Advanced Materials

Introduction to Computer Graphics

Yu-Ting Wu

(with some slides borrowed from Prof. Yung-Yu Chuang)

The Rendering Equation

- Proposed by Kajiya [1986]

reflected radiance

emitted radiance

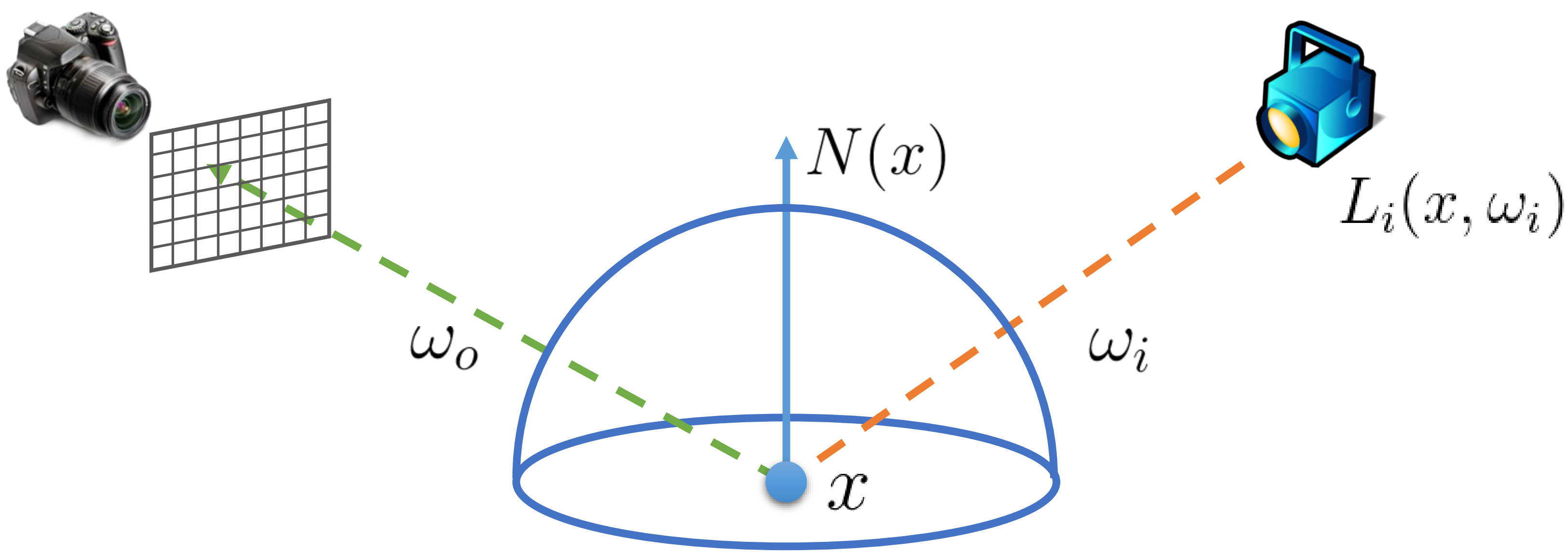
incident radiance

geometry term

$$L(x,\omega_o) = L_e(x,\omega_o) + \int_{\Omega} L_i(x,\omega_i) f_r(x,\omega_o \leftarrow \omega_i)(N(x)\cdot\omega_i)d\omega_i$$

bidirectional reflectance distribution function (BRDF)

Integral of all directions

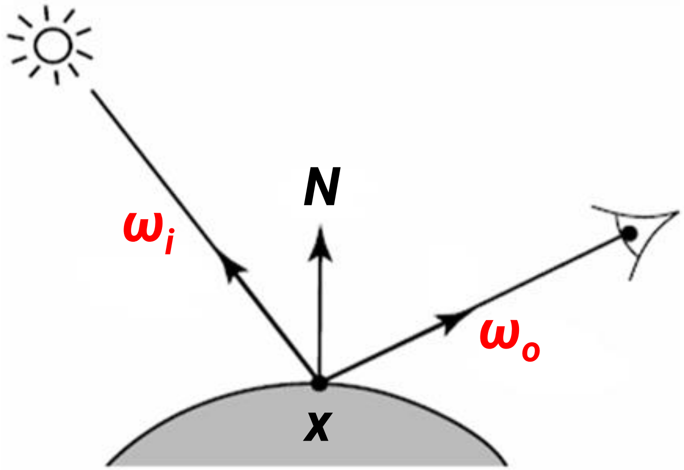


Formal Material Representation

- In **Physically-based Rendering (PBR)**, the characteristic of a material is usually defined by ***Bidirectional Reflectance Distribution Function (BRDF)***

$$f_r(x, \omega_o \leftarrow \omega_i)$$

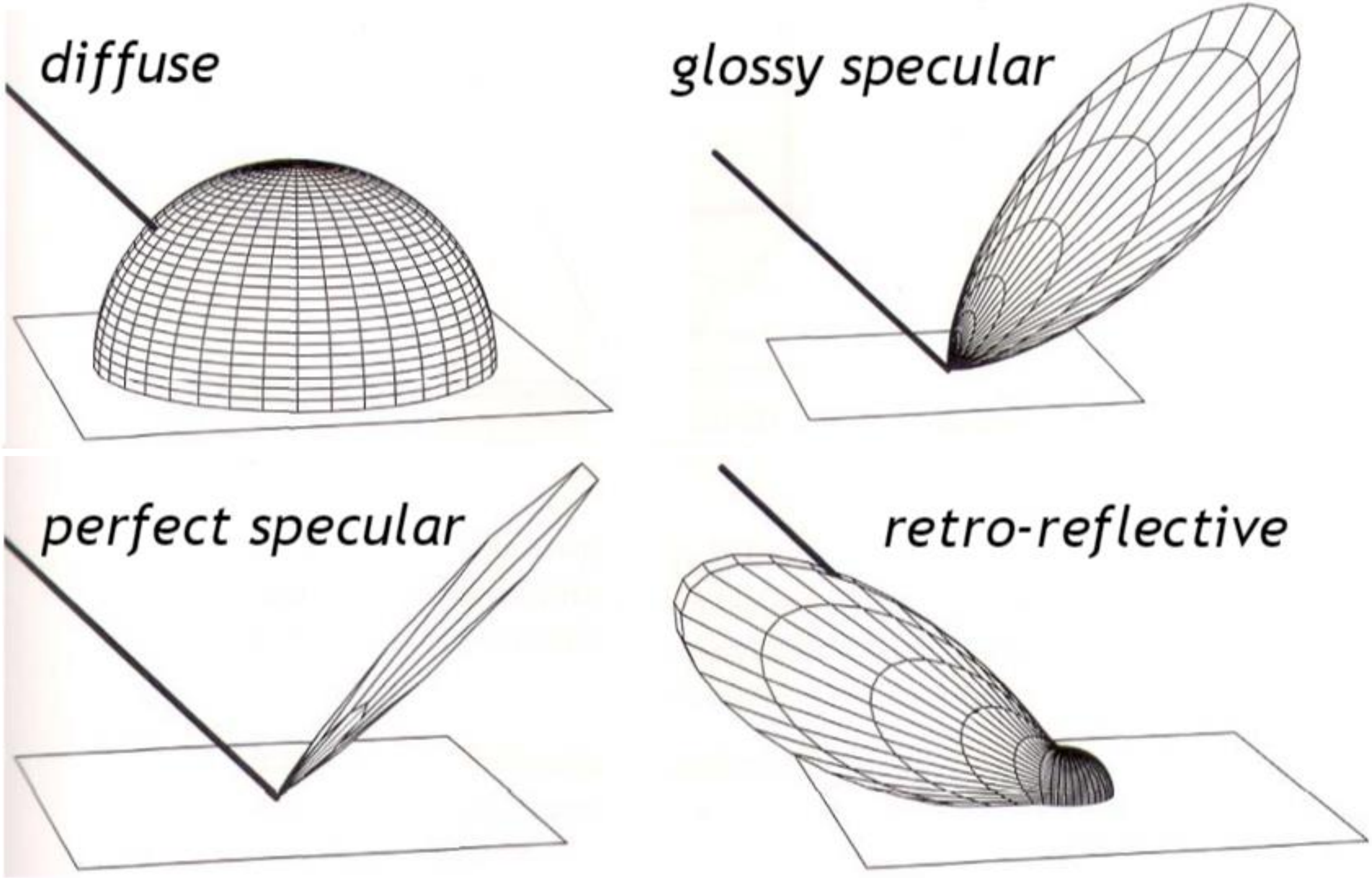
  - Describe how much light (**ratio**) coming from $\boldsymbol{\omega_i}$ will reflect toward $\boldsymbol{\omega_o}$ at point $\boldsymbol{x}$



A good representation should have

- Accuracy
- Expressiveness
- Speed

Reflection Categories



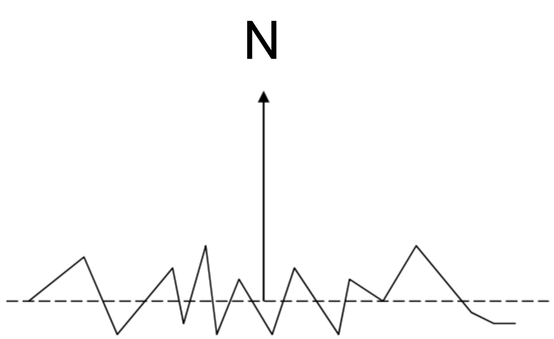
Classification of BRDF

- **Phenomenological models**
  - Qualitative approach
  - Models with intuitive parameters
  - Examples are Phong and Blinn-Phong lighting models
- **Geometric optics**
  - Microfacet models
- **Measured data**
  - Usually described in tabular form or coefficients of a set of basis functions

