$\Lambda_b \to p\mu\nu$ Form Factors

William Sutcliffe

Imperial College London

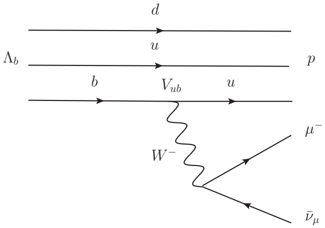
July 23, 2013

$\Lambda_b \to p\mu\nu$ Form Factors

The tree-level matrix element for this decay may be written as:

$$\mathcal{M} = -i\frac{G_F}{\sqrt{2}} V_{ub} H_\nu \overline{u}_\mu \gamma^\nu (1-\gamma_5) v_{\nu_\mu}$$

where $H_\nu = < N^+(p', s')|\overline{u}\gamma_\nu(1-\gamma_5)b|\Lambda_b^0(p,s) >$



- In general for baryon to baryon V-A transitions: $H_\nu = \overline{u}_N(p')[F_1^V\gamma_\nu + F_2^V v_\nu + F_3^V v'_\nu - (F_1^A\gamma_\nu + F_2^A v_\nu + F_3^A v'_\nu)\gamma_5]u_{\Lambda_b}(p)$
- This convention is used by EvtGen. Current papers presenting predictions for $\Lambda_b \to p$ form factors use different conventions.

For instance A. Khodjamirian et al. in arXiv:1108.2971 (LCSR predictions) use:

dimensionless. The hadronic matrix elements with the vector and axial-vector transition currents contain three form factors each:

$$\langle \Lambda_c(P-q)|\,\bar{c}\,\gamma_\mu\,u\,|N(P)\rangle = \bar{u}_{\Lambda_c}(P-q)\left\{f_1(q^2)\,\gamma_\mu + i\frac{f_2(q^2)}{m_{\Lambda_c}}\,\sigma_{\mu\nu}q^\nu + \frac{f_3(q^2)}{m_{\Lambda_c}}\,q_\mu\right\}u_N(P)\,, \tag{11}$$

$$\langle \Lambda_c(P-q)|\,\bar{c}\,\gamma_\mu\gamma_5\,u\,|N(P)\rangle = \bar{u}_{\Lambda_c}(P-q)\left\{g_1(q^2)\,\gamma_\mu + i\frac{g_2(q^2)}{m_{\Lambda_c}}\,\sigma_{\mu\nu}q^\nu + \frac{g_3(q^2)}{m_{\Lambda_c}}\,q_\mu\right\}\gamma_5 u_N(P)\,. \tag{12}$$

Meanwhile W. Detmold, S. Meinel et al. in arXiv:1306.0446 (LQCD predictions) use the form:

The $\Lambda_b \to p$ matrix elements of the vector and axial vector $b \to u$ currents are parametrized in terms of six independent form factors (see, e.g., Ref. [8]). In leading-order heavy-quark effective theory (HQET), which becomes exact in the limit $m_b \to \infty$ and is a good approximation at the physical value of m_b, only two independent form factors remain, and the matrix element with arbitrary Dirac matrix Γ in the current can be written as [8–10]

$$\langle N^+(p',s')|\, \bar{u}\Gamma Q\, |\Lambda_Q(v,s)\rangle = \overline{u}_N(p',s')\, [F_1 + \not{v}\, F_2]\, \Gamma\, u_{\Lambda_Q}(v,s). \tag{3}$$

- Use HQET under which only 2 form factors are required.

Also include next to first order loop corrections:

where A_μ is the hadronic matrix element

$$A_\mu = \langle N^+(p',s')|\,\bar{u}\gamma_\mu(1-\gamma_5)b\,|\Lambda_b(p,s)\rangle. \tag{31}$$

Because we have computed the form factors in HQET, we need to match the QCD current $\bar{u}\gamma_\mu(1-\gamma_5)b$ in Eq. (31) to the effective theory. This gives (at leading order in $1/m_b$)

$$A_\mu = \sqrt{m_{\Lambda_b}}\langle N^+(p',s')|\big(c_\gamma\,\bar{u}\gamma_\mu Q + c_v\,\bar{u}v_\mu Q - c_\gamma\,\bar{u}\gamma_\mu\gamma_5 Q + c_v\,\bar{u}v_\mu\gamma_5 Q\big)|\Lambda_Q(v,s)\rangle, \tag{32}$$

where Q is the static heavy-quark field, and to one loop, the matching coefficients are given by [40]

$$c_\gamma = 1 - \frac{\alpha_s(\mu)}{\pi}\left[\frac{4}{3} + \ln\left(\frac{\mu}{m_b}\right)\right], \tag{33}$$

