$\Lambda_b \to p$ form factors

Stefan Meinel

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1 Relativistic form factors

The authors of Ref. [1] define in their Eq. (73)

$$\begin{aligned}
\langle N^+(p')|\overline{u}\,\gamma^\mu\, b|\Lambda_b(p)\rangle &= \overline{u}_N(p')\left[\tilde{F}_1^V\,\gamma^\mu + \tilde{F}_2^V\,v^\mu + \tilde{F}_3^V\,v'^\mu\right]u_{\Lambda_b}(p), &(1)\\
\langle N^+(p')|\overline{u}\,\gamma^\mu\gamma_5\, b|\Lambda_b(p)\rangle &= -\overline{u}_N(p')\left[\tilde{F}_1^A\,\gamma^\mu + \tilde{F}_2^A v^\mu + \tilde{F}_3^A v'^\mu\right]\gamma_5\,u_{\Lambda_b}(p), &(2)
\end{aligned}$$

where $v^\mu = p^\mu/m_{\Lambda_b}$, $v'^\mu = p'^\mu/m_N$.

2 HQET form factors

In Ref. [2] we use HQET for the b quark, and calculate the form factors F_1 and F_2, which are defined as

$$\langle N^+(p')|\,\bar{u}\Gamma Q\,|\Lambda_Q(v)\rangle = \overline{u}_N(p')\,[F_1 + \not{v}\,F_2]\,\Gamma\,u_{\Lambda_Q}(v). \tag{3}$$

3 Relativistic form factors in terms of HQET form factors

First, we match the QCD vector and axial vector currents to HQET:

$$\begin{aligned}
\overline{u}\,\gamma^\mu\, b &= c_\gamma\,\bar{u}\gamma^\mu Q + c_v\,\bar{u}v^\mu Q, &(4)\\
\overline{u}\,\gamma^\mu\gamma_5\, b &= c_\gamma\,\bar{u}\gamma^\mu\gamma_5 Q - c_v\,\bar{u}v^\mu\gamma_5 Q, &(5)
\end{aligned}$$

where

$$\begin{aligned}
c_\gamma &= 1 - \frac{\alpha_s(\mu)}{\pi}\left[\frac{4}{3} + \ln\left(\frac{\mu}{m_b}\right)\right], &(6)\\
c_v &= \frac{2}{3}\frac{\alpha_s(\mu)}{\pi}. &(7)
\end{aligned}$$

(Here we set $\mu = m_b$). Inserting the matched vector current into Eq. (3), we have

$$\begin{aligned}
\langle N^+(p')|\,\bar{u}\gamma^\mu b\,|\Lambda_b(p)\rangle &= \overline{u}_N(p')\,[F_1 + \not{v}\,F_2]\,(c_\gamma\gamma^\mu + c_v v^\mu)\,u_{\Lambda_b}(p)\\
&= \overline{u}_N(p')\,[c_\gamma F_1\gamma^\mu + c_\gamma F_2\,(-\gamma^\mu\not{v} + \gamma^\mu\not{v} + \not{v}\gamma^\mu) + c_v F_1 v^\mu + c_v F_2 v^\mu\not{v}\,]\;u_{\Lambda_b}(p)\\
&= \overline{u}_N(p')\left[c_\gamma F_1\gamma^\mu + c_\gamma F_2\left(-\gamma^\mu\not{v} + v_\nu(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu)\right) + c_v(F_1+F_2)v^\mu\right]\;u_{\Lambda_b}(p)\\
&= \overline{u}_N(p')\,[c_\gamma F_1\gamma^\mu + c_\gamma F_2\,(-\gamma^\mu\not{v} + 2v^\mu) + c_v(F_1+F_2)v^\mu]\;u_{\Lambda_b}(p)\\
&= \overline{u}_N(p')\,[c_\gamma F_1\gamma^\mu + c_\gamma F_2\,(-\gamma^\mu + 2v^\mu) + c_v(F_1+F_2)v^\mu\,]\;u_{\Lambda_b}(p)\\
&= \overline{u}_N(p')\left[c_\gamma(F_1-F_2)\gamma^\mu + \Big(c_v F_1 + (2c_\gamma + c_v)F_2\Big)\,v^\mu\right]\;u_{\Lambda_b}(p). &(8)
\end{aligned}$$

