103-1 Final Exam, CSIE, NTPU

Advanced Algorithms (高等演算法)

Date: 2015/1/13 Class: 資工碩1 ID: 710383112 Name: 張景[illegible]

1. (15%) Given a subroutine Biased_Rand( ) that outputs 1 with probability p and 0 with probability $1 - p$, where $0 < p < 1$ and p is unknown, do the following tasks.
   (a) Design an algorithm Unbiased_Rand( ) that returns 1 with probability 1/2 and 0 with probability 1/2.
   (b) Prove that your algorithm is correct.
   (c) What is the expected running time of your algorithm as a function of p?

2. (10%) Use dynamic programming to find a Longest Common Subsequence of the two sequences X = (A, B, C, B, D, A, B) and Y = (B, D, C, A, B, A).

3. (15%) Given 4 matrices A_1 (with dimension 30×35), A_2 (35×15), A_3 (15×5), A_4 (5×10), we want to compute the matrix-chain product $A_1A_2A_3A_4$. It is known that different ways to parenthesize the product may need different numbers of scalar multiplications.
   (a) List all possible ways in which we can parenthesize the product $A_1A_2A_3A_4$.
   (b) Find the optimal parenthesization to minimize the number of scalar multiplications.
   (c) Find the minimal number of scalar multiplications needed to compute the product $A_1A_2A_3A_4$.

4. (10%) How to develop a greedy algorithm for a given problem? State your method step by step.

5. (20%) For Minimum Spanning Trees,
   (a) Describe Prim's algorithm.
   (b) Use Prim's algorithm to find (show your steps) a minimum spanning tree for the graph given in Figure 1.

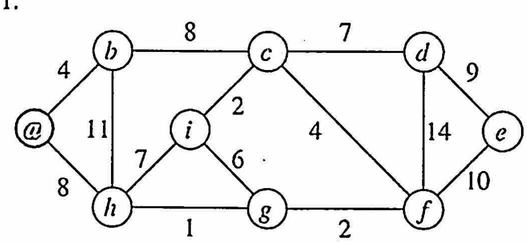


Figure 1

6. (15%) Answer the following questions.
   (a) What is the difference between worst-case analysis and amortized analysis?
   (b) What is the main idea of aggregate method?
   (c) What is the main idea of accounting method?

7. (15%) A sequence of n operations is performed on a data structure. The i-th operation costs i if i is an exact power of 2, and 1 otherwise.
   (a) Use worst-case analysis to determine the worst-case cost of an operation. (5%)
   (b) Use aggregate analysis to determine the amortized cost per operation. (5%)
   (c) Use accounting method to determine the amortized cost per operation. (5%)

記分 Score	教師簽章 Instructor Signature
81	

國立臺北大學 學年度第 學期期 考試試卷
National Taipei University Student's Answer Paper

系級/Department & Grade 資工碩 科目/Course Title 高等演算法

(該科目所屬系級)/Course Given Department 資工碩 1.

☐學士班 Bachelor Program ☑碩士班 Master Program ☐博士班 Ph.D. Program

學號/Student ID 710383112 姓名/Student Name 張景棠

任課教師/Instructor Name 張仁俊 103年 Year 1月 Month 13日 Date

(上列各項，考生須逐項填明，違者該科試卷以零分計)

Please fill out the above information, otherwise the score of this quiz will be zero.

1. (a)

```
Unbiased_Rand() {
    while (1) {
        a = Biased_Rand();
        b = Biased_Rand();
        if (a != b) return a;
    }
}
```

(b)

a	b	P
0	0	$(1-p)^2$
0	1	$p(1-p)$
1	0	$p(1-p)$
1	1	p^2

Assume Unbiased_Random has output with $p' = p(1-p)$

no output with $q' = (1-p)^2 + p^2$

For 0 or 1 output, $P = p' + q'p' + q'^2 p' + \cdots$

$= \frac{p'}{1-q'} = \frac{p(1-p)}{1-(1-2p+2p^2)} = \frac{p-p^2}{2p-2p^2}$

$= \frac{1}{2}$ ※

(c) $E_X = p' \times 1 + q'p' \times 2 + q'^2 p' \times 3 + \cdots$

$-)\ E_X q' = \qquad q'p' \times 1 + q'^2 \times p' \times 2 + \cdots$

$E_X - E_X q' = p' + q'p' + q'^2 \times p' + \cdots = \frac{1}{2}$ (as above)

$E_X = \frac{\frac{1}{2}}{1-q'} = \frac{1}{2(2p-2p^2)} = \frac{1}{4(p-p^2)}$ ※

2.

Y \ X =		A	B	C	B	D	A	B
	0	0	0	0	0	0	0	0
B	0	←0	↖1	←1	↖1	←1	←1	↖1
D	0	←0	↑1	←1	←1	↖2	←2	←2
C	0	←0	↖1	↖2	←2	←2	←2	←2
A	0	↖1	←1	↑2	←2	←2	↖3	←3
B	0	↑1	←2	←2	↖3	←3	←3	↖4
A	0	↖1	↑2	←2	↑3	←3	↖4	←↑4

