

Excerpts from

## **Chicago Price Theory**

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# Chicago Price Theory: An Introduction

## The Chicago Economics Tradition

A longstanding Chicago tradition treats economics as an empirical subject that measures, explains, and predicts how people behave. Price theory is the analytical toolkit that has been assembled over the years for the purpose of formulating the explanations and predictions, and for guiding the measurement.

The purpose of this course is to, in the tradition of Chicago's "Economics 301," help you master the tools in the kit so that you can use them to answer practical questions. Studying price theory at Chicago is "a process of immersion in those models so that they bec[o]me so intuitive to one's work that, in combination with new empirical investigation, they open[] the door to novel evaluations of market organization and government policy."<sup>1</sup>

Because price theory at Chicago has always been tethered to practical questions, this course and the course Jacob Viner taught at Chicago almost 90 years ago (Viner, 1930/2013) share some remarkable similarities. The tradition draws heavily on Alfred Marshall in, among other things, viewing human behavior in the aggregate of an industry, region, or demographic group. Market analysis is essential to price theory because experience has shown that markets enable each person to do things far differently than he would if he lived in isolation. It is no accident that price theory is named after a fundamental market phenomenon: prices.

Price theory is not primarily concerned with individual behavior; models of individuals are provided when they offer insight about the aggregate. None of this is to say that price theory only looks at average or representative agents. Indeed, a primary reason that markets transform human activity is that people have some innate differences, which the market encourages them to amplify. Heterogeneity can be important; as we see in the example of comparative advantage below, markets can amplify heterogeneity through returns to specialization.

Price theory has not been static, though. Gary Becker, who taught Economics 301 for many years and gives a couple of the lectures in the video series that accompanies this book, developed human capital analysis, as well as extending price theory to deal with discrimination, crime, the family, and other "noneconomic" behaviors. Becker and Murphy revisited the topic of complementary goods, using it to examine addictions, advertising, and social interactions.<sup>2</sup> Most important, people and businesses are in different circumstances today than in Viner's time as witnessed by the decline of agricultural employment, increased life expectancy, and the rise of information technology.

<sup>1</sup> Quoted from Ross B. Emmett's (2010, p. 2) introduction to his volume on the "Chicago School of Economics."

<sup>2</sup> See Becker (1957, 1968, 1993) and Becker and Murphy (1988, 1993, 2003).

## Price Theory is Different from Microeconomics

Although strategic behavior, such as the interactions among sellers in a market where they are few in number, has been treated with price theory (Weyl, 2018), the introductory Chicago price theory course has not emphasized it. Competition, by which we mean that buyers and sellers take prices as given and the marginal entrant earns zero profit, is emphasized in large part because, for most purposes, it is a reasonable description of most markets (Pashigian & Self, 2007). Moreover, the competitive framework is simple enough to make room for us to master additional aspects of tastes and technology – such as product quality, habit formation, social interactions, durable production inputs, and complementarities – that are important for practical problems. Monopoly models are used on those occasions when price-setting behavior is relevant (Friedman, 1966, pp. pp. 34-5; Stigler, 1972; Demsetz, 1993, p. p. 799). More generally, price theory is stingy as to the number of variables that are declared to be important in any given application.

In emphasizing markets and competition, price theory is different from microeconomics. Both typically begin with the consumer/household but price theory stresses how consumers react to prices, many times without reference to utility or even “rationality,” whereas microeconomics takes care to lay down an axiomatic foundation of the utility function and individual demand functions. Price theory then quickly gets to market equilibrium, treating related subjects such as compensating differences, tax incidence, and price controls.

Microeconomics is much more intensive in game theory, which traditionally puts somewhat more emphasis on rationality and optimizing agents. Both price and game theory model behavior as an equilibrium, but the latter typically focuses on interactions among small numbers of agents, and strives to make separate predictions for each one. The rest of the market is treated as a constant.

The typical auction model of price (Klemperer, 2004) is an example of the game-theoretic approach. That model has a fixed number of goods for sale in the auction, with little attention to how the goods were produced or how they would be used if not sold in the auction. The model has a fixed number of buyers and predicts how each buyer separately makes bids on the items for sale. Understanding why there are, say, two buyers rather than some other number, or what determines the seller’s reservation price, is considered to be an advanced topic. With its emphasis on competitive market equilibrium, basic price theory is not concerned with bid prices but rather the ultimate transaction price, aggregate quantities produced and sold, and how they are connected with costs of various kinds as well as how the good is situated in the consumer demand system.

The market-equilibrium approach says that the most important effects of policy, technical change, and other events are not necessarily found in the immediate proximity of the event. An ethanol-subsidy example, discussed below, features a subsidy that is paid only in the market for fuel that uses just a fraction of total corn production but has more price sensitive demand. The market for animal feed is unsubsidized, but corn farmers’ opportunity cost for selling animal feed is linked to the subsidized fuel

market, so much of their gain from the subsidy comes from the increase in the equilibrium price of animal feed.

Real-life situations involve an element of strategic interactions where the players in a small-scale game understand the outside options available to them in a larger market. One approach would be to simultaneously model both the strategies and market prices. Auction models could, in principle, have endogenous production, entry, and reservation values that reflect economic activity outside the auction. But the point of theory in economics or any other field is focus on important variables and leave the others to the side. As noted above, a great many markets have many buyers and many sellers, and have complementarities, taxes, habits, and other variables that need attention before getting into the strategic details for specific buyers or sellers. These are the situations when price theory is needed.

The ethanol-subsidy example also demonstrates how price theory guides measurement. Empirical studies of markets over time, or comparisons across countries or industries, must consider how to summarize a seemingly complicated reality behind each observation. Price theory shows how the appropriate approach to measurement depends on the question at hand.

Putting practical questions in a market context changes the answer. Trained economists are generally aware that market analysis is why the economic incidence of, say, a tax is different from the legal liability for paying the tax. But without price theory, economics training has too little practice in market analysis and results in policy investigations that too quickly presume that, say, the corporate-income tax primarily harms corporations or an earned income tax credit primarily benefits workers..

### Using *Chicago Price Theory* to Learn Economics

Graduate microeconomic texts often devote more pages to game theory than competitive equilibrium, and part of their competitive analysis is dedicated to confirming that an equilibrium exists as a mathematical object. To the price theorist, the toolkit's mathematical foundations and possible abstract generalizations are an interesting subject for specialists, whereas a general economic education requires seeing how the tools have been successfully applied in the past and preparing to nimbly apply them to the next practical question that we encounter. Completing a mathematical microeconomics course will not make you good at price theory; price theory skills are obtained by practicing applications of the toolkit.

Whereas many economics courses help you master models, and leave application of those models as an advanced topic, price theory immediately engages the student with applications. The book and video series together provide three or four methods of practicing applications. First, both book and videos contain chapter-length examples such as addictive goods, urban-property pricing, learning-by-doing, the consequences of prohibition, the value of a statistical life, and occupational choice. These chapters are instances of applications of price theory that were advanced with important research papers, and sometimes spawning an entire subfield of research activity, with novel and counterintuitive results.

At Chicago, both the students and instructors over the years have gotten better at price theory as a result of engaging with the homework. If you want a formula that makes you good at price theory this is it: practice. Know what tools are available to study markets, and with repetition notice the types of questions that each tool is best suited in the sense of offering a simple analysis with predictions in accordance with observation.

The Chicago homework problems are not paired with specific lectures, because part of excelling at real-world applications is knowing which price-theoretic tool is the best one to use for a particular practical problem. This book therefore provides a number of sample homework questions, but only at the end of one of the three book sections. The video series includes about a dozen of Professor Murphy's impromptu answers to student questions about current market events.

Becker and Murphy's course has always been intensive in solving applied problems, with considerable time of the instructors and advanced star graduate students devoted to formulating and helping students solve homework questions. The drafts of the book and video are now being used at Chicago to further "flip" the Price Theory classroom so that more of the student interactions with Murphy are addressing applied problems.<sup>3</sup> Price theory instructors away from Chicago now also have the opportunity to reallocate their time away from lecturing – let this book and video series help with that – and toward developing and discussing relevant and challenging applied homework questions.

