

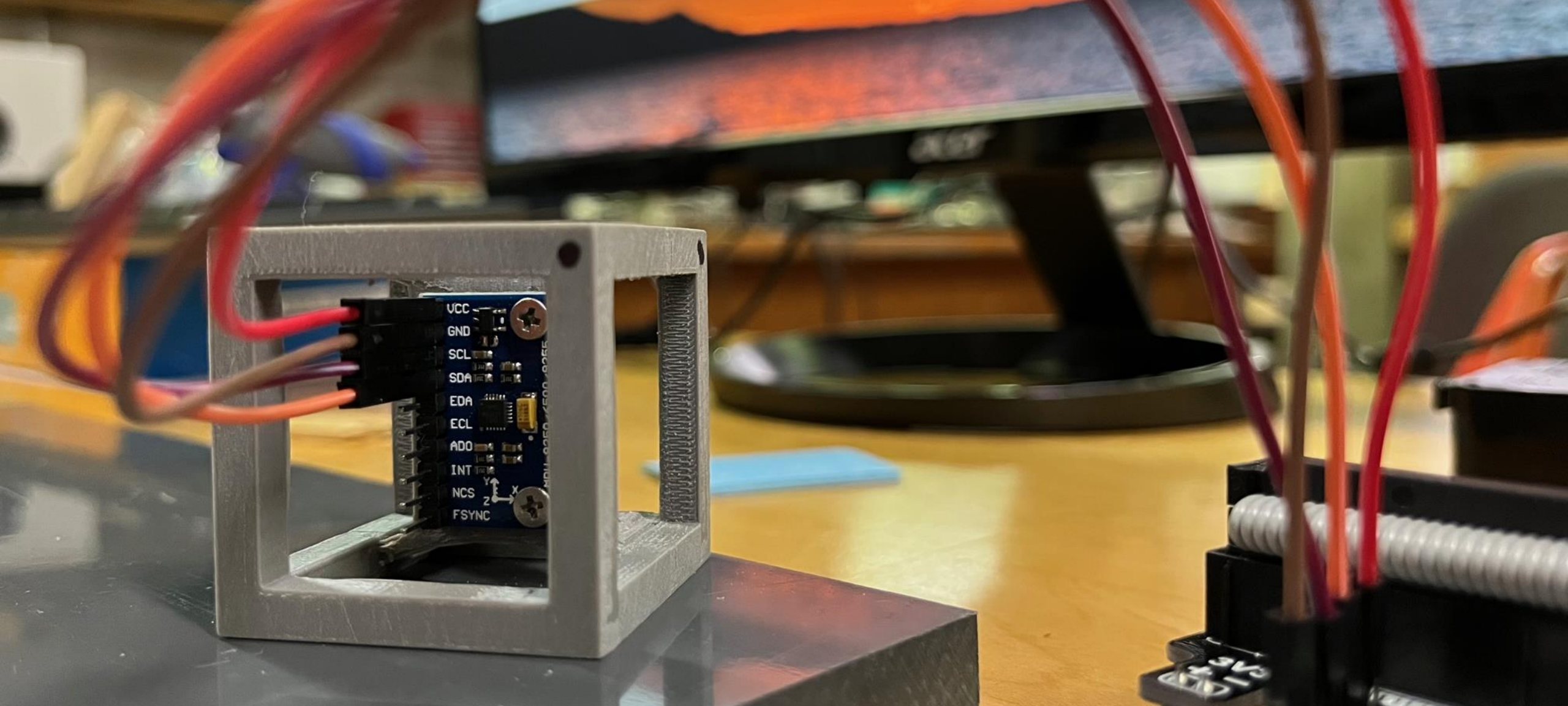
Investigating MEMS Accelerometer Calibration Techniques

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University Scholars Program
Senior Honors Capstone

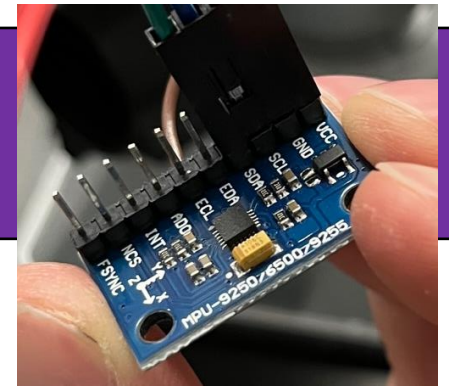
Faculty Mentor: Dr. William Slaton
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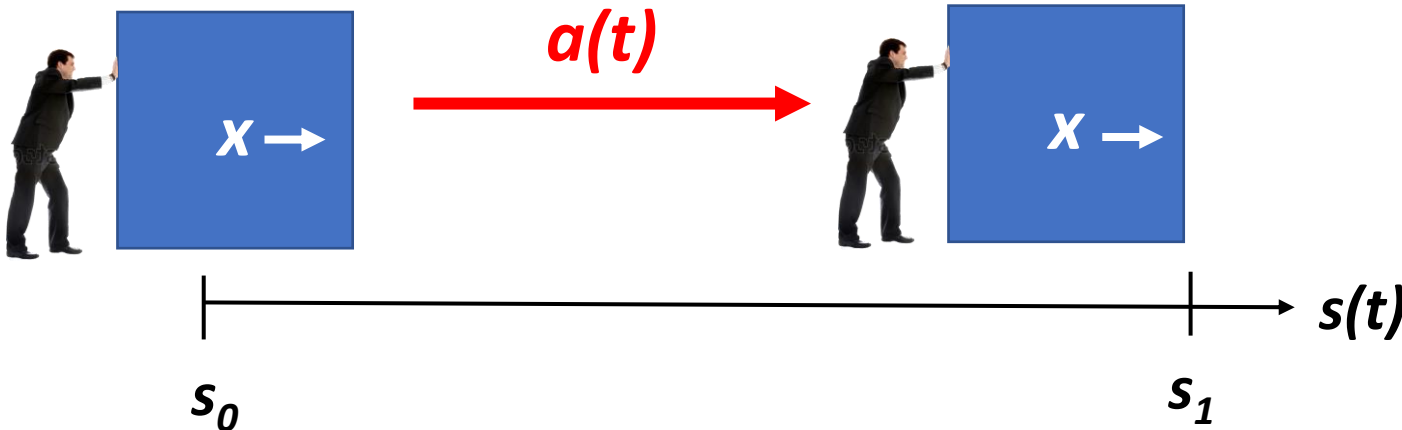


Introduction

What is an Accelerometer?



- **Accelerometer:** a sensor that measures linear acceleration
- **Robotics Application:** integrate acceleration to calculate velocity and position



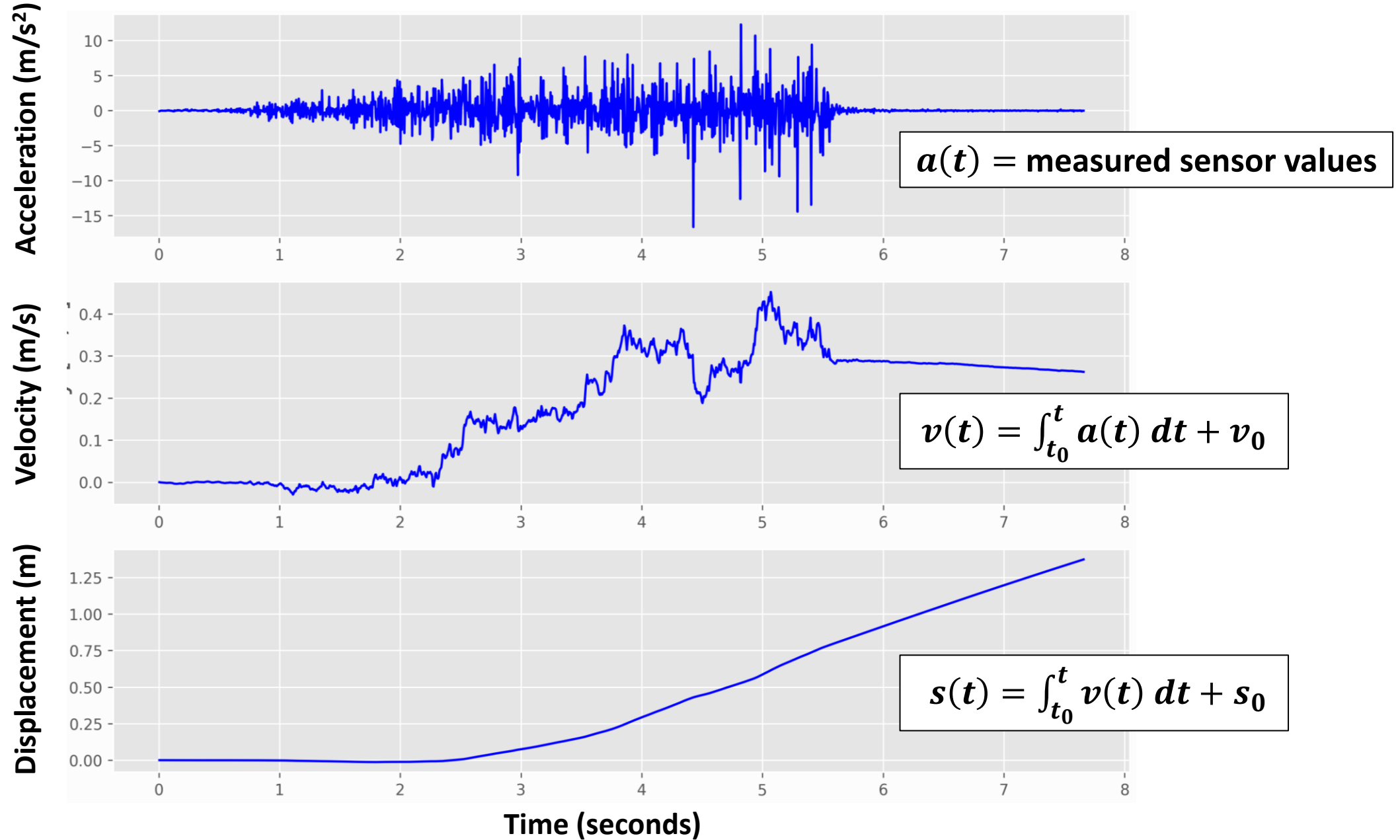
$$v(t) = \int_{t_0}^t a(t) dt + v_0$$

[2]

