104-1 Midterm, CSIE, NTPU

Advanced Algorithms (高等演算法)

Date: 2015/12/01 Class: ID: Name:

1. (10%) Please briefly describe the **Merge Sort** algorithm, and analyze its worst-case time complexity for sorting n numbers with the **Recursion Tree** method.
2. (10%) Solve each following **recurrence** for $T(n)$.
   (a) $T(n) = 4T(n/2) + n$ (3%)
   (b) $T(n) = 4T(n/2) + n^2$ (3%)
   (c) $T(n) = 4T(n/2) + n^3$ (4%)
3. (10%) 1234^{2345} **mod** 100 = ? Calculate by hand (without any calculator) efficiently.
4. (15%) Consider the following recursive function where the global variable *count* is initialized to 0 and input n is a positive integer.
   (a) Find out the asymptotic time complexity in Θ-notation. (5%)
   (b) Find out the asymptotic space complexity in Θ-notation. (5%)
   (c) Express the final value of *count* as a function of n. (5%)

```
Rec-x( n )
{
  if ( n = 1 ) or ( n = 2 ) then
    count ← count + 1
  else
  {
    Rec-x( n – 2 )
    Rec-x( n – 2 )
    Rec-x( n – 1 )
    count ← count + 1
  }
}
```

5. (10%) A **linear** consecutive-k-out-of-n system consists of n nodes arranged in a line, where the system fails if and only if some k consecutive nodes fail. A **circular** consecutive-k-out-of-n system consists of n nodes arranged on a circle, where the system fails if and only if some k consecutive nodes fail. Suppose the nodes are statistically independent and the reliability of node i is p_i for any $i \in \{1, 2, \ldots, n\}$ (node i functions with probability p_i and fails with probability $1 - p_i$). Let $R_L(i, j, k)$ denote the reliability of the linear consecutive-k-out-of-n subsystem consisting of nodes $i, i+1, i+2, \ldots, j$. $R_C(i, j, k)$ denote the reliability of the circular consecutive-k-out-of-n subsystem consisting of nodes $i, i+1, i+2, \ldots, j$.
   (a) Express $R_L(1,n,k)$ with $R_L(1,n-1,k)$ and $R_L(1,n-k-1,k)$. (5%)
   (b) Express $R_C(1,n,k)$ with some $R_L(1,?,k)$ terms. (5%)

6. (10%) What are the main differences between **Randomized Algorithms** and Non-Randomized Algorithms?
7. (15%) Let m and n be positive integers such that $1 < m < n$. Given a procedure Unbiased_RBG that outputs an unbiased random bit which will be 0 or 1 with equal probability, 1/2, please
   (a) Design an algorithm, Biased_RBG, with calls of subroutine Unbiased-RBG to output 1 with probability $m/(m+n)$ and 0 with probability $n/(m+n)$. (5%)
   (b) Prove that your algorithm is correct. (5%)
   (c) What's the expected running time of your algorithm as a function of m and n? (5%)
8. (10%) Consider the following program segment.

```
i ← 2
x ← 3
while ( i < n ) do
{
    i ← i × i
    x ← x + 1
}
```

   (a) Find the time complexity (in Θ notation) of this program segment. (5%)
   (b) Express the final value of x as a function of n. (5%)
9. (10%) Write the SELECT() algorithm to output the i-th smallest element of the input array in worst-case time complexity $O(n)$ when given an array of n elements and an index integer i.