Another way to practice applications is to do some homework before you begin the course and return to them afterwards. You will be amazed at how differently you think at the end! The six questions below are good examples:

1. Is learning by working on the job cheaper than formal schooling? [see Chapter 9]
2. What is the difference between prohibiting marijuana sales and subjecting its sales to a high tax? [see Chapter 12]
3. A great many manufacturers use machines and labor in fixed proportions. Does that mean that the wage rate has little effect on the amount of labor used in manufacturing? [see Chapter 7]
4. Does the availability of ebooks reduce the sales of physical books? [see Chapter 11]
5. When housing prices are above their long-run values and continue to rise, is that good evidence that home buyers or builders have unrealistic expectations about the future? [see Chapter 15]
6. Could a billion dollars in federal subsidies to farmers increase farm incomes by more than one billion? [this chapter]

As you work through the homework questions and the applied chapters, you will practice identifying and applying the tools of price theory. But the tools are just a means to an end, which is to understand human behavior. Most of the homework questions and applied chapters in price theory are therefore real-world

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<sup>3</sup> A "flipped classroom" is one where the students are exposed to background material or general methods at home (with a textbook or video lecture) and use their time with their instructor to practice applying the methods.

questions about human behavior, of the same kind that are addressed by professional economists every day at central banks, major corporations like amazon.com, and at regulatory agencies like the Food and Drug Administration.

Because it is useful, price theory gets applied to large number of practical questions. Each practitioner of price theory thereby builds a wealth of experience that pays dividends in subsequent applications. New problems are recognized for their relations with problems already solved. Perhaps this is why price theory is sometimes called “intuitive.”<sup>4</sup>

### Example: Ethanol-fuel Subsidies

#### *A Market “Multiplier”*

The federal government has been supporting the production of ethanol fuel with a variety of tax credits, subsidies, guarantees, etc. When the U.S. government started subsidizing ethanol fuel, the price of land used to grow corn, which is the primary ingredient in U.S. ethanol production, increased, regardless of whether the corn grown on that land actually ended up in the fuel.

Given that U.S. ethanol is primarily produced with corn, is it possible that corn farmers benefit more than \$1 billion for each \$1 billion that the federal treasury spends on that support? In other words, let’s use price theory to examine the incidence of ethanol-fuel subsidies.

Take a simple model in which corn,  $C$ , is used to make either ethanol fuel,  $E$ , or animal feed,  $F$ . We will consider demand curves  $D_E$ ,  $D_F$ , and  $D_C$ , where  $D_C$ , the market demand curve for corn, is found by adding the demands for ethanol and animal feed. A subsidy of the amount  $x$  per unit corn used in ethanol serves to increase the demand for ethanol by  $x$  units in the price dimension to  $D'_E$ . Horizontally adding the new ethanol demand curve with the stable feed demand curve, we get a new overall corn demand curve  $D'_C$ . Supply and demand for corn determine the equilibrium price of corn, which is the same regardless of how it is used. An example of our market is shown below:

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<sup>4</sup> If “intuitive” is meant as an antonym to mathematical rigor, we disagree. This book is mathematically rigorous in the sense that results are derived from fully-specified assumptions. Indeed, many of the deductive results in this book have been performed on computers, which have no ability to fill in missing implicit assumptions (the symbolic-computation representation of many of them can be found at <http://examples.economicreasoning.com/>). But the course emphasizes practice at applications – knowing what tools are available and when to use which ones – not practice at derivations.

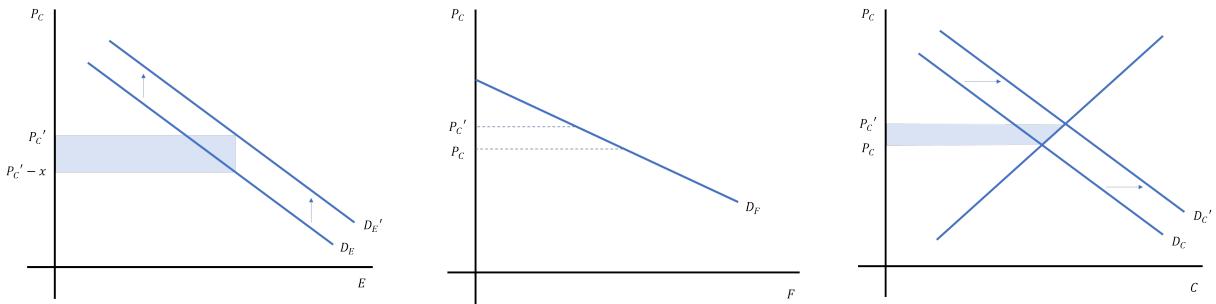


Figure 1: Can farmers gain more from an ethanol subsidy than the amount the government pays?

The result of the subsidy is that more corn is sold overall, and for a higher price ( $P'_C$  rather than  $P_C$ ). Less corn is sold for animal feed, because that demand curve is stable and the price is higher. The extra corn sales go to ethanol because the subsidy amount  $x$  more than offsets the price increase.

Our question, posed from the perspective of the figure, is whether the producer-surplus trapezoid in the market for corn (see the right-most chart) can be larger than the subsidy-expenditure rectangle in the market for ethanol (see the left-most chart).

Consider a case in which the demand for ethanol fuel is perfectly elastic and the demand for feed is strictly decreasing. The overall demand curve is flat when the price is below what the ethanol market will bear. At prices above that, all corn is sold for animal feed and none for ethanol. Putting the two together, we have an overall demand curve with a hockey-stick shape, as shown below when we adapt the previous graphs to this new setting:

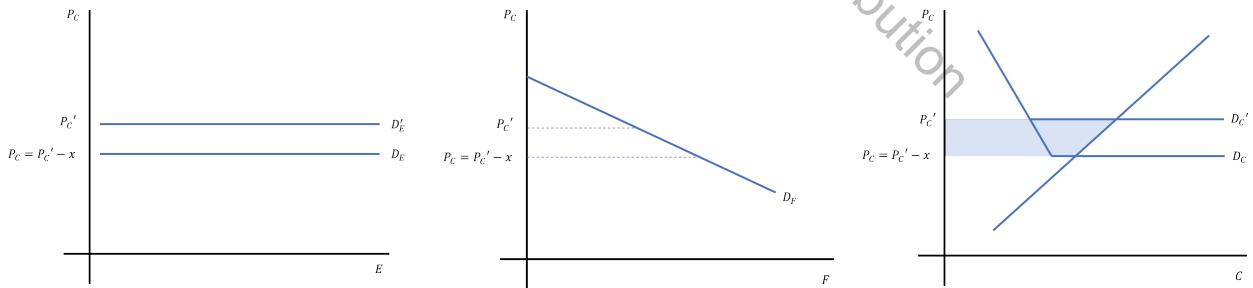
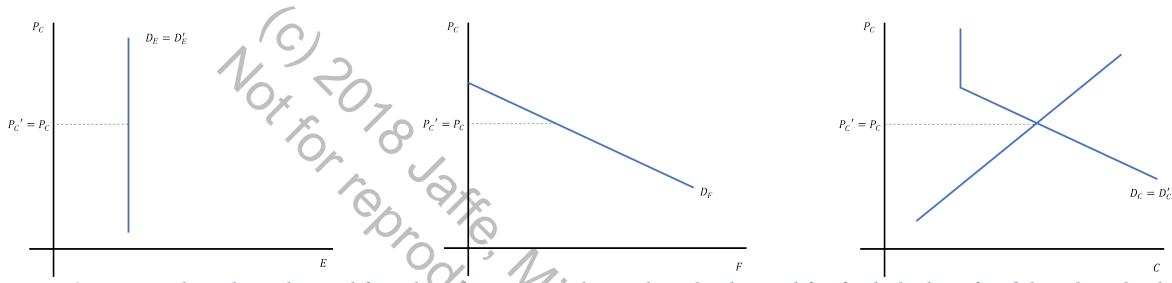


Figure 2: In a market where demand for ethanol is more elastic than the demand for feed, the benefit of the ethanol subsidy to corn farmers can exceed the amount the government spends on the subsidy.

Suppose the subsidy is \$0.10 per gallon. Then, in this market, the \$0.10 gap created between the buyer and seller price per gallon in the ethanol market gets carried over in full to the aggregate market for corn.<sup>5</sup> If the subsidy is small enough, the gain to corn farmers is larger than the amount the government is paying.<sup>6</sup> Why? Not only do corn farmers get \$0.10 more for the corn going to ethanol, which the government pays; they also get \$0.10 more for the corn going to feed, which the animal-feed buyers pay. Maybe this also helps explain why the federal government assists corn farmers with an ethanol subsidy rather than paying the farmers cash directly.

Now consider a case in which the demand for ethanol fuel is perfectly inelastic. We leave the demand for feed unchanged.



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Figure 3: In a market where demand for ethanol is more inelastic than the demand for feed, the benefit of the ethanol subsidy to farmers cannot exceed the amount the government spends on the subsidy. The ethanol demand shown above is perfectly inelastic, so the subsidy has no price impact.

Here, ethanol corn demand is perfectly inelastic, which means that, given any price, people demand the same amount. So an ethanol subsidy, which reduces the price the ethanol corn buyers see, has no effect on their demand. Because the market demand curve is just the sum of the demand curves in the ethanol and feed markets, there is likewise no effect on market demand. The corn farmers, in this case, get no surplus from the subsidy despite what the government spends on it.

