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On heavy baryon decay form factors

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Abstract. We consider in detail the consequences of the heavy quark mass limit in the weak decay of $\frac{1}{2}^+$ heavy baryons to $\frac{1}{2}^+$ and $\frac{3}{2}^+$ heavy baryon final states. We also analyze heavy baryon to light baryon transitions as well as e^+e^--annihilation into heavy baryon–antibaryon-pairs. We discuss possible approximations to the most general approach and some of their implications for future experiments.

1 Introduction

Recently, heavy flavour decays have received a great deal of attention. In [1] we demonstrated how the equal velocity assumption, arising from the heavy quark limit, can be formulated in a covariant manner using the old $U(12)$ wave functions of Delbourgo, Salam and Strathdee.

In this paper we consider in detail the form factors arising in the decay of $\frac{1}{2}^+$ heavy baryons to $\frac{1}{2}^+$ and $\frac{3}{2}^+$ heavy baryons where the underlying transition is from a heavy quark to another heavy quark. We also comment on the form factors in heavy to light baryon transitions. In Sect. 2, we review the covariant baryon wave functions for $\frac{1}{2}^+$ Λ-type and $\frac{1}{2}^+$ and $\frac{3}{2}^+$ Σ-type heavy baryons. In Sect. 3 we evaluate weak decay matrix elements for heavy baryon to heavy baryon transitions. By analyzing the spin structure of the heavy-light bound state wavefunctions we find that three independent universal form factors are needed to describe current-induced heavy baryon transitions. We discuss two approximation schemes where the number of independent form factors is reduced to two and one, respectively. We briefly comment on how our derivations are related to the Bloch–Nordsieck approximation to QCD as outlined in [3]. In Sect. 4 we discuss heavy to light baryon transitions. Section 5 is devoted to an application of these ideas to $e^+e^- \to$ heavy baryon pairs. Section 6, finally, contains a summary and our conclusions.

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2 Baryon wave functions

In a recent paper [1], we presented a covariant formulation of current-induced transitions among heavy-light hadrons in the limit where the mass of the heavy quark becomes large. In this paper, we consider in detail the baryon form factors arising in this formulation.

We begin by briefly reviewing the covariant formulation presented in [1]. In the heavy quark mass limit, the heavy quark spin is decoupled from the light degrees of freedom and the heavy quark moves like a free particle with the same velocity as the baryon [2]. Since the heavy quark is free we can embed its spin into the $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ representation of the Lorentz group in the standard way, i.e. in a four-dimensional space of Dirac indices [1, 4–7]. Thus we can write the spin and momentum part of the s-wave baryon wave function covariantly as a three index spinor $B_{\alpha\beta\gamma}(P, p_Q, k_i)$, where α is the Dirac index for the heavy quark, P is the momentum of the baryon, p_Q is the momentum of the heavy quark and k_i are the momenta of the light quarks. β, γ are Dirac indices representing the light degrees of freedom. We are ignoring flavour and colour for the moment. These will be reinstated later. One requires at least three Dirac indices to describe a baryon because of the direct product of the quark number $U(1)$ gauge group with the Lorentz group (see [1]).

To describe a baryon of definite spin we need to reduce the large number of degrees of freedom contained in $B_{\alpha\beta\gamma}$ which is a direct product of three $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ representations of the Lorentz group. Thus, to describe a spin $\frac{1}{2}$ particle we require a mixed symmetry Dirac tensor with the fully antisymmetric part excluded (see [1, 4]). The spin $\frac{3}{2}$ baryon is described by the fully symmetric Dirac tensor. Now, since α is the Dirac label for the heavy quark, which is on-shell, we must have

$$\left(\frac{\not p_Q}{m_Q} - 1\right)_\alpha^{\alpha'} B_{\alpha'\beta\gamma} = 0. \tag{1}$$

But, since in the heavy mass limit, the heavy quark moves with the same velocity as the baryon, $p_Q/m_Q = P/M$, we obtain the Bargmann–Wigner equations

$$(\not P - M)_\alpha^{\alpha'} B_{\alpha'\beta\gamma} = 0, \tag{2}$$

where m_Q and M are the masses of the heavy quark and baryon, respectively.

Then the requirements of symmetry and the Bargmann–Wigner equations (2) lead to the following wave functions:

$J^P = \frac{1}{2}^+$:

$$B_{\mathscr{A}_i;\mathscr{B}_j;\mathscr{C}_k} = \frac{1}{\sqrt{3}M}\frac{\varepsilon_{ijk}}{\sqrt{6}}\{[(\not P + M)\gamma_5 C]_{\delta\sigma} u_\alpha(P) B_{a[bc]} + \text{cycl.}(\alpha, a; \delta, b; \sigma, c)\} A^{\delta\sigma}_{\beta\gamma}(k_1, k_2)$$
$$\equiv \frac{\varepsilon_{ijk}}{\sqrt{6}} \chi^{(\frac{1}{2})}_{\alpha\delta\sigma;abc} A^{\delta\sigma}_{\beta\gamma}(k_1, k_2). \tag{3}$$