$$s(t) = \int_{t_0}^t v(t) dt + s_0$$

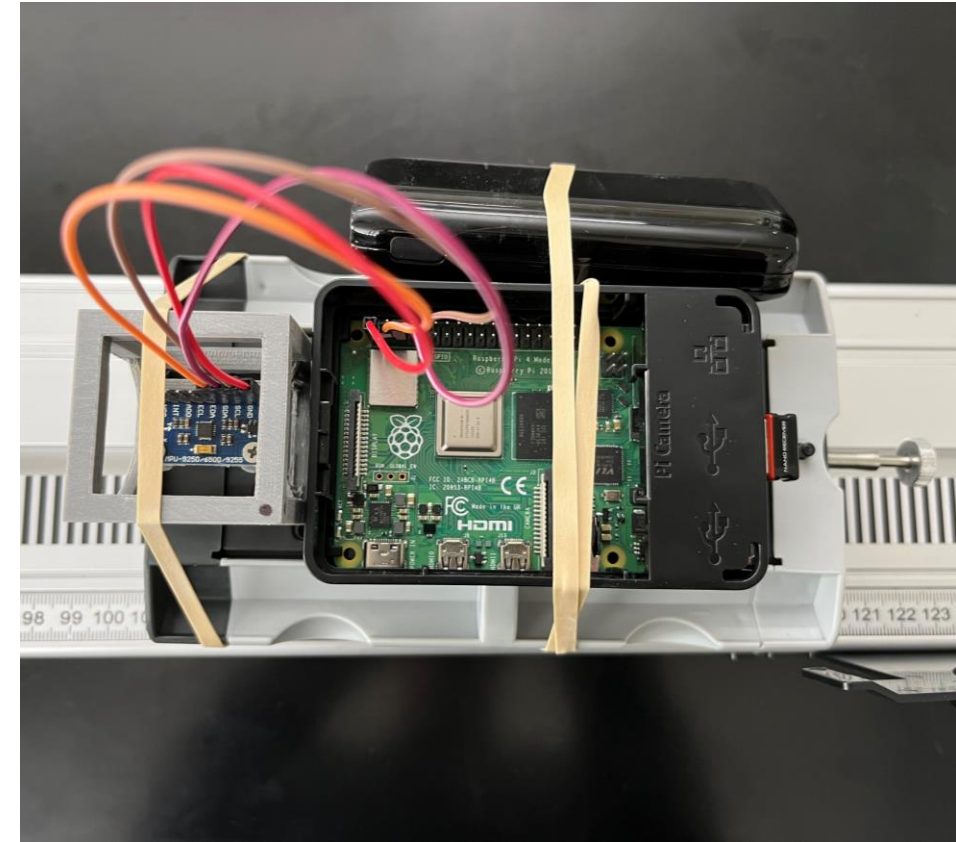
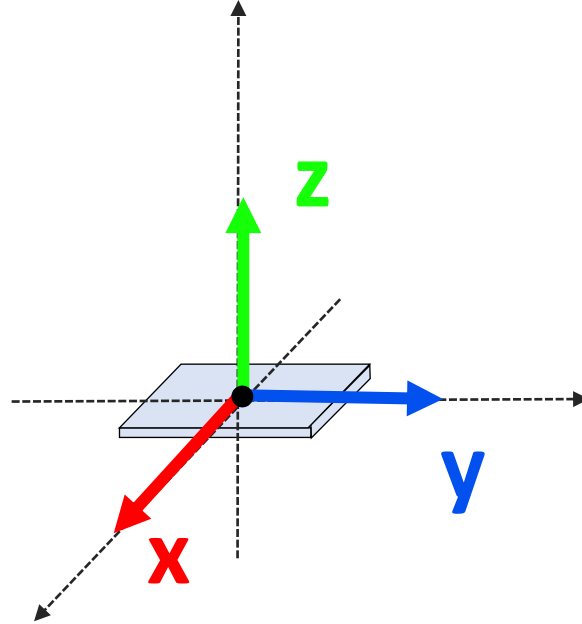
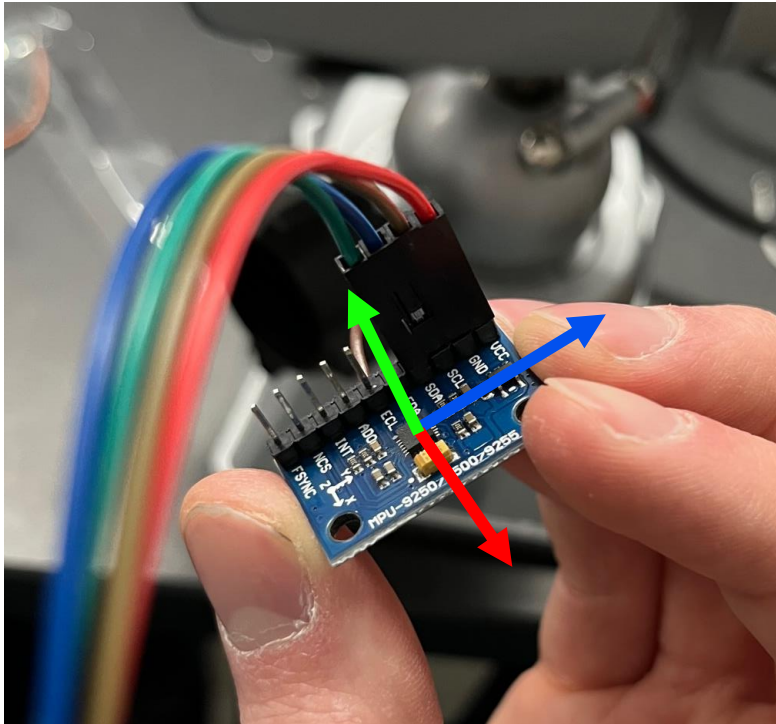
[2]

Acceleration, Velocity, and Displacement Over Time



MEMS Accelerometers

- **Micro Electromechanical Systems (MEMS)**
- Small, light, cheap sensor packages [3, 4]
- Ideal for electronics and robotics applications



Odometry and Situational Awareness

Odometry: tracking a robot's movement (displacement and rotation) over time [10]

Accurate **situational awareness** is essential in robotics navigation and operation.



Hmm... Where
am I?

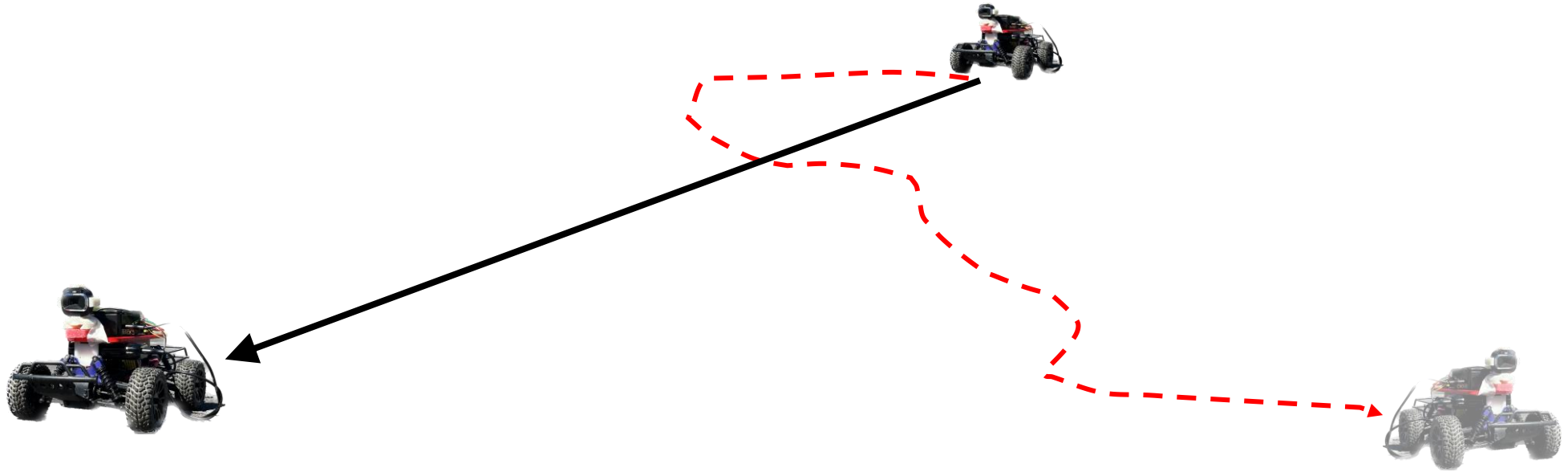
The Problem: Odometry Accuracy

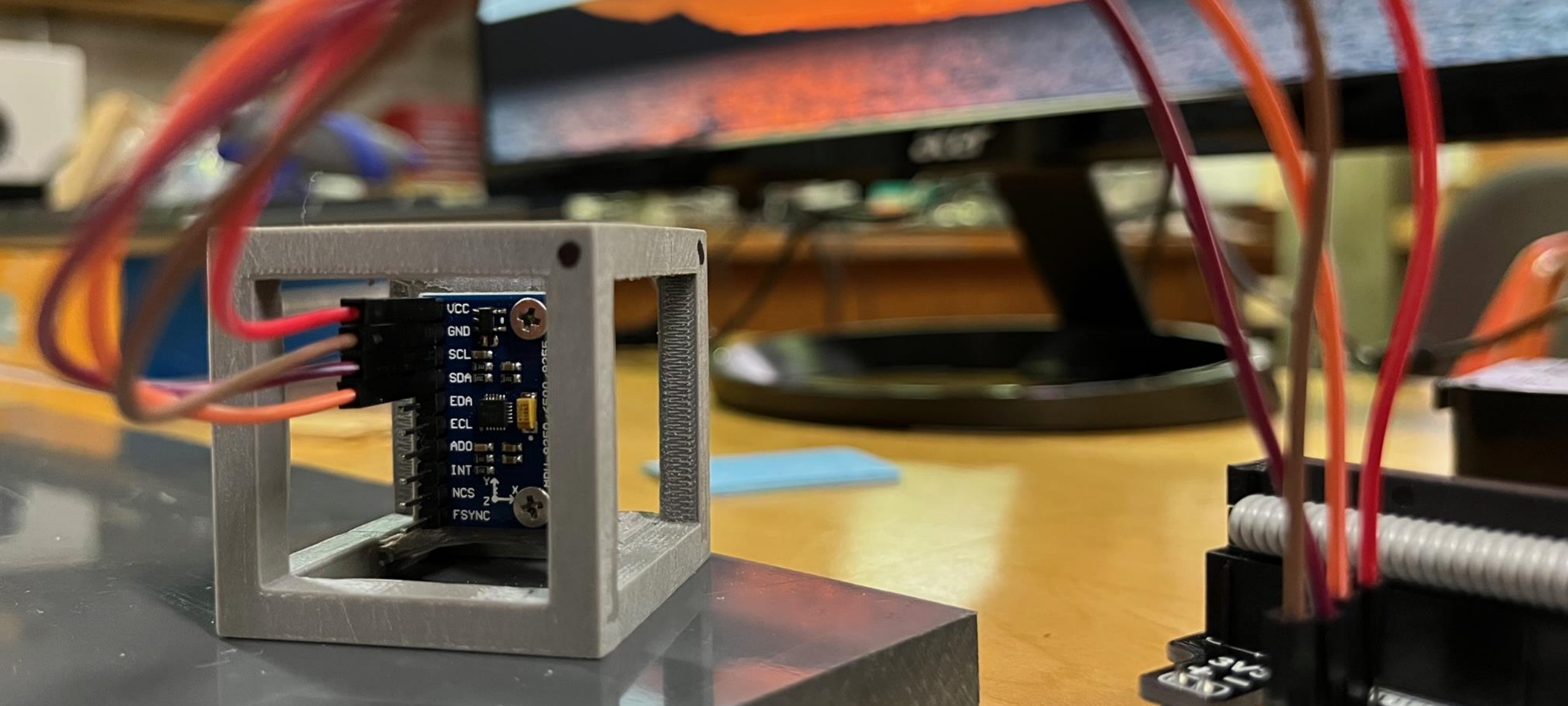
- MEMS accelerometer measurements tend to be **biased** (offset from the true value)
- Error in acceleration accumulates in displacement calculations

Displacement Drift Over Time for Different Acceleration Errors			
	t = 10 seconds	t = 30 seconds	t = 60 seconds
a = 0.001 m/s ²	0.05 meters	0.45 meters	1.8 meters
a = 0.01 m/s ²	0.5 meters	4.5 meters	18 meters
a = 0.1 m/s ²	5 meters	45 meters	180 meters

Investigating Accelerometer Calibration

- Robots rely on data accuracy for decision making
- A calibrated accelerometer provides better data, even if it's still low quality
- **Goal:** calibrate the accelerometer to achieve better displacement tracking