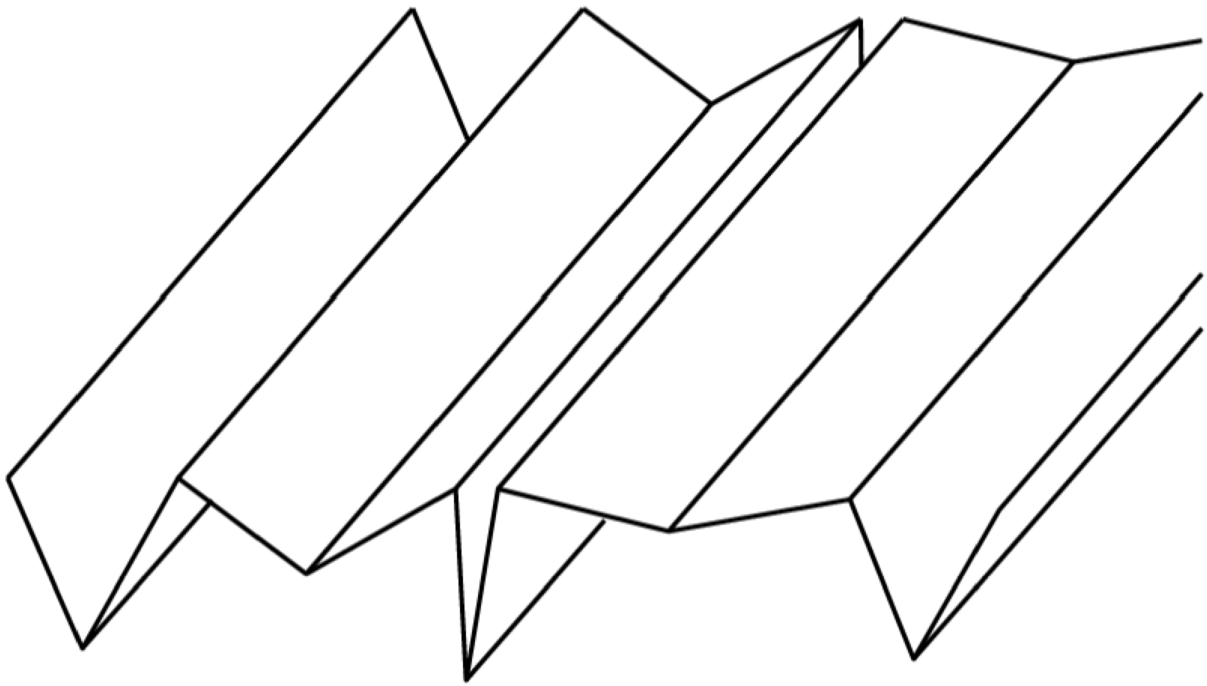
Microfacet Model

Microfacet Model

- Rough surfaces can be modeled as a collection of small **microfacets**
- The **aggregate behavior** of the small microfacets determines the scattering
- Two components for deriving a closed-form BRDF expression
  - The distribution of microfacets
  - How light scatters from the individual microfacet



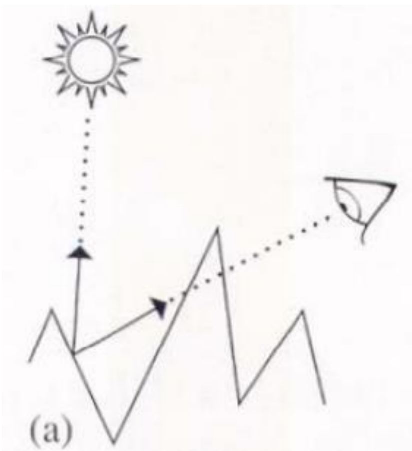
Microfacet Model (cont.)



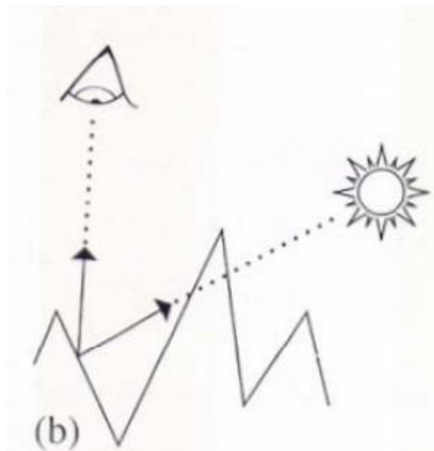
Most microfacet models assume that all microfacets make up **symmetric V-shaped** grooves so that only neighboring microfacet needs to be considered

Microfacet Model (cont.)

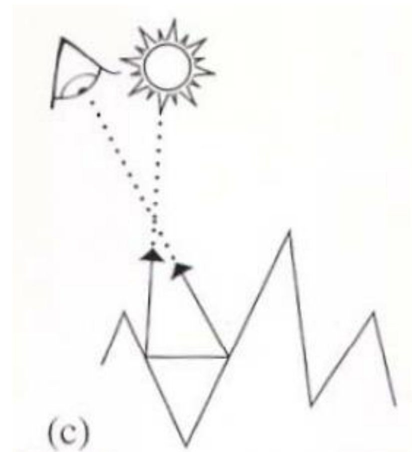
- Important geometric effects to consider



masking



shadowing

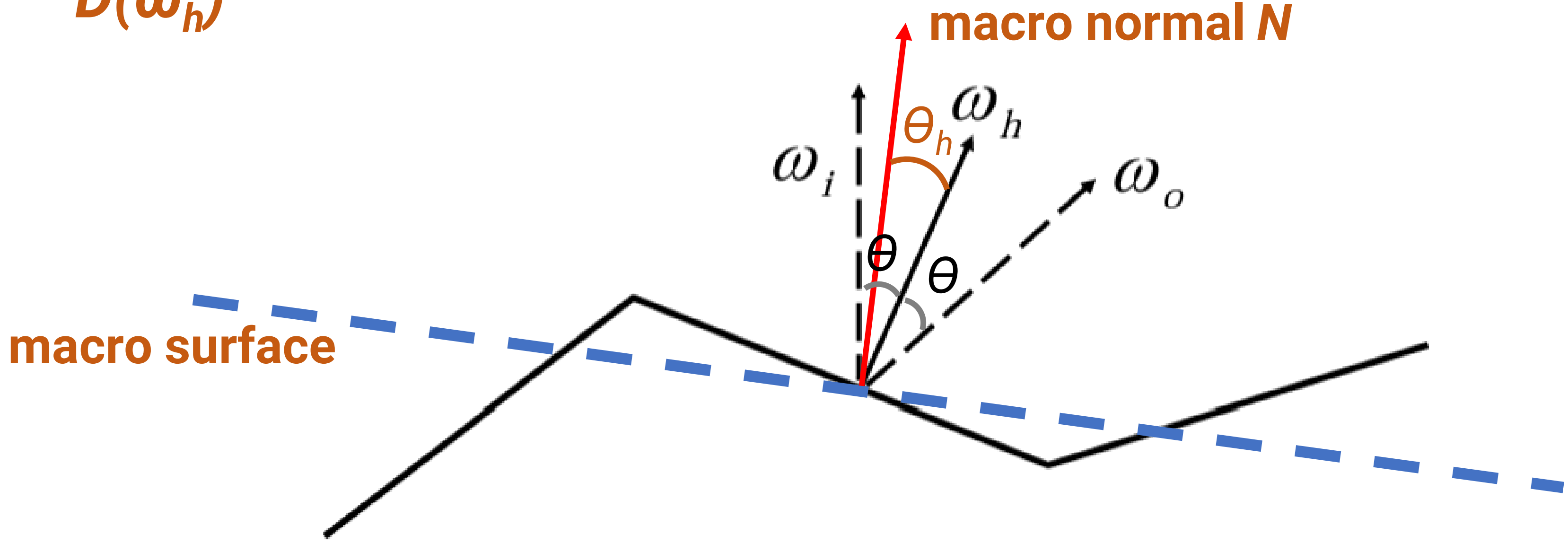


interreflection

- Particular models consider these effects with varying degrees of accuracy

Torrance-Sparrow Model

- One of the first microfacet model
- Designed to model **metallic** surfaces
- Assumption: a surface is composed of a collection of **perfectly smooth mirrored** microfacets with **distribution** $D(\omega_h)$



Torrance-Sparrow Model (cont.)

