GURLS_mkl: A PFBS-based Implementation for Multiple Kernel Learning

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1 Hierarchical Dirichlet Process

1.1 Model

Classic view:

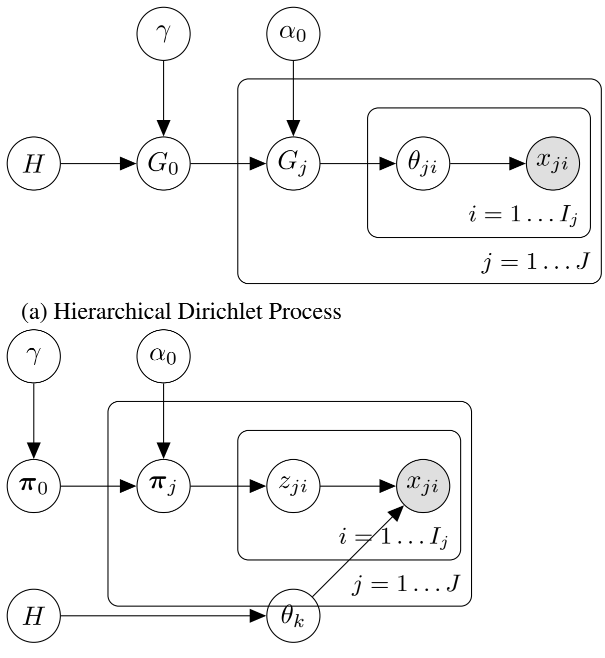
$$\begin{aligned}
G_0|\gamma, H &\sim DP(\gamma, H)\\
G_j|\alpha_0, G_0 &\sim DP(\alpha_0, G_0)\\
\theta_{ji}|G_j &\sim G_j\\
x_{ji}|\theta_{ji} &\sim F(\theta_{ji})
\end{aligned}$$

where $P \sim DP(\alpha, G)$ adopts the stick breaking representation w.p. 1:

$$P = \sum_{k=1}^{\infty} \pi_k \delta_{\phi_k} \qquad \text{where:} \qquad \pi_k \sim GEM(\alpha), \quad \phi_k \sim G$$

Alternatively, one may describe the generative processes of π_k and θ_k separately as:

$$\begin{aligned}
\boldsymbol{\pi}_0|\gamma &\sim GEM(\gamma) \qquad\qquad \theta_k|H \sim H\\
\boldsymbol{\pi}_j|\alpha_0, \boldsymbol{\pi}_0 &\sim DP(\alpha_0, \boldsymbol{\pi}_0)\\
\\
z_{ji}|\boldsymbol{\pi}_j &\sim \boldsymbol{\pi}_j\\
x_{ji}|z_{ji}, (\theta_k)_{k=1}^{\infty} &\sim F(\theta_{z_{ji}})
\end{aligned}$$



(a) Hierarchical Dirichlet Process

(b) Hierarchical Dirichlet Process

Figure 1: Hierarchical Dirichlet Process

1.2 Inference

Assuming conjugacy between H and F [1] and holding (γ, α_0) fixed,

we now describe a simplied Gibbs approach to sample parameters $(z_{ji}, m_{jk}, \boldsymbol{\pi}_0)$ from the Chinese Restaurant Franchise (see Appendix A) representation of the posterior, where the parameter z_{ji} are refered to respectively as customer-specific dish assignment, m_{jk} as dish-specific table count, and π_0 as global dish distribution. This

[1] so we can integrate out the mixture component parameters

particular method is referred to as "direct assignment" in Teh et al. [2006] since it circumvented the issue of bookkeeping for every t_{ij} (customer-specific table assignment) and k_{jt} (table-specific dish assignment) variables.

In each Gibbs iteration, denote $f_k^{-x_{ji}}(x_{ji}) = \frac{\int f(\mathbf{x}|\theta_k)h(\theta_k)d_{\theta_k}}{\int f(\mathbf{x}_{-(ji)}|\theta_k)h(\theta_k)d_{\theta_k}}$ the conditional distribution $x_{ji}|\mathbf{x}_{-(ji)}$ under $\theta = \theta_k$, and assume there are currently K dishes and T tables, we sample $(z_{ji}, m_{jk}, \boldsymbol{\pi}_0)$ iteratively as:

1. Sample $z_{ji} = k|\mathbf{z}_{-(ji)}, \mathbf{m}, \boldsymbol{\pi}_0$ from the distribution:

$$z_{ji} = k|\mathbf{z}_{-(ji)}, \mathbf{m}, \boldsymbol{\pi}_0 \propto \begin{cases} f_k^{-x_{ji}}(x_{ji}) * (n_{jk}^{-(ji)} + \alpha_0 \pi_{0,k}) & k \leq K \\ f_{K+1}^{-x_{ji}}(x_{ji}) * \alpha_0 \pi_{0,u} & k = K+1 \end{cases}$$