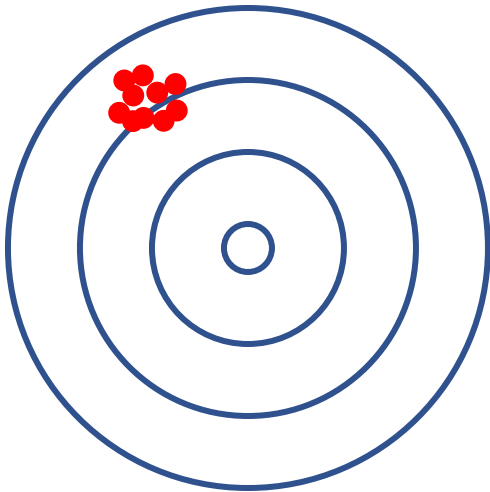


Literature Review

Types of Error

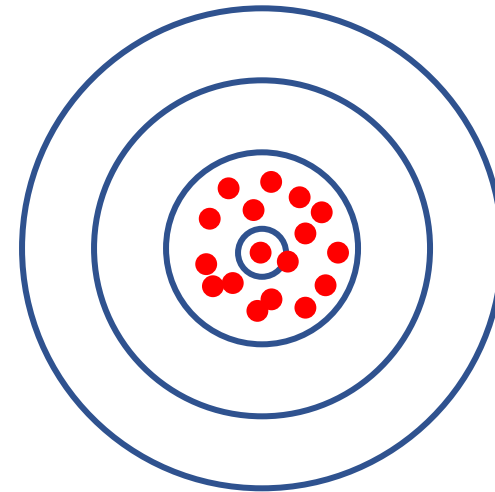
Systematic Uncertainty

- A clock that's always 2 minutes fast
- A car that wants to drift to the left

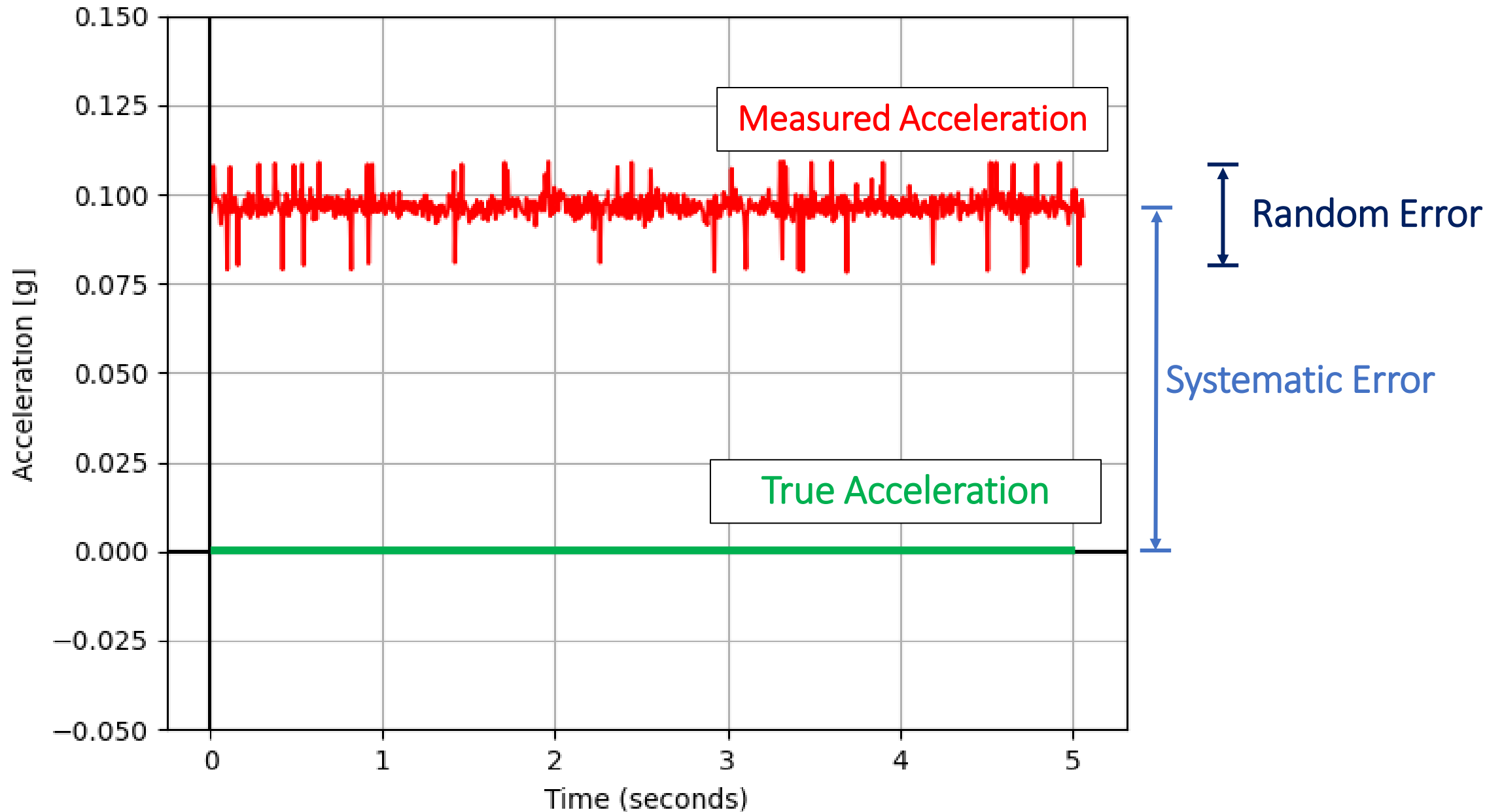


Random Uncertainty

- Unpredictable, but characterized by statistical analysis
- Random oscillations about a center

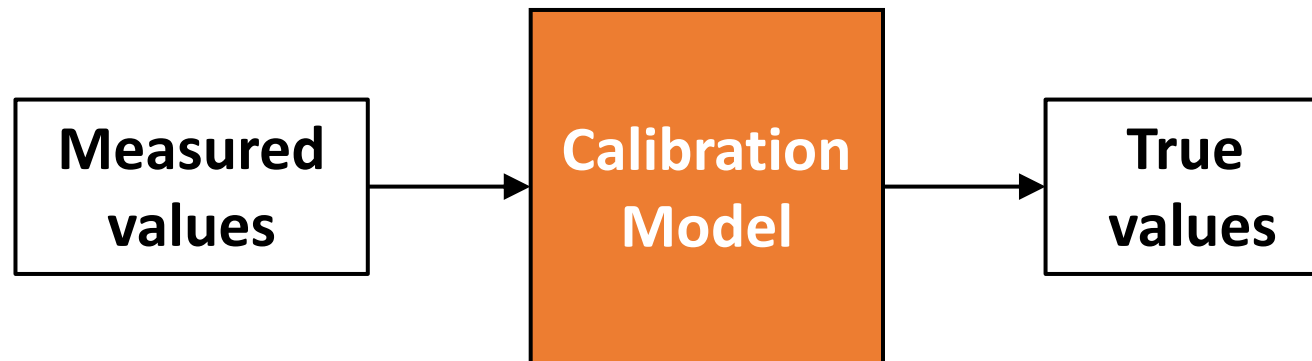


Acceleration At Rest



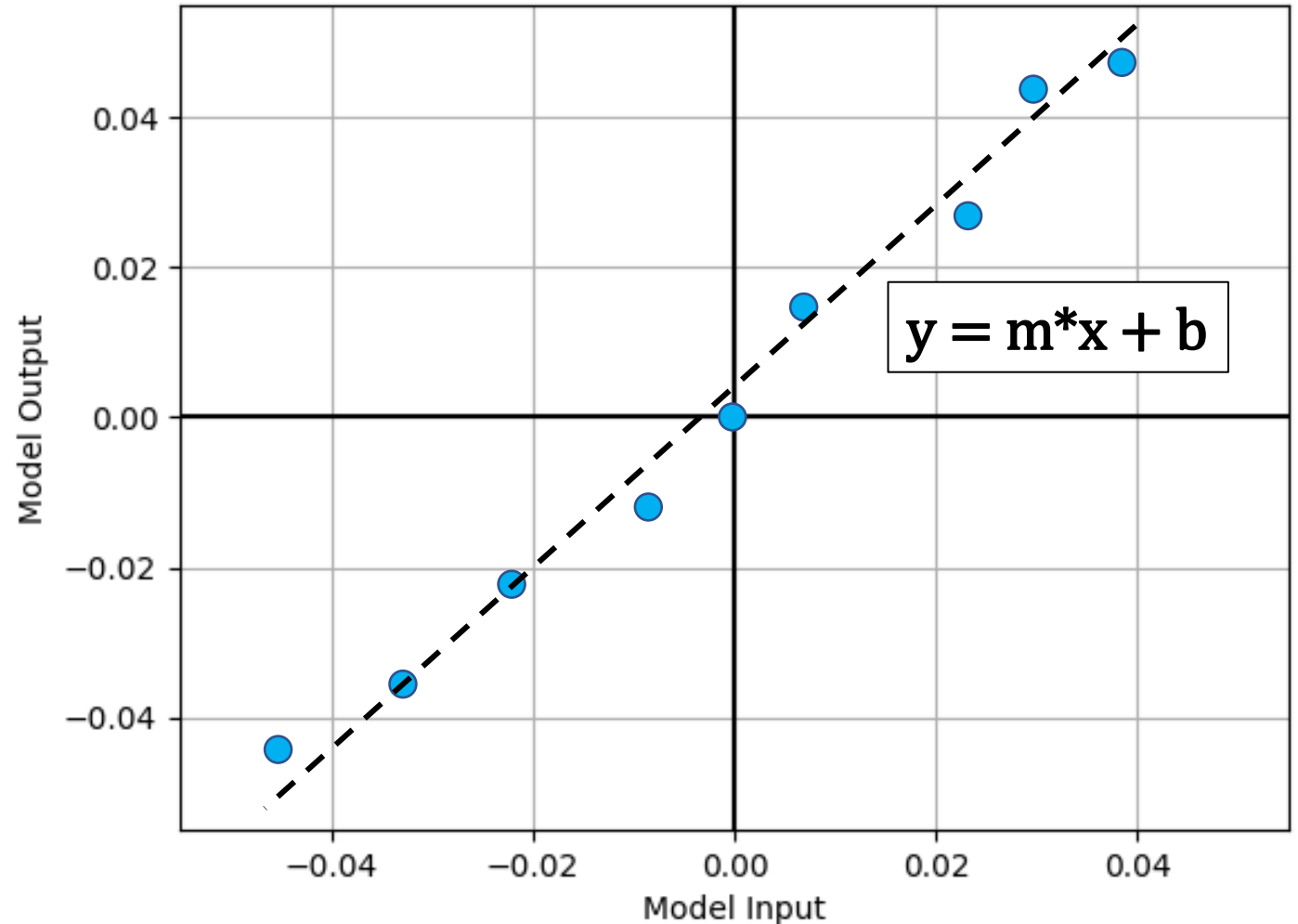
Accelerometer Error Modeling

- **Goal:** develop a calibration model that corrects sensor measurements
- Establish a mathematical relationship for the model
- Optimize the model using real data



Creating a Calibration Model

- Develop a mathematical model with **unknown parameters**
- Collect acceleration data when true acceleration is known
- Use curve fitting to optimize the model parameters



Accelerometer Error Model: Bias

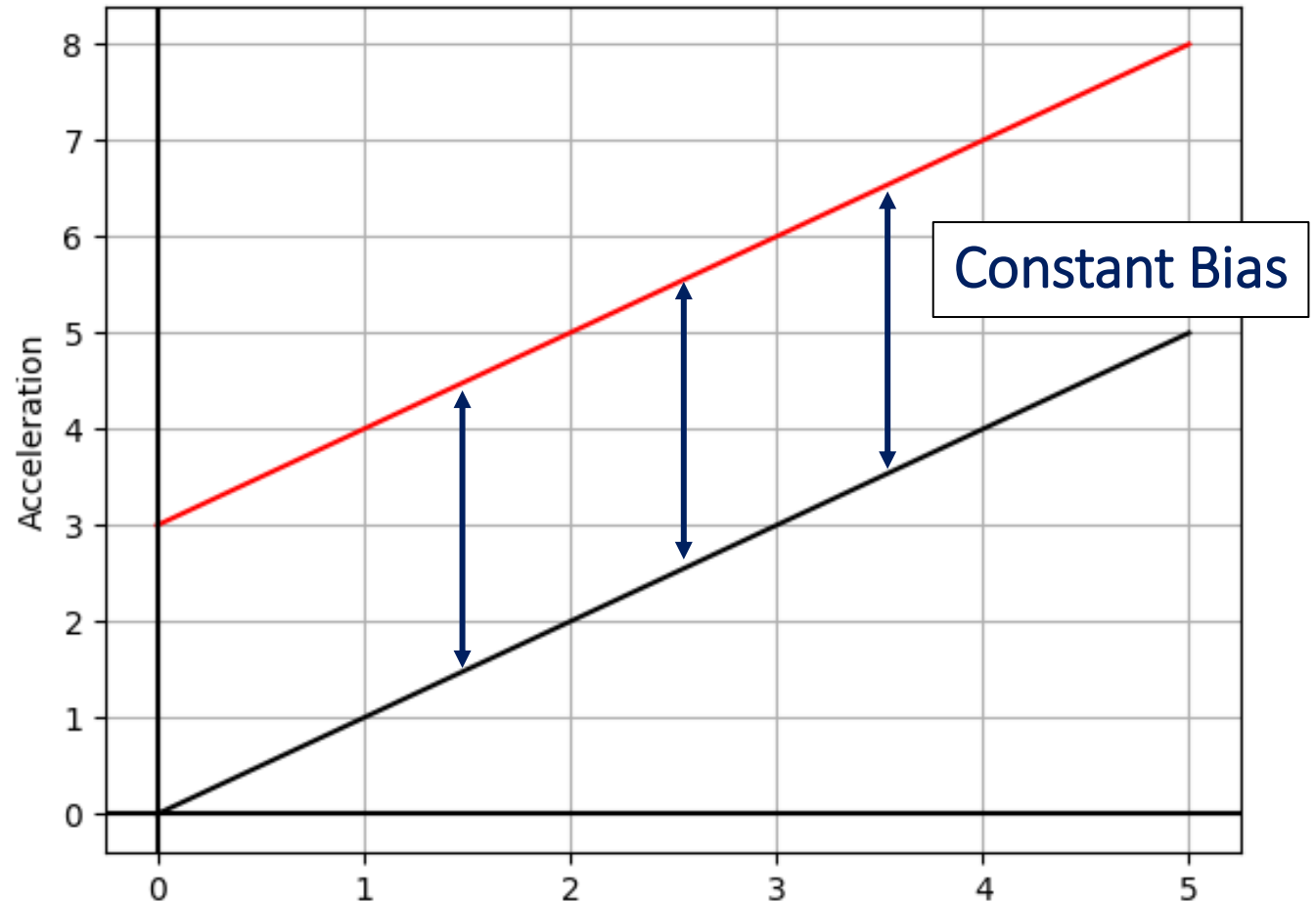
- Bias Model: measurements equals true values plus an offset [2]

