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General analysis of weak decay form factors in heavy to heavy and heavy to light baryon transitions

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We present a complete analysis of the heavy to heavy and heavy to light baryon semi-leptonic decays in the heavy quark effective theory within the framework of a Bethe–Salpeter (BS) approach and demonstrate the equivalence of this approach to other work in the field. We present in a compact form the baryon BS amplitudes which incorporate the symmetries manifest in the heavy quark limit and which also show clearly the light quark dynamics. A similar form of the BS amplitude is presented for light baryons. Using the BS amplitudes, the heavy to heavy and heavy to light semi-leptonic baryon decays are considered. As expected there is a dramatic reduction in the number of form factors. An advantage of our BS approach is demonstrated where the form factors are written as loop integrals which in principle can be calculated.

1. Introduction

In a series of recent papers [1–5] we have presented a covariant formulation of heavy meson and heavy baryon decays. This method was based on a Bethe–Salpeter formulation in the limit of the heavy quark mass going to infinity. The starting point of our investigation was the demonstration that the equal velocity assumption, arising from the heavy quark limit, can be formulated in a covariant manner using the spin–parity projectors developed by Salam et al. [6].

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In this paper we complete the project by presenting a general analysis of the weak transitions of heavy baryons to both heavy and light baryons, i.e. where the underlying transition is from a heavy quark to either another heavy quark or a light quark. In other words, we take the limit $m_{Q_i} \to \infty$ for the decaying quark while considering both the limit $m_{Q_f} \to \infty$ for the product quark and the case $m_{Q_f} \neq \infty$.

In sect. 2, we review the Bethe–Salpeter (BS) amplitudes for $\frac{1}{2}^+$ $\Lambda(\Xi)$-type and $\frac{1}{2}^+$ and $\frac{3}{2}^+$ $\Sigma(\Omega)$-type heavy and light baryons. Using symmetry properties, we show how these can be written in simple, unified and elegant forms which are the general forms within a covariant constituent quark model. We also show how the BS amplitudes are related to the wave functions for heavy baryons proposed by Georgi [12].

In sect. 3, we evaluate weak decay matrix elements for heavy to heavy transitions and write the form factors as integrals over the light quark structure functions. In sect. 4, we proceed to the heavy to light baryon weak decays, count the number of independent form factors and present the form of the decay matrix elements. We also briefly discuss the polarization of baryon weak decays. Sect. 5 contains our conclusions.

2. Baryon wave functions

We begin by briefly, once more, reviewing the Bethe–Salpeter approach to heavy baryon decays developed in refs. [1,4,5]. In the heavy quark mass limit, $m_Q \to \infty$, the heavy quark spin is decoupled from the light degrees of freedom and the heavy quark moves like a free particle with the same velocity as the baryon [7]. Since the heavy quark is free we can embed its spin into the $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ representation of the Lorentz group in the standard way, i.e. in a four-dimensional space of Dirac indices [1,8–11]. Thus we can write the spin and momentum part of the s-wave baryon wave function covariantly as a three-index spinor $B_{\alpha\beta\gamma}(P, p_Q, k_i)$, where α is the Dirac index for the heavy quark and β, γ are the Dirac indices representing the light degrees of freedom. P is the momentum of the baryon, p_Q is the momentum of the heavy quark and k_i are the momenta of the light quarks. We ignore flavour and colour for the moment. These will be incorporated later. One requires at least three Dirac indices to describe a baryon because of the direct product of the quark number U(1) gauge group with the Lorentz group (see ref. [1]).

To describe a baryon of definite spin and parity we need to reduce the large number of degrees of freedom contained in $B_{\alpha\beta\gamma}$ which is a direct product of three $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ representations of the Lorentz group. Thus, to describe a spin-$\frac{1}{2}^+$ particle we require, as a projector, a mixed symmetry Dirac tensor with the fully antisymmetric part excluded (see refs. [1,6]). The spin-$\frac{3}{2}^+$ baryon is projected out by a fully symmetric Dirac tensor. Now, since α is the Dirac index for the heavy quark, which is on-shell, we must have

$$\left(\frac{\not{p}_{Q}}{m_{Q}}-1\right)_{\alpha}^{\ \alpha'} B_{\alpha'\beta\gamma}=0. \tag{1}$$

But, since in the heavy mass limit, the heavy quark moves with the same velocity as the baryon, $p_Q/m_Q = P/M$, we obtain the Bargmann–Wigner equation

$$(\not{P}-M)_{\alpha}^{\ \alpha'} B_{\alpha'\beta\gamma}=0, \tag{2}$$

where m_Q and M are the masses of the heavy quark and baryon, respectively. Note that this equation applies only to the heavy index.

Then the requirements of symmetry and the Bargmann–Wigner equations (2) lead to the following heavy baryon wave functions:

$$J^P=\tfrac{1}{2}^+ : \quad B_{\mathscr{A}_i;\mathscr{B}_j;\mathscr{C}_k}=\frac{1}{\sqrt{3}M}\frac{\epsilon_{ijk}}{\sqrt{6}}\Big\{\big[(\not{P}+M)\gamma_5 C\big]_{\delta\sigma}u_{\alpha}(P)B_{a[bc]}$$
$$+\text{cycl.}(\alpha, a;\ \delta, b;\ \sigma, c)\Big\}A^{\delta\sigma}_{\beta\gamma}(k_1, k_2)$$
$$\equiv \frac{\epsilon_{ijk}}{\sqrt{6}}\chi^{(1/2)}_{\alpha\delta\sigma;abc}A^{\delta\sigma}_{\beta\gamma}(k_1, k_2), \tag{3}$$

$$J^P=\tfrac{3}{2}^+ : \quad B_{\mathscr{A}_i;\mathscr{B}_j;\mathscr{C}_k}=\frac{1}{\sqrt{3}M}\frac{\epsilon_{ijk}}{\sqrt{6}}\Big\{\big[(\not{P}+M)\gamma_\mu C\big]_{\delta\sigma}u^{\mu}_{\alpha}(P)B_{\{abc\}}$$
$$+\text{cycl.}(\alpha, a;\ \delta, b;\ \sigma, c)\Big\}A^{\delta\sigma}_{\beta\gamma}(k_1, k_2)$$
$$\equiv \frac{\epsilon_{ijk}}{\sqrt{6}}\chi^{(3/2)}_{\alpha\delta\sigma;abc}A^{\delta\sigma}_{\beta\gamma}(k_1, k_2), \tag{4}$$

where $\mathscr{A}=(\alpha, a)$, etc. represents the Dirac and flavour indices α and a, respectively. In eqs. (3) and (4) the heavy quark index is denoted by $\mathscr{A}$. The colour labels, finally, are denoted by i, j, k. $u_\alpha(P)$ is the usual Dirac spinor and $u^\mu_\alpha(P)$ is the spin-$\frac{3}{2}$ Rarita–Schwinger spinor. $B_{a[bc]}$ and $B_{\{abc\}}$ are the mixed symmetric and fully symmetric flavor wave functions. The projectors $\chi^{(1/2)}_{\alpha\delta\sigma;abc}$ and $\chi^{(3/2)}_{\alpha\delta\sigma;abc}$ are fully symmetric under the simultaneous interchange of flavour and Dirac indices. $A^{\delta\sigma}_{\beta\gamma}$ is a fourth-rank Dirac tensor function of the light quark momenta k_1, k_2. It transforms like a scalar under the Lorentz group so that it does not change the spin–parity structure represented by the projectors $\chi^{(1/2)}_{\alpha\delta\sigma;abc}$ and $\chi^{(3/2)}_{\alpha\delta\sigma;abc}$.

Recall that in the Λ_Q and Ξ_Q baryons the light diquark is in a spin $s=0$ state while in the Σ_Q, Σ^*_Q and Ω_Q, Ω^*_Q baryons the light diquarks are in $s=1$ states.