In general, corn farmers can benefit more than the amount the government spends on the subsidy only if the demand for ethanol is more elastic than the demand for feed. This is the empirically likely case, given that there are corn-free ways to make fuel that is essentially the same from the fuel consumer's perspective but it is not as easy to switch to alternative animal feeds. Moreover, the supply of land to growing corn may be inelastic in the short run (but probably elastic in the long run).

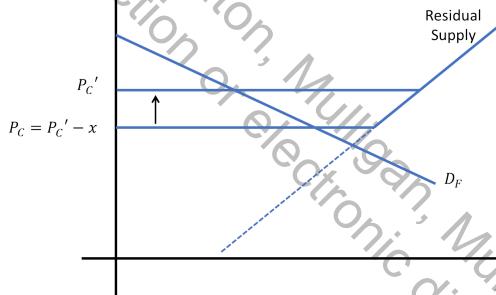
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<sup>5</sup> Here we assume that, absent the subsidy, there is a strictly positive amount of corn going to ethanol production. This assumption is visible in the chart because in that the supply curve always intersects the flat part of overall demand.

<sup>6</sup> For large subsidies, the comparison is ambiguous because a large amount of corn may be drawn into the ethanol market and therefore require additional government revenue to finance the subsidy program. See also the right-hand Figure 2 where some of the subsidy is paid to marginal supply that receives a net benefit of strictly less than  $x$ .

How can we think about this intuitively? Think about price discrimination. Normally, we want to charge the low price to the people with elastic demand and the high price to people with the relatively inelastic demand. The ethanol subsidy looks like price discrimination precisely when the demand for ethanol is price elastic relative to feed because it pushes the ethanol price down relative to the feed price. Corn farmers can gain substantially in this scenario relative to spreading the same subsidy dollars across all corn sales.

We can also look at the equilibrium from the feed market perspective. Possible feed demand curves are already drawn in middle panels of Figure 1, Figure 2 and Figure 3. The feed supply curve is a residual supply curve: the horizontal difference between overall corn supply curve and the ethanol demand curve. The more elastic is ethanol demand, the more elastic the residual supply is. In the perfectly-elastic case introduced in Figure 2, nothing is supplied to the feed market when prices are below the ethanol demand curve (all of the corn goes to ethanol) – and coincides with the overall supply curve at prices above that (no corn goes to ethanol). Figure 4 therefore draws a supply curve that is horizontal at quantities in between the price axis and the overall supply curve.



*Figure 4: The supply of corn to feed usage is a residual supply curve. It is shifted up by the subsidy in the ethanol market. The case shown here corresponds to horizontal ethanol demand.*

The ethanol subsidy  $x$  shifts up the residual supply curve by the amount  $x$  and raises the price the feed buyers pay for corn by  $x$ . The revenue that corn farmers gain in the feed market could easily exceed the revenue they gain in the subsidized market (ethanol) because (i) ethanol gets a minority of corn production and (ii) more important, ethanol demand is much more price sensitive than feed-corn demand.

The main idea here is that because we have a market, the subsidy on ethanol has an effect broader than the initial subsidy. The price of corn going into animal feed will also increase.

## Price Theory Guides Measurement

In many labor, health, and other markets with large amounts of subsidies or taxes, there is a big difference between the price paid by buyers and the price received by sellers because one of the parties is paying a tax or receiving a subsidy. In these cases price theory makes it obvious that the proper measurement of price depends on whether buyer or seller behavior is to be explained.

In our ethanol-subsidy example, some buyers pay less than others. The use of the various prices for empirical analysis depends on the question at hand. For the purposes of predicting the amount of government revenue to subsidize corn sales, what matters is the quantity-weighted average subsidy in the market. That is the average of zero on feed corn and the subsidy rate on ethanol corn, weighted by the quantity of corn going to each use.

For the purposes of measuring the price impact, the quantity weights need to be adjusted for the price sensitivity of the buyers. In the neighborhood of no subsidy, the price-impact formula is the product of three terms:<sup>7</sup>

$$\frac{dP_C}{dx} = \theta \frac{\frac{E}{C} \frac{pD'_E/E}{pD'_C/C}}{\theta} = \frac{D'_c}{D'_c - S'}$$

where  $x$  is the subsidy rate,  $S'$  is the slope of the supply curve and  $\theta$  is the usual incidence parameter indicating how each unit of a uniform subsidy would raise the price received by sellers. As a matter of algebra we could further simplify the formula but we keep the three terms separate in order to discuss their economic interpretation. The second term in the price impact formula is the quantity-weight term and recognizes that only a fraction ( $E/C$ ) of the corn supplied goes to ethanol. The third term, with a price elasticity for both its numerator and denominator, adjusts for any difference between the ethanol demand elasticity and the overall demand elasticity. The third term ranges from zero when ethanol demand is completely inelastic (Figure 3) to  $C/E > 1$  when ethanol demand is infinitely elastic (Figure 2); it would be one if both types of buyers were equally price elastic.<sup>8</sup>

In other words, the units sold to more-price-elastic buyers count more than the units sold to less-price-elastic buyers. In our example with one type of buyer that is subsidized and the less price-sensitive type of buyer that is not, the price-sensitivity adjusted weighted average subsidy exceeds the pure quantity-weighted average, which is why the corn farmers can gain more than the treasury spends on the subsidy.

<sup>7</sup> To derive this formula, totally differentiate the equilibrium condition  $D_E(p - x) + D_F(p) = S(p)$  and solve for  $dP/dx$ . Multiply both numerator and denominator by  $pE/C$  and use the fact that  $D'_E + D'_F = D'_C$ .

<sup>8</sup> As shown in Figures 5 and 6, the price impact itself ranges from zero to one.

The analysis above refers to a subsidy rate that is small in comparison with the price. With larger subsidies we need to consider, for example, that the three terms in the formula vary with the level of the subsidy, which is essentially the price-index problem whose solutions are discussed in Chapter 5.

### Example: Acquired Comparative Advantage

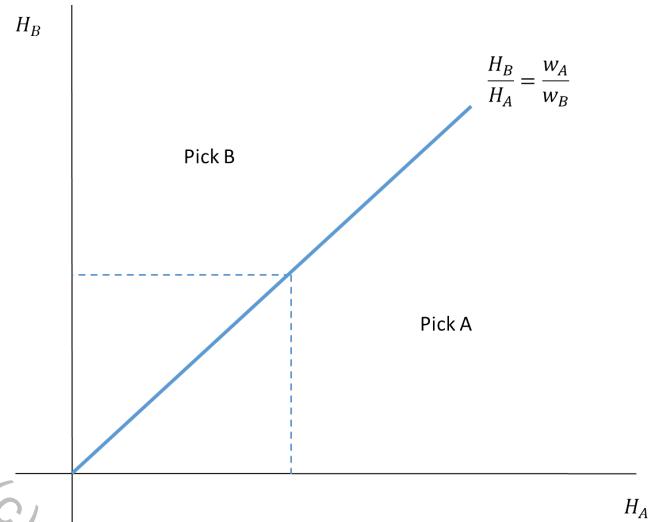
With its emphasis on markets, price theory frequently highlights comparative advantage, which is about economic progress obtained through specialization and trade. The specialization made possible by markets helps explain where people live and work (Becker & Murphy, 1992); why economies grow (Smith, 1776/1904, pp. Book I, Chapter I); why men are different from women (Becker G. S., Human Capital, Effort, and the Sexual Division of Labor, 1985), but less so recently (Mulligan & Rubinstein, Selection, Investment, and Women's Relative Wages over Time, 2008); and much more.

We examine the acquisition of comparative advantage in a simple market setup with two tasks,  $A$  and  $B$ . An individual has human capital for those tasks  $H_A$  and  $H_B$ . Whatever task he picks, he is paid a wage per unit human capital:  $w_A$  or  $w_B$  as appropriate. This will mean total income for an individual from task  $A$  is  $Y_A = w_A H_A$  and from task  $B$  is  $Y_B = w_B H_B$ . The maximum income that the individual can earn is

$$Y = \max\{w_A H_A, w_B H_B\}$$

which is obtained by picking task  $A$  if  $w_A H_A > w_B H_B \Leftrightarrow \frac{w_A}{w_B} > \frac{H_B}{H_A}$ , picking task  $B$  if  $\frac{w_A}{w_B} < \frac{H_B}{H_A}$  and picking either task if the two ratios are equal. This is comparative advantage because his task choice depends on the relative amounts of human capital that he has, not the absolute amount.

Figure 5 illustrates the choice in the  $[H_A, H_B]$  plane by drawing a solid task-indifference ray showing all of the configurations of human capital that someone could have and be indifferent between the two tasks.

