N^2 t_0^2 \mathcal{H}_{+106,+162,+2420}(x_i) \right. \\
& \left. - M_B^2 t_0^2 \mathcal{H}_{+18,+2013,+64}(x_i) + m_N^2 t_0^2 \beta \mathcal{H}_{+106,+162,-2416}(x_i) - M_B^2 t_0^2 \beta \mathcal{H}_{+183,+2033,-64}(x_i) \right] \\
& - 2M_B^2 \mathcal{H}_{12}(x_i)(-1+\beta) \left(m_N m_b t_0 + m_N^2(-1+t_0)(-1+4t_0) + t_0 Q^2 - 3Q^2 t_0^2 + t_0 s(s_0, Q^2) - 2t_0^2 s(s_0, Q^2) \right) \\
& + 2x_2 m_N^2 \mathcal{H}_{16}(x_i)(-1+\beta) \left(m_N^2(-1+t_0) + m_N m_b t_0 - t_0 M_B^2 - t_0 Q^2 + Q^2 t_0^2 - t_0 s(s_0, Q^2) \right) \\
& + 2x_2 m_N^2 \mathcal{H}_{10}(x_i)(-1+\beta) \left(3m_N^2(-1+t_0) - m_N m_b t_0 - 3t_0(M_B^2 + Q^2(1-t_0)) - 3t_0 s(s_0, Q^2) \right) + 4x_2 m_N^2 \mathcal{H}_{24}(x_i) \\
& \left(-m_N^2(5+4\beta)(-1+t_0) - m_N m_b(-1+\beta)t_0 + (5+4\beta)t_0 \left(M_B^2 + Q^2(1-t_0) + s(s_0, Q^2) \right) \right) \Big] \Big\} \Big\} \\
& \quad (A.1)
\end{aligned}$$

and

$$\begin{aligned}
f_2(Q^2) = & \frac{1}{2\sqrt{\lambda_{\Lambda_b}}} e^{m_{\Lambda_b}^2/M_B^2} \left\{ \int_{t_0}^1 dx_2 \int_0^{1-x_2} dx_1 e^{-s(x_2, Q^2)/M_B^2} \frac{1}{2\sqrt{6}x_2} \left[(-1+\beta)\mathcal{H}_{+11,+5}(x_i) + \mathcal{H}_{+17}(x_i)(-1+6\beta) \right] \right. \\
& + \int_{t_0}^1 dx_2 \int_0^{1-x_2} dx_1 \int_{t_0}^{x_2} dt_1 e^{-s(t_1, Q^2)/M_B^2} \left\{ \frac{m_N^2}{M_B^4 t_1^2 2\sqrt{6}} \left[-m_N m_b (-1+\beta) x_2 \mathcal{H}_{+10,-16,+24_2}(x_i) + 2\mathcal{H}_{22}(x_i) \right. \right. \\
& \left. \left(m_N m_b (-1+\beta) x_2 + (Q^2 + s(t_1, Q^2))(-1+\beta) x_2 + m_N^2 (1-\beta + \beta x_2 + 5x_2) \right) \right] + \frac{m_N^2}{M_B^4 t_1 2\sqrt{6}} \left[m_N m_b (-1+\beta) \right. \\
& \mathcal{H}_{+10,-16,+24_2}(x_i) - 2\mathcal{H}_{22}(x_i) \left(m_N m_b (-1+\beta) + (Q^2 + s(t_1, Q^2))(-1+\beta) - \left(3Q^2 + s(t_1, Q^2)(2+\beta) \right) x_2 \right. \\
& \left. \left. + m_N^2 (5+\beta + 2x_2 + \beta x_2) \right) \right] + \frac{m_N^2}{M_B^2 t_1 4\sqrt{6}} \left[\mathcal{H}_{-14,+15_5,+18_6,-8_2,+20_{10},+21_{20},-22_4,+36_{23},+4_4,+6_4,+8_3,-9_7}(x_i) \right. \\
& \left. - 4\mathcal{H}_{12}(x_i)(-1+\beta) + \beta \mathcal{H}_{+14,-15_5,+18_6,-2_4,+20_{34},+21_{16},-22_8,+4_8,-6_4,-8_3,+9_7}(x_i) - 8x_2(2+\beta)\mathcal{H}_{22}(x_i) \right] \\
& + \frac{m_N^4}{M_B^4 t_1^3 \sqrt{6}} x_2 (-1+\beta) \mathcal{H}_{22}(x_i) + \frac{m_N}{M_B^2 t_1^2 2\sqrt{6}} \left(m_b (-1+\beta) \mathcal{H}_{-12,+18,+20,6}(x_i) + 2m_N x_2 (1+2\beta) \mathcal{H}_{22}(x_i) \right) \\
& + \frac{m_N^2}{M_B^4 \sqrt{6}} \mathcal{H}_{22}(x_i) \left(-3Q^2 + m_N^2 (2+\beta) - s(t_1, Q^2)(2+\beta) \right) + \frac{2m_N^2}{M_B^2 \sqrt{6}} (2+\beta) \mathcal{H}_{22}(x_i) \left. \right\} \\
& + \int_{t_0}^1 dx_2 \int_0^{1-x_2} dx_1 e^{-s_0/M_B^2} \left\{ (t_0 - x_2) \left[\frac{m_N^4 t_0^2}{(Q^2 + m_N^2 t_0^2)^3 \sqrt{6}} + \frac{m_N^2}{(Q^2 + m_N^2 t_0^2)^2 2\sqrt{6}} \right] \left[m_N m_b t_0 (-1+\beta) \right. \right. \\
& \mathcal{H}_{+10,-16,+24,2}(x_i) + 2\mathcal{H}_{22}(x_i) \left(-m_N m_b t_0 (-1+\beta) - Q^2 t_0 (-1+\beta + 3t_0) + m_N^2 \left(1-\beta - t_0(5+\beta) + (2+\beta)t_0^2 \right) \right. \\
& \left. \left. - t_0 \left(-1+\beta + (2+\beta)t_0 \right) s(s_0, Q^2) \right) \right] + \frac{m_N}{(Q^2 + m_N^2 t_0^2) M_B^2 t_0 4\sqrt{6}} \left[4m_N \mathcal{H}_{22}(x_i)(t_0 - x_2) \left(-m_N m_b (-1+\beta) t_0 \right. \right. \\
& \left. \left. + m_N^2 (1-\beta - 5t_0 - \beta t_0 + 2t_0^2 + \beta t_0^2) + t_0 \left(Q^2 - Q^2(\beta + 3t_0) + M_B^2 (-1-2\beta + 2\beta t_0 + 4t_0) + s(s_0, Q^2) \right. \right. \right. \\
& \left. \left. \left. - (\beta + 2t_0 + \beta t_0) s(s_0, Q^2) \right) \right) \right] + t_0 \left(m_b M_B^2 \mathcal{H}_{-20_2,-6_2,+20_2,+6_2}(x_i) + m_N^2 m_b t_0 \mathcal{H}_{-10_2,+16_2,-24_4}(x_i) \right. \\
& + M_B^2 m_b t_0 \mathcal{H}_{-14,+15_5,-2_8,+20_{10},+21_{20},+23_{36},+4_4,+6_4,+8_3,-9_7}(x_i) + m_N^2 m_b \beta t_0 \mathcal{H}_{+10_2,-16_2,+24_4}(x_i) \\
& + M_B^2 m_N \beta t_0 \mathcal{H}_{+14,-15_5,-2_4,+20_{34},+21_{16},+23_{56},+4_8,-6_4,-8_3,+9_7}(x_i) - 2M_B^2 \mathcal{H}_{12}(x_i)(-1+\beta)(m_b + 2m_N t_0) \\
& \left. \left. + 2M_B^2 \mathcal{H}_{18}(x_i) \left(m_b (-1+\beta) + 3m_N t_0 (1+\beta) \right) - 2m_N^2 m_b x_2 (-1+\beta) \mathcal{H}_{+10,-16,+24_4}(x_i) \right) \right] \right\}, \quad (A.2)
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{H}(x_i) &= \mathcal{H}(x_1, x_2, 1 - x_1 - x_2), \\
s(y, Q^2) &= (1-y)m_N^2 + \frac{(1-y)}{y} Q^2 + \frac{m_b^2}{y}, \quad (A.3)
\end{aligned}$$

