$\Lambda_b \to p$ form factors: Hussain et al. definition vs Khodjamirian et al. definition

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1 Definition 1

Hussain et al. [1] define in their Eq. (73)

$$\langle N^+(p')|\overline{u}\,\gamma^\mu\, b|\Lambda_b(p)\rangle \;=\; \overline{u}_N(p')\left[\tilde{F}_1^V\,\gamma^\mu + \tilde{F}_2^V\,v^\mu + \tilde{F}_3^V\,v'^\mu\right]u_{\Lambda_b}(p), \tag{1}$$

$$\langle N^+(p')|\overline{u}\,\gamma^\mu\gamma_5\, b|\Lambda_b(p)\rangle \;=\; -\overline{u}_N(p')\left[\tilde{F}_1^A\,\gamma^\mu + \tilde{F}_2^A v^\mu + \tilde{F}_3^A v'^\mu\right]\gamma_5\, u_{\Lambda_b}(p), \tag{2}$$

where $v^\mu = p^\mu/m_{\Lambda_b}$, $v'^\mu = p'^\mu/m_N$.

2 Definition 2

Khodjamirian et al. [2] define

$$\langle \Lambda_b(P-q^{(\mathrm{K})})|\bar{b}\,\gamma_\mu\,u|N(P)\rangle \;=\; \bar{u}_{\Lambda_b}(P-q^{(\mathrm{K})})\left\{f_1^{(\mathrm{K})}\,\gamma_\mu + i\frac{f_2^{(\mathrm{K})}}{m_{\Lambda_b}}\,\sigma_{\mu\nu}q^{(\mathrm{K})\nu} + \frac{f_3^{(\mathrm{K})}}{m_{\Lambda_b}}\,q_\mu^{(\mathrm{K})}\right\}u_N(P)\,, \tag{3}$$

$$\langle \Lambda_b(P-q^{(\mathrm{K})})|\bar{b}\,\gamma_\mu\gamma_5\,u|N(P)\rangle \;=\; \bar{u}_{\Lambda_b}(P-q^{(\mathrm{K})})\left\{g_1^{(\mathrm{K})}\,\gamma_\mu + i\frac{g_2^{(\mathrm{K})}}{m_{\Lambda_b}}\,\sigma_{\mu\nu}q^{(\mathrm{K})\nu} + \frac{g_3^{(\mathrm{K})}}{m_{\Lambda_b}}\,q_\mu^{(\mathrm{K})}\right\}\gamma_5 u_N(P). \tag{4}$$

Using our notation for the momenta ($q = p - p' = -q^{(\mathrm{K})}$), this becomes

$$\langle \Lambda_b(p)|\bar{b}\,\gamma_\mu\,u|N(p')\rangle \;=\; \bar{u}_{\Lambda_b}(p)\left\{f_1^{(\mathrm{K})}\,\gamma_\mu - i\frac{f_2^{(\mathrm{K})}}{m_{\Lambda_b}}\,\sigma_{\mu\nu}q^\nu - \frac{f_3^{(\mathrm{K})}}{m_{\Lambda_b}}\,q_\mu\right\}u_N(p')\,, \tag{5}$$

$$\langle \Lambda_b(p)|\bar{b}\,\gamma_\mu\gamma_5\,u|N(p')\rangle \;=\; \bar{u}_{\Lambda_b}(p)\left\{g_1^{(\mathrm{K})}\,\gamma_\mu - i\frac{g_2^{(\mathrm{K})}}{m_{\Lambda_b}}\,\sigma_{\mu\nu}q^\nu - \frac{g_3^{(\mathrm{K})}}{m_{\Lambda_b}}\,q_\mu\right\}\gamma_5 u_N(p'). \tag{6}$$