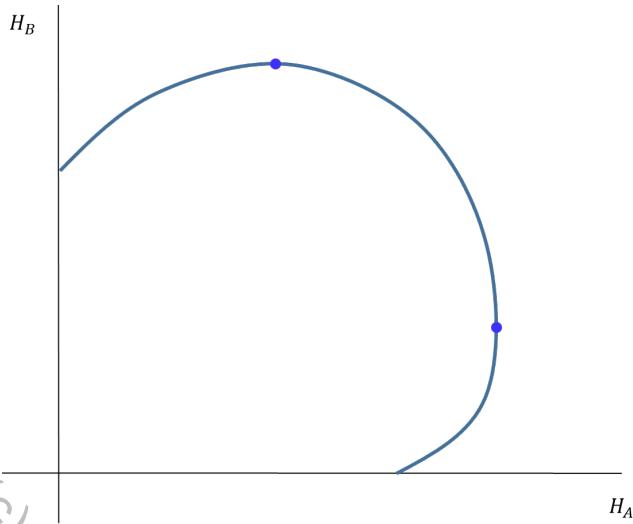


*Figure 5: Supply and demand will rotate the task-indifference ray until the right number of workers is in each task.*

There is demand for tasks  $A$  and  $B$ , which in equilibrium has to match up with the available human capital and the aforementioned incentives for workers to choose one task rather than the other. This happens with wage adjustments. If there were a lot of demand for  $A$ , then Figure 5's task-indifference ray has to be steep so that lots of workers choose task  $A$  and few choose  $B$ . In other words,  $w_A/w_B$  would be greater than one.

Now, assume we have reached the equilibrium, so that  $w_A/w_B$  reflects market supply and demand. Then for any point on the line, every person directly below and directly left must be earning the same income. See the dashed lines in Figure 5. This is because each person on the dashed line above the task-indifference ray has the same level of  $H_B$  and his  $H_A$  does not matter because he does not use it. Each person on the dashed line below the task-indifference ray has the same level of  $H_A$  and his  $H_B$  does not matter because he does not use it. Let's call the union of the two dashed lines an indifference curve for the worker.

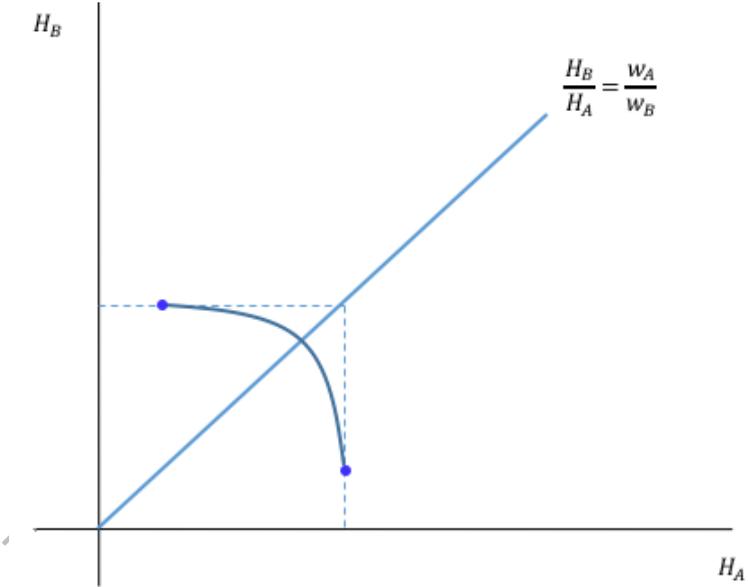
Now, let's allow each agent to choose their human capital. For example, they are considering whether to be a good plumber versus being a good carpenter. The opportunity set for human capital could have an interesting shape, as depicted in Figure 6. Consider the point associated with the maximum level of  $H_B$ . As it is depicted, this person will have some positive level of  $H_A$ . This reflects an underlying story that the tasks  $A$  and  $B$  require some of the same abilities. Thus, if I choose to be a good plumber, that doesn't mean that I end up with zero human capital as a carpenter.



*Figure 6: The opportunity set for selecting human capital. The agent with maximum human capital for task A still has positive human capital for task B.*

Note further, in this graph, that the economically relevant region of the opportunity set lies between the two points and we can erase the parts of the curve close to the axes because no one would choose a human capital pairing left of the top point or below the right point. On the erased regions, the agent could be better at both tasks!

Now let's put the opportunity set together with the worker's indifference curves, as in Figure 7. We can even have everyone identical in the sense that they all have the same opportunity curve to choose from. Nevertheless, specialization is optimal behavior. Being equally good at tasks  $A$  and  $B$  is worse than being very good at just one task because you have acquired a lot of human capital that you do not use.



*Figure 7: Specialization. Agents maximize their human capital at task A or task B.*

We started this picture by indicating the types of workers (that is, configurations of human capital) who are indifferent between the two tasks. But now we have shown that people will not choose to be those types of workers. Because the human capital is acquired, it is not an equilibrium for people to be indifferent between the two tasks.<sup>9</sup>

The equilibrium requires that both tasks are performed, so some people specialize in  $A$  and the others specialize in  $B$ . People who are identical in the sense of having the same opportunities open to them end up being different.

You might say that it is a coin flip exactly who goes toward task  $A$  and who goes toward task  $B$ , and I would agree if people were precisely identical. But in reality, people have somewhat different opportunities open to them: in Figure 6 and Figure 7, that means somewhat different opportunity curves. Some of the opportunity curves may be relatively steep and others relatively flat. Then just a small difference among people in the slope of the curve will decide who specializes in what. Specialization in the marketplace turns small differences into large differences.

## Outline of the Course

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<sup>9</sup> This simple model abstracts from timing, uncertainty and other factors. In the more general case, the market may induce some people to be on the task-indifference ray because, at the time that they acquire skills, they do not know which task they will end up doing. But even in this case, it will not make sense for everyone to be near that ray: some of them can be confident that they will be doing a particular task and thereby specialize in it.

Three economic themes are repeatedly encountered when human behaviors are viewed through the lens of economic theory: substitution effects, market equilibrium, and durable goods. Each of these is a section of the course presenting the classic model and then going through some important applications such as price indices, learning by doing, and house prices.

Section I on prices and substitution effects is written from the perspective of consumer theory. We see little need to explicitly treat firms here, merely for the sake of repetition. The theory of substitution effects is the foundation of price and quantity indices (Chapter 4), which are among the most widely used tools for the economic measurement. The distinction between short- and long-run demand, examined in Chapter 5, has a number of immediate and nontrivial applications such as habits and addictions. Chapter 6 looks at a bit of “behavioral economics” from the perspective of the Marshallian demand curve.

Once we have consumers, the purpose of bringing in firms is to have markets (Section II), which are the primary emphasis of the course. Here we begin with Adam Smith’s compensating differences, as further developed by Sherwin Rosen in his publications and teaching price theory at Chicago.<sup>10</sup> Without saying much yet about production, this allows us to get results for urban economics and the accumulation of human capital.

One of the lessons of compensating differences is to be wary of purported “free lunches.” The learning-by-doing application is of significant intrinsic interest, but was also one of Becker and Rosen’s favorite demonstrations of a consequence of market competition that reappears in a great many applications ranging from health insurance to industrial organization to taxation.

Firms are carefully examined toward the end of Section II. This completes the foundation of the “industry model” (a.k.a., supply and demand), thereby opening up a huge range of applications. One application with particularly surprising results is the consequence of prohibiting trade in specific goods such as illegal narcotics, which is the subject of Chapter 12. Exclusive dealing, quantity discounts, and other pricing practices are also readily examined once we have consumers and firms together, as we show in Chapter 13. The final chapter of Section II extends the industry model to have more than two production factors, which is helpful for examining durable goods as in Section III.

Section III looks at changes over time. It begins by defining durable goods and extending the industry model to have both a capital-rental market and a capital-purchase market. This brings us pretty close to the adjustment-cost model of investment (Chapter 15) and the neoclassical growth model (Chapter 16). These are usually considered “macroeconomics” topics but, as factor supply and demand repeated over time, the two models should not be omitted from price theory. Most important, price theory treats durable goods because durability is an important feature in many practical questions.

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<sup>10</sup> See Smith (1776/1904) and Rosen (1986).

The final three chapters look at important applications of the durable-goods models such as capital-income-tax incidence, the determination of labor's share of national income, and investments in health.