$J^P = \frac{3}{2}^+$:

$$B_{\mathscr{A}_i;\mathscr{B}_j;\mathscr{C}_k} = \frac{1}{\sqrt{3}M}\frac{\varepsilon_{ijk}}{\sqrt{6}}\{[(\not P + M)\gamma_\mu C]_{\delta\sigma} u^\mu_\alpha(P) B_{\{abc\}} + \text{cycl.}(\alpha, a; \delta, b; \sigma, c)\} A^{\delta\sigma}_{\beta\gamma}(k_1, k_2)$$
$$\equiv \frac{\varepsilon_{ijk}}{\sqrt{6}} \chi^{(\frac{3}{2})}_{\alpha\delta\sigma;abc} A^{\delta\sigma}_{\beta\gamma}(k_1, k_2), \tag{4}$$

where $\mathscr{A} \mathrel{\hat{=}} (\alpha, a)$ etc. represents the Dirac and flavour labels α and a, respectively. In (3) and (4) the heavy quark label is denoted by $\mathscr{A}$. The colour labels, finally, are denoted by i, j, k. $u_\alpha(P)$ is the usual Dirac spinor and $u^\mu_\alpha(P)$ is the spin $\frac{3}{2}$ Rarita–Schwinger spinor. $B_{a[bc]}$ and $B_{\{abc\}}$ are the mixed symmetric and fully symmetric flavour wave functions. The projections $\chi^{(\frac{1}{2})}_{\alpha\delta\sigma;abc}$ and $\chi^{(\frac{3}{2})}_{\alpha\delta\sigma;abc}$ are fully symmetric under the simultaneous interchange of flavour and Dirac indices. $A^{\delta\sigma}_{\beta\gamma}$ is a fourth rank Dirac tensor function of the light quark momenta k_1, k_2. It transforms like a scalar under the Lorentz group so that it does not change the spin-parity structure represented by the projectors $\chi^{(\frac{1}{2})}_{\alpha\delta\sigma;abc}$ and $\chi^{(\frac{3}{2})}_{\alpha\delta\sigma;abc}$. For example, $A^{\delta\sigma}_{\beta\gamma}$ can be $(\not k_1)^\delta_\beta (\not k_2)^\sigma_\gamma$ but not $(\gamma_5)^\delta_\beta (\not k_1)^\sigma_\gamma$, etc. In the simplest case $A^{\delta\sigma}_{\beta\gamma}$ could be chosen to be a scalar function as in [8], but to start with we shall discuss the most general situation.

In the limit of a large heavy quark mass, the mass of the baryon tends to the mass of the heavy quark. Thus in (3) and (4) the mass of the $(\frac{1}{2}^+)$ $\Lambda_Q(Q[qq'])$ and $\Sigma_Q(Q\{qq'\})$ type baryons and the $(\frac{3}{2}^+)$ Σ^*_Q baryons are degenerate at this scale. In fact, if we ignore the mass difference of the s and u, d quarks relative to the scale of the heavy quark mass then the $\Omega_Q(Qss)$ and Ω^*_Q are also degenerate in mass with the above baryons.

Recall that in the Λ_Q-type baryons the light diquark is in a spin $s = 0$ state while in the Σ_Q, Σ^*_Q and Ω_Q, Ω^*_Q baryons the light diquarks are in $s = 1$ states. We can rewrite the spin $\frac{1}{2}^+$ wave function, (3), in a form which clearly distinguishes the $(\frac{1}{2}^+)$ Λ_Q-type and the $\Sigma_Q(\Omega_Q)$-type baryons:

$$B_{\mathscr{A}_i;\mathscr{B}_j;\mathscr{C}_k} = \frac{1}{\sqrt{3}M}\frac{\varepsilon_{ijk}}{\sqrt{6}}\{\tfrac{3}{2}[(\not P + M)\gamma_5 C]_{\delta\sigma} u_\alpha(P) B_{a[bc]} + \tfrac{1}{2}[[(\not P + M)\gamma_5 C]_{\alpha\delta} u_\sigma(P) - [(\not P + M)\gamma_5 C]_{\sigma\alpha} u_\delta(P)][B_{c[ab]} - B_{b[ca]}]\} A^{\delta\sigma}_{\beta\gamma} \tag{5}$$
$$= \frac{\varepsilon_{ijk}}{\sqrt{6}}\{\chi^{\Lambda}_{\alpha\delta\sigma;abc} + \chi^{\Sigma}_{\alpha\delta\sigma;abc}\} A^{\delta\sigma}_{\beta\gamma}. \tag{6}$$

In (5), the first term clearly represents the Λ-type baryon. It is anti-symmetric in the light $\delta - \sigma$ indices as required from the fact that the light diquark is in an antisymmetric $s = 0$ state. The flavour function $\frac{3}{2}B_{a[bc]}$ is also in an antisymmetric state in the indices b, c which corresponds to $I = 0$ when the indices b and c represent u and d quarks. The last two terms together represent the Σ-type baryon which is symmetric in the light indices corresponding to the symmetric nature of the $s = 1$ diquark state. The combination $\frac{1}{2}[B_{c[ab]} - B_{b[ca]}]$ is simply the symmetric $I = 1$ flavour wave function when b, c represent u and d quarks.

3 Heavy baryon weak decays

The transition of a heavy baryon to another heavy baryon via a flavour changing weak current, as shown in Fig. 1, is given by the expression

$$\langle B_2(P_2)|J^{V-A}_\mu|B_1(P_1)\rangle = \int d^4k_1 d^4k_2 \bar{A}^{\beta\gamma}_{\delta\sigma} \bar{\chi}^{\alpha\delta\sigma;abc}(P_2) W^{a'}_a [\gamma_\mu(1-\gamma_5)]^{\alpha'}_\alpha (\not k_1 - m_1)^{\beta'}_\beta \cdot (\not k_2 - m_2)^{\gamma'}_\gamma \chi_{\alpha'\delta'\sigma';a'bc}(P_1) A^{\delta'\sigma'}_{\beta'\gamma'}(k_1, k_2). \tag{7}$$

Here J^{V-A}_μ is the flavour changing weak $(V-A)$-current★ and $W^{a'}_a$ is the appropriate Kobayashi–Maskawa [9] matrix element for the transition from the heavy quark Q_a to another heavy quark Q'_a.