2. Sample $m_{jk} = m|\mathbf{z}, \mathbf{m}_{-(jk)}, \boldsymbol{\pi}_0$, by setting $m_{jk} = \sum_i I(t_{ji} = t_{new}|k_{jt_{new}} = k)$, we can sample t_{ji} from:

$$t_{ji} = t|k_{jt} = k, \mathbf{t}_{-(ji)}, \boldsymbol{\pi}_0 \propto \begin{cases} n_{jt}^{-(ji)} & t \leq T \\ \alpha_0 \pi_{0,k} & t = T+1 \end{cases}$$

or as in Fox [2009], sample $I(t_{ji} = t_{new}|k_{jt_{new}} = k)$ directly from:

$$Bern\Big(\frac{\alpha_0 \pi_{0,k}}{n_{jk} + \alpha_0 \pi_{0,k}}\Big)$$

3. Sample $\boldsymbol{\pi}_0$ from distribution:

$$\boldsymbol{\pi}_0 \sim Dir(m_1, \dots, m_K, \gamma)$$

Algorithm 1 HDP, Gibbs Sampler through Direct Assignment

```
procedure hdp_gibbs_ds(K, y, (τ, μ, σ))
    α^0 = 0
    for p = 1 to MAX_ITER do
        α_0^p = (1 − μ/σ) α^{p−1} − 1/(σn) (K α^{p−1} − y)
        α^p = S_{τ/σ}(K, α_0^p)
    end for
    return f^{MAX_ITER} = (α^{MAX_ITER})^T k
end procedure
```

1.3 Application: Clustering Hierarchical Gaussian Data

Consider mixture of Gaussian data $\mathbf{x} = \{\mathbf{x}_1, \dots, \mathbf{x}_K\}$ with $\mathbf{x}_k \overset{iid}{\sim} MVN(\boldsymbol{\theta}_{k,2\times1}, \mathbf{I}_{2\times2})$ with unknown mean $\boldsymbol{\theta}$. Assuming diffused Gaussian prior $\boldsymbol{\theta} \sim N(\mathbf{0}, \sigma^2 \mathbf{I})$, the form of likelihood F and base measure H are:

$$f(x_{ji}|\boldsymbol{\theta}_k) \propto exp(-\frac{1}{2}(x_{ji} - \boldsymbol{\theta}_k)^T (x_{ji} - \boldsymbol{\theta}_k))$$
$$h(\boldsymbol{\theta}_k) \propto exp(-\frac{1}{2\sigma^2}\boldsymbol{\theta}_k^T \boldsymbol{\theta}_k)$$

Then $f_k^{-x_{ji}}(x_{ji})$ should be:

$$f_k^{-x_{ji}}(x_{ji}) \sim N(\frac{n_k^{-(ji)}\sigma^2}{n_k^{-(ji)}\sigma^2 + 1}\bar{\mathbf{x}}_k^{-(ji)}, (1 + \frac{\sigma^2}{n_k^{-(ji)}\sigma^2 + 1})\mathbf{I})$$

2 HDP for Hidden Markov Model

A Chinese Restaurant Franchise

A hierarchical analogy of Chinese Restaurant Process, the Chinese Restaurant Franchise offers a convenient scheme to sample from the posterior of cluster-specific θ's in HDP. This process draw below analogy:

- H as the dish distribution for all possible dishes in the world, with the types of possible dishes being $(\theta_k)_{k=1}^{\infty}$.
- $G_0 \sim DP(\gamma, H)$ as the dish distribution for the franchise
- $G_j \sim DP(\alpha_0, G_0)$ as the dish distribution for restaurant j in the franchise
- $\psi_{jt} \sim G_0$ as the dish served at table t in restaurant j.
  $k_{jt} \sim \pi_0$ as the index of dish choice for this table.
- $\theta_{ji} \sim G_j$ as the dish will be enjoyed by customer i in restaurant j.
  $t_{ji} \sim \pi_j$ as the index of table choice for this customer.

Integrating over G_j, the sampling scheme for subject-specific dish $\theta_{ji} \sim G_j$ is:

$$\theta_{ji}|\boldsymbol{\theta}_{j(-i)}, \alpha_0, G_0 \sim \sum_{k=1}^{K} \frac{n_{jt.}}{\alpha_0 + n_{j..}} \delta_{\psi_{jt}} + \frac{\gamma}{n_{j..} + \gamma} G_0$$

Integrating over G_0, the sampling scheme for table-specific dish $\psi_{jt} \sim G_0$ is:

$$\psi_{jk}|\Psi_{j(-k)}, \gamma, H \sim \sum_{k=1}^{K} \frac{m_{.k}}{\gamma + m_{..}} \delta_{\theta_k} + \frac{\gamma}{m_{..} + \gamma} H$$

B References

References

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