$$\begin{bmatrix} a'_x \\ a'_y \\ a'_z \end{bmatrix} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$$

x-axis: $a'_x = a_x + b_x$

y-axis: $a'_y = a_y + b_y$

z-axis: $a'_z = a_z + b_z$



Accelerometer Error Model: Bias and Scale Factors

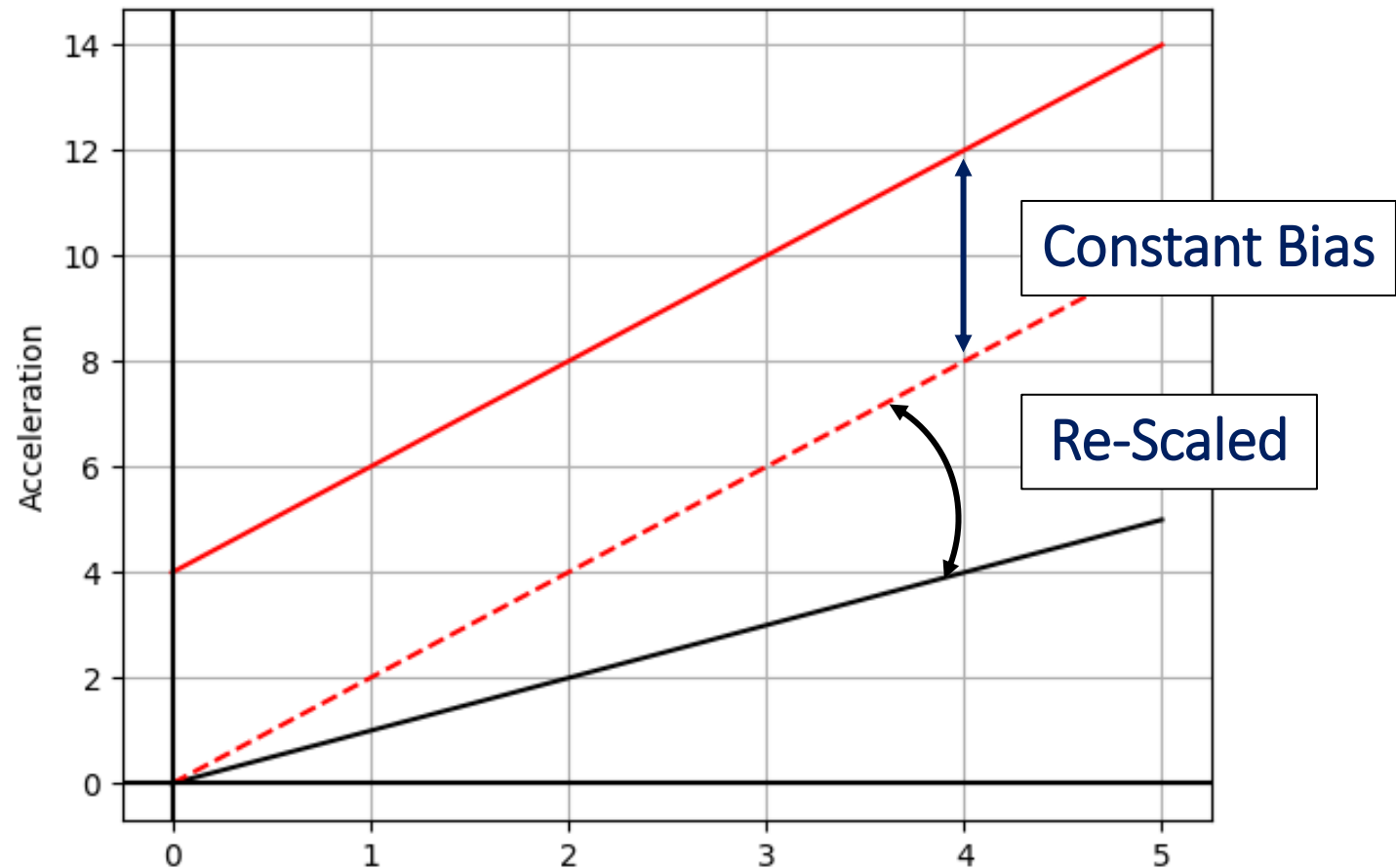
- Measured values equals true values that are rescaled and shifted [14]

$$\begin{bmatrix} a'_x \\ a'_y \\ a'_z \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & S_z \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$$

x-axis: $a'_x = S_x a_x + b_x$

y-axis: $a'_y = S_y a_y + b_y$

z-axis: $a'_z = S_z a_z + b_z$



Accelerometer Error Model: Bias, Scale Factors, and Misalignment

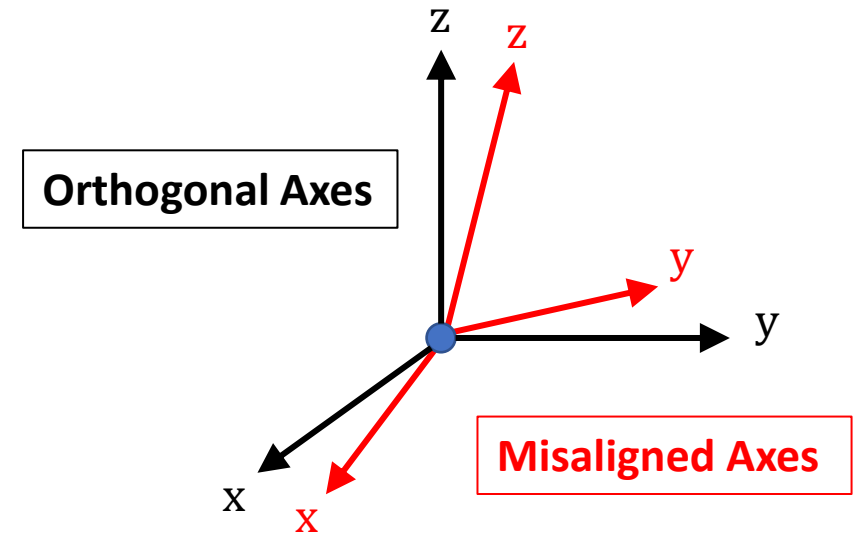
- Accommodates for any misalignments between the sensor axes [14-17]

$$\underbrace{\begin{bmatrix} a'_x \\ a'_y \\ a'_z \end{bmatrix} = \begin{bmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}}_{\text{Accelerometer Error Model}}$$

x-axis: $a'_x = S_{xx}a_x + S_{xy}a_y + S_{xz}a_z + b_x$

y-axis: $a'_y = S_{yx}a_x + S_{yy}a_y + S_{yz}a_z + b_y$

z-axis: $a'_z = S_{zx}a_x + S_{zy}a_y + S_{zz}a_z + b_z$



Accelerometer Error Models

- Parameters to Optimize:

Model 1: b_x, b_y, b_z

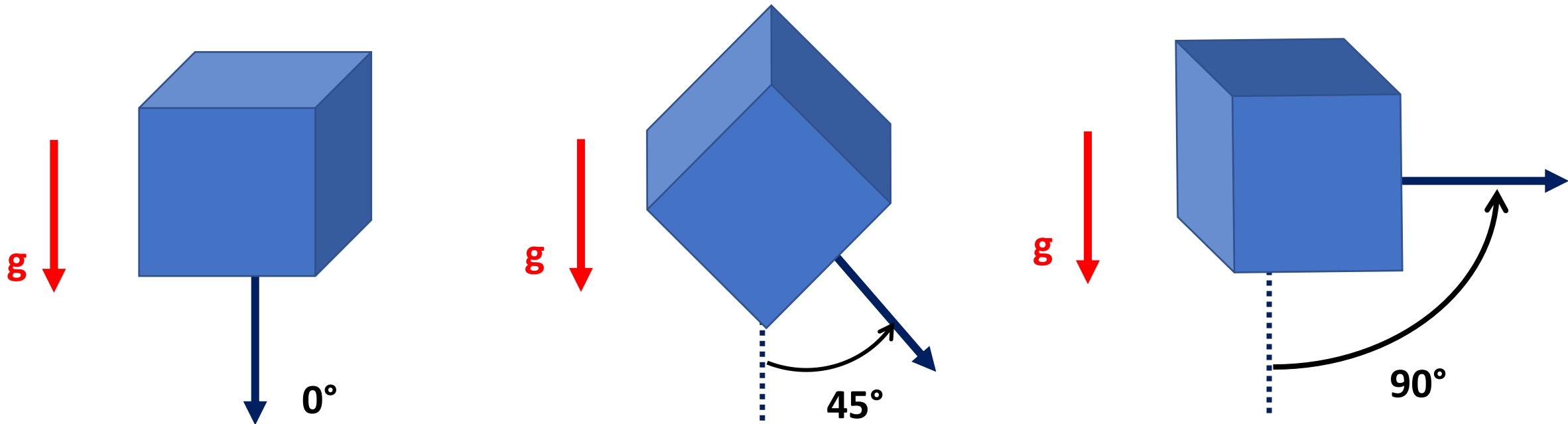
Model 2: $S_x, b_x, S_y, b_y, S_z, b_z$

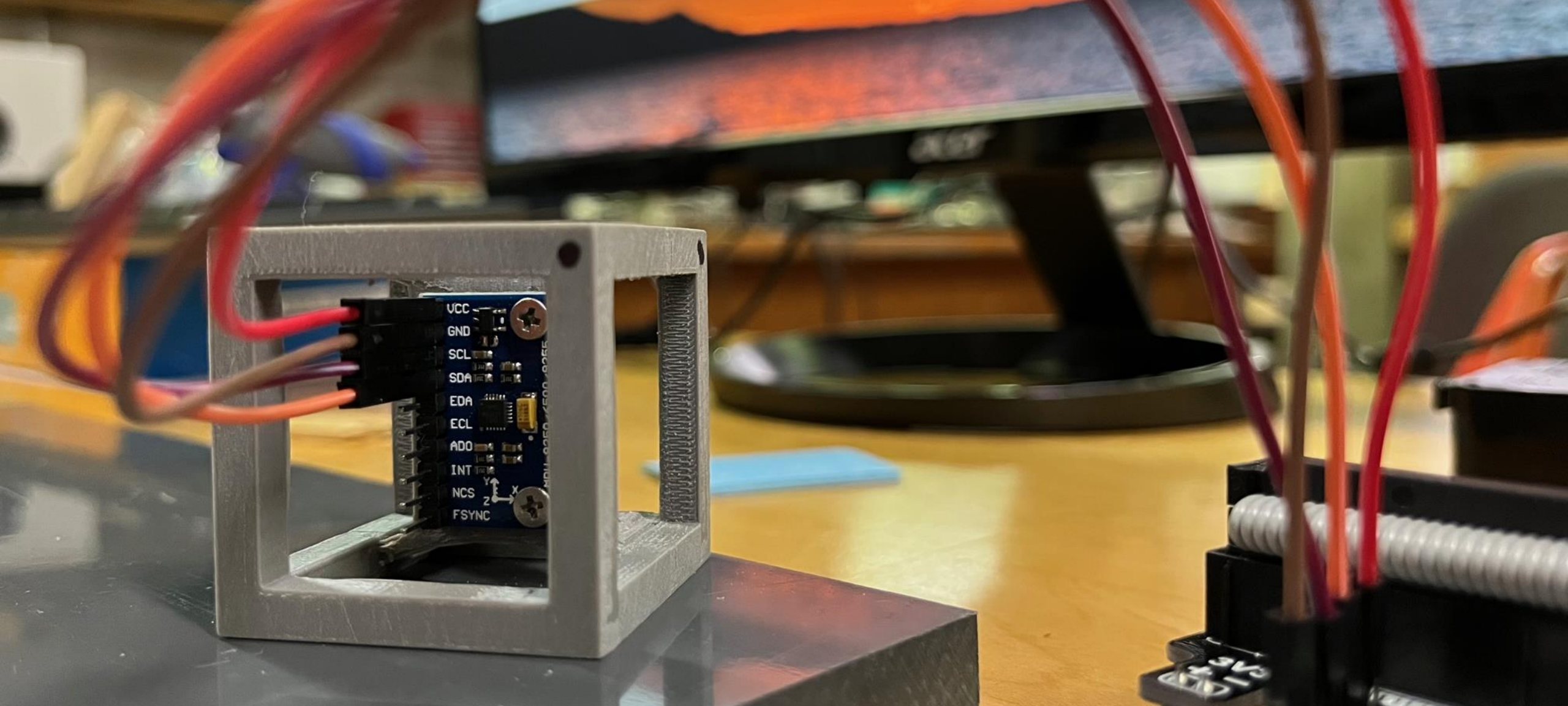
Model 3: $S_{xx}, S_{xy}, S_{xz}, b_x, S_{yx}, S_{yy}, S_{yz}, b_y, S_{zx}, S_{zy}, S_{zz}, b_z$

