Learning to Learn

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Learning to Learn

Introduction

- No universally agreed upon definition.
- Possible definitions:
  - Replace hand-crafted components of learning algorithms with learned ones.
  - Have an outer learning algorithm that seeks to optimise an inner algorithm.
- e.g.
  - Learning to optimise.
  - Few-shot learning.
- Synonymous with Meta-Learning.

Learning to Optimise

Learning as Optimisation

- (Old & well-known) idea: View learning as an optimisation problem.
- $$\theta^* = \operatorname*{argmin}_{\theta} f(\theta)$$

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- MLE: $f(\theta) := -\log p(\mathcal{D} \mid \theta)$
- VB: $f(\theta) := \mathcal{KL}\left[q_\theta(\cdot) \;||\; p(\cdot \mid \mathcal{D})\right]$

Learning to Optimise

Iterative Optimisation

- (Old & well-known) idea: Pick a function g where

$$\theta_{t+1} = g\left(f, \theta_{1:t}\right)$$

- Construct g s.t. $\theta_t \to \theta^*$ as t becomes large. (Hopefully)

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- Observation: g is hand-crafted (up to a couple of free parameters).

Learning to Optimise

Learning to Iteratively Optimise

- (New-ish) idea: Learn g.
- e.g. Parameterise g

$$\theta_{t+1} = g_{\varphi}(f, \theta_{1:t})$$

and learn φ from data. (data = optimisation problems)

Learning to Optimise

"Learning step size controllers for robust neural network training" [Daniel et al., 2016]

Learn a controller for the step-size of SGD [Daniel et al., 2016]:

$$g_{\varphi}\left(f, \theta_{1:t}\right) = \theta_t - \underbrace{\exp\left(\varphi^T \hat{\phi}_t\right)}_{\text{replaces } \eta} \nabla f\left(\theta_t\right), \quad \hat{\phi}_t = \phi\left(\theta_t, \hat{\phi}_{t-1}\right),$$

where ϕ is a hand-crafted vector-valued basis function.

Advantages

- Low memory footprint
- Small number of parameters

Disadvantages

- Hand-engineered features
- No second-order info. used

Learning to Optimise

"Learning to Learn by Gradient Descent by Gradient Descent" [Andrychowicz et al., 2016]

$$g_\varphi\left(f, \theta_{1:t}\right) = \theta_t - \underbrace{r_\varphi\left(h_t\right)}_{\substack{\text{replaces} \\ \eta \nabla f(\theta_t)}}, \quad h_t = H_\varphi\left(h_{t-1}, \nabla f\left(\theta_{t-1}\right)\right).$$

Advantages

- Flexible
- Variable dimensionality

Disadvantages

- Large memory footprint
- No coupling

Learning to Optimise

"Learning to Optimize" [Li and Malik, 2016]

Learn an MLP autoregressor m_φ:

$$g_\varphi\left(f, \theta_{1:t}\right) = m_\varphi\left(\theta_t, f\left(\theta_{t-H:t}\right), \nabla f\left(\theta_{t-H:t}\right)\right),$$

for some $H \in \mathbb{N}$.

Advantages

- Flexible
- Coupling between variables

Disadvantages

- Large memory footprint
- Not flexible in no. dims.

Learning to Optimise

"Learning to Learn without Gradient Descent by Gradient Descent" [Chen et al., 2017]

Let g_φ be an RNN r_φ with hidden state h_t given by

$$g_\varphi\left(f, \theta_{1:t}\right) = r_\varphi\left(h_t\right), \quad h_t = H_\varphi\left(h_{t-1}, \theta_{t-1}, f\left(\theta_{t-1}\right)\right).$$

Advantages

- Flexible parametrisation
- Coupling between dims

Disadvantages

- Large memory footprint
- Requires ∇f during training
- Fixed dimensionality.

Learning to Optimise

"Learning to Learn without Gradient Descent by Gradient Descent" [Chen et al., 2017]

- EI training criterion:

$$L_{\mathsf{EI}}(\theta) = -\mathbb{E}_{f, y_{1:T-1}} \underbrace{\left[\sum_{t=1}^{T} \mathsf{EI}\left(\theta_t \mid y_{1:t-1}\right) \right]}_{\text{Computed via GP}}$$

Learning to Optimise

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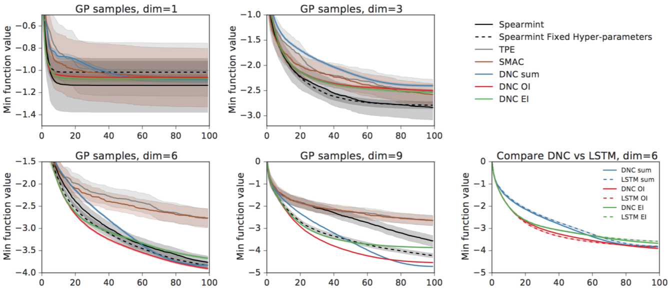

Figure 3. Average minimum observed function value, with 95% confidence intervals, as a function of search steps on functions sampled from the training GP distribution. Left four figures: Comparing DNC with different reward functions against Spearmint with fixed and estimated GP hyper-parameters, TPE and SMAC. Right bottom: Comparing different DNCs and LSTMs. As the dimension of the search space increases, the DNC's performance improves relative to the baselines.

Learning to Optimise

Conclusion

- Learning to optimise is conceptually appealing.
- Empirical results seem promising.
- Memory requirements large relative to simple methods.
  - Probably a careful trade-off required between flexibility and overhead.