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## Part I: Prices and Substitution Effects

### Chapter 1 Utility Maximization and Demand

#### Utility Maximization

We develop the analysis of substitution effects with consumer theory, and leave producer theory until the section on market equilibrium. Rather than taking the axiomatic approach that is typically in microeconomics textbooks, we follow the more practical approach found in most applied work. That is, we start with a utility function defined over a set of goods  $X_1, \dots, X_N$ , denoted  $U(X_1, \dots, X_N)$ . We assume that each of these goods is being purchased in a market. Later, we think about goods produced at home, purchased in another way, or goods a person is endowed with. We further assume you are purchasing these goods at prices  $P_1, \dots, P_N$ . Finally, we assume the agent has income  $M$ . Note that we need units to be consistent. If income  $M$  is income per week, then consumption  $X$  is consumption per week. These are both flows.

We will usually think about consumers having some budget constraint

$$\sum_{i=1}^N X_i P_i \leq M$$

That is, the agent cannot spend more than his income. Later, in other models, we will think about time constraints. Now, we consider a problem of maximizing utility subject to a budget constraint

$$\max U(X_1, \dots, X_N)$$

$$s.t. \sum_{i=1}^N X_i P_i \leq M$$

One of the toughest parts of the theory is that we don't have much *ex ante* information about what utility looks like. This will be much different than production, where we see the inputs to production and then see how much output those inputs make. We therefore want to minimize the role of utility in the analysis.

We set up the Lagrangian

$$L = U(X_1, \dots, X_N) + \lambda \left[ M - \sum_{i=1}^N X_i P_i \right]$$

The problem is solved by taking the derivative of the Lagrangian with respect to the  $X$ 's and  $\lambda$ . This procedure gives the first-order conditions

$$\frac{\partial U}{\partial X_1} - \lambda P_1 = 0$$

...

$$\frac{\partial U}{\partial X_N} - \lambda P_N = 0$$

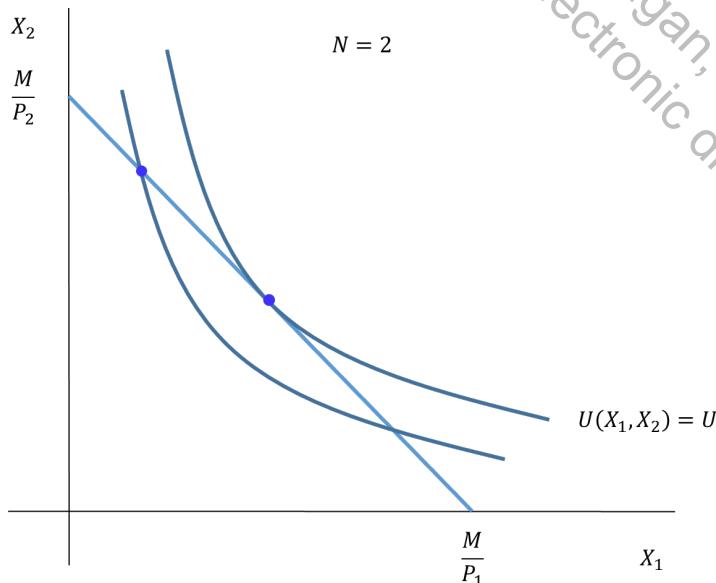
$$\sum_{i=1}^N X_i P_i = M$$

It follows that

$$\frac{\frac{\partial U}{\partial X_i}}{\frac{\partial U}{\partial X_j}} = \frac{P_i}{P_j}$$

What is the right-hand side? This tells us that to get one unit of good  $i$ , we **must give up**  $P_i/P_j$  units of good  $j$ . More generally, we call  $P_i/P_j$  the cost of good  $i$  (in units of  $j$ ). What is the left-hand side? This tells us the amount we're **willing to give up** of good  $j$  to get a unit of good  $i$ . More generally, we call the left-hand side the value of good  $i$  (in units of  $j$ ). So all this expression really says is that to be at an optimal point, it must be the case that the cost of consuming more of good  $i$  equals your value for consuming more of good  $i$ . This expression is commonly explained as "marginal benefit must equal marginal cost."

See Figure 1-1 for a graphical depiction of the optimum in the two-good case. The tangency point is the consumption choice that equates the utility cost of shifting between the two goods with the budget cost. The second point shows the indifference curve crossing the budget constraint, which means that utility can be improved by shifting to good 1 without violating the budget.



*Figure 1-1: The point given by the intersection of the budget constraint with the indifference curve labeled by  $U^*$  is the optimum. The second point depicted is not an optimum because the agent is still getting more marginal utility per dollar for units of good  $X_1$  than  $X_2$ .*

The reason the proportional relationship of prices and marginal utility is important is the fact that we can measure prices directly. We can infer something about preferences from observed behavior. The assumption that people are maximizing gives us a lot of insight into preferences, which we cannot directly measure. We do not have a theory of preferences that tells us what people like before they choose certain products, so instead we use an empirical theory of preferences: people like what they choose.

The marginal benefit and cost equation can also be written as

$$\frac{\frac{\partial U}{\partial X_i}}{P_i} = \frac{\frac{\partial U}{\partial X_j}}{P_j} = \lambda$$

This version says that, at the optimum, the utility per dollar on the margin must be equal for all goods. This makes sense intuitively. Suppose the marginal utility per dollar was higher for good  $i$  than good  $j$ . Then you'd be better off by giving up some of good  $j$  and consuming more of good  $i$ . This expression also tells us that  $\lambda$  is the marginal utility per dollar. Importantly, we don't really need to worry about how that dollar is spent. We can talk about value independent of what a person actually does. For example, suppose dentists are considering adding fluoride to the water supply to reduce cavities. It may be that once fluoride is in the water, people stop brushing their teeth and they get as many or more cavities as before. Dentists might say that the policy backfired and people didn't benefit. The economist would say: sure, people benefited; now they don't have to brush their teeth. It doesn't matter how they choose to take the benefit of fluoride in the water supply – by getting fewer cavities or by not having to brush their teeth.

We can say something else about  $\lambda$ . Since our first-order conditions give

$$\frac{\partial U}{\partial X_i} = \lambda P_i$$

For every  $i$ , we have that, at the optimum, marginal utilities are proportional to prices, and that proportion is  $\lambda$ . So if we can measure the prices people face, we can indirectly measure their marginal utilities because the marginal utilities are proportional. Let's say the price of good 10 is five times the price of good 6, so that  $P_{10} = 5P_6$ . This tells us that a unit of good 10 is worth five times as much as good 6 on the margin. This is true for everyone in the market consuming goods 6 and 10. It's not just a "market value."

Now let's consider two states of the world. In state 1, people consume goods  $X_1^*, \dots, X_N^*$  at prices  $P_1, \dots, P_N$  and have income  $M$ . Now we perturb everything slightly. That is, change consumption by  $dX_1, \dots, dX_N$ , prices by  $dP_1, \dots, dP_N$ , and income by  $dM$ . Then how does utility change? That is, what is  $dU$ ? We know  $dU = dU(X_1, \dots, X_N) = \sum_{i=1}^N \frac{\partial U}{\partial X_i} dX_i = \lambda \sum_{i=1}^N P_i dX_i$ . The key is that  $P_i dX_i$  is observable because we know the prices and the changes in consumption. Then we know  $\frac{dU}{\lambda} = \sum_{i=1}^N P_i dX_i$ . But the left-hand side is just the dollar value of how much better off the consumer is. What's at the core of this analysis? Because people are optimizing, the marginal utilities of goods are proportional to their prices.

Similarly, we can consider how utility changes over time. Consider time-varying consumption  $X_1(t), \dots, X_N(t)$ . Then  $\frac{\partial U}{\partial t} = \sum_{i=1}^N \frac{\partial U}{\partial X_i} \frac{dX_i}{dt} = \sum_{i=1}^N \lambda P_i \frac{dX_i}{dt}$ , where in the last step we once again use the fact that marginal utilities are proportional to their prices. Thus, the dollar value of the change in utility is

$$\frac{\partial U}{\lambda} = \sum_{i=1}^N P_i \frac{dX_i}{dt}$$

In terms of Figure 1-1's tangency illustration, we can think of this formula as measuring the utility change from one optimum to another according to the extra income required to achieve the new utility level. We return to this measurement approach when we look at price and quantity indices in Chapter 4.

## Theory of Demand

The solutions to the first-order conditions,  $X_1^* = X_1^M(P_1, \dots, P_N, M), \dots, X_N^* = X_N^M(P_1, \dots, P_N, M)$ ,  $\lambda^* = \lambda(P_1, \dots, P_N, M)$ , are known as demand equations. They're a particular type of equation system called the Marshallian demand equations, named after Alfred Marshall.

Often applied work begins with demand equations rather than explicitly deriving them from utility functions. But not every set of equations relating prices to the quantities purchased by an individual are consistent with utility maximization; our purpose here is to show what is special about demand equations.