We can rewrite the spin-$\frac{1}{2}^+$ wave function (3) in a form which clearly distinguishes the ($\frac{1}{2}^+$) Λ_Q (Ξ_Q)-type and the $\Sigma_Q(\Omega_Q)$-type baryons:

$$\begin{aligned} B_{\mathscr{A}_i;\mathscr{B}_j;\mathscr{C}_k} = & \frac{1}{\sqrt{3}M}\frac{\epsilon_{ijk}}{\sqrt{6}}\Big\{\tfrac{3}{2}\big[(\not{P}+M)\gamma_5 C\big]_{\delta\sigma}u_\alpha(P)B_{a[bc]} \\ & +\tfrac{1}{2}\Big[\big[(\not{P}+M)\gamma_5 C\big]_{\alpha\delta}u_\sigma(P) \\ & -\big[(\not{P}+M)\gamma_5 C\big]_{\sigma\alpha}u_\delta(P)\Big]\big[B_{c[ab]}-B_{b[ca]}\big]\Big\}A^{\delta\sigma}_{\beta\gamma} \\ \equiv & \frac{\epsilon_{ijk}}{\sqrt{6}}\Big\{\chi^{\Lambda(\Xi)}_{\alpha\delta\sigma;abc}+\chi^{\Sigma(\Omega)}_{\alpha\delta\sigma;abc}\Big\}A^{\delta\sigma}_{\beta\gamma}. \end{aligned} \tag{5}$$

In eq. (5), the first term clearly represents the $\Lambda_Q(\Xi_Q)$-type baryon. It is antisymmetric in the light δ, σ indices as required from the fact that the light diquark is in an antisymmetric $s=0$ state. The flavour function $B_{a[bc]}$ is also in an antisymmetric state in the indices b, c which corresponds to $I=0$ when the indices b and c represent u- and d-quarks. The last two terms together represent the $\Sigma_Q(\Omega_Q)$-type baryon which is symmetric in the light indices corresponding to the symmetric nature of the $s=1$ diquark state. The combination $\frac{1}{2}[B_{c[ab]}-B_{b[ca]}]$ is simply the symmetric $I=1$ flavour wave function when b, c represent u- and d-quarks.

Within the covariant constituent quark model, eqs. (4) and (5) are the natural ansätze, satisfying the symmetry requirements, for the Fourier transform of the baryon Bethe–Salpeter amplitude

$$B=\langle 0|T\big(\psi_\alpha(x_1)\psi_\beta(x_2)\psi_\gamma(x_3)\,|B\rangle \tag{6}$$

where the ψ's represent the quark fields and $|B\rangle$ is a baryon state. Diagrammatically our ansätze for the heavy baryons correspond to the decomposition shown in fig. 1. The χ's are spin–parity projectors constructed from the direct product of three $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ representations of the Lorentz group [1,6].

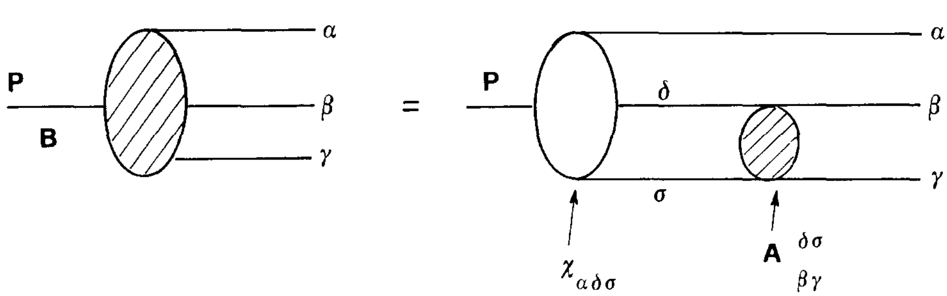

Fig. 1. Ansatz for heavy baryon Bethe–Salpeter amplitude. $\chi_{\alpha\delta\sigma}$ is the appropriate spin projector.

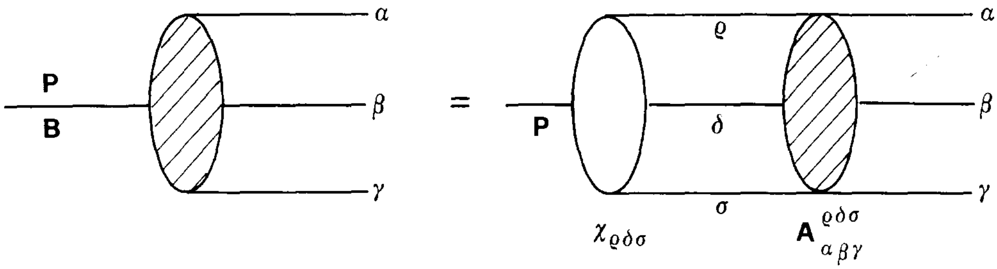

Fig. 2. Ansatz for light baryon Bethe–Salpeter amplitude.

However, we do not have such a large symmetry in writing the Bethe–Salpeter wave functions of light baryons. These light baryon wave functions do not necessarily satisfy the Bargman–Wigner equations on any of the indices nor is there decoupling of the spin of one quark from the other two quarks. But, we do have at our disposal the spin–parity projectors. With the help of these projectors we see that, within the covariant constituent quark model, the natural ansatz for the Bethe–Salpeter wave function of light baryons is

$$B_{\mathscr{A}_i;B_jC_k}=\frac{\epsilon_{ijk}}{\sqrt{6}}\chi_{\rho\delta\sigma;abc}A^{\rho\delta\sigma}_{\alpha\beta\gamma}(k_1,k_2,k_3), \tag{7}$$

where $\chi_{\rho\delta\sigma;abc}$ represents either the $\frac{1}{2}^+$ or $\frac{3}{2}^+$ projectors as in eqs. (4) and (5). Here $A^{\rho\delta\sigma}_{\alpha\beta\gamma}$ (k_1, k_2, k_3) is a sixth-rank Dirac tensor function of the three light quark momenta k_1, k_2, k_3 and transforms like a scalar under the Lorentz group. The ansatz (7) is represented diagrammatically in fig. 2.

It is easy to see that the expression (7) returns to eqs. (4) and (5), if we take the limit that one of the three quarks is very heavy and its spin decouples from the other two quarks. We suppose that the quark with index α is decoupled, then

$$A^{\rho\delta\sigma}_{\alpha\beta\gamma}=\delta^{\rho}_{\alpha}A^{\delta\sigma}_{\beta\gamma}, \tag{8}$$

and eq. (7) becomes

$$B_{\mathscr{A}_i;\mathscr{B}_j;\mathscr{C}_k}=\frac{\epsilon_{ijk}}{\sqrt{6}}\chi_{\alpha\delta\sigma;abc}A^{\delta\sigma}_{\beta\gamma}, \tag{9}$$

which is just the heavy baryon wave function

In terms of the symmetric Dirac matrices, $C^{-1}\gamma_\mu$ and $C^{-1}\sigma_{\mu\nu}$, the $\Sigma_Q(\Omega_Q)$-type $\frac{1}{2}^+$ baryon wave function, eq. (5), can be written in a much simpler, suggestive form. The projector $\chi^{\Sigma(\Omega)}_{\alpha\delta\sigma}$ is symmetric under $\delta\leftrightarrow\sigma$ interchange and thus projects out the symmetric (under $\delta\leftrightarrow\sigma$) piece, $A_{S\beta\gamma}^{\delta\sigma}$, of $A^{\delta\sigma}_{\beta\gamma}$. Thus $A_{S\beta\gamma}^{\delta\sigma}$ can be written as

$$A_{S\beta\gamma}^{\delta\sigma}(k_1,k_2)=(\phi_\mu)_{\beta\gamma}(C^{-1}\gamma^\mu)^{\delta\sigma}+(\phi_{\mu\nu})_{\beta\gamma}(C^{-1}\sigma^{\mu\nu})^{\delta\sigma}, \tag{10}$$

where the ϕ's are second-rank Dirac tensor functions of k_1, k_2 whose Lorentz transformation properties are indicated by the indices μ, ν.