In (7)

$$\bar{A}^{\beta\gamma}_{\delta\sigma} = (\gamma_0)^{\delta'}_\delta (\gamma^T_0)^\beta_{\beta'} A^{\dagger\beta'\gamma'}_{\delta'\sigma'} (\gamma_0)^\gamma_{\gamma'} (\gamma^T_0)^{\sigma'}_\sigma$$

and

$$\bar{\chi}^{\alpha\delta\sigma;abc} = \chi^{\dagger\alpha'\delta'\sigma';abc} (\gamma_0)^\alpha_{\alpha'} (\gamma_0)^\delta_{\delta'} (\gamma_0)^\sigma_{\sigma'}. \tag{8}$$

Here χ can be either of the two spin-parity projectors $\chi^{(\frac{1}{2})}$ or $\chi^{(\frac{3}{2})}$ defined in (3) and (4), respectively.

Clearly, the projectors $\chi(P_1)$ and $\bar{\chi}(P_2)$ do not depend on the integration variables and can thus be taken out of the integral (7). One then has

$$\langle B_2(P_2)|J^{V-A}_\mu|B_1(P_1)\rangle = \bar{\chi}^{\alpha\delta\sigma;abc}(P_2) W^{a'}_a [\gamma_\mu(1-\gamma_5)]^{\alpha'}_\alpha \chi_{\alpha'\delta'\sigma';a'bc}(P_1) \cdot \int d^4k_1 d^4k_2 \bar{A}^{\beta\gamma}_{\delta\sigma} (\not k_1 - m_1)^{\beta'}_\beta (\not k_2 - m_2)^{\gamma'}_\gamma A^{\delta'\sigma'}_{\beta'\gamma'}(k_1, k_2). \tag{9}$$

Most generally the integral in (9) is a fourth rank Dirac tensor, $I^{\delta'\sigma'}_{\delta\sigma}$, which can only depend on the external momenta and which must be a scalar under Lorentz transformations. Because of the projectors $\chi(P_1)$ and $\bar{\chi}(P_2)$ all factors of $\not P_1$ and $\not P_2$ in $I^{\delta'\sigma'}_{\delta\sigma}$ can be replaced by the respective masses. Thus the integral in general can only be a linear combination of $\mathbb{1} \otimes \mathbb{1}, \gamma_\mu \otimes \gamma^\mu, \gamma_5 \otimes \gamma_5, \gamma_5\gamma_\mu \otimes \gamma_5\gamma^\mu, \sigma_{\mu\nu} \otimes \sigma^{\mu\nu}$, which we call S, V, P, A and T couplings of the spins of the two spectator systems.

However, because of the symmetry present in the Λ and Σ type projectors, (5), we do not have five independent form-factors. The five Dirac tensors can be divided into

★ It is straightforward to include renormalization effects for the heavy quark current in (7) via leading log summation techniques [2]. They lead to scale and velocity dependent logarithmic corrections to the bare current in (7)

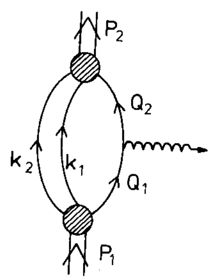

Fig. 1. Two-loop Feynman diagram for current-induced baryon transitions. The Q_i and the k_i denote heavy and light quark momenta. The hatched vertices are determined from the bound state wave functions of the baryons

three antisymmetric and two symmetric Dirac tensors under Fierz exchange:

$$\text{Antisymmetric:}\quad \begin{array}{l} 2S-\frac{1}{2}T+A \\ V+A \\ S-V-P. \end{array} \tag{10}$$

$$\text{Symmetric:}\quad \begin{array}{l} 2S+V-A-2P \\ 3S+\frac{1}{2}T+3P. \end{array} \tag{11}$$

The three antisymmetric Dirac tensors (10) contribute to the $\Lambda_{Q_1}\to\Lambda_{Q_2}$ decay. However, it is easy to see from the general form of the trace in (9) that there is just one form factor regardless of the composition of the three antisymmetric spinor tensors. For the $\Lambda_{Q_1}\to\Lambda_{Q_2}$ transition one then has (including the flavour factors):

$$\begin{aligned}&\langle\Lambda_{Q_2}(P_2)|J_\mu^{V-A}|\Lambda_{Q_1}(P_1)\rangle\\ &\quad=\bar{u}^\alpha(\gamma_\mu(1-\gamma_5))_\alpha^{\alpha'}u_{\alpha'}\int \mathrm{d}^4k_1\,\mathrm{d}^4k_2[C^{-1}\gamma_5(\not{P}_2+M_2)]^{\delta\sigma}\\ &\quad\cdot\bar{A}^{\beta\gamma}_{\delta\sigma}(\not{k}_1-m_1)_\beta^{\beta'}(\not{k}_2-m_2)_\gamma^{\gamma'}A^{\delta'\sigma'}_{\beta'\gamma'}[(\not{P}_1+M_1)\gamma_5C]_{\delta'\sigma'}\\ &\quad=F_\Lambda(q^2)\bar{u}(P_2)\gamma_\mu(1-\gamma_5)u(P_1),\end{aligned}\tag{12}$$

where $F_\Lambda(q^2)$ includes contribution from all three antisymmetric spinor tensors. Note that identical results hold for the transition $\Xi_{Q[us]}\to\Xi_{Q'[us]}$ where the $[us]$ light quark system is in a scalar diquark state.

