A New Heuristic for Scalable Quantum(-inspired) Annealing on Practical Quadratic Assignment Problems with Block-structural Properties

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Abstract-As quantum computing gains traction, there is a significant effort in applying it to real-world problems. One of the approaches is to use quantum annealing for the Quadratic Assignment Problem (QAP), which in turn has a wide range of applications. However, practical QAPs are often large because the size of a OAP increases quadratically with respect to the number of "items to be assigned". As a result, state-of-theart quantum(-inspired) annealers, with their limited number of qubits, lack scalability. This paper proposes a new heuristic that enables quantum(-inspired) annealers to solve QAPs of size linear to the number of qubits, provided a certain block-structural property is satisfied. Such property is frequently observed in practical settings such as warehouse allocation. The heuristic is implemented on Fujitsu Digital Annealer (DA), a dedicated quantum-inspired CMOS device that implements annealing. Experiments are performed on standard OAPLIB datasets and randomly generated block-structural QAP instances, and performance is compared against conventional methods such as software simulated annealing. Results demonstrate that the new heuristic 1) produces solutions of comparable quality on standard QAPs, 2) produces superior solutions for block-structural QAPs as well as for warehouse allocation and 3) DA achieves substantial speedup compared to conventional methods. Therefore, via the new heuristic, quantum(-inspired) annealers can effectively solve block-structural QAPs of large sizes and can be applied to practical problems.

Index Terms—component, formatting, style, styling, insert

I. INTRODUCTION

Quadratic Assignment Problem (QAP) is a well-known combinatorial optimisation problem with a wide range of applications, such as backboard wiring, statistical analysis, placement of electronic components, etc. QAP can be visualised as a flow between each pair of facilities and a distance between each pair of locations. The aim is to find an assignment of the facilities to the locations such that the sum of all products of flows and their corresponding distances is minimised, subject to the constraints that each facility is assigned to exactly one location and each location contains exactly one facility. Formally, if flow is given as the matrix (f_{ij}) and distance is given as the matrix (d_{kl}) , then the objective function to minimise is

the problem of assigning n facilities to n locations. There is

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} f_{ij} d_{kl} x_{ik} x_{jl}$$

subject to

$$\sum_{k=1}^{n} x_{ik} = 1 \qquad \forall 1 \le i \le n$$

$$\sum_{i=1}^{n} x_{ik} = 1 \qquad \forall 1 \le k \le n$$
(2)

$$\sum_{i=1}^{n} x_{ik} = 1 \qquad \forall 1 \le k \le n \tag{2}$$

 x_{ik} is the binary decision variable representing whether facility i goes to location k.

OAP is computationally difficult because it is NP-hard. Except for very small instances (n;15?) which can be solved with exact algorithms, most practical instances are intractable to exact algorithms and various heuristics are used to produce solutions of decent quality with short computation time. For

Some of the heuristics listed above are amenable to hardware acceleration. For example, ().

Identify applicable funding agency here. If none, delete this.

Quantum(-inspired) annealer is a relatively new kind of hardware that could accelerate QAP. A Quantum annealer works by introducing a problem Hamiltonian, which contains biases and couplings between qubits, to the initial Hamiltonian[]. The energy stay at minimum throughout the annealing process. At the end of annealing, the system is in the eigenstate of the problem Hamiltonian and the qubits are in their classical states

In order to enable quantum annealing of a QAP, the problem must first be converted into a Quadratic Unconstrained Binary Optimisation (QUBO) form[], which effectively means constraints (??) and (??) have to be subsumed as part of the objective function in some way. It is customary to do this by encoding the constraints as quadratic penalty terms which augment the objective function. It can be shown that for QUBOs, the optimal solution to the augmented objective function also minimises the original objective function [].

However, state-of-the-art quantum annealers lack scalability for OAPs of practical sizes. Whereas OAPs in research literature are often experimental in size, ranging from several tens of variables to at most over two hundred variables, practical OAPs can be way larger. For instance, a warehouse can have hundreds of locations, and the number of decision variables in a derived QAP is easily in the range of 10^4 . More importantly, the number of decision variables is $O(n^2)$ with respect to the number of items to be assigned. Meanwhile, modern quantum annealers only support a small number of qubits, with each qubit corresponding to a decision variable. The D-Wave quantum annealer uses over 2000 qubits, but its chimera architecture, where qubits are arranged in connected clusters of 8. limits the maximum number of nodes in a fully-connected problem graph to around 64 []. Increasing this number requires the graph to be sparser, but actual graph density depends on application and sparsity is not guaranteed. Quantum-inspired technologies are better off but not fully capable. For example, Fujitsu Digital Annealer (DA) is a quantum-inspired CMOS ASIC which implements annealing, and it supports over 8000 variables with full connectivity. This means a naive QAP implementation has a maximum size of nearly 90 variables. However, this is still too small to satisfy the high demands of problem size in practical QAPs.

This paper deals with QAPs having a particular block-structure. In particular, the set of variables in the distance matrix can be split into intervals such that distance between locations in the same interval is relatively small, and distance between locations in different intervals is larger. This pattern is observed in our case study of warehouse management, where locations are organised into columns which correspond with intervals. It is easy to travel within a column, but crossing over to another column will require traversing longer distances.

This paper presents a novel divide-and-conquer heuristic that takes advantage of the above block-structure. Combining a maximum k-cut on frequency matrix with a minimum k-cut on distance matrix, the original QAP can be meaningfully divided into sub-problems and each sub-problem will have a size that is O(n) to the number of items. This makes the overall QAP

amenable to solving on a quantum device.

This paper is organised as follows. Section 2 describes different aspects of related work. Section 3 elaborates on QAP and gives a more precise definition of the block-structural property. Section 4 explains the new divide-and-conquer heuristic. Section 5 describes experimental setup and computational results. Section 6 discusses some issues and points to future research. Section 7 concludes.

II. RELATED WORK

A. Decomposition Methods

There are two levels at which a problem can be decomposed. The first level is higher at QAP, while the second is the more primitive QUBO.

B. Use of Quantum Devices

III. EASE OF USE

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$$a + b = \gamma \tag{3}$$

Be sure that the symbols in your equation have been defined before or immediately following the equation. Use "(??)", not "Eq. (??)" or "equation (??)", except at the beginning of a sentence: "Equation (??) is . . ."

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An excellent style manual for science writers is [?].

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TABLE I
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Head	Table column subhead	Subhead	Subhead
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a Comple of a Table footnote			

^aSample of a Table footnote.

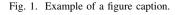


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