$$c_v = \frac{2}{3}\frac{\alpha_s(\mu)}{\pi}. \tag{34}$$

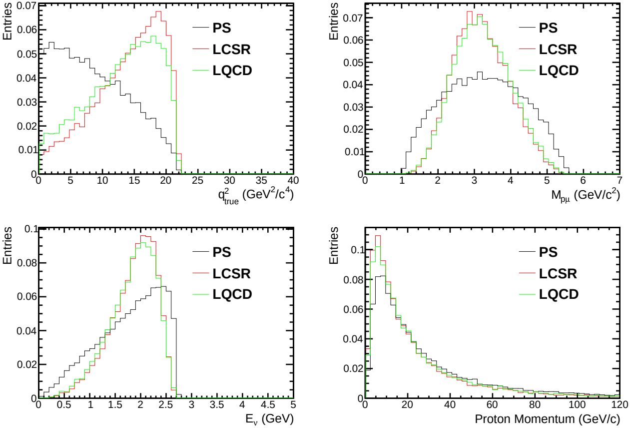
Here we set $\mu = m_b$. We can now use Eq. (3) to express the matrix element A_μ in terms of the form factors F_1 and F_2:

$$A_\mu = \bar{u}_N(p',s')\Big(F_1 + \not{v}F_2\Big)\Big(c_\gamma\gamma_\mu + c_v\,v_\mu - c_\gamma\,\gamma_\mu\gamma_5 + c_v\,v_\mu\gamma_5\Big)\sqrt{m_{\Lambda_b}}u_{\Lambda_Q}(v,s). \tag{35}$$

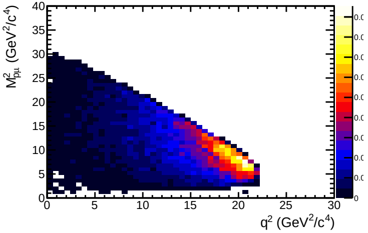
- Need to match the form factors calculated in arXiv:1108.2971 (LCSR predictions) and arXiv:1306.0446 (LQCD predictions) to convention used by EvtGen.
- Contacted Stefan Meinel (theorist at MIT) and he kindly provided me with a number of derivations relating the various from factors.

Generator level MC

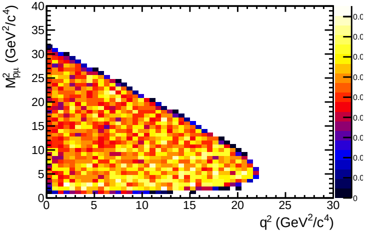
Now compare generator level distributions for LQCD, LCSR and Phase Space predictions:



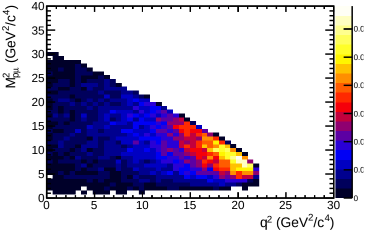
LCSR



Phase Space

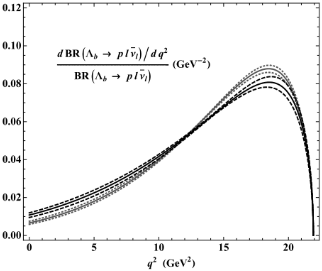


LQCD

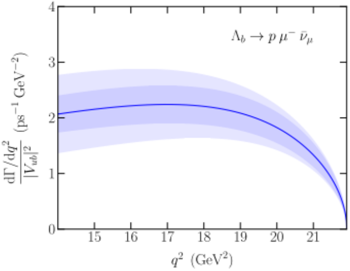


- Consistency check for these distributions?
- Can at least compare the q^2 distributions with the predicted differential rate, which is derived in terms of the various form factors in both papers.

LCSR, arXiv:1108.2971
A. Khodjamirian et al.



LQCD, arXiv:1306.0446
W. Detmold et al.