The two symmetric Dirac tensors contribute to the $\Sigma_{Q_1}\to\Sigma_{Q_2}$ and $\Sigma_{Q_1}\to\Sigma^*_{Q_2}$ decays. Therefore, in general, there will be two independent form factors. Instead of choosing the two symmetric combinations (11) it is sufficient to consider any two of the five spin couplings. The contributions from the others are just linear combinations of these two as the three zero combinations (10) show. The symmetry of the wavefunction projects out the relevant part. The simplest and physically motivated (see below) choices are S and V. Denoting the independent form factor functions associated with these two contributions as $f(q^2)$ and $g(q^2)$ we find the matrix element, for example, for $\Sigma_b\to\Sigma_c$ to be

$$\begin{aligned}&\langle\Sigma_c(P_2)|J_\mu^{V-A}|\Sigma_b(P_1)\rangle\\ &\quad=-\frac{V_{bc}}{6}\bar{u}(P_2)\Bigg[\{(w+1)f+2gw\}\gamma_\mu(1-\gamma_5)\\ &\qquad-4(f+g)\left(\frac{P_{1\mu}}{M_1}+\frac{P_{2\mu}}{M_2}\right)+4g\left(\frac{P_{1\mu}}{M_1}-\frac{P_{2\mu}}{M_2}\right)\gamma_5\Bigg]u(P_1),\end{aligned}\tag{13}$$

where $w=P_1\cdot P_2/M_1M_2$ and V_{bc} is the $b\to c$ Kobayashi–Maskawa matrix element.

Similarly for $\Sigma_b\to\Sigma_c^*$ one has:

$$\begin{aligned}&\langle\Sigma_c^*(P_2)|J_\mu^{V-A}|\Sigma_b(P_1)\rangle\\ &\quad=\frac{1}{\sqrt{3}}V_{bc}\bar{u}^\nu(P_2)\Bigg[\{f(w+1)+2g\}g_{\mu\nu}(1+\gamma_5)\\ &\qquad-(f+g)\frac{P_{1\nu}}{M_1}\gamma_5\gamma_\mu-g\frac{P_{1\nu}}{M_1}\gamma_\mu\\ &\qquad-f\frac{P_{1\nu}P_{2\mu}}{M_1M_2}(1+\gamma_5)\Bigg]u(P_1).\end{aligned}\tag{14}$$

An explicit calculation shows that the same two forms (13) and (14) also describe $\Omega_b\to\Omega_c$ and $\Omega_b\to\Omega_c^*$ decays, respectively, where the $\{ss\}$-light diquark system in the Ω's is in a vector diquark state.

Two dynamical approximations to the most general ansatz (9) come to mind:

1. Decoupling of orbital and spin degrees of freedom of the light quark systems:

$$A^{\gamma\delta}_{\alpha\beta}(k_1,k_2)=A(k_1,k_2)\delta^\gamma_\alpha\delta^\delta_\beta.\tag{15}$$

From the general form (9) one deduces that such an ansatz leads to the contribution of only two of the 5 possible spinor forms coupling the spectator spins, namely to S and V leading to the form factor contributions denoted by f and g. In this approximation the Λ-type form factor is related to the same universal form factors f and g appearing in the Σ-type form factor expressions (13) and (14):

$$F_\Lambda(w)=\tfrac{1}{2}[(w+1)f+(4-2w)g].\tag{16}$$

2. Independent quark motion

$$A^{\delta\sigma}_{\beta\gamma}(k_1,k_2)=A_1(k_1)^\delta_\beta A_2(k_2)^\sigma_\gamma.\tag{17}$$

In this approximation the overlap integral (9) factorizes into two pieces. Noting again that any contributions proportional to $\not{P}_1,\not{P}_2$ and $\not{P}_1\not{P}_2$ can be replaced by the respective masses due to the spin-parity projectors χ in (9) one arrives at the simple final result that there is no spin coupling among the spectator quark systems. The resultant form factors are then obtained from (13), (14) and (16) by setting $g=0$. Note also that this approximation corresponds to the results of using the $U(12)$ approach [4] and the quark model approach of [8]. One attractive feature of this approximation is the result that the baryon form factor can be considered as the square of the meson form factor [3]. We have used this approximation, with $g=0$, in two previous papers [10, 11] to make detailed numerical predictions for charm and bottom semi-leptonic decays.

