1. (10%) Consider the following program segment. (10%)考慮以下項目部分。

-3

x-2

while (i<n) do

i-ixi x←x+

(a) Find the time complexity (in e notation) of this program segment. (4%)

(a)求該程序段的時間複雜度(用e表示)。

(b) Find the final value of x as a function of n. (6%)

(b)求x作爲n的函數的最終值。

2. (10%) The n-th Fibonacci number can be calculated with the following recursive function.

Assume the time costs of all integer arithmetic (addition, subtraction, multiplication, division) operations are equal. First, find the time complexity of the function. Next, rewrite the function for sub-linear time efficiency and show your complexity analysis.

(10%)第n個斐波那契數可以用以下遞歸函數計算

假設所有整數算術運算(加、減、乘、除)的時間代價都相等。 首先，求函數的時間複雜度。 接下來，重寫函數的次線性時間效率，並顯示您的複雜性分析。

Fibo(n)

if (n=1) or (n=2) then o)t else

return 1

return Fibo(n-1) +Fibo(n-2)

3. (10%) Consider the following recursive function where the global variable count is initialized to 0 and input n is a positive integer考慮下面的遞歸函數，其中全局變量count初始化爲0，輸入n爲正整數

(a) Find the asymptotic time complexity in e-notation. (5%)

(a)求e符號的漸近時間複雜度。

(b) Find the final value of count as a function of n. (5%)

(b)求count作爲n的函數的最終值(5%)

4. (10%) A linear consecutive-k-out-of-n system (k <n) consists of n nodes arranged in a line, where the system fails if and only if some k consecutive nodes fail. Suppose the nodes are statistically independent and the reliability of node i is p, for any iE {1, 2... (node i functions with probability p, and fails with probability 1-p). Let R(i, j, k) denote the reliabilitv of the linear consecutive-k-out-of-n subsvstem consisting of nodes i, it1, i+2......

(10%)一個線性連續k / n系統(k <n)由n個節點組成，當且僅當k個連續節點失效時，系統失效。 假設節點是統計獨立的，節點i的可靠性爲p，對於任意iE{1,2，… (節點i運行的概率爲p，失敗的概率爲1-p)。 設R(i, j, k)表示由節點i, it1, i+2 ......組成的線性k- in -n子svstem的可靠性

(a) Express R(1, n, k) with R(1, n-1,k), R(1, n-2, k), R(1, n-3, k), . and R(1, n-k, k).

(a)表達(1,n, k)和R (1, n, k)、R (1, n, k)、R (1, n, k)。 和R(1, n-k, k)

(b) Express R(1, n, k) with R(1, n-1, k) and R(1, n-k-1, k)

(b)用R(1, n-1, k)和R(1, n-k-1, k)表示R(1, n, k)

5. (12%) Given a subroutine Biased Rand() that outputs I with probability p and 0 with probability 1-p, where 0<p<1 and p is unknown, do the following tasks. 給定一個子程序偏置Rand()，輸出概率爲p的I和概率爲1-p的0，其中0<p<1且p未知，執行以下任務。

(a) Design an algorithm Unbiased Rand() that returns 1 with probability 1/2 and O with probability 1/2. (4%)(a)設計一個無偏Rand()算法，返回1的概率爲1/2,O的概率爲1/2。 (4%)

(b) Prove that your algorithm is correct. (4%)(b)證明你的算法是正確的。 (4%)

(c) What is the expected running time of your algorithm as a function of p? (4%)(c)你的算法的預期運行時間是p的函數? (4%)

6. (12%) Given positive integers m and n such that 0 <m< n and a subroutine Unbiased Rand() that outputs 1 with probability 1/2 and O with probability 1/2. do the following tasks.(12%)給定正整數m和n，且0 <m< n，和一個子程序無偏Rand()，輸出1的概率爲1/2，輸出O的概率爲1/2。 執行以下任務

(a) Design an algorithm Biased Rand() that returns 1 with probability min and 0 with probability 1-m/n. (4%)

(a)設計一個有偏Rand()算法，返回1的概率最小，0的概率爲1-m/n。 (4%)

(b) Prove that your algorithm is correct. (4%)

(b)證明你的算法是正確的。 (4%)

(c) What is the expected running time of your algorithm as a function of m and/or n? (4%)

(c)作爲m和/或n的函數，你的算法的預期運行時間是多少? (4%)

7. (10%) Given n distinct unsigned integers where each integer has b bits, we want to sort these n integers into increasing order by Counting Sort or Radix Sort. 給定n個不同的無符號整數，每個整數有b位，我們想通過計數排序或基數排序將這n個整數按遞增順序排序。

(a) What is the (worst-case) time complexity of Counting Sort? (5%)

(a)計數排序的(最壞情況)時間複雜度是多少? (5%)

(b) What is the (worst-case) time complexity of Radix Sort? (5%)

(b)基數排序的(最壞情況)時間複雜度是多少? (5%)

8. (10%) For the radix-sort algorithm爲radix-sort算法

(a) Describe the primary steps of applying radix-sort to the nine numbers: 329, 457, 657, 838,436, 720, 355, 123, 132. (5%)

(a)描述對9個數字應用radix排序的主要步驟:329,457,657,838,436, 720, 355, 123, 132. (5%)

(b) How to sort n integers in the range O to n3-1 in O(n) time? (5%)

(b)如何在O(n)時間內對O到n'-1範圍內的n個整數排序? (5%)

9.(16%) Given 59 distinct numbers, please answer the following questions. 給出59個不同的數字，請回答以下問題

(a) In the worst case, how many comparisons at least do you need to find the maximum number? Describe your algorithm with a brief proof. (5%)

(a)在最壞的情況下，你至少需要進行多少次比較才能找到最大值? 用一個簡單的證明來描述你的算法。 (5%)

(b) In the worst case, how many comparisons at least do you need to find both the maximum and minimum numbers? Describe your algorithm with a brief proof. (5%)

(b)在最壞的情況下，你至少需要多少次比較才能同時找到最大值和最小值? 用一個簡單的證明來描述你的算法。 (5%)

(c) In the worst case, how many comparisons at least do you need to find the second largest number? Describe your algorithm with a brief proof. (6%)

(c)在最壞的情況下，你至少需要多少次比較才能找到第二大的數? 用一個簡單的證明來描述你的算法。 (6%)

4.(15%) Use Master Theorem to solve the following recurrences for T(n) 利用主定理求解T(n)

(a) T(n)= 3 T(n/9)+ n

(b) T(n)=3 T(n/9)+n0.5log2(n)

(C) T(n) =3 T(n/9)+ log2(n)

5.(10%) Given a subroutine Unbiased Rand() that outputs 1 with probability 1/2 and O with probability 1/2, do the following tasks. 給定一個子程序無偏Rand()，輸出1的概率爲1/2，輸出O的概率爲1/2，執行以下任務

(a) Design an algorithm Biased Rand() that returns 1 with probability 4/5 and O with probability 15. (5%)

(a)設計一個有偏差的Rand()算法，返回1的概率爲4/5,O的概率爲15。(5%)

(b) Prove that your algorithm is correct and find its expected running time. (5%)

(b)證明你的算法是正確的，並找到它的預期運行時間。 (5%)

9.(10%) Use dynamic programming to find a Longest Common Subsequence of the two sequences X=(A, B, C, A, B, D, A, B) and Y =(B, D, C, A, B, A).