Learning to Learn

Data-efficient learning

- Problem: ‘Insufficient’ training data
- Use related data
- k-shot learning
  - Image classification
  - Sentence completion

Learning to Learn

k-shot learning

$$\text{Large}\,\tilde{\mathcal{D}} = \{\underline{\tilde{u}}_i, \tilde{y}_i\}_{i=1}^{\tilde{N}},\ \tilde{y}_i \in \{1, ..., \tilde{C}\}$$
$$\text{Small}\,\mathcal{D} = \{\underline{u}_i, y_i\}_{i=1}^{kC},\ y_i \in \{\tilde{C}+1, ..., \tilde{C}+C\}$$

Learning to Learn
k-shot learning

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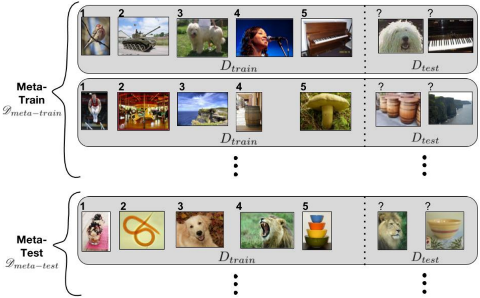
- LSTM meta-learner
- Learning on learnt features

Learning to Learn

"Optimisation as a model for few-shot learning" [Ravi and Larochelle, 2017]

$$\theta_t = \quad \theta_{t-1} - \alpha_t \nabla_{\theta_{t-1}} \mathcal{L}_t$$
$$c_t = f_t \odot c_{t-1} + i_t \odot \tilde{c}_t$$

- Learn f_t, i_t, c_0



Learning to Learn

Learning on features

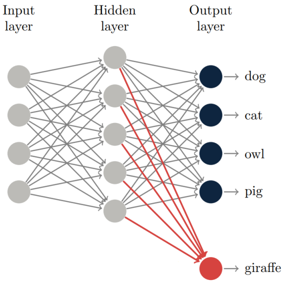
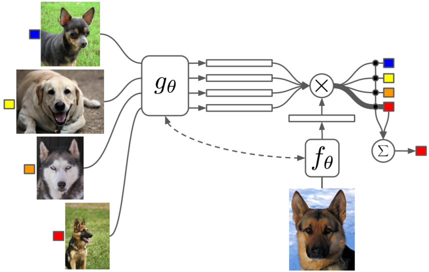


Figure 1: Learning on features [Burgess et al., 2016]

- Train a Neural Network classifier on $\tilde{\mathcal{D}}$
  - Train on embedded features obtained from last hidden layer
- Common baseline: Nearest neighbour matching

Learning to Learn

"Matching Networks for One Shot Learning" [Vinyals et al., 2016]



$$a(\hat{x}, x_i) = \frac{e^{c(f(\hat{x},S),g(x_i,S))}}{\sum_{j=1}^{k} e^{c\left(f(\hat{x},S),g(x_j,S)\right)}}$$

$$\hat{y} = \sum_{i=1}^{k} a(\hat{x}, x_i) y_i$$

Learning to Learn

"Discriminative k-shot learning using probabilistic models" [Bauer et al., 2017]

- Bayesian approach on softmax weights
- Found single Gaussian worked best

$$p(\mathrm{W}|\mathcal{D}, \tilde{\mathcal{D}}) \propto \mathcal{N}\left(\mathrm{W}|\mu^{\mathrm{MAP}}, \Sigma^{\mathrm{MAP}}\right) \prod_{n=1}^{N} p(y_n|\mathbf{x}_n, \mathrm{W})$$

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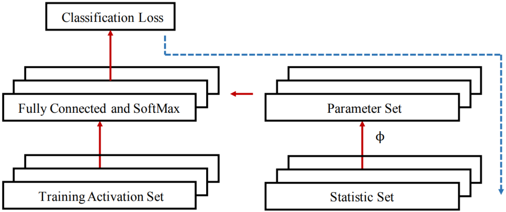
Method	1-shot	5-shot
ResNet-34 + Isotropic Gaussian (ours)	$\mathbf{56.3 \pm 0.4\%}$	$\mathbf{73.9 \pm 0.3\%}$
Matching Networks (reimplemented, 1-shot)	$46.8 \pm 0.5\%$	-
Matching Networks (reimplemented, 5-shot)	-	$62.7 \pm 0.5\%$
Meta-Learner LSTM (Ravi & Larochelle, 2017)	$43.4 \pm 0.8\%$	$60.6 \pm 0.7\%$
Prototypical Nets (1-shot) (Snell et al., 2017)	$49.4 \pm 0.8\%$	$65.4 \pm 0.7\%$
Prototypical Nets (5-shot) (Snell et al., 2017)	$45.1 \pm 0.8\%$	$68.2 \pm 0.7\%$

Learning to Learn

"Few-Shot Image Recognition by Predicting Parameters from Activations" [Qiao et al., 2017]



$$P(y_i|x_i) = \frac{e^{\mathbf{a}(x_i)\cdot\phi\left(\mathbb{E}_S[\mathbf{s}_{y_i}]\right)}}{Z}$$

- Statistic set (1st moment)
- At k-shot train, each sample is new category

Learning to Learn

Potential future research

- Similarities between classes (eg animals)
  - Attempted with GMMs [Bauer et al., 2017]
- Combine feature-learning with learning on features for general k-shot
  - Currently ‘fine-tuning’ is heuristic

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