Let us now discuss the normalization of the form factors f and g at $q^2_{\max}$ or $w=1$. At $q^2_{\max}$ the heavy daughter quark is at rest in the rest-frame of the decaying quark. In the limit of heavy quark mass the interaction of the light quarks with the heavy quark is independent of the mass and flavour of the heavy quark which acts as a static colour source [2]. Because of this there is complete overlap of the light degrees of freedom at $q^2_{\max}$ and, therefore, it is clear that the form factors which survive at $q^2_{\max}$ should be normalized to unity. This of course means that the unique form factor $F_\Lambda(q^2)$ in Λ decays is normalized to 1. For Σ_b and Ω_b decays it is easy to see from (13) that in the limit $q^2 = q^2_{\max}(w=1)$, only the combination $f+g$ survives. By analyzing the zero component of the vector current as described below one finds the normalization condition

$$(f+g)(q^2_{\max}) = 1. \tag{18}$$

Put in terms of group theory, the normalization at $q^2_{\max}$ is a consequence of the fact that the zero component of the heavy quark vector current generates an $SU(F)$ symmetry in the limit of the heavy quark masses becoming large, where F is the number of heavy flavours. The generators of this symmetry are

$$Q_i = \int d^3x\, \bar{q}(x)\gamma_0 \frac{\lambda_i}{2} q(x), \tag{19}$$

where λ_i is the $F \times F$ adjoint matrix representation of $SU(F)$, $i = 1, \ldots, F^2-1$, and $q(x)$ are the local, heavy quark fields.

It is quite instructive to decompose the transition of the diquark system from the velocity $v_1 = P_1/M_1$ to the velocity $v_2 = P_2/M_2$ in terms of it's s-wave and d-wave components. The diquark excitation may be viewed as a process where a $({}^3P_0)$-spurion with J^P quantum numbers 0^+ and a $(s=0,1)$-diquark make a transition to a $(s=0,1)$-diquark with the J^P quantum numbers $0^+ + 0^+ \to 0^+$ and $0^+ + 1^+ \to 1^+$. For the Λ-type transition the form factor $F_\Lambda(w)$ in (12) is nothing but the s-wave transition amplitude of the scalar diquark.

For the Σ-type transition one has

$$\begin{pmatrix} F_s \\ F_d \end{pmatrix} = \frac{1}{6}\begin{pmatrix} 3(w+1) & 2w+4 \\ 0 & -2\sqrt{2}(w-1) \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix}, \tag{20}$$

where F_s and F_d denote the s-wave and d-wave amplitudes for the vector diquark transitions. f and g are the amplitudes arising from the S and V couplings, respectively. As $q^2 \to q^2_{\max}$ the d-wave amplitude vanishes, as expected, with the second power of the CM momentum $((w-1) \propto p^2)$. We have chosen our s-wave amplitude normalization such that, at $w=1$, $F_s = f+g = 1$.

Alternatively, the diquark transition can be parametrized in terms of longitudinal and transverse transition amplitudes F_L and F_T describing the helicity $0 \to 0$ and helicity $1 \to 1$ transitions of the vector diquark system. Let us define these as

$$\begin{aligned} F_L &= F_s - \sqrt{2}F_d, \\ F_T &= F_s + \frac{1}{\sqrt{2}}F_d, \end{aligned} \tag{21}$$

and

$$\begin{pmatrix} F_L \\ F_T \end{pmatrix} = \frac{1}{2}\begin{pmatrix} w+1 & 2w \\ w+1 & 2 \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix}, \tag{22}$$

and at $w=1$, $F_L = F_T = f+g=1$.

It is useful to rewrite the $\frac{1}{2}^+ \to \frac{1}{2}^+$ and $\frac{1}{2}^+ \to \frac{3}{2}^+$ transitions in terms of the standard covariants as defined, for example, in [1] and [10]. The $\frac{1}{2}^+ \to \frac{1}{2}^+$ matrix element, including the Λ-type decay, can be written as:

$$\begin{aligned} &\langle B_2, \tfrac{1}{2}^+ | J_\mu^{V-A} | B_1, \tfrac{1}{2}^+ \rangle \\ &= \frac{1}{4M_1M_2}\bar{u}(P_2)[\gamma_\mu\{f(b(M_1+M_2)^2 - b'q^2) \\ &\quad + g(b(q^2+4M_1M_2) + b'(q^2 - 2(M_1-M_2)^2))\} \\ &\quad + i(f+g)(M_1+M_2)(b-b')\sigma_{\mu\nu}q^{\nu} \\ &\quad - (f+g)(M_1-M_2)(b-b')q_\mu \\ &\quad - \gamma_\mu\gamma_5\{f((M_1+M_2)^2-q^2)b' \\ &\quad + g(bq^2 + b'(q^2 - 2(M_1-M_2)^2 + 4M_1M_2))\} \\ &\quad + g(b-b')((M_1-M_2)i\sigma_{\mu\nu}q^\nu\gamma_5 \\ &\quad - (M_1+M_2)q_\mu\gamma_5)]u(P_1), \end{aligned} \tag{23}$$

where the flavour factors b and b' are given by

$$\begin{aligned} b &= \tfrac{1}{3}W_c^{c'}(\tfrac{3}{2}\bar{B}_2^{c[ab]}B_{1c'[ab]} + 3\bar{B}_2^{a[bc]}B_{1a[bc']}), \\ b' &= \tfrac{1}{3}W_c^{c'}(\tfrac{5}{2}\bar{B}_2^{c[ab]}B_{1c'[ab]} - \bar{B}_2^{a[bc]}B_{1a[bc']}). \end{aligned} \tag{24}$$

In the language of antisymmetric F-type and symmetric D-type couplings, the combination b in (23) is F-type and the combination $b' - \frac{2}{3}b$ is D-type. Tables of the relevant flavour coupling factors can be found in [10]. For the Λ-type cases one has $b = b' = 1$ and for the Σ-type cases $b=1$ and $b' = -\frac{1}{3}$. With a little algebra one then recovers (12) and (13) from (23). We have written (23) using standard covariants in order to be able to use the decay formula of [10].