Substituting eq. (10) into eq. (5) we find for the $\Sigma_Q(\Omega_Q)$-type baryon wave function (ignoring the flavour and colour part for the moment)

$$[(\gamma^\mu + v^\mu)\gamma_5 u]_\alpha (\phi^\Sigma_\mu)_{\beta\gamma}, \tag{11}$$

with

$$\phi^\Sigma_\mu = -4\phi_\mu + 8iv^\nu \phi_{\nu\mu} \tag{12}$$

and $v^\mu = p^\mu/M$ is the velocity of the heavy baryon.

Further from eq. (10) one can show that

$$4(\phi_\mu)_{\beta\gamma} = A^{\delta\sigma}_{\beta\gamma}(\gamma_\mu C)_{\sigma\delta}, \qquad 8(\phi_{\mu\nu})_{\beta\gamma} = A^{\delta\sigma}_{\beta\gamma}(\sigma_{\mu\nu} C)_{\sigma\delta}. \tag{13}$$

Here we have dropped the subscript S from A as the antisymmetric part is, in any case, annihilated in eq. (13).

With the help of eq. (13) we can now write eq. (12) in a compact form,

$$(\phi^\Sigma_\mu)_{\beta\gamma} = -[(\not{v} + 1)\gamma_\mu C]_{\delta\sigma} A^{\delta\sigma}_{\beta\gamma}. \tag{14}$$

A similar exercise for the $\frac{3}{2}^+$ baryon leads to the collapse of the three terms in eq. (4) into just one form for the $\Sigma^*_Q(\Omega^*_Q)$-type baryon wave function

$$u^\mu_\alpha(-3\phi^\Sigma_\mu)_{\beta\gamma}, \tag{15}$$

where ϕ^Σ_μ is the same as eq. (14) due to the heavy quark symmetry.

We can now write all the heavy baryon wave functions in compact form, including colour and flavour factors.

2.1. HEAVY BARYON WAVE FUNCTIONS

(i) $\Sigma_Q(\Omega_Q)$, spin $\frac{1}{2}^+$, symmetric light diquark:

$$\frac{\epsilon_{ijk}}{\sqrt{6}} \frac{1}{2\sqrt{3}} [B_{c[ab]} - B_{b[ca]}][(\gamma^\mu + v^\mu)\gamma_5 u]_\alpha (\phi^\Sigma_\mu)_{\beta\gamma}, \tag{16}$$

with $(\phi^\Sigma_\mu)_{\beta\gamma}$ as in eq. (14).

(ii) $\Sigma^*_Q(\Omega^*_Q)$, spin $\frac{3}{2}^+$, symmetric light diquark:

$$\frac{\epsilon_{ijk}}{\sqrt{6}} \frac{1}{\sqrt{3}} B_{\{abc\}} u^\mu_\alpha(-3\phi^\Sigma_\mu)_{\beta\gamma}. \tag{17}$$

(iii) $\Lambda_Q(\Xi_Q)$, spin $\frac{1}{2}^+$, antisymmetric light diquark.

The $\Lambda_Q(\Xi_Q)$ part of the wave function in eq. (5) can also be written in terms of the velocity as

$$\frac{\epsilon_{ijk}}{\sqrt{6}}\frac{\sqrt{3}}{2}B_{a[bc]}u_\alpha(\phi^\Lambda)_{\beta\gamma} \tag{18}$$

with

$$(\phi^\Lambda)_{\beta\gamma}=[(\not{v}+1)\gamma_5 C]_{\delta\sigma}A^{\delta\sigma}_{\beta\gamma}. \tag{19}$$

In eqs. (16)–(19), a, α, are the heavy quark indices.

In eq. (19), $[(\not{v}+1)\gamma_5 C]$ projects out the antisymmetric (under $\delta\to\sigma$) piece, $A_{\Lambda}{}^{\delta\sigma}_{\beta\gamma}$, of $A^{\delta\sigma}_{\beta\gamma}$. Thus A_Λ can be expanded in terms of the antisymmetric Dirac matrices, C^{-1}, $C^{-1}\gamma_5$ and $C^{-1}\gamma_\mu\gamma_5$ as

$$A_{\Lambda}{}^{\delta\sigma}_{\beta\gamma}(k_1,k_2)=\phi_{\beta\gamma}(C^{-1})^{\delta\sigma}+(\phi_5)_{\beta\gamma}(C^{-1}\gamma_5)^{\delta\sigma}+(\phi_{\mu5})_{\beta\gamma}(C^{-1}\gamma_\mu\gamma_5)^{\delta\sigma}, \tag{20}$$

where the ϕ's are second-rank Dirac tensor functions of k_1, k_2. Thus $(\phi^\Lambda)_{\beta\gamma}$ in eq. (19) can also be written as

$$(\phi^\Lambda)_{\beta\gamma}=-4(\phi_5)_{\beta\gamma}+4v^\mu(\phi_{\mu5})_{\beta\gamma} \tag{21}$$

with

$$4(\phi_5)_{\beta\gamma}=A^{\delta\sigma}_{\beta\gamma}(\gamma_5 C)_{\sigma\delta},\qquad 4(\phi_{\mu5})_{\beta\gamma}=A^{\delta\sigma}_{\beta\gamma}(\gamma_5\gamma_\mu C)_{\sigma\delta}. \tag{22}$$

Several comments on the heavy baryon wave functions are in order here:

(1) These BS wave functions clearly make explicit the decoupling of the heavy quark spin from the dynamics in the limit $m_Q\to\infty$. A consequence of this decoupling is that we have the same ϕ^Σ_μ in eqs. (16) and (17). In other words ϕ^Σ_μ depends only on the dynamics of the diquark, independent of the spin orientation of the heavy quark.

(2) In our formulae, the quantities $A^{\delta\sigma}_{\beta\gamma}$, and consequently ϕ^Λ and ϕ^Σ_μ, represent the light quark dynamics in the baryons, fig. 1. In our approach it is, in principle, possible to calculate these functions given a certain dynamical scheme.

(3) The reader may notice the similarity between these wave functions (BS amplitudes) and the ones proposed by Georgi [12] in his tensor methods for heavy baryons. In fact, eqs. (16)–(18) establish the connection between our Bethe–Salpeter approach [1–5] and Georgi's method for doing heavy baryon physics. However, our BS wave functions have the advantage that the baryon structure is transparent and the light quark physics has also been taken into account explicitly. Note the similarities between the Λ-type and $\Sigma(\Sigma^*)$-type wave functions in the

approach where the spin-zero and spin-one nature of the diquarks are clearly demonstrated in eqs. (16) and (18), respectively. Further, as mentioned above the quantities ϕ are in principle calculable given some dynamical scheme. We will see, in subsequent sections, that this calculability carries over to form factors, whereas for Georgi, the wave functions are multiplied by unknown functions whose relation to the underlying dynamics is unclear and hence the form factors are not calculable.

In a similar fashion the light baryon wave function (7) can be written in a simpler form which makes the calculation of matrix elements much easier.