- Described by
  - Microfacet distribution *D*
  - Geometric attenuation *G*
  - Fresnel reflection *F*

$$f_r(\omega_o \leftarrow \omega_i) = \frac{D(\omega_h)G(\omega_i,\omega_o)F(\omega_i,\omega_h)}{4\cos\theta_i\cos\theta_o}$$

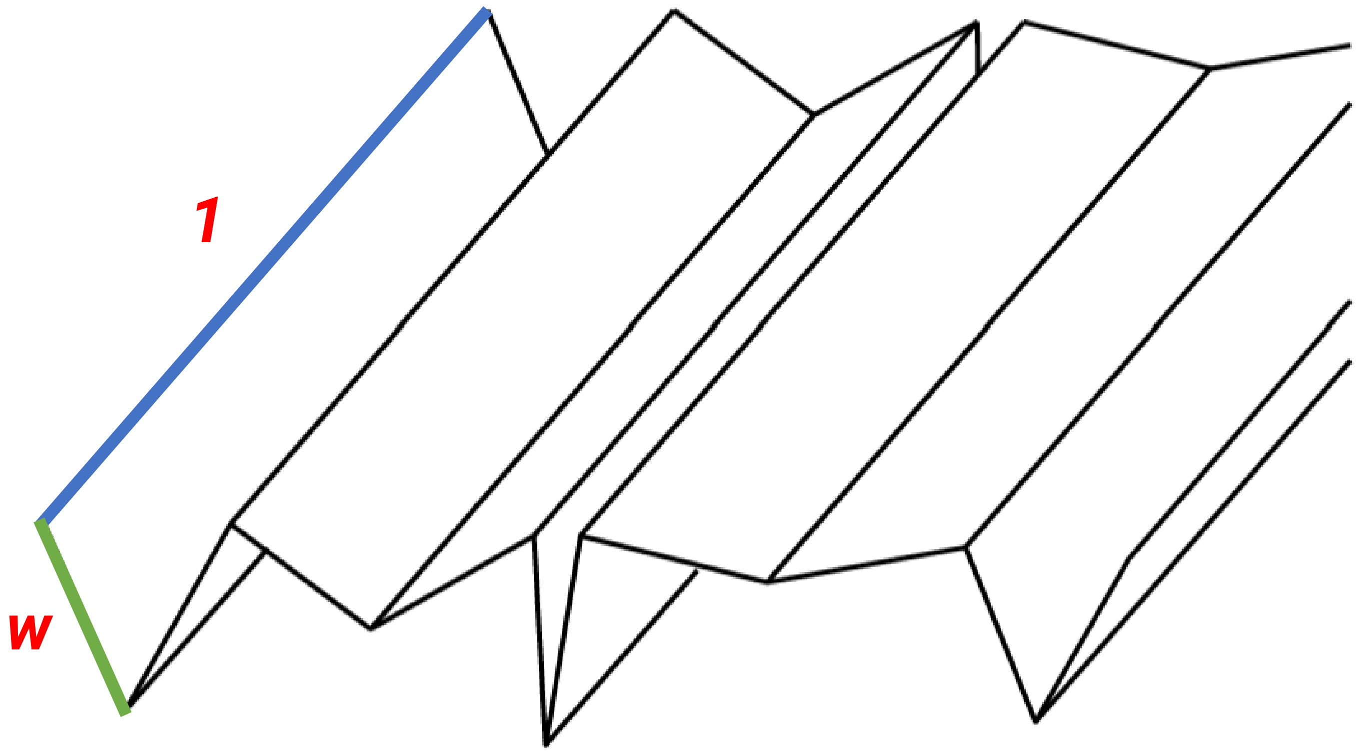
Torrance-Sparrow Model (cont.)

- Described by
  - Microfacet distribution *D*
  - **Geometric attenuation *G***
  - Fresnel reflection *F*

$$f_r(\omega_o \leftarrow \omega_i) = \frac{D(\omega_h)\boxed{G(\omega_i, \omega_o)}F(\omega_i, \omega_h)}{4\cos\theta_i\cos\theta_o}$$

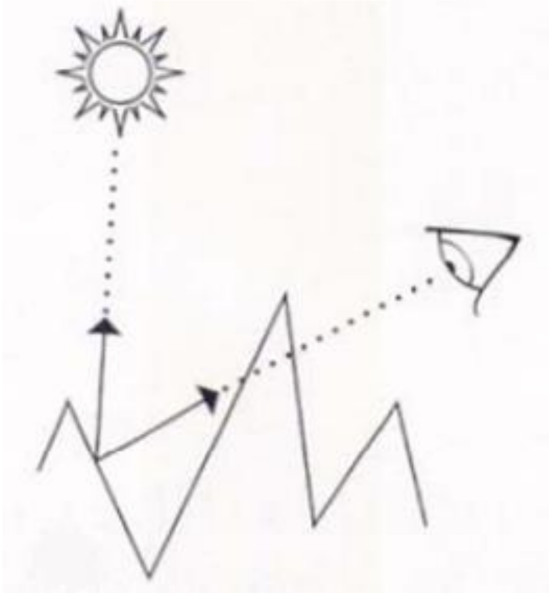
Torrance-Sparrow Model (cont.)

- Configuration

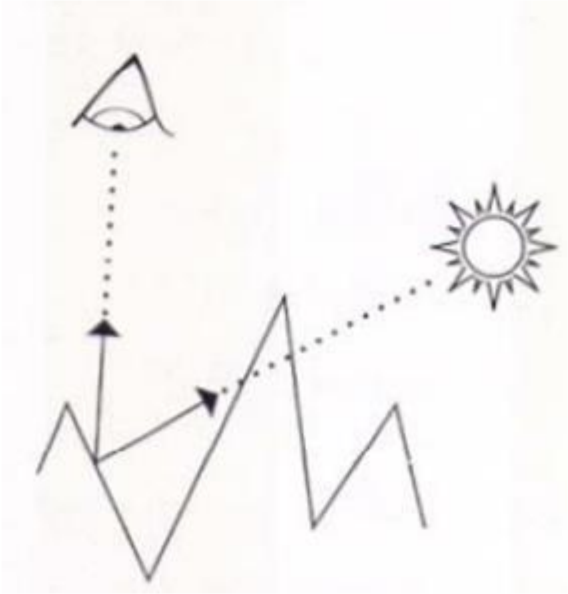


Torrance-Sparrow Model (cont.)

- Geometry attenuation factor



masking



shadowing

$$G = \frac{\textit{facet area that is both visible and illuminated}}{\textit{total facet area}}$$

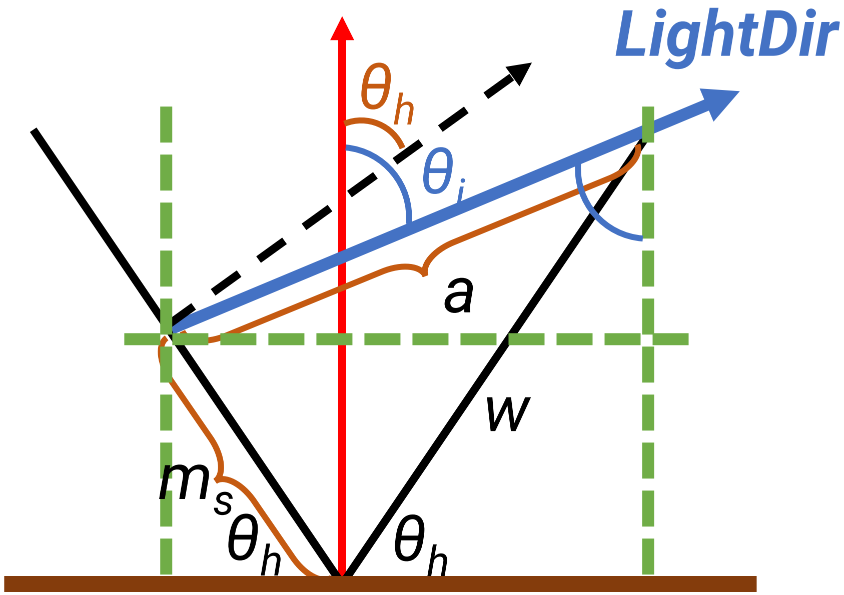
Torrance-Sparrow Model (cont.)

- Shadowing term

$$\boxed{1 - \frac{m_s}{w}}$$

$$a\sin\theta_i = w\cos\theta_h + m_s\cos\theta_h \qquad \times \cos\theta_i$$

$$a\cos\theta_i = w\sin\theta_h - m_s\sin\theta_h \qquad \times -\sin\theta_i$$



$$\frac{m_s}{w} = -\frac{\cos(\theta_h + \theta_i)}{\cos(\theta_h - \theta_i)}$$

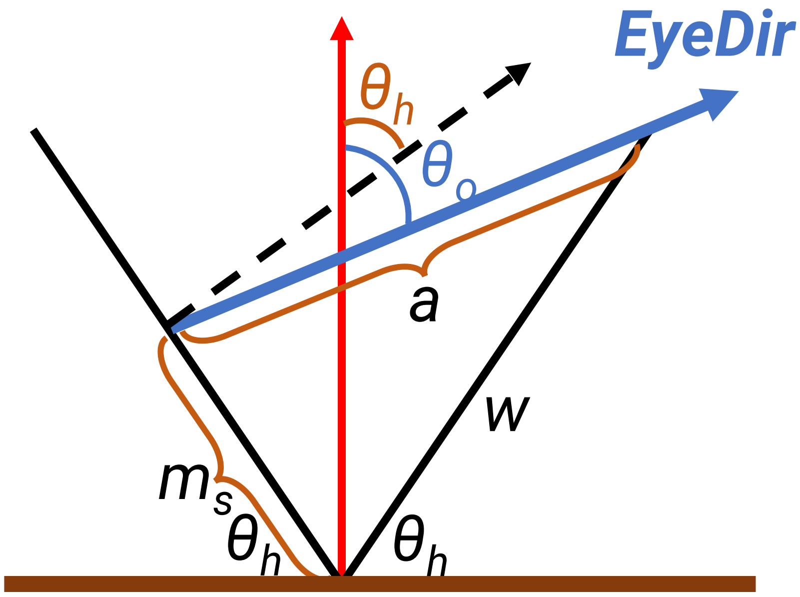
$$1 - \frac{m_s}{w} = \frac{2\cos\theta_h\cos\theta_i}{\cos(\theta_h - \theta_i)}$$

Torrance-Sparrow Model (cont.)