We emphasize that, in the general case, (23) has to be separately considered for the Λ-type and Σ-type configuration with their respective form factors F_Λ and f, g. The Λ-type form factor is related to the Σ-type form factor only in the approximation (15) as indicated in (16) and (23).

Similarly the $\frac{1}{2}^+ \to \frac{3}{2}^+$ transition, (14), can be written as

$$\begin{aligned} &\langle B_2, \tfrac{3}{2}^+ | J_\mu^{V-A} | B_1, \tfrac{1}{2}^+ \rangle \\ &= \frac{b^*}{4M_1M_2}\bar{u}^\nu(P_2)[g_{\nu\mu}(1+\gamma_5) \\ &\quad \cdot\{f((M_1+M_2)^2 - q^2) + 4gM_1M_2\} \\ &\quad + 2(f+g)P_{1\nu}\gamma_\mu M_2 - 2fP_{1\nu}P_{2\mu}(1+\gamma_5) \\ &\quad - 2gP_{1\nu}\gamma_\mu\gamma_5M_2]\gamma_5u(P_1), \end{aligned} \tag{25}$$

where the flavour coupling coefficient b^* is given by

$$b^* = 2W_c^{c'}\bar{B}^{\{abc\}}B_{a[bc']}, \tag{26}$$

and where $b^* = 2/\sqrt{3}$ for $\Sigma_b \to \Sigma_c^*$ and $\Omega_b \to \Omega_c^*$.

For the $\frac{3}{2}^+ \to \frac{3}{2}^+$ transition, finally, one finds

$$\begin{aligned} &\langle B_2, \tfrac{3}{2}^+ | J_\mu^{V-A} | B_1, \tfrac{3}{2}^+ \rangle \\ &= \frac{b^{**}}{4M_1M_2}\bar{u}^\alpha(P_2)\gamma_\mu(1-\gamma_5)u^\beta(P_1) \\ &\quad \cdot[\{f((M_1+M_2)^2-q^2) + 4M_1M_2g\}g_{\alpha\beta} - 2fP_{1\alpha}P_{2\beta}], \end{aligned} \tag{27}$$

where

$$b^{**} = 3W_c^{c'}\bar{B}^{\{abc\}}B_{\{abc\}}, \tag{28}$$

and where $b^{**} = 1$ for e.g. $\Omega_b^* \to \Omega_c^*$. The $\frac{3}{2}^+ \to \frac{3}{2}^+$ transition (27) will be needed later on for the discussion of the crossed process, namely for e^+e^--production of an $\frac{3}{2}^+\frac{3}{2}^+$ pair

It is easy to show that at $q^2_{max}(w=1)$ the zero component of the vector current transitions are given by

$$\langle B_2, \tfrac{1}{2}^+ | J_0^V | B_1, \tfrac{1}{2}^+ \rangle = b(f+g)\bar{u}(P_2)u(P_1) \tag{29}$$

$$\langle B_2, \tfrac{3}{2}^+ | J_0^V | B_1, \tfrac{1}{2}^+ \rangle = b^*(f+g)\bar{u}_0(P_2)\gamma_5 u(P_1) \tag{30}$$

and

$$\langle B_2, \tfrac{3}{2}^+ | J_0^V | B_1, \tfrac{3}{2}^+ \rangle = b^{**}(f+g)\bar{u}^\alpha(P_2)u^\beta(P_1)g_{\alpha\beta}. \tag{31}$$

Equations (29), (30) and (31) exhibit the correct normalization conditions due to the $SU(F)$ symmetry as discussed after (17).

A direct method for obtaining these results from QCD is to go to the Bloch–Nordsieck limit [14]. In [3] we presented the details of this approach. If one simply inserts the χ^Λ and χ^Σ projectors as described in that paper the transition integrals devolve to those of (7), for the same process shown in Fig. 1. The analysis is, of course, identical. Actually the results of a Bloch–Nordsieck calculation will always be identical to that of the wave function approach provided one chooses the same projection χ to define the states. This will also be true in the sequel and we will not mention it again, but the reader might like to check this.

In a recent paper, [12], Isgur and Wise have calculated the Λ_b and Ω_b weak decay matrix elements in the limit of heavy quark mass using different techniques than ours. Our results agree. Our Λ-type form factor F_Λ is trivially related to their ξ form factor $F_\Lambda = \xi$. Our Σ-type form factors are related to the Isgur–Wise form factors η and ι through $f = -\eta$ and $g = \eta + \iota$. The same results have also been obtained in [13, 19, 20] using yet other techniques. We believe that our approach has the advantage of remaining closer to the underlying physics than the derivations [12, 13, 19, 20]. This has allowed us to discuss some physics motivated approximations to the most general solution.

4 Heavy to light transitions

In heavy to light transitions, e.g. $\Lambda_c \to \Lambda_s$, we do not have such a large symmetry. However, the symmetry present in the heavy baryon wave function allows us to reduce the number of form factors even in this case. It is easy to show that in the case of $\Lambda_c \to \Lambda_s$ transitions we now have two independent form factors.