2.2. LIGHT BARYON WAVE FUNCTIONS

(i) Σ-type, $\frac{1}{2}^+$, e.g. q{ud}:

$$\frac{\epsilon_{ijk}}{\sqrt{6}}\frac{1}{2\sqrt{3}}\left[B_{c[ab]}-B_{b[ca]}\right]\left[\gamma^{\mu}+v^{\mu}\right)\gamma_5 u\right]_{\rho}\left(\phi^{\Sigma}_{\mathrm{L}\mu}\right)^{\rho}_{\alpha\beta\gamma}, \tag{23}$$

with

$$\left(\phi^{\Sigma}_{\mathrm{L}\mu}\right)^{\rho}_{\alpha\beta\gamma}=-\left[(\not{v}+1)\gamma_{\mu}C\right]_{\delta\sigma}A^{\rho\delta\sigma}_{\alpha\beta\gamma} \tag{24}$$

Also one can write

$$\phi^{\Sigma}_{\mathrm{L}\mu}=-4\phi_{\mathrm{L}\mu}+8iv^{\nu}\phi_{\mathrm{L}\nu\mu}, \tag{25}$$

where

$$4\left(\phi_{\mathrm{L}\mu}\right)^{\rho}_{\alpha\beta\gamma}=A^{\rho\delta\sigma}_{\alpha\beta\gamma}\left(\gamma_{\mu}C\right)_{\sigma\delta},\qquad 8\left(\phi_{\mathrm{L}\mu\nu}\right)^{\rho}_{\alpha\beta\gamma}=A^{\rho\delta\sigma}_{\alpha\beta\gamma}\left(\sigma_{\mu\nu}C\right)_{\sigma\delta} \tag{26}$$

are the symmetric components of $A^{\rho\delta\sigma}_{\alpha\beta\gamma}$. Here the subscript L denotes light.
(ii) Σ^*-type, $\frac{3}{2}^+$, e.g. q{ud}:

$$\frac{\epsilon_{ijk}}{\sqrt{6}}\frac{1}{2\sqrt{3}}B_{\{abc\}}u^{\mu}_{\rho}\left(-3\phi^{\Sigma^*}_{\mathrm{L}\mu}\right)^{\rho}_{\alpha\beta\gamma}. \tag{27}$$

Although $\phi^{\Sigma^*}_{\mathrm{L}\mu}$ in eq. (27) is given by an equation analogous to eq. (24), it is not in general true that $\phi^{\Sigma}_{\mathrm{L}\mu}$ and $\phi^{\Sigma^*}_{\mathrm{L}\mu}$ are the same since here the spin of the q-quark does not decouple from the other two quarks.
(iii) Λ-type, $\frac{1}{2}^+$, e.g. q[ud]:

$$\frac{\epsilon_{ijk}}{\sqrt{6}}\frac{1}{2\sqrt{3}}B_{a[bc]}u_{\rho}\left(\phi^{\Lambda}_{\mathrm{L}}\right)^{\rho}_{\alpha\beta\gamma}, \tag{28}$$

with

$$(\phi_{\mathrm{L}}^{\Lambda})^{\rho}_{\alpha\beta\gamma} = [(\not{v}+1)\gamma_5 C]_{\delta\sigma} A^{\rho\delta\sigma}_{\alpha\beta\gamma}. \tag{29}$$

Analogous to eq. (21) we have

$$\phi_{\mathrm{L}}^{\Lambda} = -4\phi_{\mathrm{L}5} + 4v^{\mu}\phi_{\mathrm{L}\mu5}, \tag{30}$$

with

$$4(\phi_{\mathrm{L}5})^{\rho}_{\alpha\beta\gamma} = A^{\delta\sigma}_{\beta\gamma}(\gamma_5 C)_{\sigma\delta}, \qquad 4(\phi_{\mathrm{L}\mu5})^{\rho}_{\alpha\beta\gamma} = A^{\delta\sigma}_{\beta\gamma}(\gamma_5\gamma_{\mu} C)_{\sigma\delta}. \tag{31}$$

Here the indices a, α refer to the light quark q.

The BS amplitudes for the light baryons, eqs. (23), (27) and (28) represent, in concise form, the general wave functions within a covariant constituent quark model. They clearly demonstrate that here the spins of the quarks are not decoupled. Also, it is easy to see that the limiting case of $m_{\mathrm{Q}} \to \infty$ can be reached simply by a restriction of the functions ϕ_{L} appearing in eqs. (23)–(31):

$$(\phi_{\mathrm{L}})^{\rho}_{\alpha\beta\gamma} \to \delta^{\rho}_{\alpha}(\phi)_{\beta\gamma}. \tag{32}$$

Alternatively, the functions ϕ appearing in the heavy baryon wave functions, eqs. (16)–(18), can be considered as the leading terms in a $1/m_{\mathrm{Q}}$ expansion of the corresponding functions ϕ_{L}.

3. Heavy to heavy baryon semi-leptonic decays

The matrix elements for the transition of a heavy baryon to another heavy baryon via a flavour changing weak current * are calculated from fig. 3, using the BS amplitude of sect. 2 [4,5]. We first consider the heavy Λ decays. It is easy to show that the matrix element is

$$\langle \Lambda_{\mathrm{Q}_2}(P_2) \,|\, J_{\lambda}^{V-A} \,|\, \Lambda_{\mathrm{Q}_1}(P_1)\rangle = V_{12} F_{\Lambda}(\omega) \bar{u}_2 \gamma_{\lambda}(1-\gamma_5) u_1, \tag{33}$$

with

$$F_{\Lambda}(\omega) = \int \mathrm{d}^4k_1\, \mathrm{d}^4k_2 (\bar{\phi}^{\Lambda})^{\beta\gamma} (\not{k}_1 - m_1)^{\beta'}_{\beta} (\not{k}_2 - m_2)^{\gamma'}_{\gamma} (\phi^{\Lambda})_{\beta'\gamma'}, \tag{34}$$

$$(\bar{\phi}^{\Lambda})^{\beta\gamma} = (\gamma_0^{\mathrm{T}})^{\beta}_{\beta'} (\phi^{\Lambda\dagger})^{\beta'\gamma'} (\gamma_0)^{\gamma}_{\gamma'}. \tag{35}$$

* It is straightforward to include renormalization effects for the heavy quark current in the matrix element (33) via leading log summation technique [7]. They lead to scale- and velocity-dependent logarithmic corrections to the bare quark current.

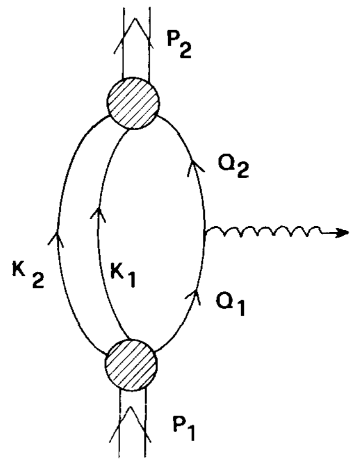

Fig. 3. Feynman diagram for current-induced baryon transitions. The Q_i and the k_i denote heavy and light quark momenta. The hatched vertices are determined from the bound-state wave functions of the baryons.

V_{12} is the appropriate Kobayashi–Maskawa matrix element. Eq. (33) implies that there is only one form factor for heavy to heavy Λ-type decays. Note that identical results hold for the transitions $\Xi_{Q[us]} \to \Xi_{Q'[us]}$ where the [us] light quark system is in an antisymmetric diquark state.

As shown in refs. [2,5] this single form factor is normalized to unity at $q^2_{\max}$ or $\omega = 1$ where $\omega = v_1 \cdot v_2$, v_1^μ and v_2^μ being the incoming and outgoing four-velocities, respectively.