- Masking term

$$\boxed{1 - \frac{m_v}{w}}$$

$$a\sin\theta_o = w\cos\theta_h + m_s\cos\theta_h \quad \times\cos\theta_o$$
$$a\cos\theta_o = w\sin\theta_h + m_s\sin\theta_h \quad \times -\sin\theta_o$$



$$1 - \frac{m_v}{w} = \frac{2\cos\theta_h\cos\theta_o}{\cos(\theta_h - \theta_o)}$$

Torrance-Sparrow Model (cont.)

- Geometry attenuation factor

$$G = \frac{\textit{facet area that is both visible and illuminated}}{\textit{total facet area}}$$

$$G = \min\left(1 - \frac{m_s}{w}, 1 - \frac{m_v}{w}\right) = \min\left(\frac{2\cos\theta_h \cos\theta_i}{\cos(\theta_h - \theta_i)}, \frac{2\cos\theta_h \cos\theta_o}{\cos(\theta_h - \theta_o)}\right)$$

Torrance-Sparrow Model (cont.)

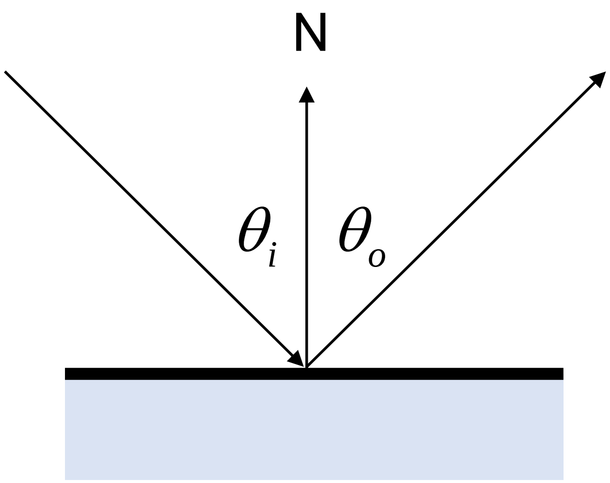
- Described by
  - Microfacet distribution *D*
  - Geometric attenuation *G*
  - **Fresnel reflection *F***

$$f_r(\omega_o \leftarrow \omega_i) = \frac{D(\omega_h)G(\omega_i, \omega_o)\boxed{F(\omega_i, \omega_h)}}{4\cos\theta_i\cos\theta_o}$$

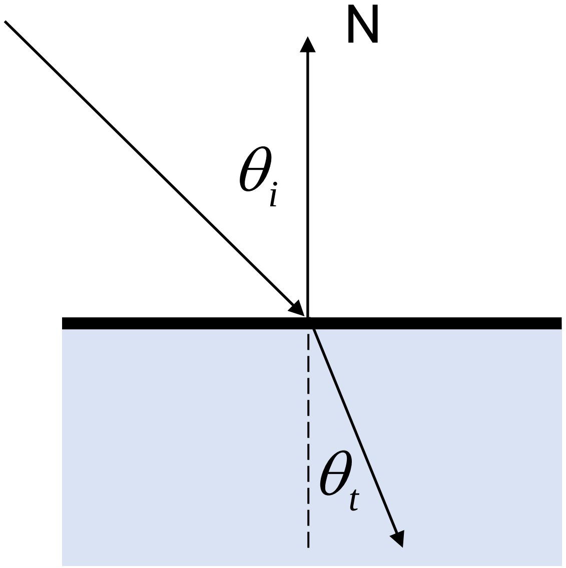
Torrance-Sparrow Model (cont.)

- Real-world surface has both **reflection** and **transmission**
  - Perfect specular reflection: $\theta_i = \theta_o$
  - Perfect specular transmission: $\underline{\eta_i}\sin\theta_i = \underline{\eta_t}\sin\theta_t$ (Snell's law)

index of refraction



perfect reflection



perfect transmission

Torrance-Sparrow Model (cont.)

- **Reflectivity** and **transmissiveness**: fraction of incoming light that is reflected or transmitted
  - Usually **view dependent**
  - Hence, the reflectivity is not a constant and should be corrected by the **Fresnel equation**

- Fresnel equation
  - Related to the wave's electric field
  - S polarization and P polarization

    https://en.wikipedia.org/wiki/Fresnel_equations

Torrance-Sparrow Model (cont.)

- Different properties for dielectrics and conductors

$$r_{\parallel} = \frac{\eta_t \cos\theta_i - \eta_i \cos\theta_t}{\eta_t \cos\theta_i + \eta_i \cos\theta_t}$$

$$r_{\perp} = \frac{\eta_i \cos\theta_i - \eta_t \cos\theta_t}{\eta_i \cos\theta_i + \eta_t \cos\theta_t}$$

Fresnel reflectance for **dielectrics**

$$r_{\parallel}^2 = \frac{\left(\eta^2 + k^2\right)\cos^2\theta_i - 2\eta\cos\theta_i + 1}{\left(\eta^2 + k^2\right)\cos^2\theta_i + 2\eta\cos\theta_i + 1}$$

$$r_{\perp}^2 = \frac{\left(\eta^2 + k^2\right) - 2\eta\cos\theta_i + \cos^2\theta_i}{\left(\eta^2 + k^2\right) + 2\eta\cos\theta_i + \cos^2\theta_i}$$

Fresnel reflectance for **conductors**

$$F_r(\omega_i) = \frac{1}{2}\left(r_{\parallel}^2 + r_{\perp}^2\right)$$

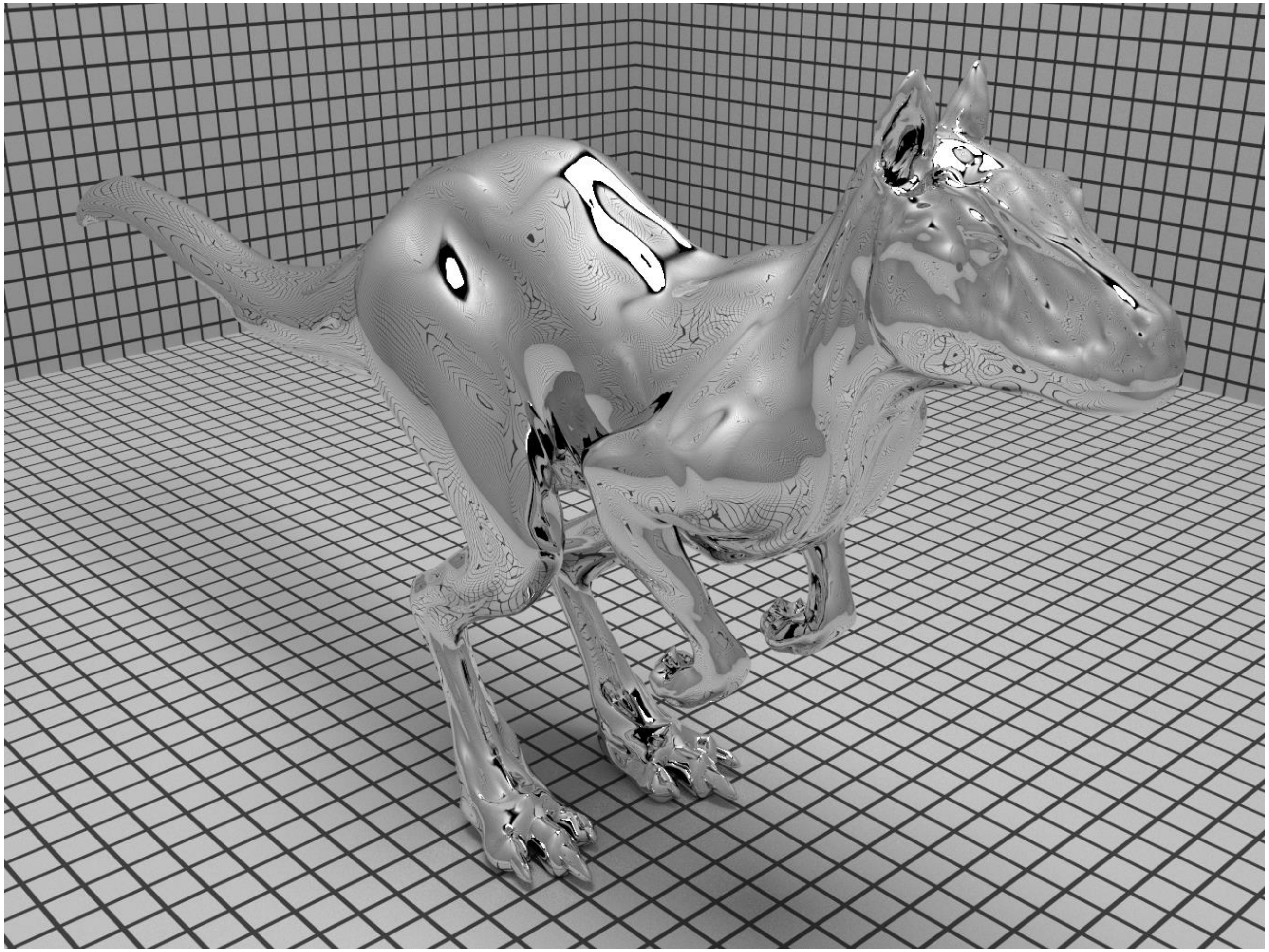
assume light is unpolarized

Torrance-Sparrow Model (cont.)

