

Q1 (Exercise 3.7)

$$\begin{aligned}
p(w|\mathbf{t}) &\propto p(\mathbf{t}|X, w, \beta)p(w) \propto \prod_{n=1}^N N(t_n|w^T \phi(x_n), \beta^{-1}) \times N(w|m_0, S_0) \\
&\propto \exp\left(-\frac{\beta}{2}(\mathbf{t} - \Phi w)^T(\mathbf{t} - \Phi w)\right) \exp\left(-\frac{1}{2}(w - m_0)^T S_0^{-1}(w - m_0)\right) \\
&= \exp\left(-\frac{1}{2}(\beta \mathbf{t}^T \mathbf{t} - \beta w^T \Phi^T \mathbf{t} - \beta \mathbf{t}^T \Phi w + \beta w^T \Phi^T \Phi w + w^T S_0^{-1} w - w^T S_0^{-1} m_0 \right. \\
&\quad \left. + m_0^T S_0^{-1} m_0)\right) \\
&= \exp\left(-\frac{1}{2}(w^T(\beta \Phi^T \Phi + S_0^{-1})w - (S_0^{-1} m_0 + \beta \mathbf{t} \Phi^T)^T w - w^T(S_0^{-1} m_0 + \beta \mathbf{t} \Phi^T) \right. \\
&\quad \left. + \beta \mathbf{t}^T \mathbf{t} + m_0^T S_0^{-1} m_0)\right) \\
&= \exp\left(-\frac{1}{2}(w^T S_N^{-1} w - S_N^{-1} m_N^T w - w^T S_N^{-1} m_N + \beta \mathbf{t}^T \mathbf{t} + m_0^T S_0^{-1} m_0)\right) \\
&= \exp\left(-\frac{1}{2}(w - m_N)^T S_N^{-1}(w - m_N)\right) \\
&\quad \times \exp\left(-\frac{1}{2}(\beta \mathbf{t}^T \mathbf{t} + m_0^T S_0^{-1} m_0 - m_N^T S_N^{-1} m_N)\right) \\
&= N(w|m_N, S_N)
\end{aligned}$$

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