Note that the Marshallian demand function allows the agent to alter consumption of all goods in response to the price change; for example, we can consider the effect of changing  $P_j$  on good 1, i.e.  $\partial X_1^M / \partial P_j$ . In taking these to the data, we might get differences between short run and long run responses because it takes time for people to adjust in reality.

Utility maximization places some restrictions on these Marshallian demand equations. These restrictions are most easily expressed in terms of demand elasticities. First,  $\epsilon_{ii}$ , the elasticity of demand for good  $i$  with respect to the price of good  $i$  is given by

$$\epsilon_i = \epsilon_{ii} = \frac{\% \Delta X_i}{\% \Delta P_i} = \frac{P_i}{X_i} \frac{\partial X_i^M}{\partial P_i} = \frac{\partial X_i^M}{X_i} / \frac{\partial P_i}{P_i}$$

This is also called own-price elasticity and is often shortened from  $\epsilon_{ii}$  to  $\epsilon_i$ . It gives us the percent change in demand for good  $i$  for each 1% increase in the price of good  $i$ . We also have the cross-price elasticity,

$$\epsilon_{ij} = \frac{P_j}{X_i} \frac{\partial X_i^M}{\partial P_j}$$

Which gives the percent increase in demand for good  $i$  for each 1% increase in the price of good  $j$ . Typically, if  $\epsilon_{ij} > 0$  – demand for good  $i$  increases when the price for good  $j$  increases – we say that goods  $i$  and  $j$  are substitutes. If  $\epsilon_{ij} < 0$ , goods  $i$  and  $j$  are complements.

Lastly, the income elasticity of demand for good  $i$  is

$$\eta_i = \frac{M}{X_i} \frac{\partial X_i^M}{\partial M}$$

This gives us the percent change in demand for good  $i$  in response to each 1% change in income. For  $\eta_i > 0$ , we say  $i$  is normal. It's natural to think about an income elasticity of 1, since it corresponds to a 10% increase in income resulting in scaling up consumption of everything by 10%. This is the threshold between luxury and necessity goods: for  $\eta_i < 1$ ,  $i$  is a necessity; for  $\eta_i > 1$ ,  $i$  is a luxury. This intuition also suggests that the average income elasticity is one:

$$\sum_{i=1}^N s_i \eta_i = 1$$

Where  $s_i$  is the share of good  $i$  in total spending (a.k.a., good  $i$ 's “budget share”). Note that the shares and elasticities are evaluated at a particular set of prices and a particular income. The fact that the share-weighted income elasticity of demand is one is sometimes known as “Engel aggregation.”

We can see mathematically that the average income elasticity is one by (a) differentiating the budget constraint with respect to income, (b) rewriting each income-effect term with the corresponding income elasticity, (c) rewriting each price term with the corresponding spending share, and (d) canceling like terms in numerators and denominators:

$$1 = \sum_i \frac{\partial X_i^M}{\partial M} P_i = \sum_i \left( \frac{X_i^M \eta_i}{M} \right) \left( s_i \frac{M}{X_i^M} \right) = \sum_i \eta_i s_i$$

We have two more constraints on elasticities, namely

$$\sum_i s_i \epsilon_{ij} = -s_j$$

And

$$\sum_j \epsilon_{ij} + \eta_i = 0$$

The first, known as the “adding up” constraint (for the Marshallian demand system), is a sum over goods.<sup>11</sup> It can be proved by differentiating the budget constraint with respect to the  $j$ th price and, as before, rewriting demand slopes in terms of elasticities and prices and quantities in terms of shares:

$$\sum_i \frac{\partial X_i^M}{\partial P_j} P_i + X_j^M = 0 \Leftrightarrow \sum_i \left( \frac{X_i^M}{P_j} \epsilon_{ij} \right) \left( s_i \frac{M}{X_i^M} \right) = -\frac{M}{P_j} s_j \Leftrightarrow \sum_i s_i \epsilon_{ij} = -s_j$$

Since quantities are not negative, this constraint is another way of expressing a law of demand.<sup>12</sup> Increasing the price of good  $j$  may increase the Marshallian demand for good  $j$  (specifically, when good  $j$  is a Giffen good) but then it must, weighted by prices, reduce the demand for other goods even more.

The second elasticity constraint, known as “homogeneity” (for a Marshallian demand function), is a sum over prices merely saying that increasing income and all prices by the same proportion has no effect on choices, expressed in terms of elasticities. Take, for example, a strictly positive demand for good  $i$  in response to a  $\mu \neq 0$  proportion change in prices and income:

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<sup>11</sup> It is also sometimes known as Cournot aggregation in order to distinguish it from the aforementioned Engel aggregation, which can also be referenced as “adding up.” Hereafter we use “adding up” only to refer to Cournot aggregation.

<sup>12</sup> See also Chapter 3 and Becker's (1962) demonstration of a law of demand that relies only on the household budget constraint.

$$0 = \sum_j \frac{\partial X_i^M}{\partial P_j} P_j \mu + \frac{\partial X_i^M}{\partial M} M \mu = \mu X_i^M \left( \sum_j \epsilon_{ij} + \eta_i \right)$$

What's an example of an inferior good? Classic examples include potatoes and Kraft Mac and Cheese. But is food inferior? In general, it is not. As income increases, people buy "more" food. But what do we mean by "more?" We mean  $FOOD = \sum_{i \in FOOD} P_i X_i$ , and then  $dFOOD = \sum_{i \in FOOD} P_i dX_i$ . We're not measuring more food in calories, pounds, etc.; we're measuring it by weighting the consumption by prices. This is useful because  $P_i$  gives us insight into the agent's value for  $X_i$ . We could easily have total calories or total pounds of food consumption decreasing while our price-based measure of food increases.

Note also that whether goods are normal or inferior is highly dependent on the level of income. In many areas of the world, Walmart expects demand for its products to increase when income rises. In the U.S., on the other hand, consumers facing higher income might be more likely to stop shopping at Walmart and shop elsewhere instead. Figure 1-2 illustrates with an Engel curve – a graph of income versus quantity demanded – for a good that is normal at lower incomes and inferior at relatively high incomes. If the good were shopping at Walmart, then the peak demand would happen at the income where people tend to shift their shopping to "fancier" stores.

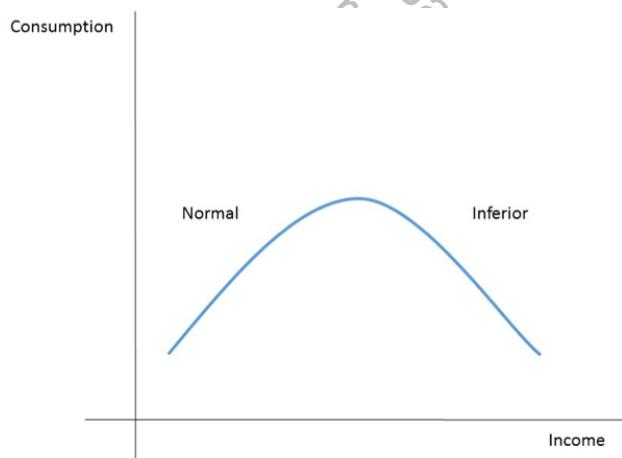
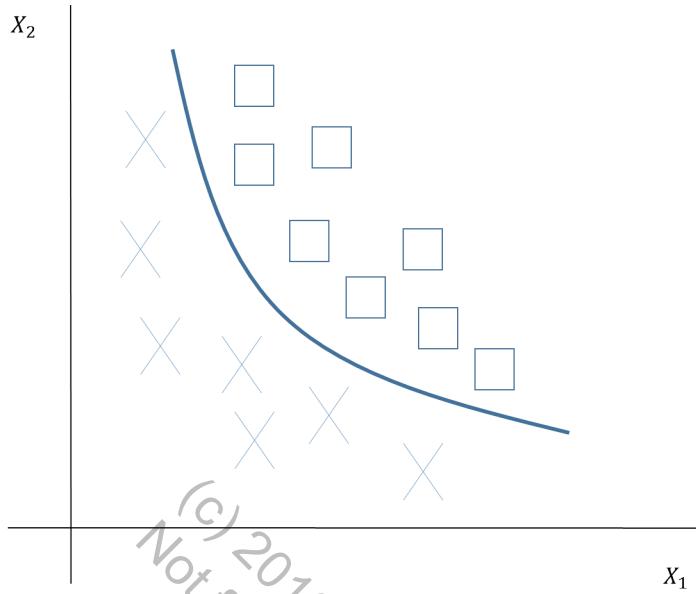


Figure 1-2: Goods can be normal over certain income ranges and inferior over others.

The analysis so far has taken  $U$  is just a black box. But take cars and gasoline, for example. Are they represented as complements in the utility function because of psychology? No, not at all. Cars and gasoline are complements because they are both needed for transportation. It has to do with technology much more than it has to do with preferences.