For LQCD:

the spinor $u_{\Lambda_b}(p,s)$ has the standard relativistic normalization. A straightforward calculation then gives the following differential decay rate,

$$\frac{d\Gamma}{dq^2} = \frac{|V_{ub}|^2 G_F^2}{768\pi^3 q^6 m_{\Lambda_b}^5}(q^2 - m_\ell^2)^2 \sqrt{((m_{\Lambda_b}+m_N)^2 - q^2)((m_{\Lambda_b}-m_N)^2-q^2)}$$
$$\times \left[(4c_\gamma^2 + 4c_\gamma c_v + 2c_v^2) m_\ell^2 \mathcal{F}\mathcal{I} + \left(2c_\gamma^2(\mathcal{I} + 3q^2 m_{\Lambda_b}^2) + c_v(2c_\gamma + c_v)(\mathcal{I} - 3q^2 m_{\Lambda_b}^2) \right) q^2 \mathcal{F} + 4c_\gamma(c_\gamma + c_v)\mathcal{K} \right], \quad (36)$$

where we have defined the combinations

$$\mathcal{F} = ((m_{\Lambda_b}+m_N)^2 - q^2)F_+^2 + ((m_{\Lambda_b}-m_N)^2 - q^2)F_-^2, \quad (37)$$
$$\mathcal{I} = m_{\Lambda_b}^4 - 2m_N^2(m_{\Lambda_b}^2 + q^2) + q^2 m_{\Lambda_b}^2 + m_N^4 + q^4, \quad (38)$$
$$\mathcal{K} = (2m_\ell^2 + q^2)((m_{\Lambda_b}+m_N)^2 - q^2)((m_{\Lambda_b}-m_N)^2 - q^2)(m_{\Lambda_b}^2 - m_N^2 + q^2)F_+F_-, \quad (39)$$

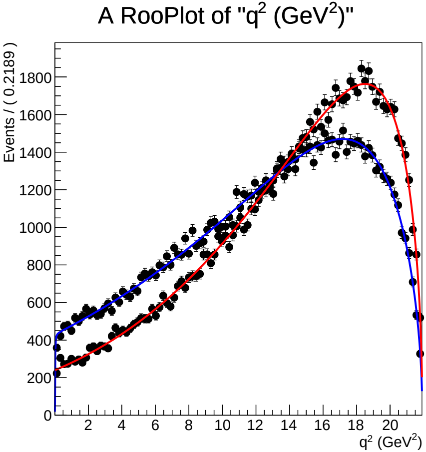
For LCSR:

$$\frac{d\Gamma}{dq^2}(\Lambda_b \to p l \nu_l) = \frac{G_F^2 m_{\Lambda_b}^3}{192\pi^3}|V_{ub}|^2 \lambda^{1/2}(1,r^2,t)\Big\{ [(1-r)^2 - t][(1+r)^2 + 2t]|f_1(q^2)|^2$$
$$+[(1+r)^2 - t][(1-r)^2 + 2t]|g_1(q^2)|^2 - 6t[(1-r)^2 - t](1+r)f_1(q^2)f_2(q^2)$$
$$-6t[(1+r)^2 - t](1-r)g_1(q^2)g_2(q^2) + t[(1-r)^2 - t][2(1+r)^2 + t]|f_2(q^2)|^2$$
$$+t[(1+r)^2 - t][2(1-r)^2 + t]|g_2(q^2)|^2 \Big\}, \quad (63)$$

where $r = m_N/m_{\Lambda_b}$, $t = q^2/m_{\Lambda_b}^2$ and $\lambda(a,b,c) = a^2+b^2+c^2-2ab-2ac-2bc$. Substituting the form factors (62) and integrating over q^2 we obtain the total branching fraction

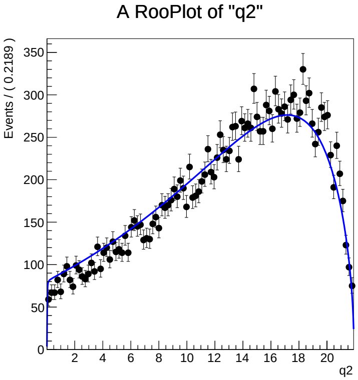
- Make 2 new RooPdf classes for LCSR and LQCD differential BF predictions.
- Can then fit these to the generator level q^2 distribution.
- Only leave normalisation free.

- Generate 100,000 MC toys using the theoretically predicted differential BF.
- Here LCSR (red) and LQCD (blue):

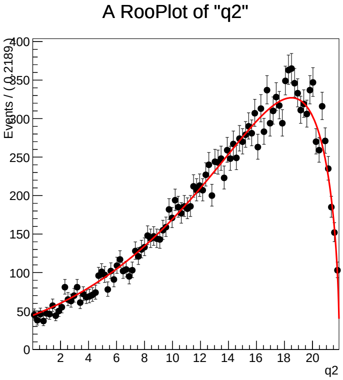


Fit generator level data:

LQCD



LCSR



Conclusion

- Thanks to Stefan Meinel have relations between the form factors defined by EvtGen and those defined in papers.
- Plots seem to make sense.
- However, LQCD fit is not perfect. Further checks + greater statistics required.
- Need to get changes into EvtGen soon so I can get new and improved MC