Here we have used the Dirac equation $\not{v}u_{\Lambda_b} = u_{\Lambda_b}$. By comparing Eqs. (8) and (1), we see that, in HQET,

$$\begin{aligned}
\tilde{F}_1^V &= c_\gamma(F_1 - F_2), &(9)\\
\tilde{F}_2^V &= c_v F_1 + (2c_\gamma + c_v)F_2, &(10)\\
\tilde{F}_3^V &= 0. &(11)
\end{aligned}$$

Similarly, for the the axial vector current, we have

$$\begin{aligned}
\langle N^+(p')|\, \bar{u}\gamma^\mu\gamma_5 b\, |\Lambda_b(p)\rangle &= \overline{u}_N(p')\, [F_1 + \not{v}\, F_2]\, (c_\gamma\gamma^\mu - c_v v^\mu)\gamma_5\, u_{\Lambda_b}(p) \\
&= \overline{u}_N(p')\, [c_\gamma F_1\gamma^\mu + c_\gamma F_2\, (-\gamma^\mu\not{v} + \gamma^\mu\not{v} + \not{v}\gamma^\mu) - c_v F_1 v^\mu - c_v F_2 v^\mu\not{v}\,]\, \gamma_5\, u_{\Lambda_b}(p) \\
&= \overline{u}_N(p')\, \big[c_\gamma F_1\gamma^\mu + c_\gamma F_2\, \big(-\gamma^\mu\not{v} + v_\nu(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu)\big) - c_v(F_1 - F_2)v^\mu\big]\, \gamma_5\, u_{\Lambda_b}(p) \\
&= \overline{u}_N(p')\, [c_\gamma F_1\gamma^\mu + c_\gamma F_2\, (-\gamma^\mu\not{v} + 2v^\mu) - c_v(F_1 - F_2)v^\mu]\, \gamma_5\, u_{\Lambda_b}(p) \\
&= \overline{u}_N(p')\, [c_\gamma F_1\gamma^\mu + c_\gamma F_2\, (+\gamma^\mu + 2v^\mu) - c_v(F_1 - F_2)v^\mu\,]\, \gamma_5\, u_{\Lambda_b}(p) \\
&= \overline{u}_N(p')\, \Big[c_\gamma(F_1 + F_2)\gamma^\mu + \Big((2c_\gamma + c_v)F_2 - c_v F_1\Big)\, v^\mu\Big]\, u_{\Lambda_b}(p). \qquad (12)
\end{aligned}$$

By comparing Eqs. (12) and (2), we see that, in HQET,

$$\tilde{F}_1^A = -c_\gamma(F_1 + F_2), \qquad (13)$$

$$\tilde{F}_2^A = c_v F_1 - (2c_\gamma + c_v)F_2, \qquad (14)$$

$$\tilde{F}_3^A = 0. \qquad (15)$$

The form factors F_1 and F_2 in Ref. [2] are written as functions of $E_N - m_N$, where E_N is the energy of the proton in the Λ_b rest frame. In terms of q^2, we have

$$E_N - m_N = \frac{m_{\Lambda_b}^2 + m_N^2 - q^2}{2m_{\Lambda_b}} - m_N. \qquad (16)$$

References

[1] F. Hussain, D.-S. Liu, M. Kramer, J. G. Körner, and S. Tawfiq, Nucl. Phys. B **370**, 259 (1992).

[2] W. Detmold, C.-J. D. Lin, S. Meinel, and M. Wingate, arXiv:1306.0446 [hep-lat].

[3] E. Eichten and B. R. Hill, Phys. Lett. B **234**, 511 (1990).