This theory we've developed is not a heuristic or a tautology. For instance, suppose we know a person chose to consume  $X_1, \dots, X_N$  at prices  $P_1, \dots, P_N$  and  $\sum P_i X_i = M$ . Then we know that person would prefer that point to  $\widehat{X}_1, \dots, \widehat{X}_N$ , where  $\sum P_i \widehat{X}_i < M$  because the bundle of  $\widehat{X}$ 's was originally affordable but was not chosen. Further, by looking at real choices, we can back out indifference curves, as in Figure 1-3.



*Figure 1-3: Boxes denote choices preferred over bundles on the indifference curve, and crosses denote choices not preferred over bundles on the indifference curve.*

It is important that the choices are “real,” however. Do not ask an alcoholic how much he drinks because he will likely underestimate his consumption. Look at choices people actually make in the marketplace. Those are the choices depicted in Figure 1-3.

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## Chapter 2 Cost Minimization and Demand

Chapter [1] derives the Marshallian demand curves for a consumer with income  $M$  choosing among  $N$  goods, which are repeated below for the reader's convenience.

$$X_1 = X_1^M(P_1, \dots, P_N, M)$$

...

$$X_N = X_N^M(P_1, \dots, P_N, M)$$

Note that there are no quantities in the Marshallian demand functions. Any price change analyzed with this system involves changes in the quantities consumed of every good. A change in the price of toys not only affects the number of toys purchased for each child, but the number of children! In turning to the data, it is important to ask what is really varying: it may be that some of the quantities are held fixed, in which case there is a quantities-constant system (more on this later) to use for the analysis.

### The Cost Function

Consider a cost-minimization problem, the dual of the utility maximization problem. This problem is

$$\begin{aligned} & \min \sum_{i=1}^N X_i P_i \\ & \text{s.t. } U(X_1, \dots, X_N) = \bar{U} \end{aligned}$$

Instead of looking at the choices that maximize the level of utility that can be obtained with a given income, we now look at the choices that minimize the expenditure needed to achieve a given level of utility. For the maximization problem examined in Chapter 1, the budget constraint was fixed and we looked for the consumption choices that attain the highest indifference curve reachable with that budget. In the minimization problem, we fix the indifference curve and find the consumption choices that attain that utility using the least possible income, as in Figure 2-1. These procedures land us at the same optimum point.

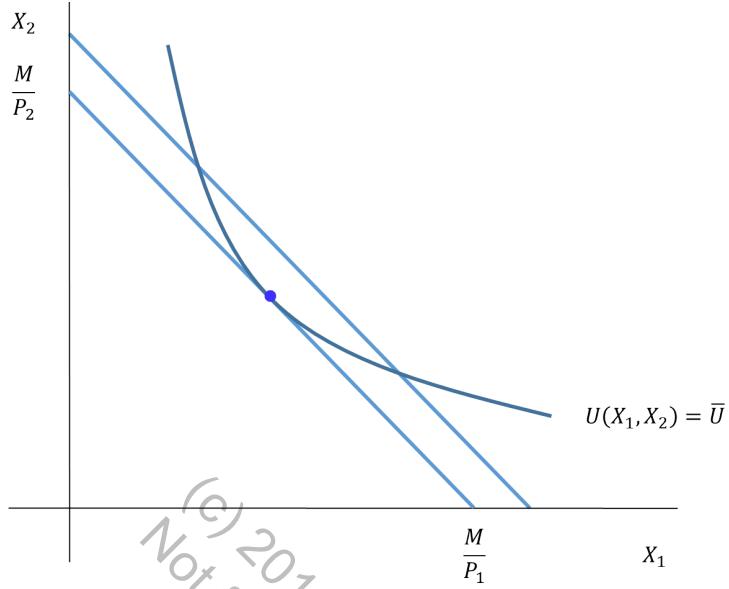


Figure 2-1: The primal and dual approaches have the same consumer first-order conditions. Finding the highest indifference curve for a given budget constraint is closely related to finding the lowest budget constraint for a given indifference curve.

The Lagrangian for this problem is

$$L = \sum_{i=1}^N X_i P_i + \mu [\bar{U} - U(X_1, \dots, X_N)]$$

By taking derivatives and setting them equal to 0, we get the first-order conditions

$$P_1 = \mu \frac{\partial U}{\partial X_1}$$

...

$$P_N = \mu \frac{\partial U}{\partial X_N}$$

$$\bar{U} = U(X_1, \dots, X_N)$$

Except for the last one, these are the same conditions as before; we just have  $\mu = 1/\lambda$ . The first-order conditions from cost minimization give the Hicksian demand functions

$$X_1^* = X_1^H(P_1, \dots, P_N, \bar{U})$$

...

$$X_N^* = X_N^H(P_1, \dots, P_N, \bar{U})$$

These choices resulting from the cost minimization problem are no different than the choices we got from the maximization problem. That is  $X_1^* = X_1^H(P_1, \dots, P_N, \bar{U}) = X_1^M(P_1, \dots, P_N, M)$ . The only difference is that now we're indexing by the level of utility achieved rather than the level of income required.

As economists, we want to use this theory to make predictions. One result from the cost minimization problem that is particularly useful is the cost function, the minimum cost needed to achieve a level of utility  $\bar{U}$  given prices  $P_1, \dots, P_N$ ,

$$C(P_1, \dots, P_N, \bar{U}) = \min \sum_{i=1}^N X_i P_i \text{ s.t. } U(X_1, \dots, X_N) = \bar{U}$$

The cost function has a few properties that make it a useful tool for solving problems in economics. It is homogeneous of degree 1 in prices. If all prices double, the cost doubles. It is nondecreasing in prices; that is,  $\frac{\partial C}{\partial P_i} \geq 0$ . Furthermore, the partial derivative of the cost function is the Hicksian demand curve,  $\frac{\partial C}{\partial P_i} = X_i^H(P_1, \dots, P_N, \bar{U})$ . The cost function is also concave in prices<sup>13</sup>. Finally, cost is increasing in utility, i.e.  $\frac{\partial C}{\partial U} > 0$ . To summarize:

### Properties of the Cost Function

1.  $C$  is homogeneous of degree 1 in prices
2.  $\frac{\partial C}{\partial P_i} \geq 0 \forall i$
3.  $\frac{\partial C}{\partial P_i} = X_i^H(P_1, \dots, P_N, \bar{U})$
4.  $C$  is concave
5.  $\frac{\partial C}{\partial U} > 0$

We prove properties (3) and (4) intuitively.

Let's consider a unidimensional case, as in Figure 2-2. In our experiment, we move the price of good  $j$  from some initial price  $P_j^0$ , while all other prices  $P_1^0, \dots, P_{j-1}^0, P_{j+1}^0, \dots, P_N^0$  are held constant. Consider the point  $(P_j^0, C(P_1^0, \dots, P_N^0, \bar{U}))$ , and suppose we consider the "monkey" solution where, as the price of good  $j$  changes, we do not change our consumption bundle at all. Then we'd move straight along a line that is linear in  $P_j$ . If we're consuming ten units of good  $j$ , then for every dollar increase in  $P_j$ , we spend ten more dollars. The slope of that line equals  $X_j^H(P_1^0, \dots, P_N^0, \bar{U})$ . It's just the demand bundle, so it's also equal to the Marshallian quantity at those prices and the associated level of income. But we know only the point  $(P_j^0, C(P_1^0, \dots, P_N^0, \bar{U}))$  is optimal on the line wth slope  $X_j^H(P_1^0, \dots, P_N^0, \bar{U})$  because the optimal bundle adjusts as  $P_j$  changes. Other optimal points are therefore below the line associated with the "monkey" strategy we've considered – a point on the line gives a feasible cost for achieving  $\bar{U}$  and the minimum cost must below or equal to any feasible cost. . But how concave is the line associated with the optimal cost? It is more concave the more room for adjustment there is.

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<sup>13</sup> Recall, if  $\alpha \in [0,1]$  and  $F$  is concave, then  $F(\alpha X_1 + (1 - \alpha)X_2) \geq \alpha F(X_1) + (1 - \alpha)F(X_2)$ .

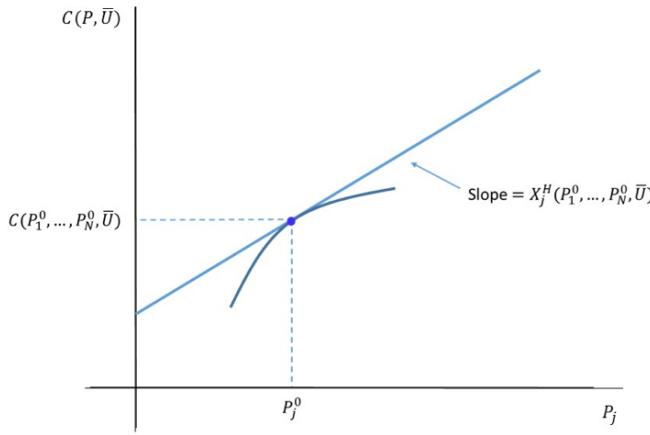


Figure 2-2: Graphical proof of the concavity and derivative properties of the cost function. The line depicts the “monkey” solution of just buying the same bundle as the price of good  $j$  differs from  $P_j^0$ , and the curve depicts the cost function.