- Collect measured acceleration and true acceleration
- Use least-squares optimization to fit the data to the models and determine the model parameters
- Test the optimized models on new data

Common Data Collection Procedures

- Rotate the sensor to different orientations
- Gravity is the only acceleration measured
- Use known rotation angles to calculate true gravity





Methodology

Electronic Component Setup

- MPU-9250 IMU Breakout Board (HiLetgo [19])

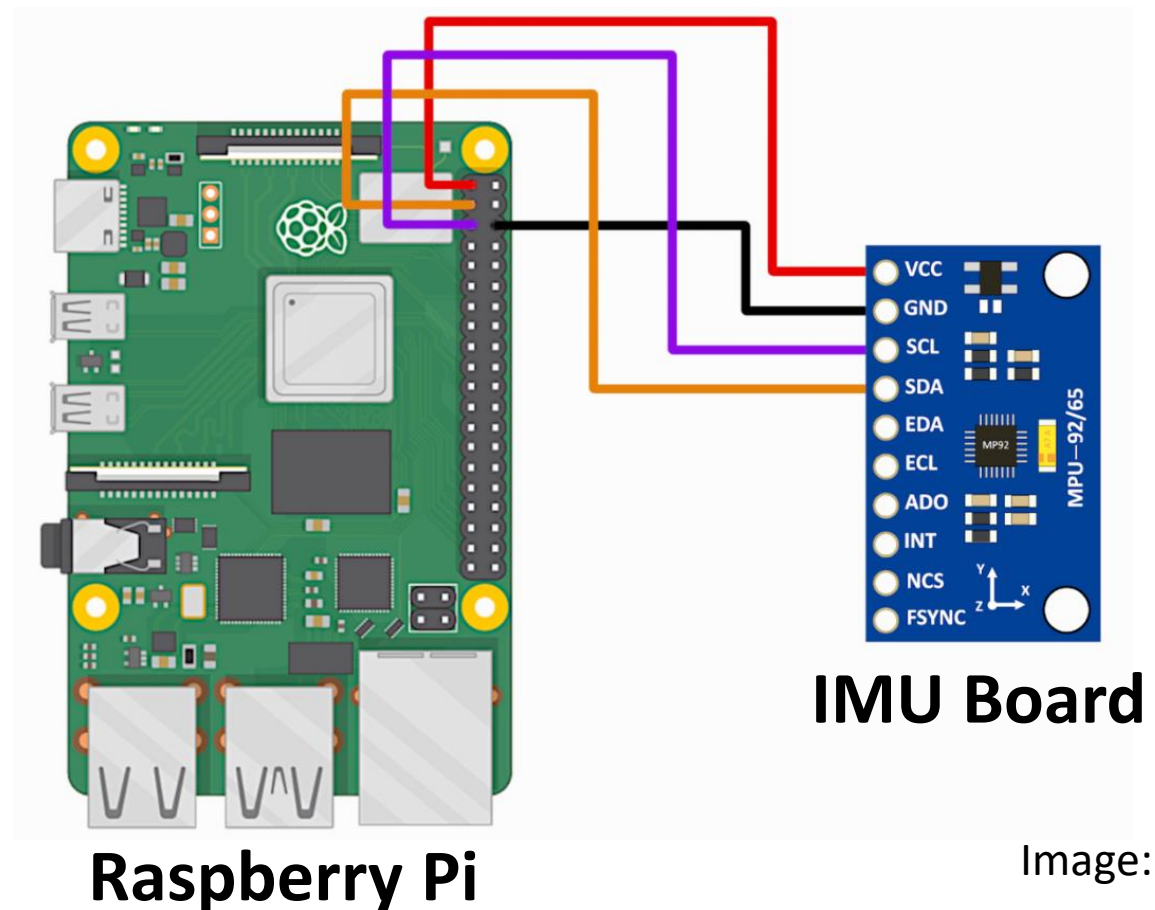
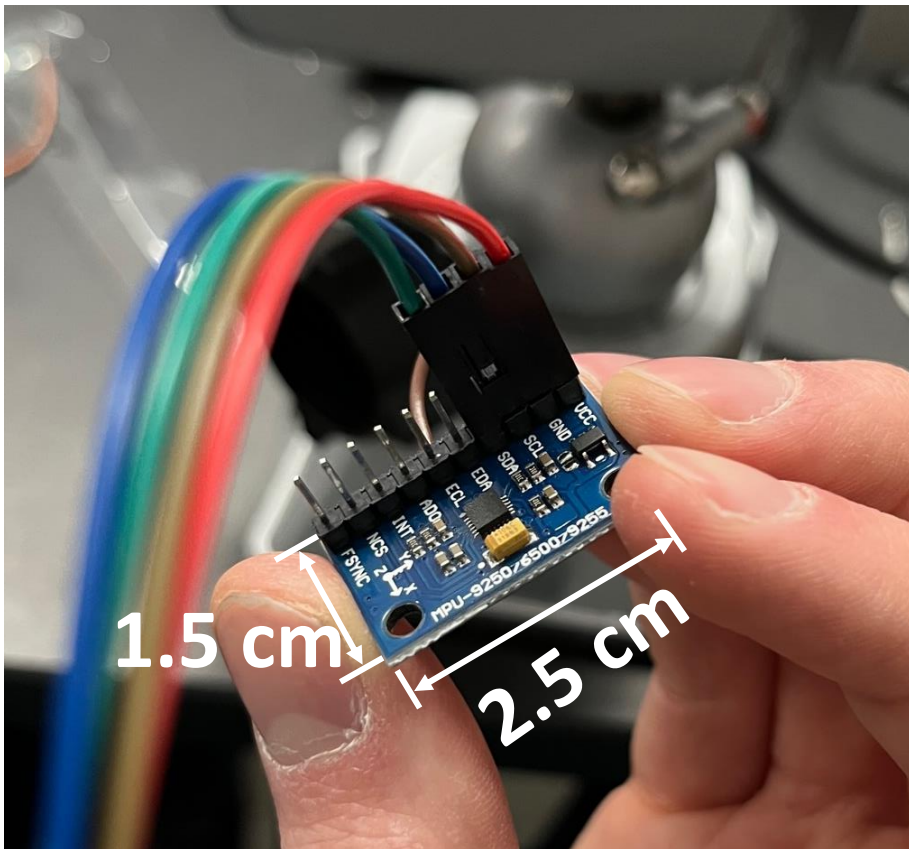
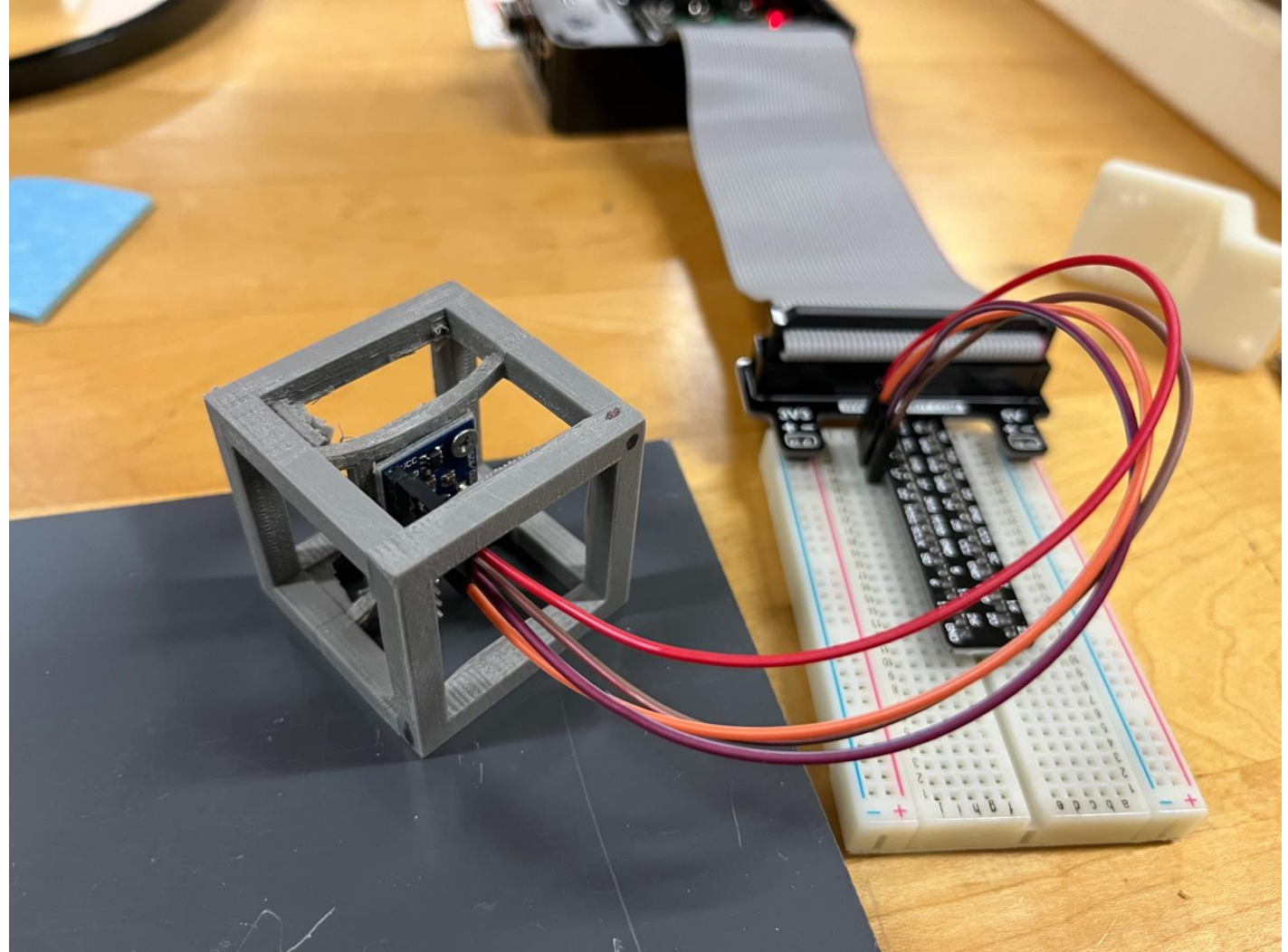
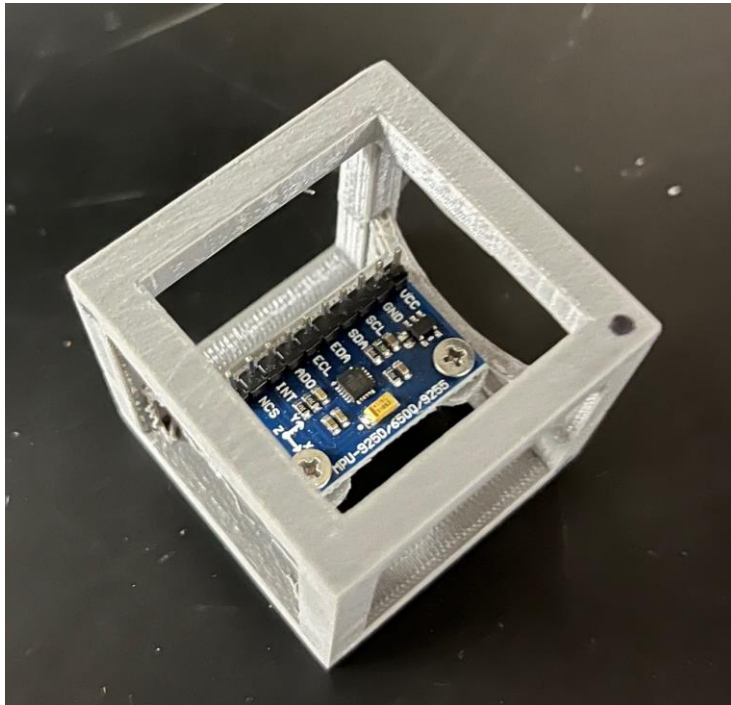