- Indices of refraction

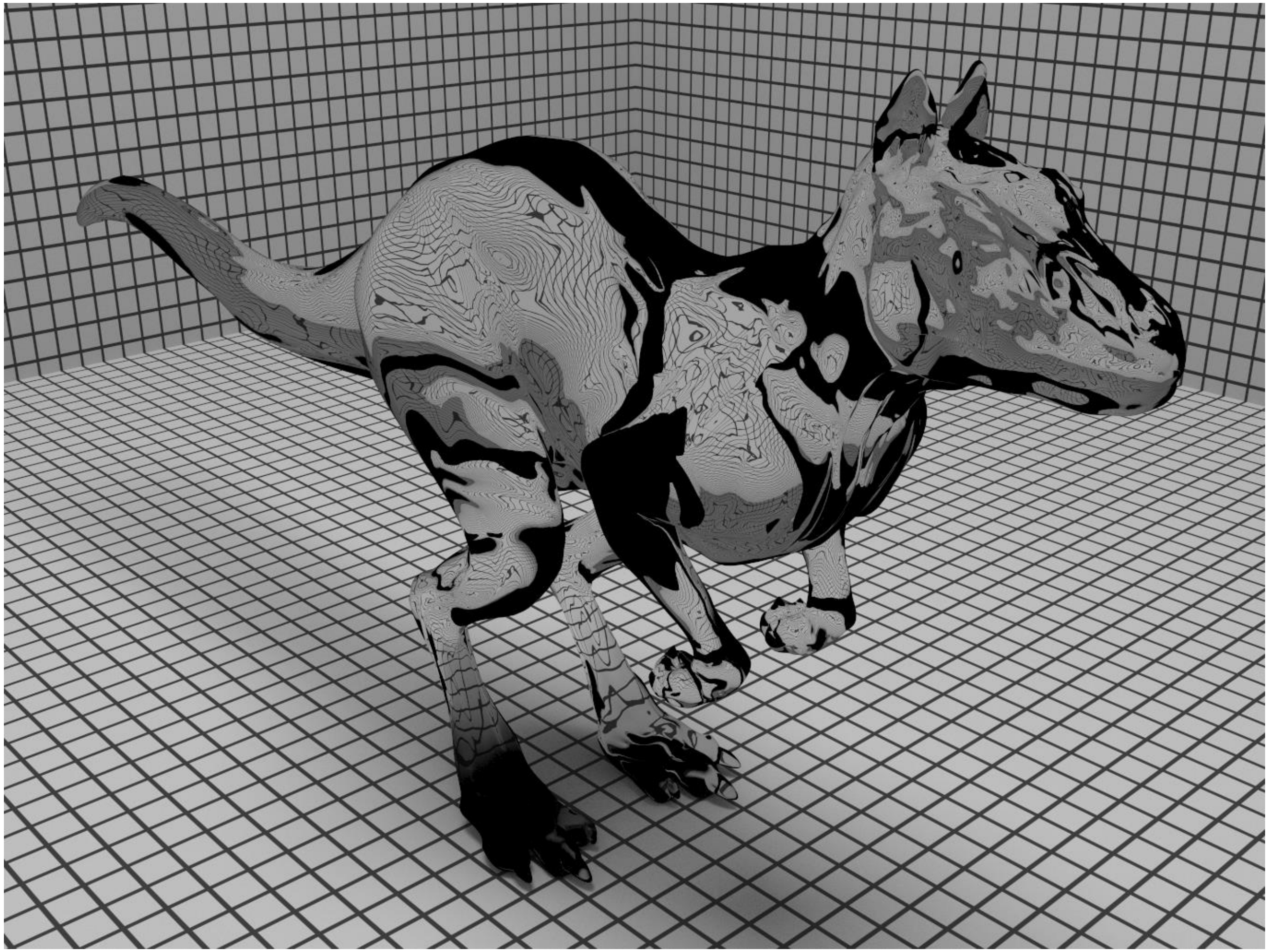
medium	Index of refraction
Vaccum	1.0
Air at sea level	1.00029
Ice	1.31
Water (20℃)	1.333
Fused quartz	1.46
Glass	1.5~1.6
Sapphire	1.77
Diamond	2.42

Torrance-Sparrow Model (cont.)



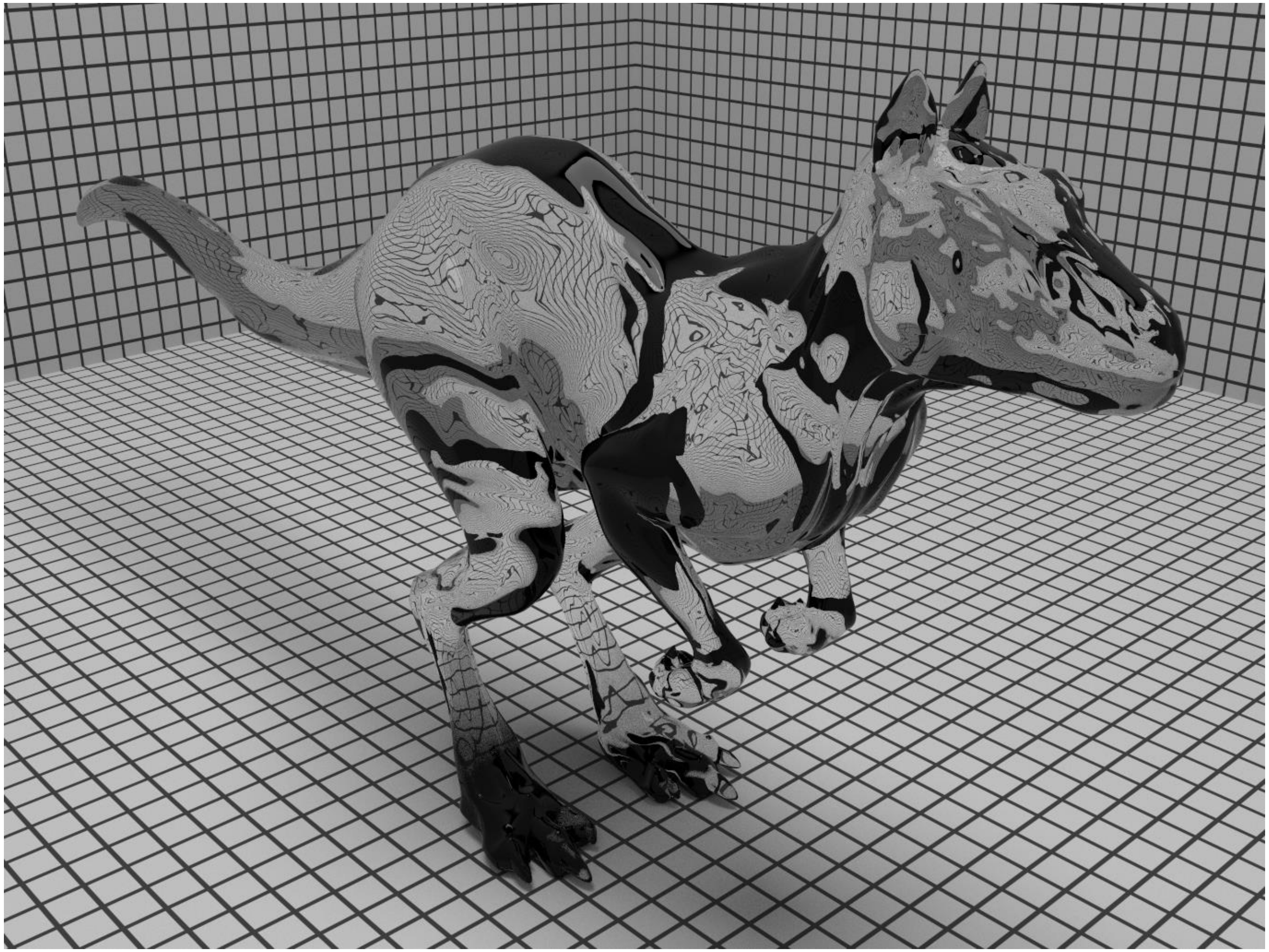
perfect specular refraction

Torrance-Sparrow Model (cont.)



perfect specular transmission (refraction)

Torrance-Sparrow Model (cont.)



Fresnel modulation

Torrance-Sparrow Model (cont.)