∴ LCS = BCBA ※

or BDAB

3. (a) $A_1 A_2 A_3 A_4$

$A_1 (A_2 A_3) A_4$

$A_1 A_2 (A_3 A_4)$

$A_1 (A_2 (A_3 A_4))$

$A_1 ((A_2 A_3) A_4)$

(第二面) Page2

3. (a) $A_1A_2A_3A_4$

$A_1(A_2A_3)A_4$

$A_1A_2(A_3A_4)$

$A_1(A_2(A_3A_4))$

$A_1((A_2A_3)A_4)$

(c) $A_1: 30\times35$ $A_2: 35\times15$ $A_3: 15\times5$ $A_4: 5\times10$

$A_1A_2A_3A_4 = 30\times35\times15 + 30\times15\times5 + 30\times5\times10$

$= 15750 + 2250 + 1500 = 19500$

$A_1(A_2A_3)A_4 = 35\times15\times5 + 30\times35\times5 + 30\times5\times10$

$= 2625 + 5250 + 1500 = 9375$

$A_1A_2(A_3A_4) = 15\times5\times10 + 30\times35\times15 + 30\times15\times10$

$= 750 + 15750 + 4500 = 21000$

$A_1(A_2(A_3A_4)) = 15\times5\times10 + 35\times15\times10 + 30\times35\times10$

$= 750 + 5250 + 10500 = 16500$

$A_1((A_2A_3)A_4) = 35\times15\times5 + 35\times5\times10 + 30\times35\times10$

$= 2625 + 1750 + 10500 = 14875$

Min : $A_1(A_2A_3)A_4 = 9375$ ※

(b) 同上.

4. { optimal substructure
greedy choice property

For each step, choose the minimal cost to develop the total cost minimized with greedy choice property.

5. (a) Maintain V − A as a priority queue Q.

$Q \leftarrow V$

$key[v] \leftarrow \infty$ for all $v \in Q$

$key[s] = 0$ for every arbitrary s.

While $Q \neq \emptyset$

for each $u \leftarrow$ EXTRACT-MIN(Q)

if $key[v] < w(u, v)$

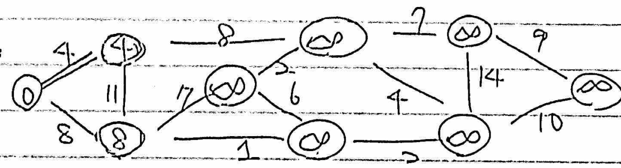
do $key[u] = w(u, v)$

$\pi[v] = u$.

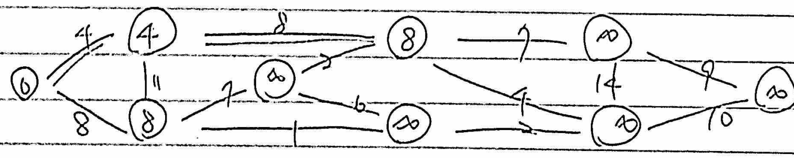
$\{(v, \pi[v])\}$ forms the MST.

(b)

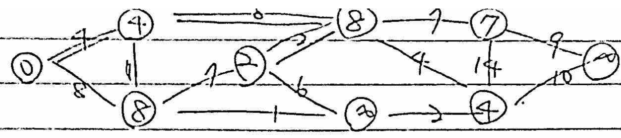
Step 1 :



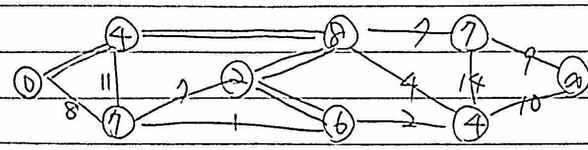
Step 2 :



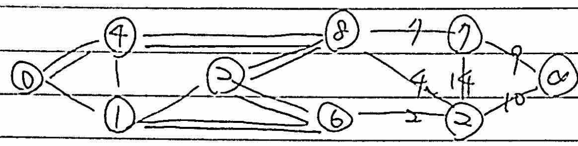
Step 3 :



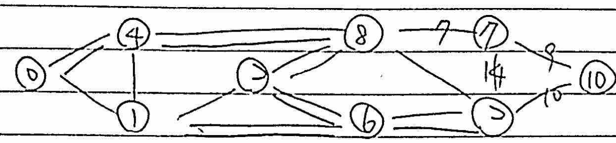
Step 4 :



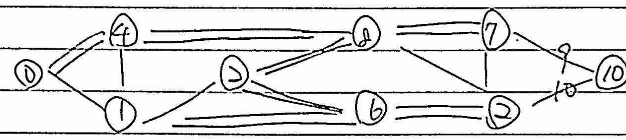
Step 5 :



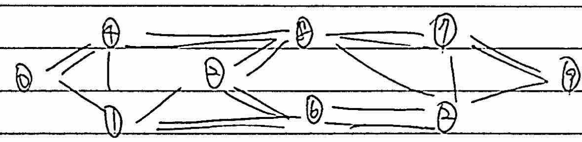
Step 6 :



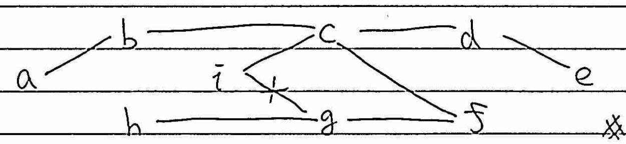
Step 7 :



Step 8 :



MST :



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6. (a) { Worst-case analysis : a guarantee, but estimation usually too high
amortized analysis : a guarantee, estimation is closed to actually cost.

(b) According the change of state, find average cost of worst-case.

(c) Pay first, additional one credit makes amortized cost ≤ actually cost.

7. (a) If nth operation need n times, $n \times n = n^2$, $O = (n^2)$

-15 (b) /

(c) /