Image: [22]

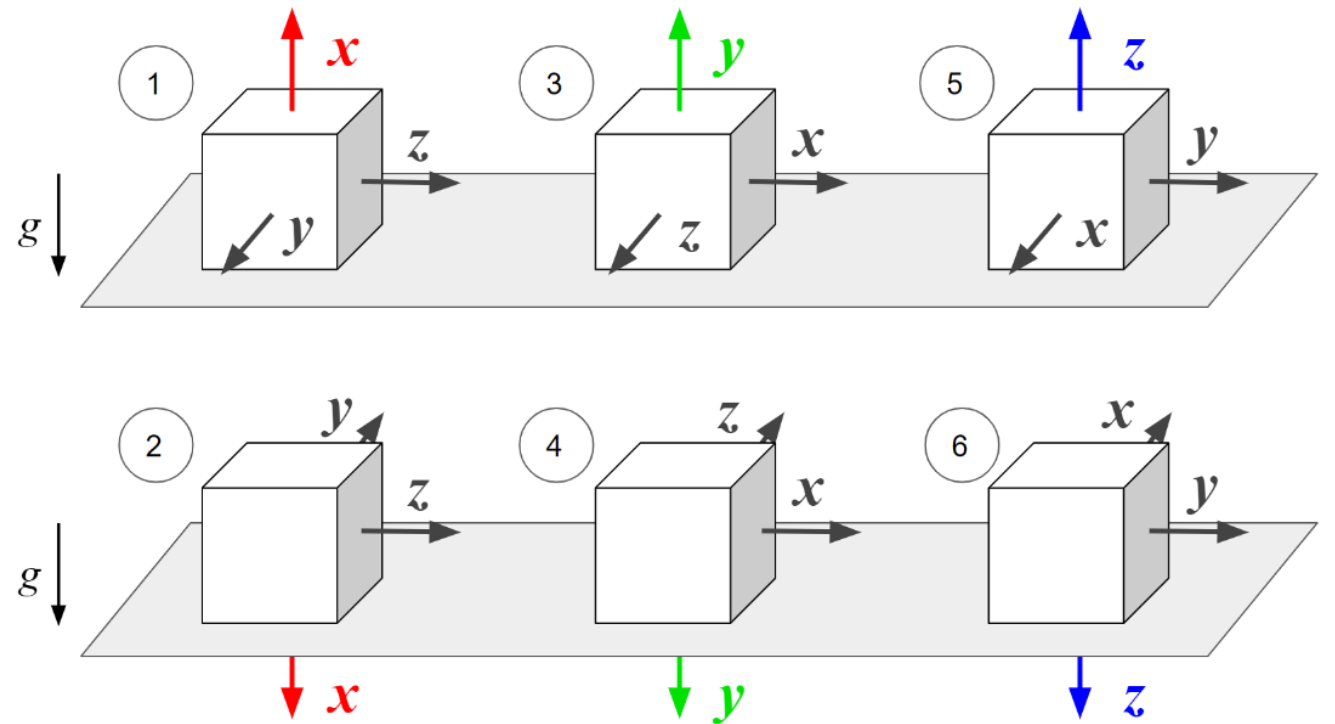
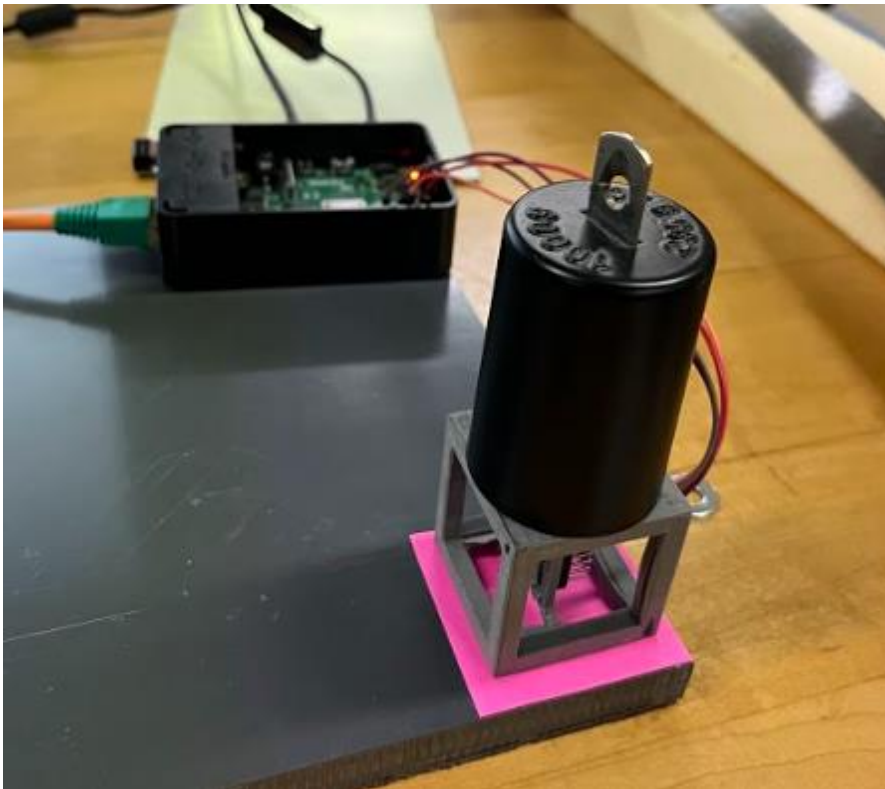
Level Calibration Cube

- 3D printed calibration cube [23]
- Allows easy 90° rotations
- Sides sanded flat



Six-Position Data Collection

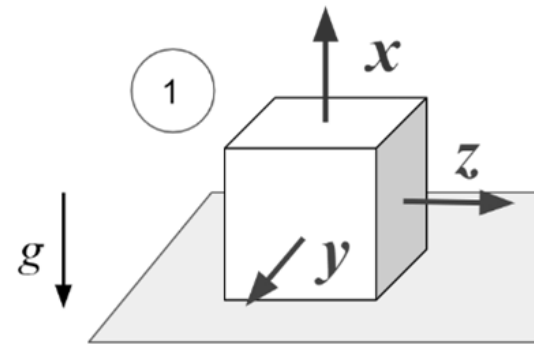
- Accelerometer at rest on a table
- Rotate so one axis is parallel with gravity and record data



Six-Position Data Collection

- Python script for collecting data, optimizing model parameters, and testing the models
- Collection process:
 1. Collect at one orientation for 30 seconds (180 measurements per second)
 2. Average the data and save with a ground truth label
 3. Rotate the sensor to the next orientation and repeat
- Collect data in **three trials**

$$\begin{bmatrix} (x_m, x) \\ (y_m, y) \\ (z_m, z) \end{bmatrix}_1, \begin{bmatrix} (x_m, x) \\ (y_m, y) \\ (z_m, z) \end{bmatrix}_2, \begin{bmatrix} (x_m, x) \\ (y_m, y) \\ (z_m, z) \end{bmatrix}_3, \dots$$

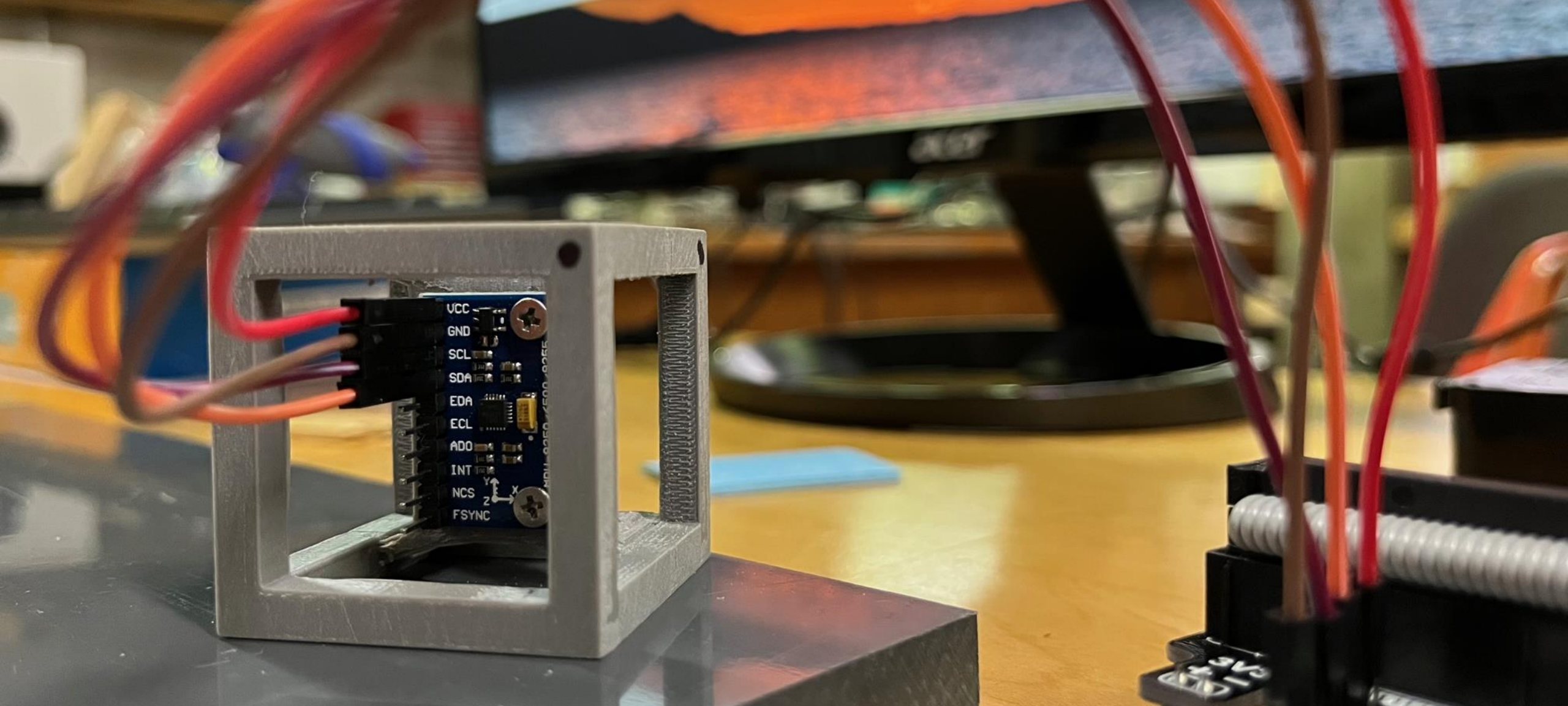


Calibration Models Tested

$$\text{Model 1: } \begin{bmatrix} a'_x \\ a'_y \\ a'_z \end{bmatrix} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$$

$$\text{Model 2: } \begin{bmatrix} a'_x \\ a'_y \\ a'_z \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & S_z \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$$

$$\text{Model 3: } \begin{bmatrix} a'_x \\ a'_y \\ a'_z \end{bmatrix} = \begin{bmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$$