- Described by
  - **Microfacet distribution *D***
  - Geometric attenuation *G*
  - Fresnel reflection *F*

$$f_r(\omega_o \leftarrow \omega_i) = \frac{\boxed{D(\omega_h)}G(\omega_i, \omega_o)F(\omega_i, \omega_h)}{4\cos\theta_i\cos\theta_o}$$

How many micro surfaces have this orientation

Commonly used distributions: Beckmann, GGX

$$D(\omega_h) = \frac{\alpha^2}{\pi\Big((\mathbf{n} \cdot \boldsymbol{\omega}_h)^2 (\alpha^2 - 1) + 1\Big)^2}$$

Torrance-Sparrow Model (cont.)

- Put it all together

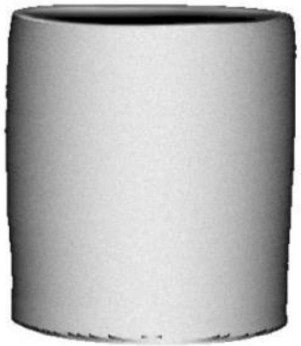
measured

Blinn-Phong

Cook-Torrance (microfacet)

Oren-Nayar Model

- Many real-world materials such as concrete, sand and cloth are not real Lambertian
  - Specifically, rough surfaces generally appear brighter as the illumination direction approaches the viewing direction



Lambertian model

real image

- Assumption: a surface is composed of a collection of **perfectly Lambertian** grooves whose orientation angles follow a Gaussian distribution

Oren-Nayar Model (cont.)

$$f_r(\omega_o \leftarrow \omega_i) = \frac{\rho}{\pi}(A + B\max(0, \cos(\phi_i - \phi_o))\sin\alpha\tan\beta)$$

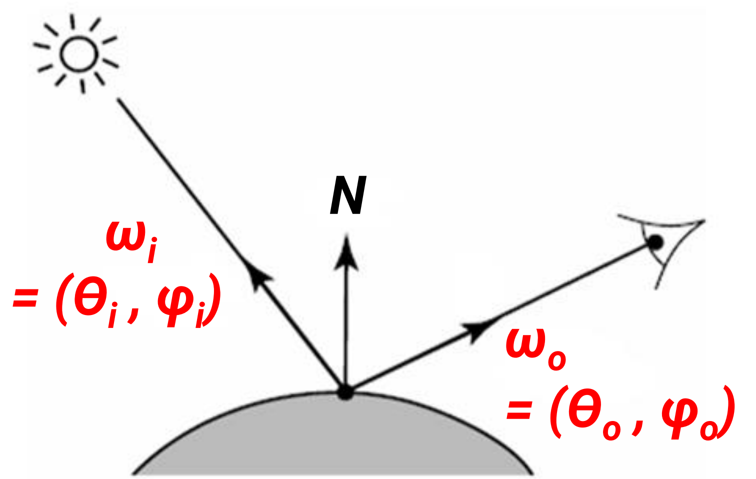
$\boxed{\sigma}$ the standard deviation of Gaussian

$$A = 1 - \frac{\boxed{\sigma}^2}{2(\sigma^2 + 0.33)}$$

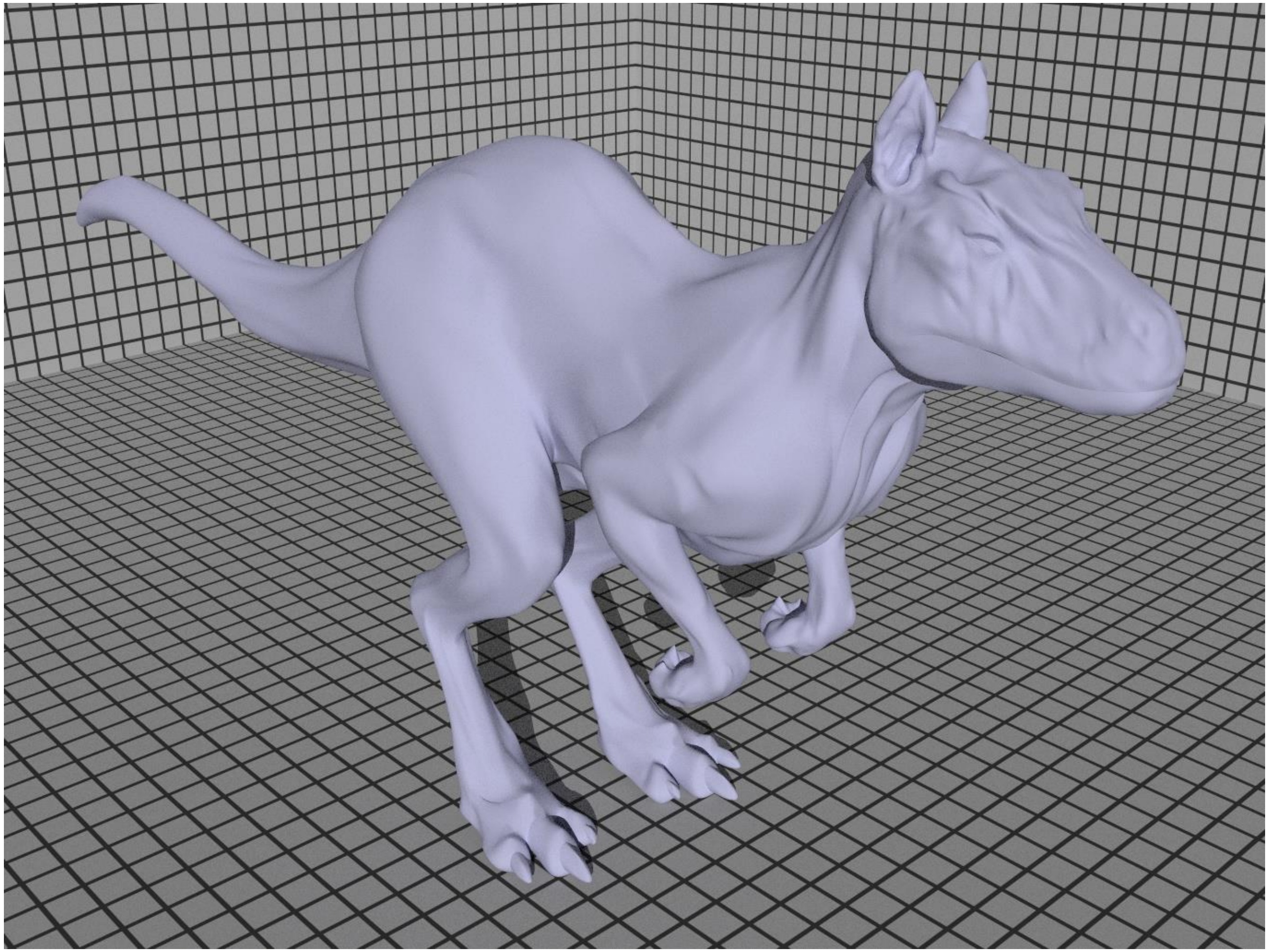
$$B = \frac{0.45\sigma^2}{\sigma^2 + 0.09}$$

