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任課教師/Instructor Name ____ 年 Year ___ 月 Month ___ 日

(上列各項，考生須逐項填明，違者該科試卷以零分計)

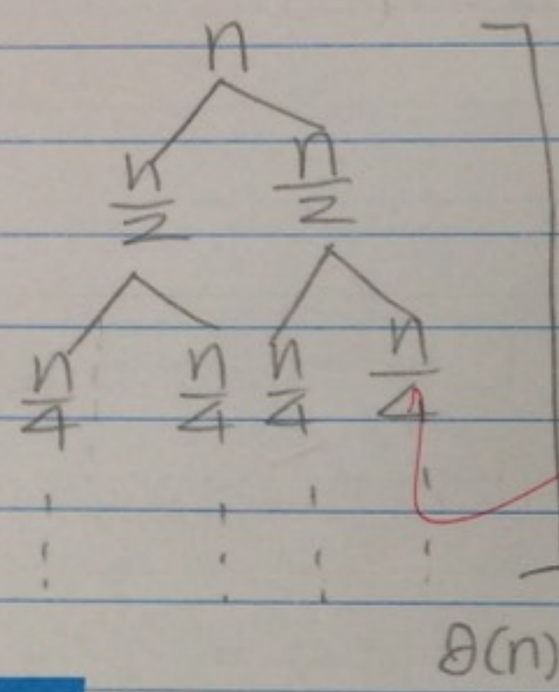
Please fill out the above information, otherwise the score of this quiz will be zer

① A[1,...,n]

1. if n=1 done

2. 遞迴 Sort $A[1,\ldots,\lceil \frac{n}{2} \rceil]$ and $A[\lceil \frac{n}{2} \rceil + 1,\ldots,n]$

3. Merge Sort 2 lists



$\therefore \Theta(n\log n)$ #

② (a) $f(n)=n$, by CASE 1: $n^{\log_2 4-\varepsilon} = n^{2-\varepsilon}$, 令 $\varepsilon=1$

則 $T(n)=\Theta(n^2)$

(b) $f(n)=n^2$, by CASE 2: $n^{\log_2 4}\lg^k n = n^2\lg^k n$ 令 $k=0$

則 $T(n)=\Theta(n^2\lg n)$ #

(c) $f(n)=n^3$, by CASE 3: $n^{\log_2 4+\varepsilon} = n^{2+\varepsilon}$, 令 $\varepsilon=1$, 且 $4f(\frac{n}{2}) \le c f(n)$

則 $T(n)=\Theta(n^3)$ #

③ $(1234) \bmod 100 \equiv (34) \bmod 100 = ((34^2) \bmod 100) \times 34) \bmod 100$

$= ((56^{1172} \bmod 100) \times 34) \bmod 100$, $= 56 \times 34 \bmod 100$

$= 24 \bmod 100$

#

56)

④ $T(n) = \begin{cases} 1 & n=1 \text{ or } n=2 \\ 2T(n-2) + T(n-1) + 1 & n>2 \end{cases}$

令 $x^2 = 2 + \alpha$ 則 $\alpha = -1$ or 2

$\beta = 2\beta + \beta + 1$, $B = -\frac{1}{2}$

則 $T(n) = A(2)^n + B(-1)^n - \frac{1}{2}$

令 $T(1) = 2A - B - \frac{1}{2} = 1$

$T(2) = 4A + B - \frac{1}{2} = 1$

$A = \frac{1}{2}$, $B = -\frac{1}{2}$

則 $T(n) = \frac{1}{2}(2)^n - \frac{1}{2}(-1)^n - \frac{1}{2}$

$= \frac{(2)^n - (-1)^n - 1}{2}$

(b) $= \theta(n)$

(a) $= \theta(2^n)$

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⑤

(a) $R_L(1,n,k) = \begin{cases} 1 & n<k \\ 1-q_1q_2\cdots q_k & n=k \\ R_L(1,n-1,k) - P_{n-k}\, q_{n-k+1}\cdots q_n\, R_L(1,n-k-1,k) & \end{cases}$

(b) $R_C(1,n,k) = \begin{cases} 1 & n<k \\ \sum_{i=1}^{k} q_1q_2q_3\cdots q_{i+1}p_i\, R_C(i+1, n+i-1, k) & n\ge k \end{cases}$

⑥

Randomized Algorithms：有一Input值，会隨機產生不同output值

Non-Randomized Algorithms：有一Input值，output ~~有可能~~会相同，

⑦

(a)

```
int Biased_RBG( )
{
    int m,n;
    while(true)
    {
        if (m/(m+n) < n/(m+n))
            return 0;
        else return 1;

        m <- Unbiased_RBG(m);
        n <- Unbiased_RBG(n);
    }
}
```

(b) -3

$Pr\{Output\ 1\ 1st\ itera\}: \frac{n}{m+n}\cdot(1-\frac{n}{m+n}) = \frac{mn}{m+n}$

$Pr\{output\ 0\ 1st\ itera\}: \frac{m}{m+n}\cdot(1-\frac{m}{m+n}) = \frac{mn}{m+n}$.

$Pr\{No\ output\ 1st\ itera\} = (1-2\frac{mn}{m+n}) = q$

$Pr\{Output\ 1\} = \frac{mn}{m+n} + q(\frac{mn}{m+n}) + q^2(\frac{mn}{m+n}) + \cdots$

$= \frac{\frac{mn}{m+n}}{1-(1-2\frac{mn}{m+n})} = \frac{1}{2}$ #

$Pr\{Output\ 0\} = \frac{mn}{m+n} + q(\frac{mn}{m+n}) + q^2(\frac{mn}{m+n}) + \cdots$

$= \frac{1}{2}$

(c)

$E(x) = 1\times1 + 1\times q + 1\times q^2 + \cdots$

$= \frac{1}{1-q} = \frac{1}{1-(1-2\frac{mn}{m+n})} = \frac{1}{2\frac{mn}{m+n}} = \Theta(\frac{1}{2\frac{mn}{m+n}})$

⑧ 令 k：執行次數

(a)

$k=1\quad 2^0 < n \le 2^2$

(b) 又 $k = 3 + X$

(c)

$E(x) = 1 \times 1 + 1 \times q + 1 \times q^2 + \cdots\cdots$

$= \frac{1}{1-q} = \frac{1}{1-(1-2\frac{mn}{m+n})} = \frac{1}{2\frac{mn}{m+n}} = \theta\left(\frac{1}{2\frac{mn}{m+n}}\right)$

⑧ 令 k：執行次數

(a)

$k=1 \quad 2^0 < n \le 2^1$

$k=2 \quad 2^1 < n \le 2^2$

$\vdots$

$k=n \quad 2^{k-1} < n \le 2^k$

$\downarrow \times \log_2 \log_2$

$k-1 < \log_2 \log_2 n \le k$

$\therefore \theta(\log_2 \log_2 n)$

(b) 又 $k = 3 + X$

$n = 3 + \lceil \log_2 \log_2 n \rceil$.

⑨ SELECT(i, n)

$\theta(n)$：1. Divide n elements into group of 5 找出中間元素，

$\theta(\frac{n}{5})$：2. 找出所有中間數的中間數 $\lceil \frac{n}{5} \rceil$（用遞迴）

$\theta(n)$：3. partation around the x, lets $k = \text{rank}(x)$

$\theta(\frac{3}{4}n)$：4. if $i = k$ return x

else if $i < k$ then select i-th smallest in lower part,

else select $(i-k)$th smallest in upper part.

$\therefore O(n) = T(\frac{n}{5}) + T(\frac{3}{4}n) + \theta(n) \doteq \theta(n)$