任課教師/Instructor Name________ 年 Year____ 月 Month____ 日 Date

（上列各項，考生須逐項填明，違者該科試卷以零分計）

Please fill out the above information, otherwise the score of this quiz will be zero.

1. $f(n) = O(n^{\log_b a - \varepsilon})$ $\varepsilon > 0$

   $\rightarrow \Theta(n^{\log_b a})$

   $f(n) = \Theta(n^{\log_b a} \cdot \lg^k n)$ $k > 0$

   $\rightarrow \Theta(n^{\log_b a} \cdot \lg^{k+1} n)$

   $f(n) = \Omega(n^{\log_b a + \varepsilon})$ $\varepsilon > 0$

   符合 $af(\frac{n}{b}) \le cf(n)$ 才能使用

   $\rightarrow \Theta(f(n))$.

2.

(a) $T(n) = 4T(n/2) + n$

$\rightarrow n = O(n^{\log_2 4 - \varepsilon})$, $\varepsilon = 1$

$T(n) = \Theta(n^2)$ #

(b) $T(n) = 4T(n/2) + n^2$

$\rightarrow n^2 = \Theta(n^{\log_2 4} \cdot \lg^k n)$ $k = 0$

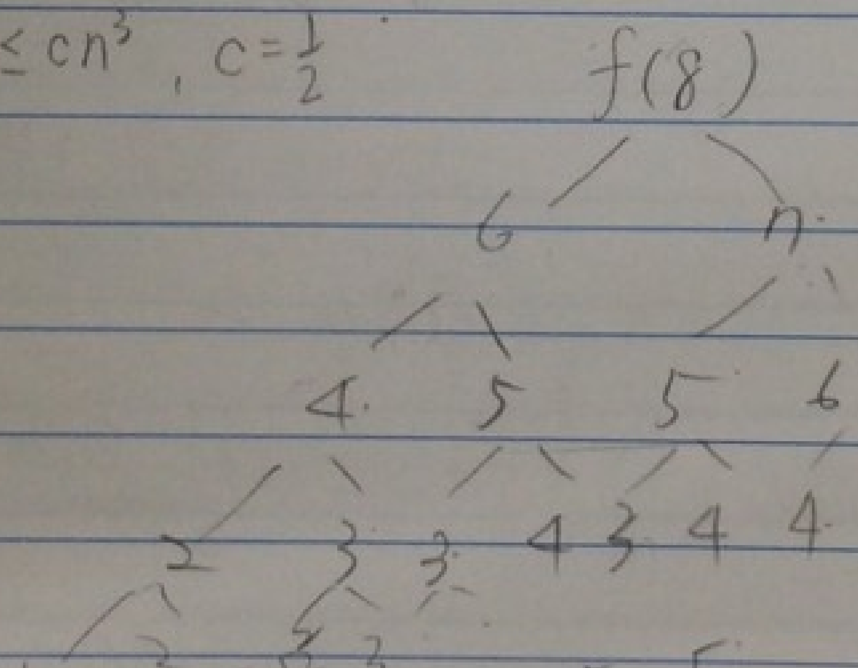
$T(n) = \Theta(n^2 \lg n)$ #

(c) $T(n) = 4T(n/2) + n^3$

$\rightarrow n^3 = \Omega(n^{\log_2 4 + \varepsilon})$, $\varepsilon = 1$

$4(\frac{n}{2})^3 \le cn^3$, $\frac{n^3}{2} \le cn^3$, $c = \frac{1}{2}$

$T(n) = \Theta(n^3)$ #



3.

$$\begin{cases} 1 & n=1, n=2 \\ T(n-1)+T(n-2) & \end{cases}$$

$t^2 - t - 1 = 0$

$t = \frac{1 \pm \sqrt{5}}{2}$

$A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$

$\frac{(1+\sqrt{5})^2}{4}$ $\quad 1 + 2 \cdot 1 \cdot \sqrt{5} + 5$ $\quad \frac{\cancel{6}+2\sqrt{5}}{\cancel{4}}$ $\quad \frac{3+\sqrt{5}}{2}$

(a) $\Theta\left(\left(\frac{1+\sqrt{5}}{2}\right)^n\right)$ worst case. #

Goldenratio

3(b)