The heavy to heavy $\Sigma(\Sigma^*)$-type $\frac{1}{2}^+ \to \frac{1}{2}^+$, $\frac{1}{2}^+ \to \frac{3}{2}^+$ and $\frac{3}{2}^+ \to \frac{3}{2}^+$ decay matrix elements can all be written in terms of one tensor function $L_{\mu\nu}$:

$$\text{(i)} \quad \tfrac{1}{2}^+ \to \tfrac{1}{2}^+: \quad \tfrac{1}{2} b \bar{u}_2 \gamma_5 (\gamma^\mu + v_2^\mu) \gamma_\lambda (1 - \gamma_5)(\gamma^\nu + v_1^\nu) \gamma_5 u_1 L_{\mu\nu}, \tag{36}$$

$$\text{(ii)} \quad \tfrac{1}{2}^+ \to \tfrac{3}{2}^+: \quad -\tfrac{1}{2} b^* \bar{u}_2^\mu (1 - \gamma_5)(\gamma^\nu + v_1^\nu) \gamma_5 u_1 L_{\mu\nu}, \tag{37}$$

$$\text{(iii)} \quad \tfrac{3}{2}^+ \to \tfrac{3}{2}^+: \quad 4 b^{**} \bar{u}_2^\mu \gamma_\lambda (1 - \gamma_5) u_1^\nu L_{\mu\nu}, \tag{38}$$

with the flavour factors

$$b = \tfrac{1}{6}\left[B^{c[a'b]} - B^{b[ca']}\right] W_{a'}^a \left[B_{c[ab]} - B_{b[ca]}\right], \qquad b^* = 2 W_{a'}^a \bar{B}^{\{a'bc\}} B_{c[ab]} \tag{39}$$

and $b^{**} = 2 W_{a'}^a \bar{B}^{\{a'bc\}} B_{\{abc\}}$. $W_{a'}^a$ is the appropriate Kobayashi–Maskawa matrix element.

Note here that

$$L_{\mu\nu} = \int \mathrm{d}^4 k_1 \, \mathrm{d}^4 k_2 \left(\bar{\phi}_\mu^\Sigma\right)^{\beta\gamma} (\not{k}_1 - m_1)_\beta^{\beta'} (\not{k}_2 - m_2)_\gamma^{\gamma'} (\phi_\nu^\Sigma)_{\beta'\gamma'} \tag{40}$$

is a tensor function of the velocities and has no Dirac indices. Thus $L_{\mu\nu}$ can in general be written as

$$L_{\mu\nu} = 4\left(-F_1 g_{\mu\nu} + F_2 v_{1\mu} v_{2\nu}\right). \tag{41}$$

We cannot use $v_{2\mu}$ or $v_{1\nu}$ as $v_{2\mu}$ annihilates both $\bar{u}_2\gamma_5(\gamma^\mu + v_2^\mu)$ and $\bar{u}_2^\mu$ whereas $v_{1\nu}$ annihilates $(\gamma^\nu + v_1^\nu)\gamma_5 u_1$. The signs and the factor of 4 have been chosen keeping in mind subsequent normalization and contact with earlier work. Hence we have only two independent form factors F_1 and F_2.

With this choice of $L_{\mu\nu}$, the matrix elements become respectively

(i) $\frac{1}{2}^+ \to \frac{1}{2}^+$, $\Sigma_{Q_1}(\Omega_{Q_1}) \to \Sigma_{Q_2}(\Omega_{Q_2})$ weak decay matrix element:

$$2b\bar{u}_2\left\{\left[\omega F_1 + (1-\omega^2)F_2\right]\gamma_\lambda(1-\gamma_5) - 2\left[F_1 + (1-\omega)F_2\right](v_{1\lambda} + v_{2\lambda})\right.$$
$$\left. + 2\left[-F_1 + (1+\omega)F_2\right](v_{2\lambda} - v_{1\lambda})\gamma_5\right\}u_1. \tag{42}$$

(ii) $\frac{1}{2}^+ \to \frac{3}{2}^+$, $\Sigma_{Q_1}(\Omega_{Q_1}) \to \Sigma^*_{Q_2}(\Omega^*_{Q_2})$ decay matrix element:

$$b^*\bar{u}_2^\mu\left\{F_1 g_{\mu\nu}(1+\gamma_5) - \tfrac{1}{2}(F_1 - \omega F_2)v_{1\mu}\gamma_\lambda(1-\gamma_5)\right.$$
$$\left. + \tfrac{1}{2}F_2 v_{1\mu}\gamma_\lambda(1+\gamma_5) - F_2 v_{1\mu}v_{2\lambda}(1+\gamma_5)\right\}u_1. \tag{43}$$

(iii) $\frac{3}{2}^+ \to \frac{3}{2}^+$, $\Sigma^*_{Q_1}(\Omega^*_{Q_1}) \to \Sigma^*_{Q_2}(\Omega^*_{Q_2})$ decay matrix element:

$$4b^{**}\bar{u}_2^\mu\gamma_\lambda(1-\gamma_5)u_1^\nu\left(-F_1 g_{\mu\nu} + F_2 v_{1\mu}v_{2\nu}\right). \tag{44}$$

These results are equivalent to our earlier calculation [5] and to the works of Georgi [12], Isgur and Wise [13] and Mannel et al. [14]. The form factors F_1 and F_2 in the present work are related to the form factors, f and g introduced in our earlier work [5] through the relations

$$F_1 = \tfrac{1}{2}(1+\omega)f + g, \qquad F_2 = \tfrac{1}{2}f. \tag{45}$$

As shown in ref. [5] one of these form factors is normalized absolutely at $q^2 = q^2_{\max}(\omega = 1)$. Following the same arguments we find

$$F_1\,|_{\omega=1} = (f+g)\,|_{\omega=1} = 1. \tag{46}$$

If one uses a renormalized heavy quark current (see footnote at beginning of this section), we still have only two form factors but the detailed Lorentz structure of the matrix element is different and depends on the choice of the renormalized current.

Before we leave the heavy to heavy sector, it is instructive to decompose the transition of the diquark system from the velocity v_1 to the velocity v_2 in terms of its s-wave and d-wave components. The diquark excitation may be viewed as a process where a $(3P_0)$-spurion with J^P quantum numbers 0^+ and a $(s=0, 1)$-diquark makes a transition to a $(s=0, 1)$-diquark with J^P quantum numbers $0^+ + 0^+ \to 0^+$ and $0^+ + 1^+ \to 1^+$. For the $\Lambda_Q(\Xi_Q)$ type transition the form factor $F_\Lambda(\omega)$, in eq. (29), is nothing but the s-wave transition amplitude of the scalar diquark.

For the $\Sigma_Q(\Omega_Q)$-type transition one has

$$\begin{pmatrix} F_s \\ F_d \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 2\omega + 4 & 2(1-\omega^2) \\ -2\sqrt{2}\,(\omega - 1) & -2\sqrt{2}\,(1-\omega^2) \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} \tag{47}$$

where F_s and F_d denote the s-wave and d-wave amplitudes for the vector diquark transitions. As $q^2 \to q^2_{max}(\omega \to 1)$, the d-wave amplitude vanishes, as expected, with the second power of the c.m. momentum $((\omega - 1) \propto p^2)$. We have chosen an s-wave amplitude normalization such that, at $\omega = 1$, $F_s = F_1 = 1$.

Alternatively, the diquark transition can be parametrized in terms of the longitudinal and transverse transition amplitudes F_L and F_T describing the helicity $0 \to 0$ and helicity $1 \to 1$ transitions of the vector diquark system.

Defining these as

$$F_L = F_s - \frac{1}{\sqrt{2}} F_d, \qquad F_T = F_s + \frac{1}{\sqrt{2}} F_d, \tag{48}$$

we find

$$\begin{pmatrix} F_L \\ F_T \end{pmatrix} = \begin{pmatrix} \omega & (1-\omega^2) \\ 1 & 0 \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} \tag{49}$$

and at $\omega = 1$, $F_L = F_T = F_1 = 1$.