Next, we compute the Dirac conjugate (recall that $\overline{\gamma^\mu} = \gamma^\mu$, $\overline{\sigma_{\mu\nu}} = \sigma_{\mu\nu}$, and $\overline{\gamma_5} = -\gamma_5$),

$$\langle N(p')|\bar{u}\,\gamma_\mu\,b|\Lambda_b(p)\rangle \;=\; \bar{u}_N(p')\left\{f_1^{(\mathrm{K})}\,\gamma_\mu + i\frac{f_2^{(\mathrm{K})}}{m_{\Lambda_b}}\,\sigma_{\mu\nu}q^\nu - \frac{f_3^{(\mathrm{K})}}{m_{\Lambda_b}}\,q_\mu\right\}u_{\Lambda_b}(p)\,, \tag{7}$$

$$-\langle N(p')|\bar{u}\,\gamma_5\gamma_\mu\,b|\Lambda_b(p)\rangle \;=\; -\bar{u}_N(p')\gamma_5\left\{g_1^{(\mathrm{K})}\,\gamma_\mu + i\frac{g_2^{(\mathrm{K})}}{m_{\Lambda_b}}\,\sigma_{\mu\nu}q^\nu - \frac{g_3^{(\mathrm{K})}}{m_{\Lambda_b}}\,q_\mu\right\}u_{\Lambda_b}(p), \tag{8}$$

and move the γ_5's:

$$\langle N(p')|\bar{u}\,\gamma_\mu\,b|\Lambda_b(p)\rangle \;=\; \bar{u}_N(p')\left\{f_1^{(\mathrm{K})}\,\gamma_\mu + i\frac{f_2^{(\mathrm{K})}}{m_{\Lambda_b}}\,\sigma_{\mu\nu}q^\nu - \frac{f_3^{(\mathrm{K})}}{m_{\Lambda_b}}\,q_\mu\right\}u_{\Lambda_b}(p)\,, \tag{9}$$

$$\langle N(p')|\bar{u}\,\gamma_\mu\gamma_5\,b|\Lambda_b(p)\rangle \;=\; \bar{u}_N(p')\left\{g_1^{(\mathrm{K})}\,\gamma_\mu - i\frac{g_2^{(\mathrm{K})}}{m_{\Lambda_b}}\,\sigma_{\mu\nu}q^\nu + \frac{g_3^{(\mathrm{K})}}{m_{\Lambda_b}}\,q_\mu\right\}\gamma_5 u_{\Lambda_b}(p). \tag{10}$$

3 Relation between definitions 1 and 2

We can rewrite Eq. (9) in the following way:

$$\begin{aligned}
&\langle N^+(p')|\overline{u}\,\gamma^\mu\,b|\Lambda_b(p)\rangle \\
&= \overline{u}_N(p')\left[f_1^{(\mathrm{K})}\,\gamma^\mu + \frac{f_2^{(\mathrm{K})}}{m_{\Lambda_b}} i\frac{i}{2}(\gamma^\mu \not{q} - \not{q}\gamma^\mu) - \frac{f_3^{(\mathrm{K})}}{m_{\Lambda_b}} q^\mu\right] u_{\Lambda_b}(p) \\
&= \overline{u}_N(p')\left[f_1^{(\mathrm{K})}\,\gamma^\mu + \frac{f_2^{(\mathrm{K})}}{m_{\Lambda_b}} i\frac{i}{2}(\gamma^\mu \not{p} - \gamma^\mu \not{p}' - \not{p}\gamma^\mu + \not{p}'\gamma^\mu) - \frac{f_3^{(\mathrm{K})}}{m_{\Lambda_b}} q^\mu\right] u_{\Lambda_b}(p) \\
&= \overline{u}_N(p')\left[f_1^{(\mathrm{K})}\,\gamma^\mu + \frac{f_2^{(\mathrm{K})}}{m_{\Lambda_b}} i\frac{i}{2}(\gamma^\mu m_{\Lambda_b} - \gamma^\mu \not{p}' - \not{p}\gamma^\mu + m_N\gamma^\mu) - \frac{f_3^{(\mathrm{K})}}{m_{\Lambda_b}} q^\mu\right] u_{\Lambda_b}(p) \\
&= \overline{u}_N(p')\left[f_1^{(\mathrm{K})}\,\gamma^\mu + \frac{f_2^{(\mathrm{K})}}{m_{\Lambda_b}} i\frac{i}{2}(\gamma^\mu m_{\Lambda_b} + \not{p}'\gamma^\mu - (\not{p}'\gamma^\mu + \gamma^\mu\not{p}') + \gamma^\mu\not{p} - (\gamma^\mu\not{p} + \not{p}\gamma^\mu) + m_N\gamma^\mu) - \frac{f_3^{(\mathrm{K})}}{m_{\Lambda_b}} q^\mu\right] u_{\Lambda_b}(p) \\
&= \overline{u}_N(p')\left[f_1^{(\mathrm{K})}\,\gamma^\mu + \frac{f_2^{(\mathrm{K})}}{m_{\Lambda_b}} i\frac{i}{2}(\gamma^\mu m_{\Lambda_b} + m_N\gamma^\mu - 2p'^\mu + \gamma^\mu m_{\Lambda_b} - 2p^\mu + m_N\gamma^\mu) - \frac{f_3^{(\mathrm{K})}}{m_{\Lambda_b}} (p^\mu - p'^\mu)\right] u_{\Lambda_b}(p) \\
&= \overline{u}_N(p')\left[\left(f_1^{(\mathrm{K})} + \frac{f_2^{(\mathrm{K})}}{m_{\Lambda_b}} i\frac{i}{2}(2m_{\Lambda_b} + 2m_N)\right)\gamma^\mu + \left(+\frac{f_2^{(\mathrm{K})}}{m_{\Lambda_b}} i\frac{i}{2}(-2) - \frac{f_3^{(\mathrm{K})}}{m_{\Lambda_b}}\right)p^\mu + \left(+\frac{f_2^{(\mathrm{K})}}{m_{\Lambda_b}} i\frac{i}{2}(-2) + \frac{f_3^{(\mathrm{K})}}{m_{\Lambda_b}}\right)p'^\mu\right] u_{\Lambda_b}(p) \\
&= \overline{u}_N(p')\left[\left(f_1^{(\mathrm{K})} - \frac{m_{\Lambda_b} + m_N}{m_{\Lambda_b}} f_2^{(\mathrm{K})}\right)\gamma^\mu + \left(f_2^{(\mathrm{K})} - f_3^{(\mathrm{K})}\right)v^\mu + \left(f_2^{(\mathrm{K})} + f_3^{(\mathrm{K})}\right)\frac{m_N}{m_{\Lambda_b}} v'^\mu\right] u_{\Lambda_b}(p), \qquad (11)
\end{aligned}$$

