A New Heuristic for Scalable Quantum(-inspired) Annealing on Practical Quadratic Assignment Problems with Block-structural Properties

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Abstract*—As quantum computing gains traction, there is a significant effort in applying it to real-world problems. One of the approaches is to use quantum annealing for the Quadratic Assignment Problem (QAP), which in turn has a wide range of applications. However, practical QAPs are often large because the size of a QAP increases quadratically with respect to the number of "items to be assigned". As a result, state-of-the-art quantum(-inspired) annealers, with their limited number of qubits, lack scalability. This paper proposes a new heuristic that enables quantum(-inspired) annealers to solve QAPs of size linear to the number of qubits, provided a certain block-structural property is satisfied. Such property is frequently observed in practical settings such as warehouse allocation. The heuristic is implemented on Fujitsu Digital Annealer (DA), a dedicated quantum-inspired CMOS device that implements annealing. Experiments are performed on standard QAPLIB datasets and randomly generated block-structural QAP instances, and performance is compared against conventional methods such as software simulated annealing. Results demonstrate that the new heuristic 1) produces solutions of comparable quality on standard QAPs, 2) produces superior solutions for block-structural QAPs as well as for warehouse allocation and 3) DA achieves substantial speedup compared to conventional methods. Therefore, via the new heuristic, quantum(-inspired) annealers can effectively solve block-structural QAPs of large sizes and can be applied to practical problems.

***Index Terms*—component, formatting, style, styling, insert**

I. Introduction

Quadratic Assignment Problem (QAP) is a well-known combinatorial optimisation problem with a wide range of applications, such as backboard wiring, statistical analysis, placement of electronic components, etc. QAP can be visualised as the problem of assigning n facilities to n locations. There is a flow between each pair of facilities and a distance between each pair of locations. The aim is to find an assignment of the facilities to the locations such that the sum of all products of flows and their corresponding distances is minimised, subject to the constraints that each facility is assigned to exactly one location and each location contains exactly one facility. Formally, if flow is given as the matrix (f_{ij}) and distance is given as the matrix (d_{kl}), then the objective function to minimise is

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} f_{ij} d_{kl} x_{ik} x_{jl}$$

subject to

$$\sum_{k=1}^{n} x_{ik} = 1 \qquad \forall 1 \leq i \leq n \tag{1}$$

$$\sum_{i=1}^{n} x_{ik} = 1 \qquad \forall 1 \leq k \leq n \tag{2}$$

x_{ik} is the binary decision variable representing whether facility i goes to location k.

QAP is computationally difficult because it is NP-hard. Except for very small instances (n¡15?) which can be solved with exact algorithms, most practical instances are intractable to exact algorithms and various heuristics are used to produce solutions of decent quality with short computation time. For example, ().

Some of the heuristics listed above are amenable to hardware acceleration. For example, ().

Identify applicable funding agency here. If none, delete this.

Quantum(-inspired) annealer is a relatively new kind of hardware that could accelerate QAP. A Quantum annealer works by introducing a problem Hamiltonian, which contains biases and couplings between qubits, to the initial Hamiltonian[]. The energy stay at minimum throughout the annealing process. At the end of annealing, the system is in the eigenstate of the problem Hamiltonian and the qubits are in their classical states.

In order to enable quantum annealing of a QAP, the problem must first be converted into a Quadratic Unconstrained Binary Optimisation (QUBO) form[], which effectively means constraints (**??**) and (**??**) have to be subsumed as part of the objective function in some way. It is customary to do this by encoding the constraints as quadratic penalty terms which augment the objective function. It can be shown that for QUBOs, the optimal solution to the augmented objective function also minimises the original objective function [].

However, state-of-the-art quantum annealers lack scalability for QAPs of practical sizes. Whereas QAPs in research literature are often experimental in size, ranging from several tens of variables to at most over two hundred variables, practical QAPs can be way larger. For instance, a warehouse can have hundreds of locations, and the number of decision variables in a derived QAP is easily in the range of 10^4. More importantly, the number of decision variables is $O(n^2)$ with respect to the number of items to be assigned. Meanwhile, modern quantum annealers only support a small number of qubits, with each qubit corresponding to a decision variable. The D-Wave quantum annealer uses over 2000 qubits, but its *chimera* architecture, where qubits are arranged in connected clusters of 8, limits the maximum number of nodes in a fully-connected problem graph to around 64 []. Increasing this number requires the graph to be sparser, but actual graph density depends on application and sparsity is not guaranteed. Quantum-inspired technologies are better off but not fully capable. For example, Fujitsu Digital Annealer (DA) is a quantum-inspired CMOS ASIC which implements annealing, and it supports over 8000 variables with full connectivity. This means a naive QAP implementation has a maximum size of nearly 90 variables. However, this is still too small to satisfy the high demands of problem size in practical QAPs.