In terms of F_L and F_T the matrix elements (42)–(44) can be written as

$$\text{(i)}\ \tfrac{1}{2}^+ \to \tfrac{1}{2}^+ :\quad 2b\bar{u}_2\bigg[F_L\gamma_\lambda(1-\gamma_5) - \frac{2}{1+\omega}(F_L + F_T)(v_{1\lambda} + v_{2\lambda}) + \frac{2}{1-\omega}(F_L - F_T)(v_{2\lambda} - v_{1\lambda})\gamma_5\bigg]u_1. \tag{50}$$

$$\text{(ii)}\ \tfrac{1}{2}^+ \to \tfrac{3}{2}^+ :\quad b^*\bar{u}_2\bigg\{F_T g_{\mu\lambda}(1+\gamma_5) + \frac{1}{2(1-\omega)}(F_L - F_T)v_{1\mu}\gamma_\lambda + \frac{1}{2(1+\omega)}(F_L + F_T)v_{1\mu}\gamma_\lambda\gamma_5 - \frac{1}{1-\omega^2}(F_L - \omega F_T)v_{1\mu}v_{2\lambda}(1+\gamma_5)\bigg\}u_1. \tag{51}$$

$$\text{(iii)}\ \tfrac{3}{2}^+ \to \tfrac{3}{2}^+:\quad 4b^{**}\bar{u}_2^{\mu}\gamma_\lambda(1-\gamma_5)u_1^{\nu}\left\{-F_{\mathrm{T}}g_{\mu\nu} + \frac{1}{1-\omega^2}(F_{\mathrm{L}} - \omega F_{\mathrm{T}})v_{1\mu}v_{2\nu}\right\}. \tag{52}$$

4. Heavy to light baryon semi-leptonic decays

Given the heavy baryon wave functions, eqs. (16)–(18), and the light baryon wave functions, eqs. (23), (27) and (28), it is straightforward to write down the matrix elements for heavy to light transitions. Following the same procedure as in sect. 3, we obtain the following matrix elements for heavy to light ($\mathrm{Q}_1 \to \mathrm{q}_2$) transitions:

(i) $\Lambda(\Xi)$-type, $\frac{1}{2}^+ \to \frac{1}{2}^+$ transitions.

$$\langle \Lambda_{q_2}(P_2) \,|\, J_\lambda^{V-A} \,|\, \Lambda_{\mathrm{Q}_1}(P_1)\rangle = V_{12}\bar{u}_2 L(v_1, v_2)\left[\gamma_\lambda(1-\gamma_5)\right]u_1, \tag{53}$$

with $L(v_1, v_2)$ a matrix given by

$$L(v_1, v_2)_\rho^{\ \alpha} = \int \mathrm{d}^4k_1\, \mathrm{d}^4k_2 \left(\bar{\phi}_{\mathrm{L}}^{\Lambda}\right)_\rho^{\ \alpha\beta\gamma}(\not{k}_1 - m_1)_\beta^{\ \beta'}(\not{k}_2 - m_2)_\gamma^{\ \gamma'}(\phi^{\Lambda})_{\beta'\gamma'}. \tag{54}$$

From Lorentz invariance, $L(v_1, v_2)$ can only be $\mathbf{1}$, $\not{v}_1$, $\not{v}_2$ or $\not{v}_1\not{v}_2$. Since $\bar{u}_2\not{v}_2 = \bar{u}_2$, the most general form for $L(v_1, v_2)$ is

$$L(v_1, v_2) = F_1^{\Lambda}(\omega) + \not{v}_1 F_2^{\Lambda}(\omega). \tag{55}$$

Thus there are, in the case of $\Lambda_{\mathrm{Q}_1} \to \Lambda_{\mathrm{q}_2}$ (e.g. $\Lambda_{\mathrm{c}} \to \Lambda_{\mathrm{s}}$) decays, only two form factors and the matrix element can be written as

$$V_{12}\bar{u}(P_2)\left[\left(F_1^{\Lambda} - F_2^{\Lambda}\right)\gamma_\lambda - \left(F_1^{\Lambda} + F_2^{\Lambda}\right)\gamma_\lambda\gamma_5 + 2v_{1\lambda}F_2^{\Lambda}(1-\gamma_5)\right]u(P_1). \tag{56}$$

The heavy to heavy case ($m_{\mathrm{q}_2} \to \infty$) is obtained by putting $F_2^{\Lambda} = 0$. This means that the leading term of F_2^{Λ} goes at least as $1/m_{\mathrm{q}_2}$ whereas F_1^{Λ} goes as

$$F_1^{\Lambda} = F_{\Lambda} + \mathrm{O}(1/m_{\mathrm{q}_2}). \tag{57}$$

(ii) $\Sigma_{\mathrm{Q}_1}(\Omega_{\mathrm{Q}_1}) \to \Sigma_{\mathrm{q}_2}(\Omega_{\mathrm{q}_2})$, $\frac{1}{2}^+ \to \frac{1}{2}^+$ transitions. Here, the decay matrix element has the form

$$\langle \Sigma_{\mathrm{q}_2}(P_2) \,|\, J_\lambda^{V-A} \,|\, \Sigma_{\mathrm{Q}_1}(P_1)\rangle$$
$$= \tfrac{1}{2}b\bar{u}_2\gamma_5(\gamma^\mu + v_2^\mu)L_{\mu\nu}\gamma_\lambda(1-\gamma_5)(\gamma^\nu + v_1^\nu)\gamma_5 u_1. \tag{58}$$

where now $L_{\mu\nu}(v_1, v_2)$ is a matrix given by

$$(L_{\mu\nu})_\rho^{\ \alpha} = \int d^4k_1\, d^4k_2 \big(\bar{\phi}^{\Sigma}_{L\mu}\big)_\rho^{\ \alpha\beta\gamma} (\not{k}_1 - m_1)_\beta^{\ \beta'} (\not{k}_2 - m_2)_\gamma^{\ \gamma'} (\phi^{\Sigma}_\nu)_{\beta'\gamma'}. \quad (59)$$

Keeping in mind that $\bar{u}_2\gamma_5(\gamma^\mu + v_2^\mu)$ is annihilated by $v_{2\mu}$ and $(\gamma^\nu + v_1^\nu)\gamma_5 u_1$ by $v_{1\nu}$, the most general expansion for $L_{\mu\nu}$ is

$$\begin{aligned} L_{\mu\nu} = 4\big(&-G_1 g_{\mu\nu} + G_2 v_{1\mu} v_{2\nu} + G_3 g_{\mu\nu} \not{v}_1 + G_4 v_{1\mu} v_{2\nu} \not{v}_1 \\ &+ G_5 v_{1\mu}\gamma_\nu + G_6 v_{1\mu}\gamma_\nu \not{v}_1 + G_7 \gamma_\mu v_{2\nu} + G_8 \gamma_\mu \gamma_\nu \\ &+ G_9 \gamma_\mu v_{2\nu} \not{v}_1 + G_{10}\gamma_\mu\gamma_\nu\not{v}_1\big) \end{aligned} \quad (60)$$

in terms of 10 Lorentz scalar functions $\{G_i\}$. However, because of the factor $\bar{u}_2\gamma_5(\gamma^\mu + v_2^\mu)$ in the matrix element (58) only four combinations appear in the matrix element which can be written as

$$2b\bar{u}_2\gamma_5\big(\bar{F}_1\gamma_\nu + \bar{F}_2 v_{2\nu} + \bar{F}_3\gamma_\nu\not{v}_1 + \bar{F}_4 v_{2\nu}\not{v}_1\big)\gamma_\lambda(1-\gamma_5)(\gamma^\nu + v_1^\nu)\gamma_5 u_1 \quad (58')$$

with

$$\begin{aligned} \bar{F}_1 &= -G_1 + \omega G_5 - G_6 + 3G_8, \\ \bar{F}_2 &= -G_1 + \omega G_2 + G_4 + 3G_7, \\ \bar{F}_3 &= G_3 - G_5 + \omega G_6 + 3G_{10}, \\ \bar{F}_4 &= G_2 + G_3 + \omega G_4 + 3G_9. \end{aligned} \quad (61)$$