$$\alpha = \max(\theta_i, \theta_o)$$

$$\beta = \min(\theta_i, \theta_o)$$

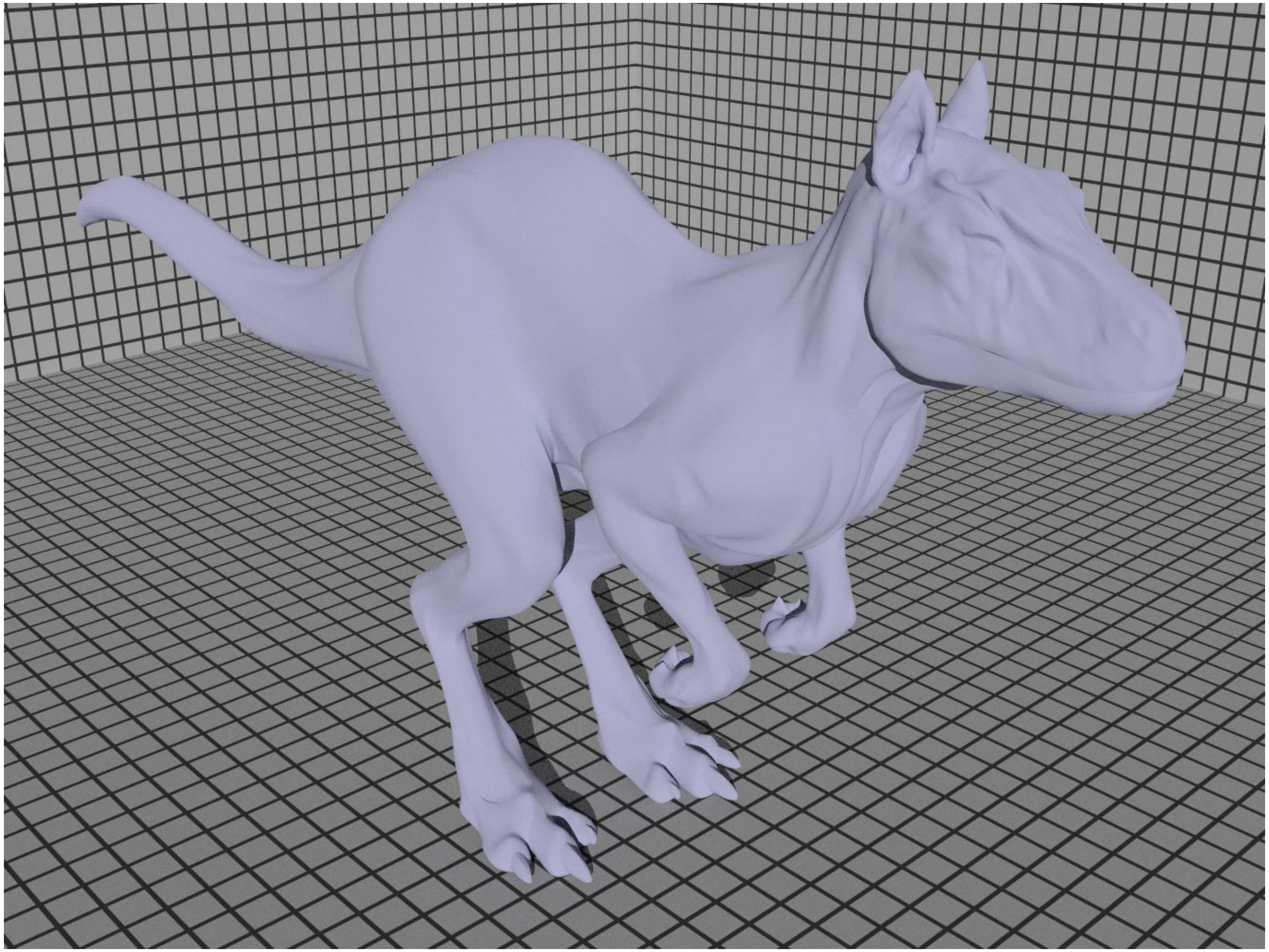


Oren-Nayar Model (cont.)



Lambertian model

Oren-Nayar Model (cont.)



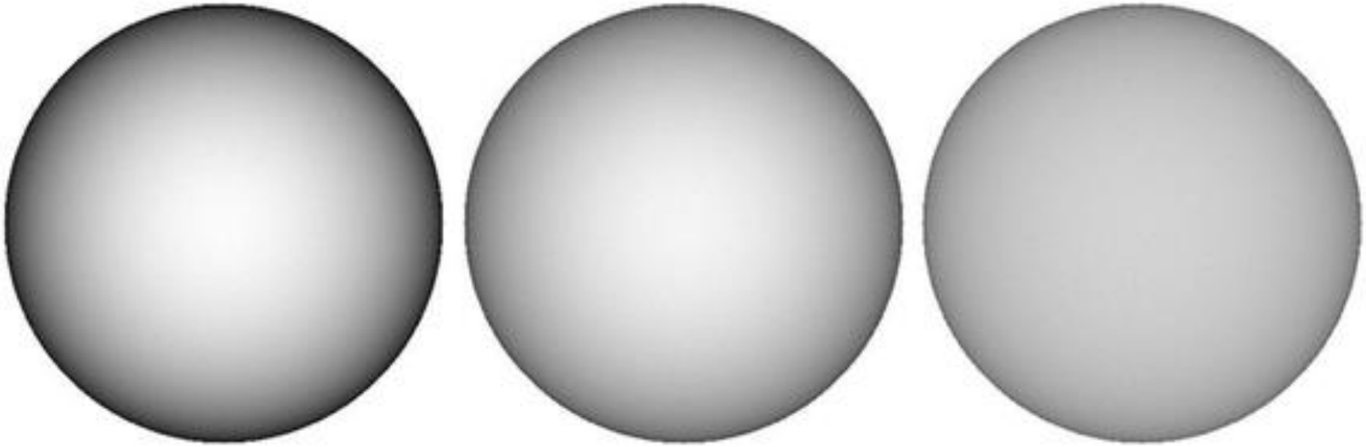
Oren-Nayar model

Oren-Nayar Model (cont.)

- When the standard deviation ***σ*** becomes zero, Oren-Nayar model is reduced to Lambertian model

$$f_r(\omega_o \leftarrow \omega_i) = \frac{\rho}{\pi}(A + B\max(0, \cos(\phi_i - \phi_o))\sin\alpha\tan\beta)$$

$$\Rightarrow \quad f_r(\omega_o \leftarrow \omega_i) = \frac{\rho}{\pi}$$



$\sigma = 0$ $\sigma = 0.1$ $\sigma = 0.3$

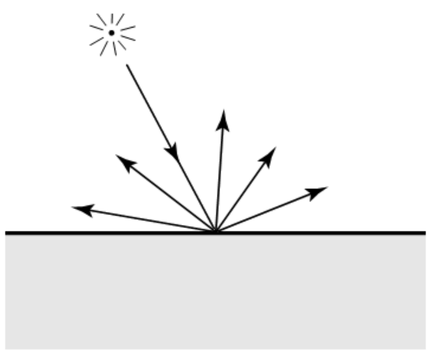
Materials Beyond BRDF

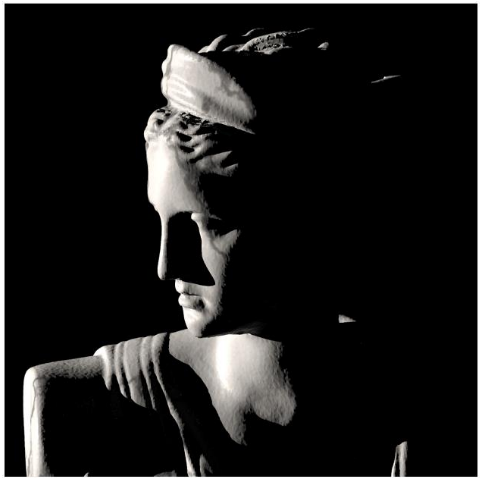
Subsurface Scattering

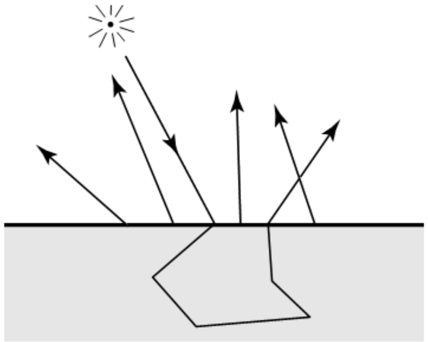
- Some materials interact with lights with a subsurface scattering process that **allows lights to enter and scatter within a medium**
- It gives objects a distinct soft look

BSSRDF

- BRDF v.s. BSSRDF



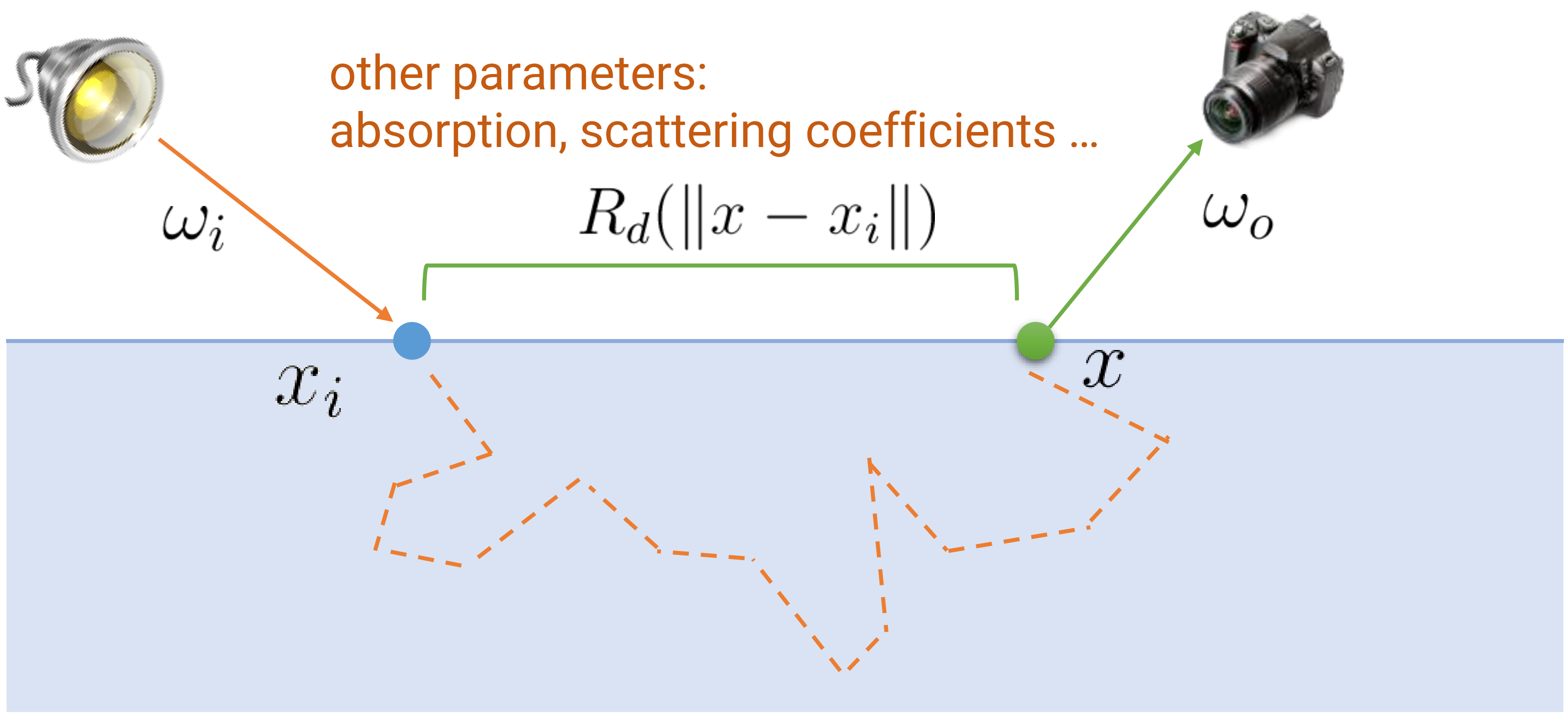




Bidirectional Reflectance Distribution Function **(BRDF)**

Bidirectional Subsurface Scattering Reflectance Distribution Function **(BSSRDF)**

