**Exercise 1**

# A Random Binary String

A random binary string contains $n$ random bits. For $1 \le i \le n$, the probability that the $i$-th random bit is 1 is $p_i$, $0 \le p_i \le 1$. That is, the probability for the $i$-th random bit to be 0 is $1 - p_i$. Given $n$, another integer $k$, $2 \le k \le n \le 200$, and $p_1, p_2, p_3, \ldots, p_n$, your program should output the probability that the random binary string **does not** contain $k$ (or more) consecutive 1's. For example, let $n$=4, $p_1$=0.9, $p_2$=0.8, $p_3$=0.7, $p_4$=0.6, and $k$=2, we have the following table.

| Random binary string, *S* | Probability, *S* occurs | *S* Contains 2 (or more) consecutive 1's |
|---|---|---|
| 0000 | 0.1×0.2×0.3×0.4=0.0024 | |
| 0001 | 0.1×0.2×0.3×0.6=0.0036 | |
| 0010 | 0.1×0.2×0.7×0.4=0.0056 | |
| 0011 | 0.1×0.2×0.7×0.6=0.0084 | Yes |
| 0100 | 0.1×0.8×0.3×0.4=0.0096 | |
| 0101 | 0.1×0.8×0.3×0.6=0.0144 | |
| 0110 | 0.1×0.8×0.7×0.4=0.0224 | Yes |
| 0111 | 0.1×0.8×0.7×0.6=0.0336 | Yes |
| 1000 | 0.9×0.2×0.3×0.4=0.0216 | |
| 1001 | 0.9×0.2×0.3×0.6=0.0324 | |
| 1010 | 0.9×0.2×0.7×0.4=0.0504 | |
| 1011 | 0.9×0.2×0.7×0.6=0.0756 | Yes |
| 1100 | 0.9×0.8×0.3×0.4=0.0864 | Yes |
| 1101 | 0.9×0.8×0.3×0.6=0.1296 | Yes |
| 1110 | 0.9×0.8×0.7×0.4=0.2016 | Yes |
| 1111 | 0.9×0.8×0.7×0.6=0.3024 | Yes |

The probability that the random binary string **does not** contain 2 or more consecutive 1's is 0.0024 + 0.0036 + 0.0056 + 0.0096 + 0.0144 + 0.0216 + 0.0324 + 0.0504 = 0.14. Note that since $n$ and $k$ can be as large as 200, your program must have an efficient algorithm in order to finish the computation in time.

**Input Format**

The input file contains several (at most 10) test cases. Each test case is given in a separated line with integers $n$, $k$, and probabilities $p_1, p_2, p_3, \ldots, p_n$. You can assume that $2 \le k \le n \le 200$ and no invalid probability value will be given as input. All integers and probabilities are separated by one or more space. The last test case will

be followed by a single line containing the integer 0.

**Output Format**

For each test case, your program should output a single line containing a floating point number to indicate the probability that the random binary string **does not** contain *k* (or more) consecutive 1's. The floating point number must be rounded to 5 digits after decimal point.

**Sample Input**

```
4 2 0.9 0.8 0.7 0.6
10 6 0 1 0.7 0.8 0.9 0.8 0.7 0.6 0.5 0.432
0
```

**Sample Output**

```
0.14000
0.66841
```