and $t_0 = t_0(s_0, Q^2)$ is the solution of the equation $s(t_0, Q^2) = s_0$, and is given as

$$t_0(s_0, Q^2) = \frac{m_N^2 - Q^2 + \sqrt{4m_N^2(m_b^2 + Q^2) + (m_N^2 - Q^2 - s_0)^2} + s_0}{2m_N^2}. \quad (A.4)$$

In calculations, the following short hand notations for the functions $\mathcal{H}_{\pm i_a, \pm j_b, \dots} = \pm a \mathcal{H}_i \pm b \mathcal{H}_j \dots$ have been used,

and \mathcal{H}_i are given in terms of the DA's as follows:

$$\begin{aligned}
\mathcal{H}_1 &= S_1 & \mathcal{H}_2 &= S_{1,-2} \\
\mathcal{H}_3 &= P_1 & \mathcal{H}_4 &= P_{1,-2} \\
\mathcal{H}_5 &= V_1 & \mathcal{H}_6 &= V_{1,-2,-3} \\
\mathcal{H}_7 &= V_3 & \mathcal{H}_8 &= -2V_{1,-5} + V_{3,4} \\
\mathcal{H}_9 &= V_{4,-3} & \mathcal{H}_{10} &= -V_{1,-2,-3,-4,-5,6} \\
\mathcal{H}_{11} &= A_1 & \mathcal{H}_{12} &= -A_{1,-2,3} \\
\mathcal{H}_{13} &= A_3 & \mathcal{H}_{14} &= -2A_{1,-5} - A_{3,4} \\
\mathcal{H}_{15} &= A_{3,-4} & \mathcal{H}_{16} &= A_{1,-2,3,4,-5,6} \\
\mathcal{H}_{17} &= T_1 & \mathcal{H}_{18} &= T_{1,2} - 2T_3 \\
\mathcal{H}_{19} &= T_7 & \mathcal{H}_{20} &= T_{1,-2} - 2T_7 \\
\mathcal{H}_{21} &= -T_{1,-5} + 2T_8 & \mathcal{H}_{22} &= T_{2,-3,-4,5,7,8} \\
\mathcal{H}_{23} &= T_{7,-8} & \mathcal{H}_{24} &= -T_{1,-2,-5,6} + 2T_{7,8},
\end{aligned} \tag{A.5}$$

where for each DA's, $X_{\pm i, \pm j, \dots} = \pm X_i \pm X_j \dots$ have also been used.