1	{ Int F[1000];
1	Int n;
1	F[0]=0;
1	F[1]=1;
1	Int i;
$n-1+1$	for i form 2 to n do
$n-1+1$	F[i] ← F[i-1]+F[i-2]

$2n+5 \rightarrow \Theta(n)$

0 $n-1$

n

n

n^{-1}

$t^2 - t - 1 = 0$

$t = \frac{1 \pm \sqrt{5}}{2}$

$A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$

(a) $\Theta\left(\left(\frac{1+\sqrt{5}}{2}\right)^n\right)$ worst-case. #

Golden ratio

4.
(a)

$$T(n) = \begin{cases} 1 & , n=1, n=2 \\ 2T(n-2) + T(n-1) + 1 & n > 2 \end{cases}$$

$x^2 - x - 2 = 0$

$x = 2$ or -1

$y = 2y + y + 1$

$-2y = 1$

$y = -\frac{1}{2}$

$A(2)^n + B(-1)^n - \frac{1}{2}$

$T(1) = 2A - B - \frac{1}{2} = 1$ — ①

+) $T(2) = 4A + B - \frac{1}{2} = 1$ — ②

$6A - 1 = 2$

$6A = 3$

$A = \frac{1}{2}$

代入①

$1 - B - \frac{1}{2} = 1$

$-B = \frac{1}{2}$

$B = -\frac{1}{2}$.

$T(n) = \frac{2^n}{2} - \frac{(-1)^n}{2} - \frac{1}{2}$

$= \frac{2^n - (-1)^n - 1}{2}$

(a) $\Theta(2^n)$

(b) $\Theta(n)$

(c) $\frac{2^n - (-1)^n - 1}{2}$ #

5.

$$R_L(1, n, K) = \begin{cases} R_L(1, n-1, K) - q_n \cdots q_{n-K+1} \cdot P_{n-K} R_L(1, n-K-1, K) & , n > K \\ 1 - q_1 \cdots q_K & , n = K \\ 1 & , n < K \end{cases}$$

$R_L(1, n, K) = P_1 R_L(1, n-1, K) + P_2 q(1, n-2, K) + P_3 q^2 (1, n-3, K)$

$= P_n q R(1, n-K-1, K)$

6.

k=1 $\quad 2^{2^0} < n \le 2^{2^1}$ $\qquad X = k+3$

k=2 $\quad 2^{2^1} < n \le 2^{2^2}$ $\qquad = \lceil \log_2 \log_2 n \rceil + 3.$

k=3 $\quad 2^{2^2} < n \le 2^{2^3}$

$\rightarrow \quad 2^{2^{k-1}} < n \le 2^{2^k}$

$k-1 < \log_2 \log_2 n \le k$

$\log_2 \log_2 n \le k \le \log_2 \log_2 n + 1$

(a) $\Theta(\log_2 \log_2 n)$

(b) $\lceil \log_2 \log_2 n \rceil + 3$

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7.

(a)

```
Int Unbiased_Rand_Bit()
{
  Int a, b;
  while (true) do
  { a ← Biased_Rand_Bit();
    b ← Biased_Rand_Bit();

    If (a≠b)
      return 1;
  }
}
```

(b) Pr{1st Interact return 1} $p(1-p)$.

Pr{1st Interact return 0} $(1-p)p$

Pr{1st no return} $1-2p(1-p) = q$

Pr{1st return 1}

$p(1-p) + qp(1-p) + q^2 p(1-p) \cdots$

$= \frac{p(1-p)}{1-q} = \frac{p(1-p)}{2p(1-p)} = \frac{1}{2}$ #

(c) Ith $\quad 1 \cdots n$

$E(X) = E(X_1) + E(X_2) + \cdots E(X_n)$

$= 1 \times p(p-1) + 1 \times q p(p-1) + q^2 p(p-1)$

$= \frac{1}{1-q} = \frac{1}{2p(p-1)}$ #

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8.

1 2 … 39

(a) 38 次. if(a>b) swap(a,b)

$\Theta(n)$

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(b)

Max

1 … n-1 … n

① 找出Maximun n-1

[1…n-1] 找出最大 n-2

2n-3 次

2·39-3 = 次

$\Theta(n)$

(c) Maximun $\quad n-1$

$n-2$

$\vdots$

Minimun $\quad 1$

$\frac{1+(n-1)\times n}{2} = \frac{n^2}{2}$

$\Theta(n^2)$