Results

Six-Position Results

Table 4: Mean of Trial 1 Static Test Data Before and After Calibration

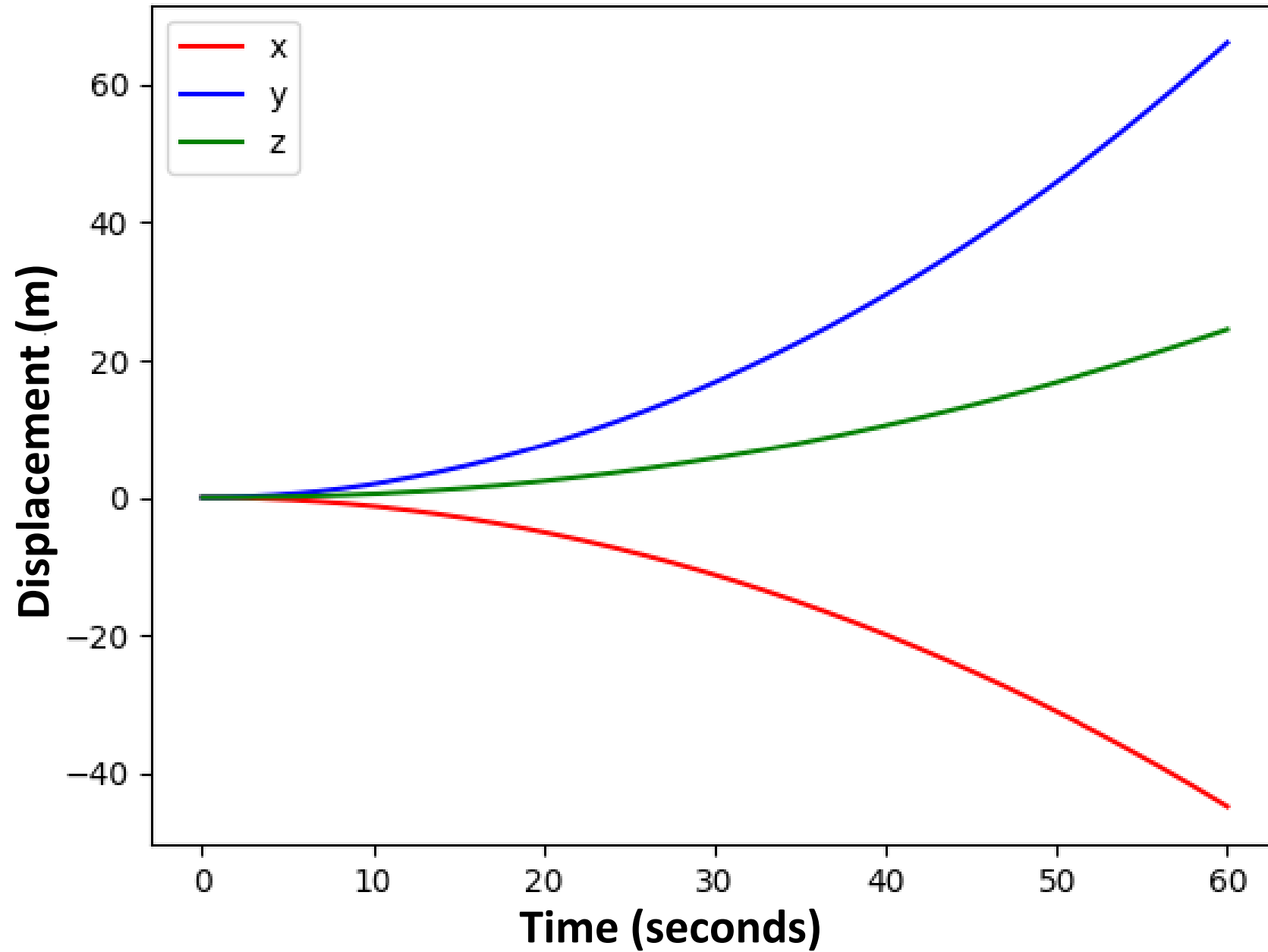
	x mean (m/s ²)	y mean (m/s ²)	z mean (m/s ²)
No Calibration	0.9617 ± 0.03%	0.6050 ± 0.09%	-2.2238 ± 0.019%
Model 1	0.0638 ± 0.5%	0.1411 ± 0.4%	0.2296 ± 0.18%
Model 2	0.0638 ± 0.5%	0.1411 ± 0.4%	0.0140 ± 3%
Model 3	-0.0252 ± 1.2%	0.0367 ± 2.2%	0.0139 ± 3%

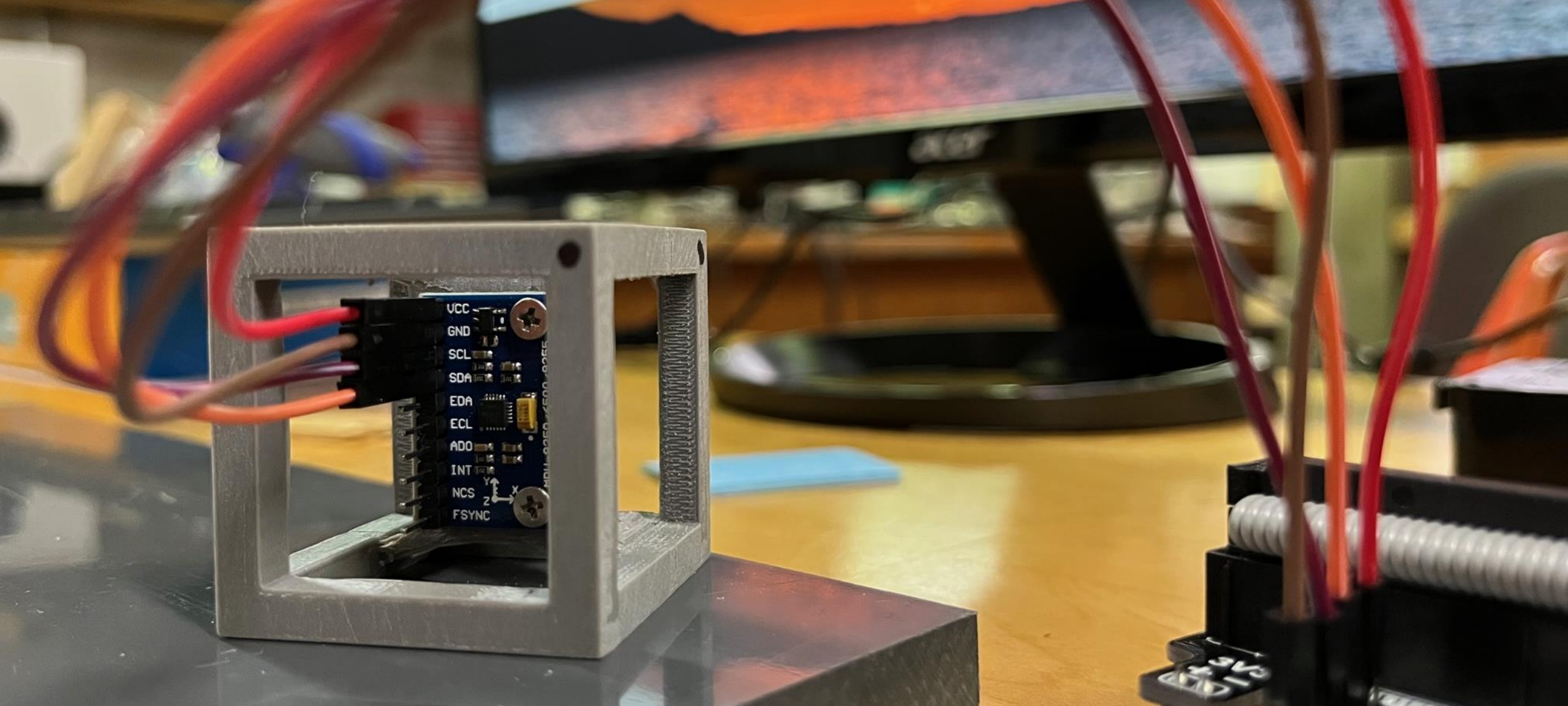
Six-Position Results

Table 5: Final Integrated Displacement of Trial 1 Static Test Data (60 seconds)

	x (m)	y (m)	z (m)
No Calibration	1731	1089	-4002
Model 1	115	254	413
Model 2	115	254	25
Model 3	-45	66	25

Displacement After Calibrating with Model 3 (Trial 1)





Bonus Experiment!

Further Calibration: Displacement Model

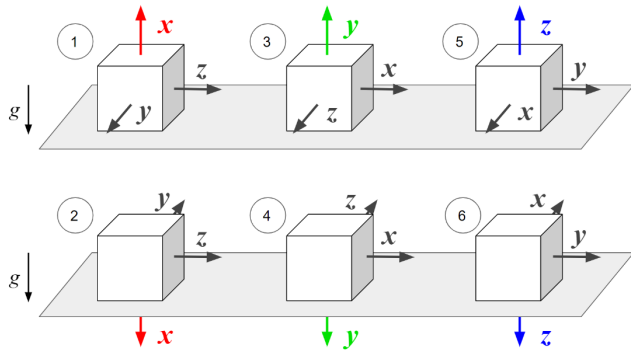
- Imperfect acceleration calibration leads to leftover displacement drift
- Model remaining drift as a polynomial [2, 13]

$$\mathbf{d}' = \mathbf{d} + \frac{1}{2} \mathbf{q}_2 t^2 + \mathbf{q}_1 t + \mathbf{q}_0$$

The diagram illustrates the displacement model equation $\mathbf{d}' = \mathbf{d} + \frac{1}{2} \mathbf{q}_2 t^2 + \mathbf{q}_1 t + \mathbf{q}_0$. A green box labeled "Measured Displacement" has a green arrow pointing to \mathbf{d}' . A blue box labeled "True Displacement" has a blue arrow pointing to \mathbf{d} . A red bracket under the error terms $\frac{1}{2} \mathbf{q}_2 t^2 + \mathbf{q}_1 t + \mathbf{q}_0$ is connected to a red box labeled "Error terms".

Final Calibration Process

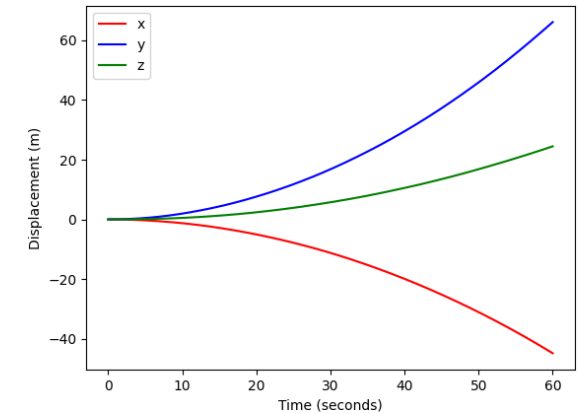
1. Six-Position Data Collection



2. Optimize Acceleration Model 3

$$\begin{bmatrix} a'_x \\ a'_y \\ a'_z \end{bmatrix} = \begin{bmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$$

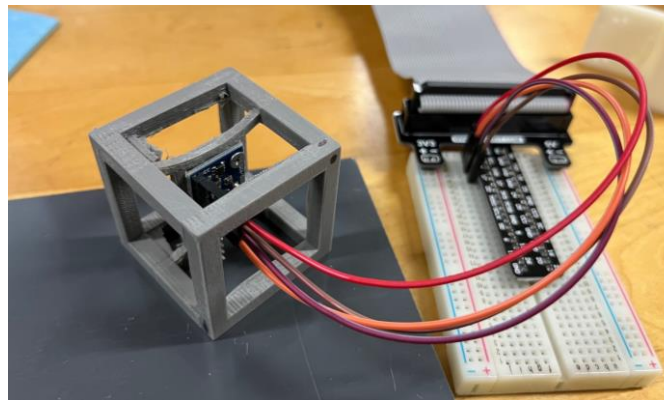
3. Calibrate and Integrate Static Data



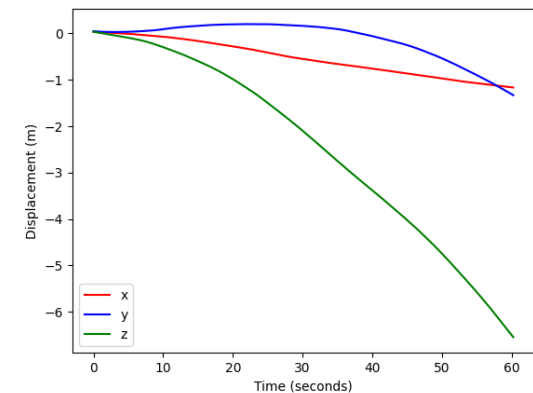
4. Optimize the Displacement Model

$$\mathbf{d}' = \mathbf{d} + \frac{1}{2} \mathbf{q}_2 t^2 + \mathbf{q}_1 t + \mathbf{q}_0$$