The wave function of the Λ_s must still be antisymmetric in the u, d labels but now it does not necessarily satisfy the Bargmann–Wigner equations. The Λ_s wave function can be taken as

$$B_{\mathscr{A}_i;\mathscr{B}_j;\mathscr{C}_k} = \frac{\varepsilon_{ijk}}{\sqrt{6}}\frac{1}{\sqrt{3M}}\frac{3}{2}B_{a[bc]}[(\not{P}+M)\gamma_5 C]_{\sigma\delta}u_\rho(P)A^{\rho\sigma\delta}_{\alpha\beta\gamma}(k_1,k_2,k_3). \tag{32}$$

Here k_i are the three light quark momenta. $A^{\rho\sigma\delta}_{\alpha\beta\gamma}(k_1,k_2,k_3)$ is an arbitrary spinor tensor which does not spoil the spin parity properties of the spin $\frac{1}{2}$ projector, i.e. it behaves like a Lorentz scalar. Here a and α are the s-quark indices. The projector is antisymmetric under $\sigma \leftrightarrow \delta$ and $b \leftrightarrow c$ respectively, where b and c represent the u and d quarks.

The rest of the analysis follows as in Sect. 3 with the Λ_c wave function as in (6). Because of the particular form of the Λ_c wave function the trace in (7) always reduces to

$$\bar{u}^\rho(P_2)L^\alpha_\rho(P_1,P_2)(\gamma_\mu(1-\gamma_5))^{\alpha'}_\alpha u_{\alpha'}(P_1). \tag{33}$$

From Lorentz invariance $L^\alpha_\rho(P_1,P_2)$ can only be $\mathbb{1}$, $\not{P}_1$, $\not{P}_2$, or $\not{P}_1\not{P}_2$. This leads to the following form for the matrix element

$$\langle \Lambda_s(P_2)| J^{V-A}_\mu |\Lambda_c(P_1)\rangle = F_1(q^2)\bar{u}\gamma_\mu(1-\gamma_5)u + F_2(q^2)\bar{u}\not{P}_1\gamma_\mu(1-\gamma_5)u. \tag{34}$$

Therefore in the case of $\Lambda_c \to \Lambda_s$ decays there are only two form factors. In fact one can now see easily that for $\Lambda_c \to \Lambda_s + anything$ we will always get an expression like (33). A similar exercise shows that in Σ-type transitions there are four independent form factors in the $\frac{1}{2}^+ \to \frac{1}{2}^+$ case and six independent form factors in the $\frac{1}{2}^+ \to \frac{3}{2}^+$ case [20].

5 e^+e^--annihilation into heavy baryon pairs

Further tests of our form factor predictions for current induced heavy baryon transitions may be obtained in the time-like region for $q^2 \geqq (M_1+M_2)^2$ where heavy baryons are pair produced in e^+e^--annihilations. In the threshold region the form factors are dominated by the nearby heavy quarkonium ($Q\bar{Q}$)-states so that one is again testing the heavy quark current which now reads $\bar{Q}_i\gamma_\mu Q_i$.

The covariant results, (22), (24) and (26), can easily be crossed into the relevant baryon–antibaryon configuration needed for the present application. In order to facilitate the computation of production cross section ratios we shall present our results in terms of the independent helicity amplitudes contributing to the annihilation processes [15, 16].

One finds (in units of e_Q (charge of the heavy quark Q))

$\underline{\Lambda_Q\bar{\Lambda}_Q}$:

$$F_{\frac{1}{2}\frac{1}{2}} = -2M\bar{F}_\Lambda \quad F_{\frac{1}{2}-\frac{1}{2}} = 2M\sqrt{w+1}\,\bar{F}_\Lambda. \tag{35}$$

$\underline{\Sigma_Q\bar{\Sigma}^*_Q}$:

$$F_{\frac{1}{2}\frac{1}{2}} = \frac{M}{9}(\bar{F}_s + \sqrt{8}\bar{F}_d)$$

$$F_{\frac{1}{2}-\frac{1}{2}} = -\frac{M}{9}\sqrt{w+1}(\bar{F}_s - \sqrt{2}\bar{F}_d). \tag{36}$$

$\underline{\Sigma^*_Q\bar{\Sigma}^*_Q}$:

$$F_{\frac{1}{2}\frac{1}{2}} = \frac{M}{9}(\sqrt{8}\bar{F}_s - \bar{F}_d)$$

$$F_{\frac{1}{2}-\frac{1}{2}} = \frac{M}{9}\sqrt{w+1}\sqrt{2}(\bar{F}_s - \sqrt{2}\bar{F}_d),$$

$$F_{\frac{1}{2}\frac{3}{2}} = \frac{M\sqrt{3}}{9}\sqrt{w+1}(\sqrt{2}\bar{F}_s + \bar{F}_d). \tag{37}$$

$\Sigma_Q^* \bar{\Sigma}_Q^*$:

$$F_{\frac{1}{2}\frac{1}{2}} = -\frac{M}{9\sqrt{2}}(\sqrt{2}\bar{F}_s - 5\bar{F}_d)$$

$$F_{\frac{1}{2}-\frac{1}{2}} = -M\sqrt{w+1}\,\tfrac{2}{9}(\bar{F}_s - \sqrt{2}\bar{F}_d),$$

$$F_{\frac{3}{2}\frac{3}{2}} = \frac{M}{3\sqrt{2}}(\sqrt{2}\bar{F}_s + \bar{F}_d)$$

$$F_{\frac{1}{2}\frac{3}{2}} = \frac{M}{9}\sqrt{\frac{3}{2}}\sqrt{w+1}(\sqrt{2}\bar{F}_s + \bar{F}_d). \tag{38}$$

The other dependent helicity amplitudes can be obtained from the above set by parity and charge conjugation (see e.g. [15, 16]).