and, similarly for Eq. (10),

$$\langle N^+(p')|\overline{u}\,\gamma^\mu\gamma_5\,b|\Lambda_b(p)\rangle = \overline{u}_N(p')\left[\left(g_1^{(\mathrm{K})} - \frac{m_{\Lambda_b} - m_N}{m_{\Lambda_b}} g_2^{(\mathrm{K})}\right)\gamma^\mu - \left(g_2^{(\mathrm{K})} - g_3^{(\mathrm{K})}\right)v^\mu - \left(g_2^{(\mathrm{K})} + g_3^{(\mathrm{K})}\right)\frac{m_N}{m_{\Lambda_b}} v'^\mu\right]\gamma_5\,u_{\Lambda_b}(p), \qquad (12)$$

By comparing Eqs. (1), (2) and (11), (12), we see that

$$\tilde{F}_1^V = f_1^{(\mathrm{K})} - \frac{m_{\Lambda_b} + m_N}{m_{\Lambda_b}} f_2^{(\mathrm{K})}, \qquad (13)$$

$$\tilde{F}_2^V = f_2^{(\mathrm{K})} - f_3^{(\mathrm{K})}, \qquad (14)$$

$$\tilde{F}_3^V = \frac{m_N}{m_{\Lambda_b}}\left(f_2^{(\mathrm{K})} + f_3^{(\mathrm{K})}\right), \qquad (15)$$

$$\tilde{F}_1^A = -g_1^{(\mathrm{K})} + \frac{m_{\Lambda_b} - m_N}{m_{\Lambda_b}} g_2^{(\mathrm{K})}, \qquad (16)$$

$$\tilde{F}_2^A = g_2^{(\mathrm{K})} - g_3^{(\mathrm{K})}, \qquad (17)$$

$$\tilde{F}_3^A = \frac{m_N}{m_{\Lambda_b}}\left(g_2^{(\mathrm{K})} + g_3^{(\mathrm{K})}\right). \qquad (18)$$

References

[1] F. Hussain, D.-S. Liu, M. Kramer, J. G. Körner, and S. Tawfiq, Nucl. Phys. B **370**, 259 (1992).

[2] A. Khodjamirian, C. Klein, T. Mannel, and Y.-M. Wang, JHEP **1109**, 106 (2011) [arXiv:1108.2971 [hep-ph]].