In terms of $\bar{F}_1$, $\bar{F}_2$, $\bar{F}_3$, $\bar{F}_4$ the matrix element (58) reduces to

$$\begin{aligned} &\langle \Sigma_{q_2}(P_2) | J_\lambda^{V-A} | \Sigma_{Q_1}(P_1)\rangle \\ &\quad = 2b\bar{u}_2\Big\{\big(-\omega\bar{F}_2 - \bar{F}_3 + \bar{F}_4\big)\gamma_\lambda(1-\gamma_5) + \big(\bar{F}_1 - \bar{F}_2 + \omega\bar{F}_4\big)\gamma_\lambda(1+\gamma_5) \\ &\qquad - 2v_{1\lambda}\big[-\bar{F}_1 - 2\bar{F}_3 + (1-\omega)\bar{F}_4\big] - 2v_{2\lambda}\big(-\bar{F}_2 + \bar{F}_4\big) \\ &\qquad + 2v_{2\lambda}\gamma_5\big(\bar{F}_2 + \bar{F}_4\big) - 2v_{1\lambda}\gamma_5\big[\bar{F}_1 - 2\bar{F}_3 + (1+\omega)\bar{F}_4\big]\Big\}u_1. \end{aligned} \quad (62)$$

Therefore for the heavy to light $\Sigma(\Omega)$-type decays there are only 4 independent form factors $\bar{F}_1$, $\bar{F}_2$, $\bar{F}_3$ and $\bar{F}_4$ which are related to the underlying dynamic through eqs. (59)–(61). The heavy to heavy limit ($m_{q_2} \to \infty$) is obtained by setting

$$\bar{F}_1 = -F_1, \quad \bar{F}_2 = -F_1 + \omega F_2, \quad \bar{F}_3 = 0, \quad \bar{F}_4 = F_2, \tag{63}$$

where F_1 and F_2 are the heavy to heavy form factors or equivalently

$$\begin{gathered} G_1 = F_1, G_2 = F_2, \\ \omega G_5 - G_6 + 3G_8 = 0. \\ G_4 + 3G_7 = 0, \\ G_3 - G_5 + \omega G_6 + 3G_{10} = 0, \\ G_3 + \omega G_4 + 3G_9 = 0. \end{gathered} \tag{64}$$

Following the discussion after eq. (47) we conclude that G_1 and G_2 have the expansion

$$G_1 = F_1 + \mathrm{O}\left(\frac{1}{m_{q_2}}\right), \qquad G_2 = F_2 + \mathrm{O}\left(\frac{1}{m_{q_2}}\right), \tag{65}$$

whereas the leading terms in the expansions of the linear combinations in eq. (64) are, at least of $\mathrm{O}(1/m_{q_2})$.

(iii) $\Sigma_{Q_1}(\Omega_{Q_1}) \to \Sigma^*_{q_2}(\Omega^*_{q_2})$, $\frac{1}{2}^+ \to \frac{3}{2}^+$ transitions. Here we have

$$\begin{aligned} &\langle \Sigma^*_{q_2}(P_2) | J_\lambda^{V-A} | \Sigma_{Q_1}(P_1) \rangle \\ &\quad = -\tfrac{1}{2} b^* \bar{u}_2^\mu L_{\mu\nu} \gamma_\lambda (1-\gamma_5)(\gamma^\nu + v_1^\nu)\gamma_5 u_1, \end{aligned} \tag{66}$$

where $L_{\mu\nu}$ is defined by

$$(L_{\mu\nu})_\rho^{\ \alpha} = \int \mathrm{d}^4 k_1\, \mathrm{d}^4 k_2 \left(\bar{\phi}^{\Sigma^*}_{L\mu}\right)_\rho^{\ \alpha\beta\gamma} (\not{k}_1 - m_1)_\beta^{\ \beta'} (\not{k}_2 - m_2))_\gamma^{\ \gamma'} (\phi^\Sigma_\nu)_{\beta'\gamma'}. \tag{67}$$

Keeping in mind the fact that $\bar{u}_2^\mu$ is annihilated by γ_μ and $v_{2\mu}$ and $(\gamma^\nu + v_1^\nu)\gamma_5 u_1$ by $v_{1\nu}$ the most general expression for $L_{\mu\nu}$ in this case is

$$\begin{aligned} L_{\mu\nu} = 4\big(&-G_1^* g_{\mu\nu} + G_2^* v_{1\mu} v_{2\nu} + G_3^* g_{\mu\nu} \not{v}_1 \\ &+ G_4^* v_{1\mu} v_{2\nu} \not{v}_1 + G_5^* v_{1\mu} \gamma_\nu + G_6^* v_{1\mu} \gamma_\nu \not{v}_1 \big). \end{aligned} \tag{68}$$

Substituting this into eq. (66) gives us

$$\begin{aligned}
&\langle \Sigma_{q_2}(P_2) \,|\, J_\lambda^{V-A} \,|\, \Sigma_{Q_1}(P_1) \rangle \\
&= b^* \bar{u}_2^\mu \big\{ G_1^* g_{\mu\lambda}(1+\gamma_5) - G_3^* g_{\mu\lambda}(1-\gamma_5) \\
&\quad - \tfrac{1}{2}(G_1^* - \omega G_2^* - G_4^* - G_6^*) v_{1\mu}\gamma_\lambda(1-\gamma_5) \\
&\quad + \tfrac{1}{2}(G_2^* + G_3^* + \omega G_4^* + G_5^*)\gamma_\lambda(1+\gamma_5) \\
&\quad - (G_2^* + G_4^* + 2G_6^*) v_{1\mu} v_{2\lambda}(1+\gamma_5) \\
&\quad + (G_3^* - \omega G_4^* + G_5^*) v_{1\lambda} v_{1\mu}(1-\gamma_5) \\
&\quad - G_4^* v_{1\mu} v_{2\lambda}(1-\gamma_5) \big\} u_1 .
\end{aligned} \tag{69}$$

Here there are 6 independent form factors. The heavy to heavy limit ($m_{q_2} \to \infty$) can be obtained by setting

$$G_1^* = F_1, \qquad G_2^* = F_2, \qquad G_3^* = G_4^* = G_5^* = G_6^* = 0, \tag{70}$$

and the $1/m_{q_2}$ expansion goes as

$$G_1^* = F_1 + \mathrm{O}(1/m_{q_2}), \qquad G_2^* = F_2 + \mathrm{O}(1/m_{q_2}), \tag{71}$$

whereas the other form factors start at $\mathrm{O}(1/m_{q_2})$. Note that the leading terms of G_1^* and G_2^* are equal to the leading terms of G_1 and G_2.

(iv) $\Sigma_{Q_1}^*(\Omega_{Q_1}^*) \to \Sigma_{q_2}^*(\Omega_{q_2}^*)$, $\frac{3}{2}^+ \to \frac{3}{2}^+$ transition. A similar analysis leads to the form of the matrix element

$$\begin{aligned}
&\langle \Sigma_{q_2}^*(P_2) \,|\, J_\lambda^{V-A} \,|\, \Sigma_{Q_1}^*(P_1) \rangle \\
&= 4b^{**} \bar{u}_2^\mu \big\{ (-G_1^* g_{\mu\nu} + G_2^* v_{1\mu} v_{2\nu})\gamma_\lambda(1-\gamma_5) \\
&\quad - (G_3^* g_{\mu\nu} + G_4^* v_{1\mu} v_{2\nu})\gamma_\lambda(1+\gamma_5) \\
&\quad + 2(G_3^* g_{\mu\nu} + G_4^* v_{1\mu} v_{2\nu}) v_{1\lambda}(1-\gamma_5) \\
&\quad + 2G_5^* v_{1\mu} g_{\nu\lambda}(1-\gamma_5) + 2G_6^* v_{1\mu} g_{\nu\lambda}(1+\gamma_5) \big\} u_1^\nu
\end{aligned} \tag{72}$$

in terms of six independent form factors.