Approximate BSSRDF with Dipole



$$S(x, \omega_o; x_i, \omega_i) = S^1(x, \omega_o; x_i, \omega_i) + S^d(x, \omega_o; x_i, \omega_i)$$

$$S^d(x, \omega_o; x_i, \omega_i) = \frac{1}{\pi} F_t(\eta, \omega_o) R_d(\|x - x_i\|) F_t(\eta, \omega_i)$$

“A Practical Model for Subsurface Light Transport”, Jensen et al. 2001

BRDF for Production

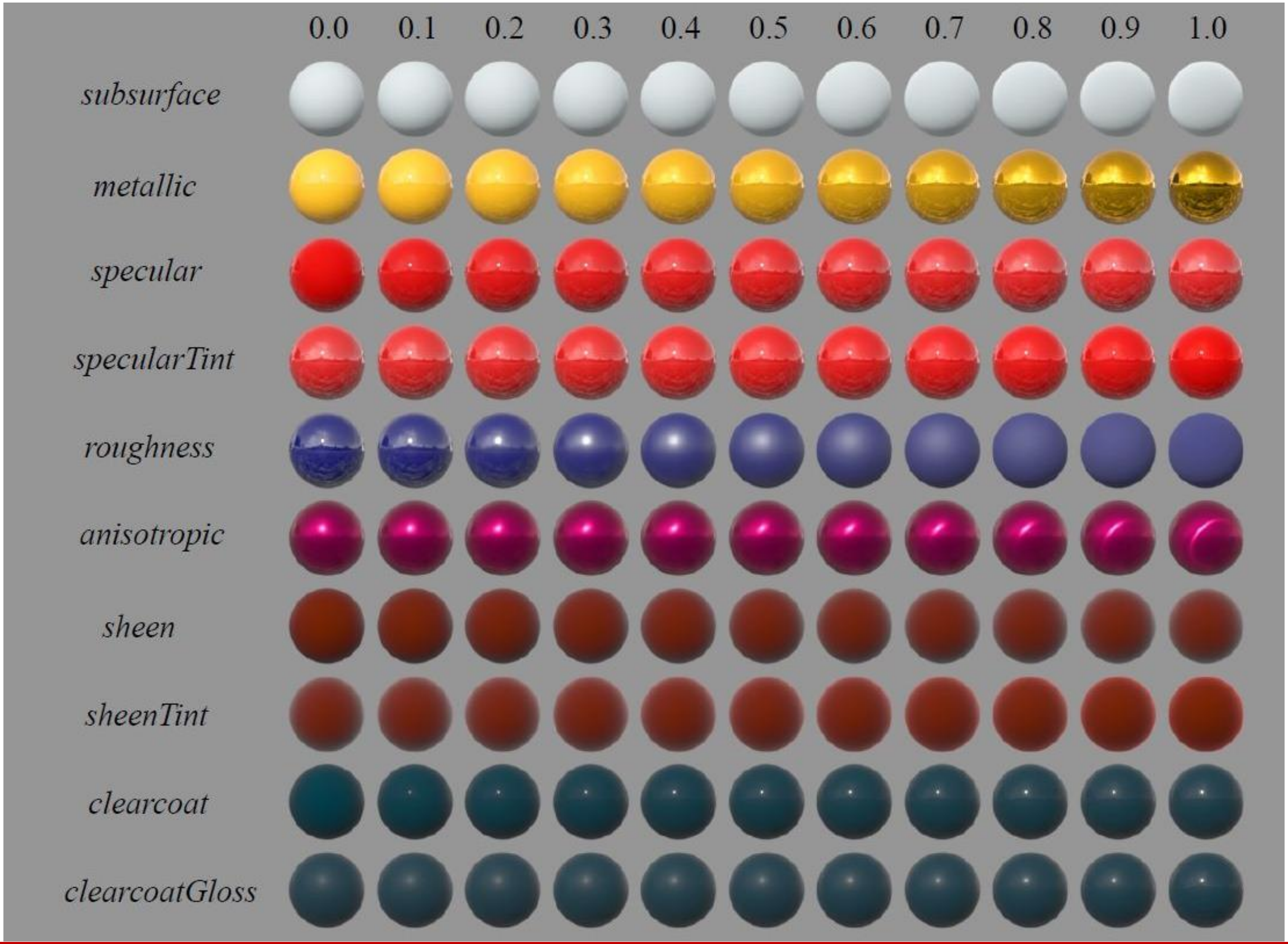
Disney Principled BRDF

- **Phenomenological models**
  - More intuitive parameters; however, not accurate
- **Geometric optics**
  - More accurate but difficult to use by artists
- **Disney Principled BRDF** would like to combine the advantages of both models!
  - Represent a physically-based model (based on the Microfacet model) with few intuitive parameters
  - Each parameter has a range between [0, 1]
  - https://disneyanimation.com/publications/physically-based-shading-at-disney/

Disney Principled BRDF (cont.)

- Proposed when producing the movie, **Wreck-It Ralph** (2012)
  - Also used by the **Unity** and **Unreal** engine

Disney Principled BRDF (cont.)



Disney Principled BRDF (cont.)

- Code: https://github.com/wdas/brdf/blob/main/src/brdfs/disney.brdf

$$
\begin{aligned}
f_{\text{disney}}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \; & (1-\sigma_m)\left(\frac{C}{\pi}\mathrm{mix}(\overset{\text{diffuse}}{\underline{f_d(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o)}}, \overset{\text{subsurface}}{\underline{f_{ss}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o)}}, \sigma_{ss}) + \overset{\text{sheen}}{\underline{f_{sh}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o)}}\right) \\
& + \frac{F_s(\theta_d) G_s(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) D_s(\boldsymbol{\omega}_h)}{4\cos\theta_i \cos\theta_o} \quad \text{specular} \\
& + \frac{\sigma_c}{4} \frac{F_c(\theta_d) G_c(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) D_c(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o)}{4\cos\theta_i \cos\theta_o} \quad \text{clearcoat}
\end{aligned}
$$

$$
\begin{aligned}
f_d(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) &= (1 + (F_{D90} - 1)(1 - \cos\theta_i)^5)(1 + (F_{D90} - 1)(1 - \cos\theta_o)^5) \\
F_{D90} &= 0.5 + 2\cos^2\theta_d \sigma_r
\end{aligned}
$$

$$
\begin{aligned}
f_{ss}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) &= 1.25(F_{ss}(1/(\cos\theta_i + \cos\theta_o) - 0.5) + 0.5) \\
F_{ss} &= (1 + (F_{ss90} - 1)(1 - \cos\theta_i)^5)(1 + (F_{ss90} - 1)(1 - \cos\theta_o)^5) \\
F_{ss90} &= \cos^2\theta_d \sigma_r
\end{aligned}
$$

$$
\begin{aligned}
f_{sh}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) &= \mathrm{mix}(\mathrm{one}, C_{tint}, \sigma_{sht})\sigma_{sh}(1 - \cos\theta_d)^5 \\
C_{tint} &= \frac{C}{\mathrm{lum}(C)}
\end{aligned}
$$

$$
\begin{aligned}
F_s(\theta_d) &= C_s + (1 - C_s)(1 - \cos\theta_d)^5 \\
C_s &= \mathrm{mix}(0.08\sigma_s \mathrm{mix}(\mathrm{one}, C_{tint}, \sigma_{st}), C, \sigma_m)
\end{aligned}
$$

$$
G_s(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = G_{s1}(\boldsymbol{\omega}_i) G_{s1}(\boldsymbol{\omega}_o)
$$

$$
D_s(\boldsymbol{\omega}_h) = \frac{1}{\pi\alpha_x\alpha_y\left(\sin^2\theta_h\left(\frac{\cos^2\phi}{\alpha_x^2} + \frac{\sin^2\phi}{\alpha_y^2}\right) + \cos^2\theta_h\right)^2}
$$

$$
\begin{aligned}
F_c(\theta_d) &= 0.04 + 0.96(1 - \cos\theta_d)^5 \\
G_c(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) &= G_{c1}(\boldsymbol{\omega}_i) G_{c1}(\boldsymbol{\omega}_o) \\
D_c(\boldsymbol{\omega}_h) &= \frac{\alpha^2 - 1}{2\pi \ln\alpha(\alpha^2\cos^2\theta_h + \sin^2\theta_h)}
\end{aligned}
$$

Any Questions?