This paper deals with QAPs having a particular block-structure. In particular, the set of variables in the distance matrix can be split into intervals such that distance between locations in the same interval is relatively small, and distance between locations in different intervals is larger. This pattern is observed in our case study of warehouse management, where locations are organised into columns which correspond with intervals. It is easy to travel within a column, but crossing over to another column will require traversing longer distances.

This paper presents a novel divide-and-conquer heuristic that takes advantage of the above block-structure. Combining a maximum k-cut on frequency matrix with a minimum k-cut on distance matrix, the original QAP can be meaningfully divided into sub-problems and each sub-problem will have a size that is $O(n)$ to the number of items. This makes the overall QAP amenable to solving on a quantum device.

This paper is organised as follows. Section 2 describes different aspects of related work. Section 3 elaborates on QAP and gives a more precise definition of the block-structural property. Section 4 explains the new divide-and-conquer heuristic. Section 5 describes experimental setup and computational results. Section 6 discusses some issues and points to future research. Section 7 concludes.

II. Related Work

A. Decomposition Methods

There are two levels at which a problem can be decomposed. The first level is higher at QAP, while the second is the more primitive QUBO.

B. Use of Quantum Devices

III. Ease of Use

A. Maintaining the Integrity of the Specifications

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Keep your text and graphic files separate until after the text has been formatted and styled. Do not number text heads—LaTeX will do that for you.

A. Abbreviations and Acronyms

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, ac, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

B. Units

- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as "3.5-inch disk drive".
- Avoid combining SI and CGS units, such as current in amperes and magnetic field in oersteds. This often leads to confusion because equations do not balance dimensionally. If you must use mixed units, clearly state the units for each quantity that you use in an equation.

- Do not mix complete spellings and abbreviations of units: "Wb/m^2" or "webers per square meter", not "webers/m^2". Spell out units when they appear in text: ". . . a few henries", not ". . . a few H".
- Use a zero before decimal points: "0.25", not ".25". Use "cm^3", not "cc".)

C. Equations

Number equations consecutively. To make your equations more compact, you may use the solidus (/), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in:

$$a + b = \gamma \tag{3}$$

Be sure that the symbols in your equation have been defined before or immediately following the equation. Use "(**??**)", not "Eq. (**??**)" or "equation (**??**)", except at the beginning of a sentence: "Equation (**??**) is . . ."

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Please use "soft" (e.g., `\eqref{Eq}`) cross references instead of "hard" references (e.g., `(1)`). That will make it possible to combine sections, add equations, or change the order of figures or citations without having to go through the file line by line.

Please don't use the `{eqnarray}` equation environment. Use `{align}` or `{IEEEeqnarray}` instead. The `{eqnarray}` environment leaves unsightly spaces around relation symbols.

Please note that the `{subequations}` environment in LaTeX will increment the main equation counter even when there are no equation numbers displayed. If you forget that, you might write an article in which the equation numbers skip from (17) to (20), causing the copy editors to wonder if you've discovered a new method of counting.

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- In American English, commas, semicolons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)
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- Be aware of the different meanings of the homophones "affect" and "effect", "complement" and "compliment", "discreet" and "discrete", "principal" and "principle".
- Do not confuse "imply" and "infer".
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- There is no period after the "et" in the Latin abbreviation "et al.".
- The abbreviation "i.e." means "that is", and the abbreviation "e.g." means "for example".

An excellent style manual for science writers is [**?**].

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a) Positioning Figures and Tables: Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables. Insert figures and tables after they are cited in the text. Use the abbreviation "Fig. **??**", even at the beginning of a sentence.

TABLE I
TABLE TYPE STYLES

Table Head	**Table Column Head**		
	Table column subhead	***Subhead***	***Subhead***
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[a]Sample of a Table footnote.

Fig. 1. Example of a figure caption.

Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an example, write the quantity "Magnetization", or "Magnetization, M", not just "M". If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write "Magnetization (A/m)" or "Magnetization {A[m(1)]}", not just "A/m". Do not label axes with a ratio of quantities and units. For example, write "Temperature (K)", not "Temperature/K".

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