The form factors appearing here are the same as the $\frac{1}{2}^+ \to \frac{3}{2}^+$ case discussed above because the heavy baryons $\Sigma_{Q_1}^*$ and Σ_{Q_1} have the same structure functions ϕ^Σ.

In ref. [2], we have related the helicity amplitudes to the invariant form factors. Utilizing those relations we find that as $q^2 \to 0$

$$H_0^V / H_0^A = 1$$

in the heavy to heavy Λ baryon decays as well as in the heavy to light Λ decay, regardless of the proportion of the two independent form factors in the latter case. This predicts that the product Λ is 100% negatively longitudinally polarized. The heavy to heavy $\Omega(\frac{1}{2} \to \frac{1}{2})$ decay also has the same polarization at $q^2 = 0$. However, this value changes in the case of heavy to light Ω decays. The result is 60% longitudinal polarization with a correction proportional to the mass difference of initial Ω with product Ω'.

5. Conclusions

The situation regarding the counting of form factors for $\frac{1}{2}^+ \to \frac{1}{2}^+$ and $\frac{1}{2}^+ \to \frac{3}{2}^+$ is summarized in fig. 4 (for heavy to heavy) and fig. 5 (for heavy to light) transitions. The general form factor structure used in the first column is

$$\langle \tfrac{1}{2}^+, P_2 | J_\lambda^{V-A} | \tfrac{1}{2}^+, P_1 \rangle = \bar{u}(P_2)\left\{\gamma_\lambda\left(\tilde{F}_1^V + \tilde{F}_1^A\gamma_5\right) + v_{1\lambda}\left(\tilde{F}_2^V + \tilde{F}_2^A\gamma_5\right)\right.$$
$$\left. + v_{2\lambda}\left(\tilde{F}_3^V + \tilde{F}_3^A\gamma_5\right)\right\}u(P_1), \qquad (73)$$

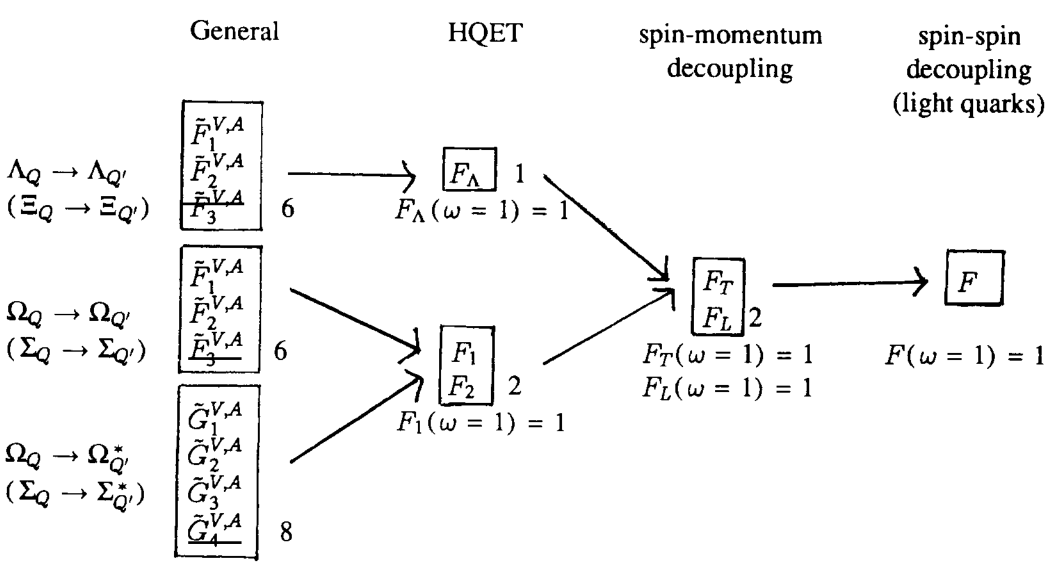

Fig. 4. Reduction of form factors in the heavy quark effective theory (HQET) for heavy to heavy transitions. Columns 3 and 4, dynamical approximation, is taken from ref. [5]. The numbers near the right-lower corner of each box are the number of independent form factors.

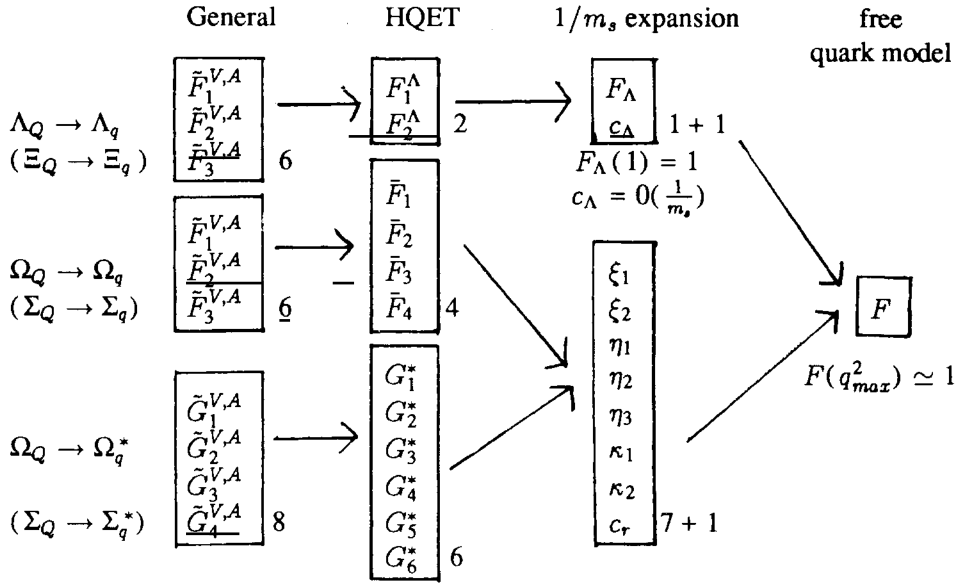

Fig. 5. Reduction of form factors in HQET for heavy to light transitions. Column 3, $1/m_s$ expansion, is taken from ref. [15] where the functions ξ_i, η_i are defined. The numbers near the right-lower corner of each box are the number of independent form factors.

$$\langle \tfrac{3}{2}^+, P_2 | J_\lambda^{V-A} | \tfrac{1}{2}^+, P_1 \rangle = \bar{u}^\nu(P_2)\Big\{ g_{\nu\lambda}\big(\tilde{G}_1^V + \tilde{G}_1^A\gamma_5\big) + v_{1\nu}\gamma_\lambda\big(\tilde{G}_2^V + \tilde{G}_2^A\gamma_5\big)$$
$$+ v_{1\nu}v_{1\lambda}\big(\tilde{G}_3^V + \tilde{G}_3^A\gamma_5\big) + v_{1\nu}v_{2\lambda}\big(\tilde{G}_4^V + \tilde{G}_4^A\gamma_5\big)\Big\}\gamma_5 u(P_2). \tag{74}$$

From figs. 4 and 5 one sees the dramatic reduction in the number of form factors in the limit $m_Q \to \infty$. It is not possible to go further with purely symmetry arguments. To make further headway it is necessary to include dynamics and to actually calculate the form factors. The present work provides a framework in which this can be done. In the BS approach advocated here the form factors, in both the heavy to heavy case and heavy to light case, are expressed as loop integrals. In principle, these integrals can be calculated given a certain dynamical scheme.

In this paper, we have presented the natural ansätze for the Bethe–Salpeter amplitudes for baryons within a covariant constituent quark model. These wave functions have allowed us to make certain predictions about the form factors occurring in semi-leptonic decays. However, these wave functions can also be used for example, in the analysis of non-leptonic decays and baryon–antibaryon production from e^+e^- annihilation [5].

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