5. Collect New Static Data



6. Calibrate Using Both Models

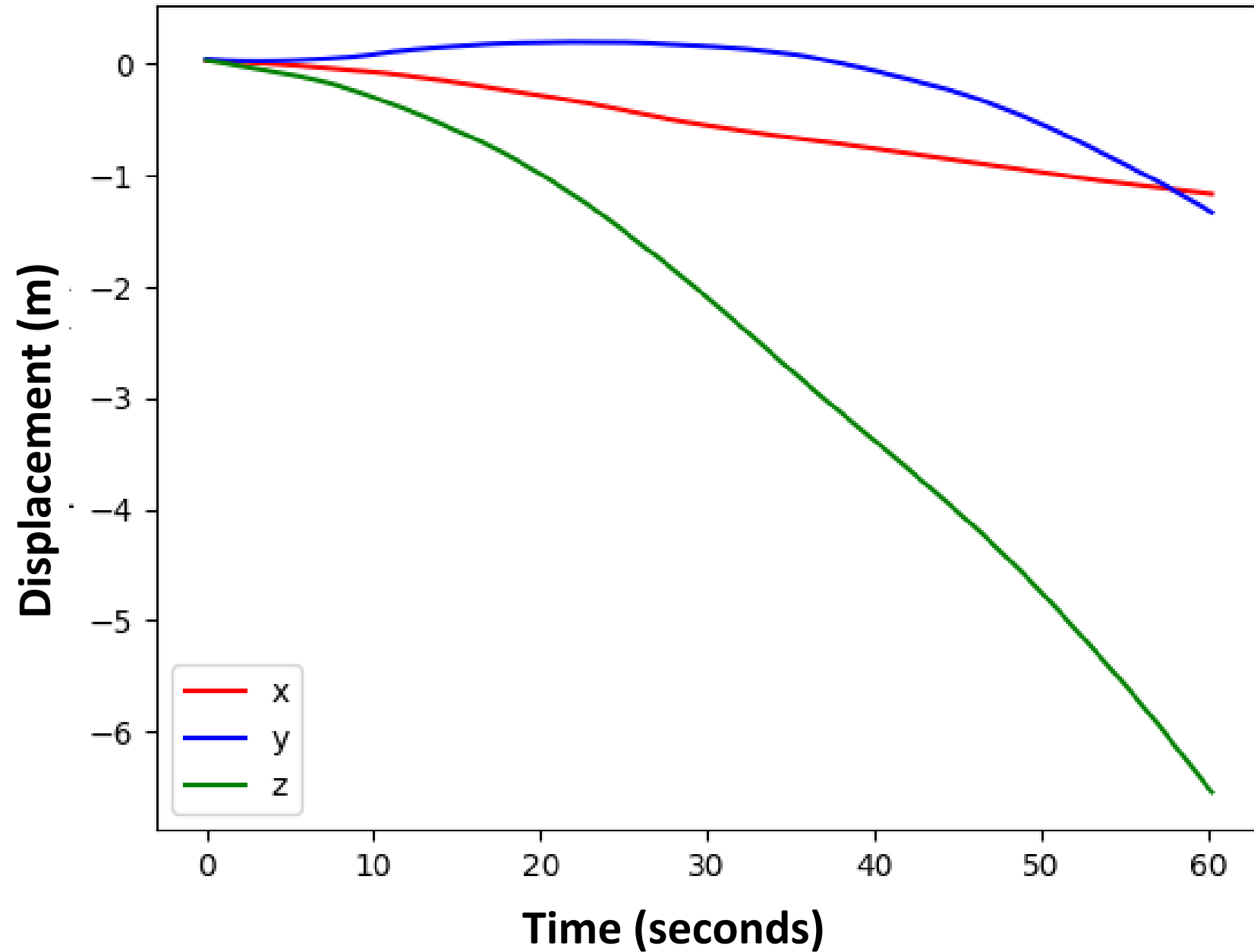


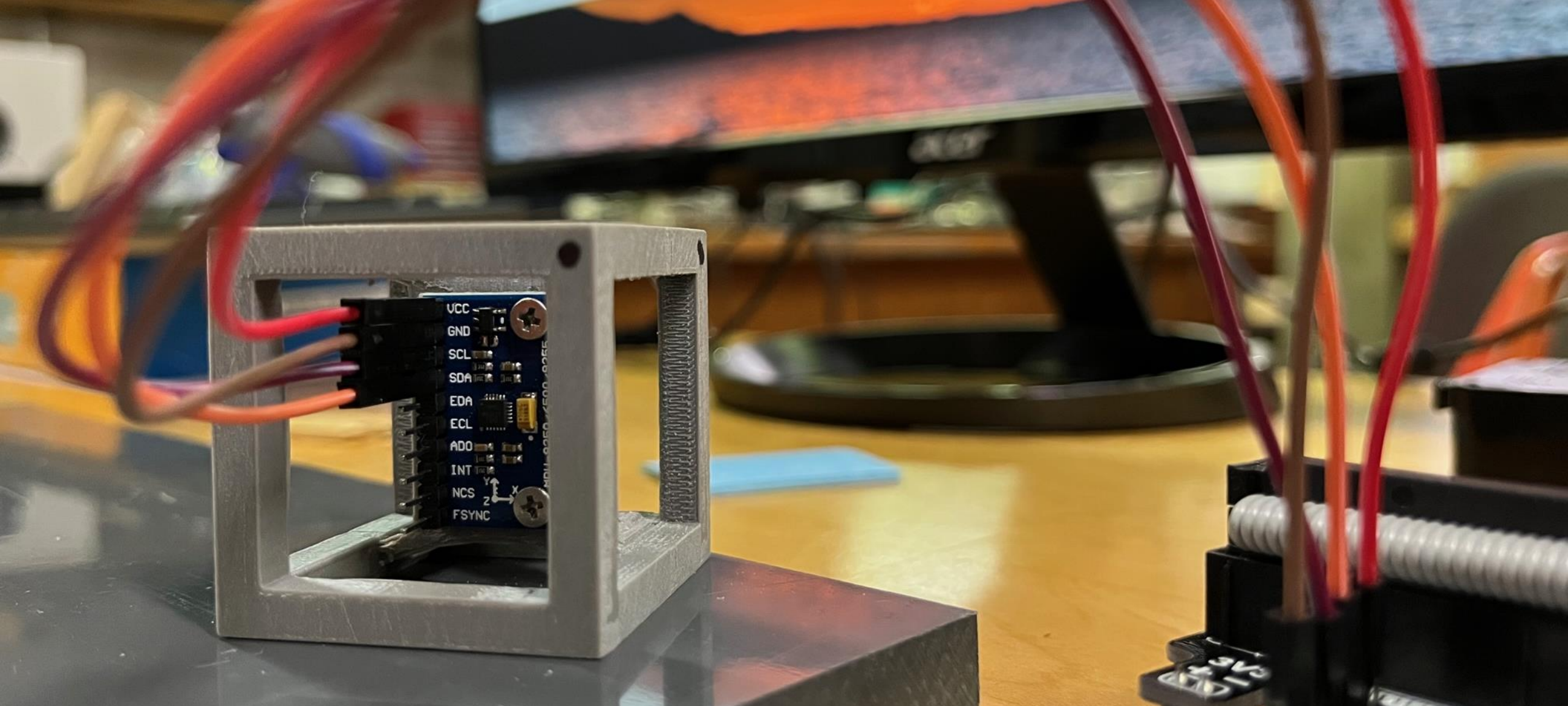
Final Calibration Results

Displacement of Final Trial Static Test Data (60 seconds)

	x (m)	y (m)	z (m)
No Calibration	1760	1061	-4014
Model 3	-35	96	25
Model 3 and Displacement Model	-1	-1	-7

Final Calibrated Displacement (Both Models)





Conclusions

Best Calibration Results

60 seconds of Acceleration Data



Direct Integration (NO Calibration)

Displacement
drifts over 1000
meters

$$\begin{bmatrix} a'_x \\ a'_y \\ a'_z \end{bmatrix} = \begin{bmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$$

Calibration Acceleration
with Model 3

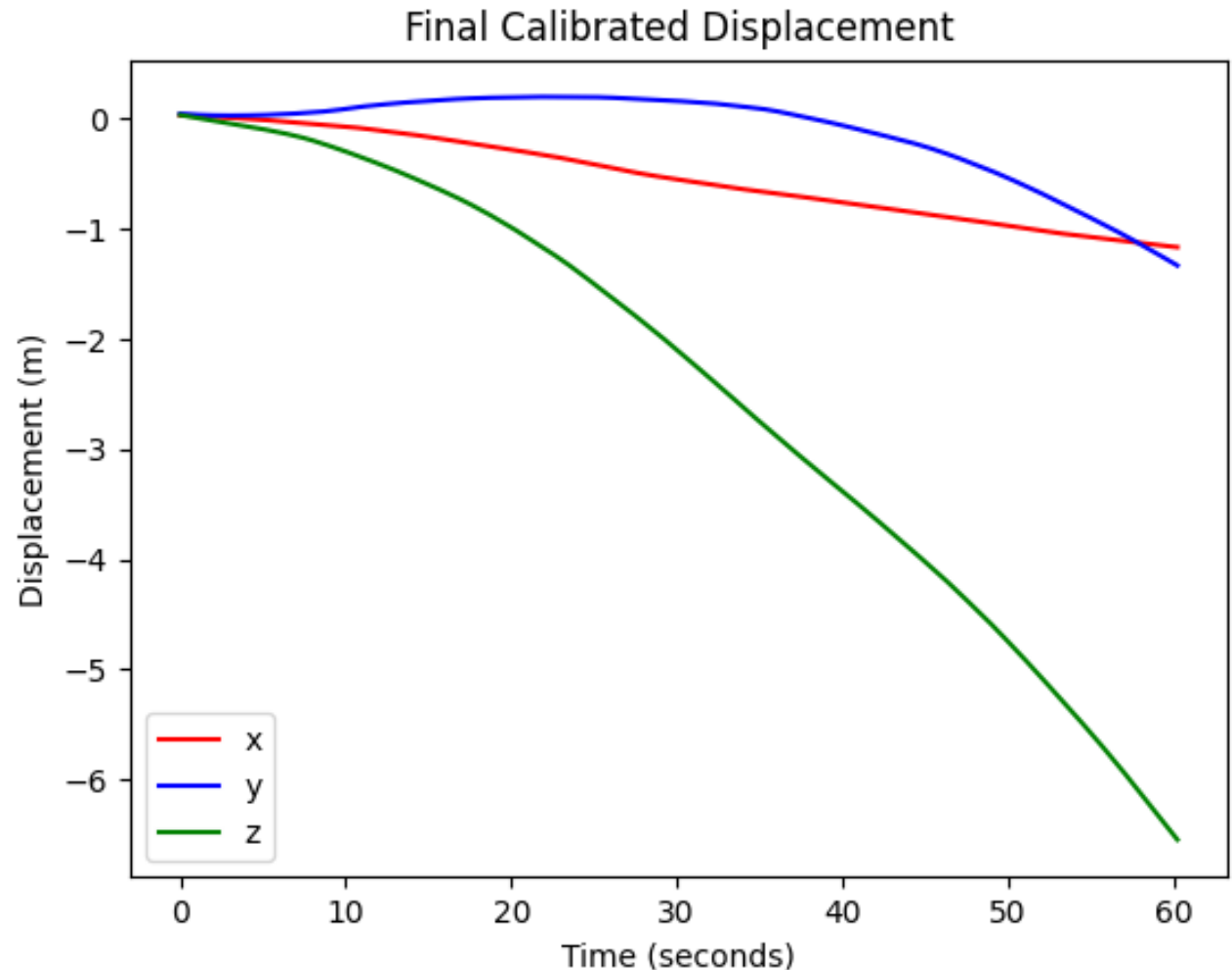
$$\mathbf{d}' = \mathbf{d} + \frac{1}{2} \mathbf{q}_2 t^2 + \mathbf{q}_1 t + \mathbf{q}_0$$

Integrate and Calibrate
Displacement

Displacement
drifts under 10
meters

Persisting Issues

- Displacement drift **always** accumulates if left uncorrected
- Longer runtime = more drift
- **Common Solution:** use GPS, camera, or radar to correct drift
- However, these sensors are not always available

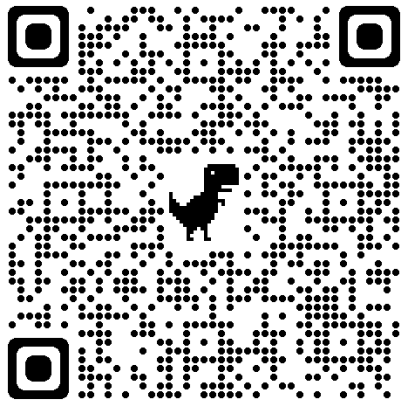


Conclusion and Future Work

- Displacement estimations improved by approximately 1000 meters after best calibration procedure
- However, drift always exists in measurements
- **Future Work:**
 - Testing calibration on a moving robot
 - Further investigation of errors in calibration
 - Investigation of alternative accelerometer calibration parameters



Thank You!



References