In (35–38) we have used the form factor combinations

$$\bar{F}_s = (w-1)f - (2w+4)g,$$
$$\bar{F}_d = 2\sqrt{2}[(w-1)f + (w-1)g], \tag{39}$$

where $\bar{F}_s$ and $\bar{F}_d$ are the s- and d-wave vector diquark pair creation amplitudes from the vacuum ($w = P_1P_2/M_1M_2$). The d-wave amplitude shows the characteristic d-wave suppression factor $(w-1) \propto p^2$ as expected. Note that we have set all heavy baryon masses equal to M in order to arrive at the simple expresssions (35)–(38)*.

Again, the Λ-type form factor can be related to the Σ-type form factors in the approximation (15) where spin and orbital degrees of freedom are assumed to be decoupled. One then finds for the s-wave amplitude

$$\bar{F}_\Lambda = \tfrac{1}{2}(-(w-1)f + (2w+4)g)), \tag{40}$$

which can be obtained from (16) by crossing ($w \to -w$). In the independent quark motion approximation (16) with $g = 0$ all pair production cross sections are severely suppressed close to threshold since then even s-wave production is proportional to $(w-1) \propto p^2$. The reason is that each quark pair is being created independently from the vacuum in this picture which then leads to a product of two p-wave suppression factors $((\sqrt{w-1})^2 \propto p^2)$.

The differential annihilation cross section reads (see e.g. [15])

$$\frac{d\sigma}{d\cos\Theta} = \frac{2\pi}{3}\frac{\alpha^2 p}{(q^2)^{\frac{5}{2}}}\left(\sum_\lambda(|F_{\lambda+1,\lambda}|^2 + |F_{\lambda-1,\lambda}|^2)\frac{3}{8}(1+\cos^2\Theta)\right.$$
$$\left. + \sum_\lambda |F_{\lambda,\lambda}|^2 \frac{3}{4}\sin^2\Theta\right), \tag{41}$$

where p is the CM momentum and Θ is the polar angle between the particle and beam direction in the CM frame. The longitudinal and transverse helicity configurations $F_{\lambda\lambda}$ and $F_{\lambda\pm1,\lambda}$ contribute to the differential cross section (41) with different angular factors as (41) shows.

Close to the physical threshold $w = 1$ one can neglect the d-wave contribution $\bar{F}_d$ as it is proportional to $(w-1) \propto p^2$. Counting all charge configurations one then obtains the annihilation cross section ratios

$$(\sigma_{\Lambda_Q\bar{\Lambda}_Q}):\sigma_{\Sigma_Q\bar{\Sigma}_Q}:\sigma_{\Sigma_Q\bar{\Sigma}_Q^*} + \sigma_{\Sigma_Q^*\bar{\Sigma}_Q}:\sigma_{\Sigma_Q^*\bar{\Sigma}_Q^*} = (27):1:16:10 \tag{42}$$

For the Σ_Q-type configurations these ratios agree with the results [16–19]*. The (bracketed) Λ_Q-type annihilation cross section is not related to the Σ_Q-type cross sections in the most general approach. However, in the approximation (15) discussed after (40) the $\Lambda_Q\bar{\Lambda}_Q$ cross section comes in quite dramatically as (42) shows.

6 Summary and conclusions

We have analyzed heavy to heavy and heavy to light current induced baryon transitions in the limit where the heavy quark and heavy baryon mass become large. There are altogether three different dynamical form factors describing the ground state $(\frac{1}{2}^+, \frac{3}{2}^+) \to (\frac{1}{2}^+, \frac{3}{2}^+)$ heavy baryon to heavy baryon transitions. Two of these form factors are normalized at maximum momentum transfer due to the heavy flavour symmetry present in the theory.

Our analysis was based on a physical picture of current-induced transitions among heavy-light baryons described by general Bethe– Salpeter (B.S.) bound state wave functions. In the B.S. approach the three independent form factors have a natural interpretation in terms of the independent spin interactions among the spectator quark systems. By performing a simple partial wave analysis of the diquark excitations these could in turn be related to the s- and d-wave excitations of the scalar and vector diquark system.

Compared to the more group theoretical settings used in [12, 13, 19, 20] to derive the same results our B.S. formalism approach has the advantage that it allows one to establish contact with existing dynamical bound state models, and in this context, to discuss physics motivated approximations to the most general analysis. In an approximation where the spectator light quark systems move independently and do not spin communicate among each other the number of independent form factors is reduced to one. In this approximation one recovers the results of the $U(12)$ ansatz of [8, 10, 11] and thereby the results of the static spectator quark model. Alternatively, we have discussed an approximation where the orbital and spin degrees of freedom of the light quark systems decouple. In this case the number of independent form factors is reduced to two. The above approximations lead to testable predictions for spectra and polarisation density matrices that go beyond the predictions of the most general approach. We hope to work out the details of these predictions in the future. For the case of the e^+e^--annihilation into heavy baryon pairs we showed that the latter approximation leads to a dramatic threshold enhancement of the $\Lambda_Q\bar{\Lambda}_Q$ annihilation channel relative to the $\Sigma_Q\bar{\Sigma}_Q$, $\Sigma_Q^*\bar{\Sigma}_Q$ and $\Sigma_Q^*\bar{\Sigma}_Q^*$ channels. Finally, we discussed heavy baryon to light baryon transitions where we found that the heavy quark limit gives a useful constraint on the Λ_Q-type transitions.

* In the $\frac{1}{2}^+ \to \frac{3}{2}^+$ case the g-type contribution is current-conserved only in this limit

* We believe, though, that Table 3 in [18] is in error

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