This also connects the cost function to the elasticity of demand. We have just considered, intuitively, why  $\frac{\partial C}{\partial P_j} = X_j^H(P_1, \dots, P_N, \bar{U})$ , but we also have that  $\frac{\partial^2 C}{\partial P_j^2} = \frac{\partial X_j^H(P_1, \dots, P_N, \bar{U})}{\partial P_j}$ . In other words, the function can only be as concave as the demand function is responsive to a change in price.

### Hicks’ Generalized Law of Demand

There’s another way to think about the concavity of the cost function. Consider two price-utility vectors with the same utility level,  $(P_1^0, \dots, P_N^0, \bar{U})$  and  $(P_1^1, \dots, P_N^1, \bar{U})$ . Assume the optimal quantities associated with these vectors are  $X_1^0, \dots, X_N^0$  and  $X_1^1, \dots, X_N^1$ . The fact that these bundles are cost minimizing tells us that

$$\sum_{i=1}^N X_i^1 P_i^0 \geq \sum_{i=1}^N X_i^0 P_i^0$$

Similarly,  $\sum_{i=1}^N X_i^0 P_i^1 \geq \sum_{i=1}^N X_i^1 P_i^1$  by the same logic. We can add these two inequalities to get

$$\sum_{i=1}^N X_i^1 P_i^0 + \sum_{i=1}^N X_i^0 P_i^1 \geq \sum_{i=1}^N X_i^0 P_i^0 + \sum_{i=1}^N X_i^1 P_i^1$$

Simplifying this condition, we get

$$\sum_{i=1}^N (X_i^1 - X_i^0)(P_i^1 - P_i^0) \leq 0$$

This is a generalized version of the law of demand, also due to Sir John Hicks. It's more general than  $\frac{\partial x_i^H}{\partial P_i} \leq 0$  because the generalized version has all of the prices changing at the same time, the price changes do not have to be infinitesimal, and the Hicksian demand curves need not have derivatives at the points of interest. On average, goods whose prices go up are consumed less (and the opposite is also true). Equivalently, the cross-good correlation between price changes and demand changes cannot be positive. So the law of demand comes right out of cost minimization. You can't say "it got cheaper so I'm going to buy less." This would be inconsistent with the idea of cost minimization.

The generalized law of demand says that the Hicksian demand system is concave. The law of the demand for individual Hicksian demand curves – that is,  $\frac{\partial x_i^H}{\partial P_i} \leq 0$  – says that, good-by-good, quantities are nonincreasing in their prices.

The cost function also shows that the cross-price effects on Hicksian demand are symmetric,  $\frac{\partial x_i^H}{\partial P_j} = \frac{\partial x_j^H}{\partial P_i}$ , since both are equivalent to  $\frac{\partial^2 C}{\partial P_i \partial P_j}$ . This is *not* a statement that the elasticities are equal; in general, the elasticities differ systematically from equality.

## Relationships between Indifference Curves and the Demand System

Cost and demand functions are related to the indifference curves. As the indifference curves become more curved, demand becomes more inelastic but the cost function becomes straighter. So the cost function is closer to linear when the indifference curves have significant curvature. The opposite is also true. Suppose we have the linear-in-prices cost function  $C(P_1, P_2, \bar{U}) = a(\bar{U})P_1 + b(\bar{U})P_2$ . What do the indifferent curves look like? They're a right angle at the point  $(a, b)$ . What if  $C(P_1, P_2, \bar{U}) = \min(aP_1, bP_2)$ ? Then the indifferent curves are straight lines.

What about someone whose indifference curves are bowed outward from the origin? See Figure 2-3. If budget lines are steep, the agent picks only the y-axis object; if they're flat, the consumer picks only the x-axis object. Empirically, these agents look just like the consumers with linear indifference curves (such as the dash line shown in the figure) because they only pick the endpoints.

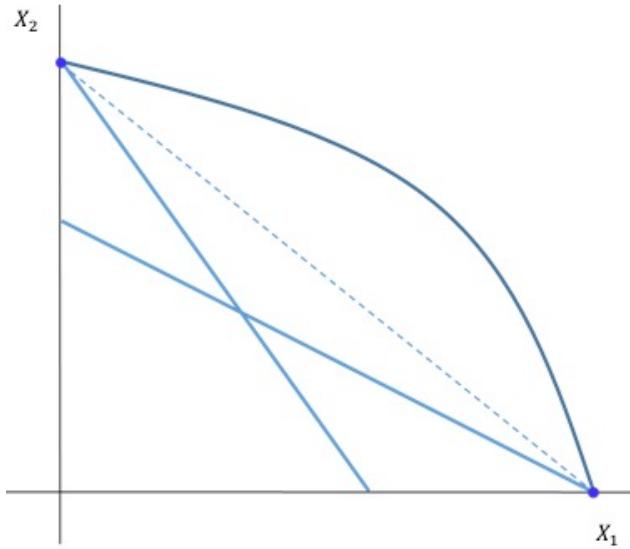


Figure 2-3: Indifference curves concave to the origin yield solutions no different than a linear indifference curve. Consumers pick optimal points at the corners depending on how steep the budget constraint is.

Consider an indifference curve that has a more typical shape but has a region concave to the origin like the previous case. See Figure 2-4. Then for certain very small price movements, the consumer makes large changes in his consumption bundle.

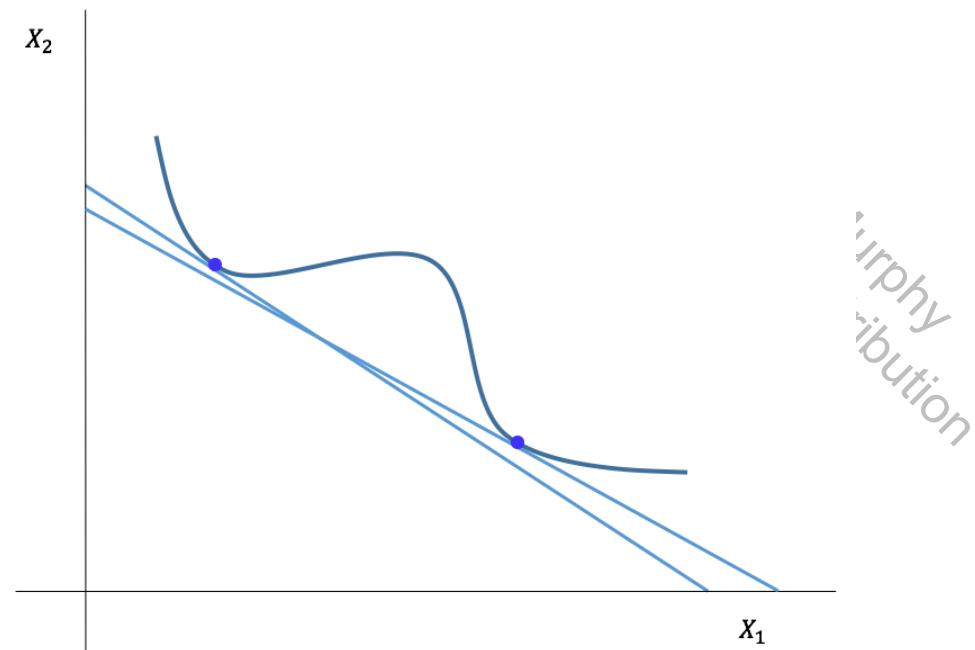


Figure 2-4: A region of nonconvex preferences creates a price threshold around which the consumer makes a large change in consumption in response to a small change in relative prices.

### Properties of Hicksian Demand Functions

The fact that the cost function is homogenous of degree 1 in prices means that each Hicksian demand function is homogenous of degree 0 in prices. That is,  $X_i^H(\alpha P_1, \dots, \alpha P_N, \bar{U}) = X_i^H(P_1, \dots, P_N, \bar{U})$ . Because these functions are equivalent for arbitrary  $\alpha$ , their derivatives are also equal, which means  $\sum_{j=1}^N \frac{\partial X_i^H}{\partial P_j} P_j = 0$ . Dividing both sides by  $X_j$  and denoting the Hicksian cross-price elasticity of good  $i$  demand with respect to  $P_j$  as  $\epsilon_{ij}^H$ , we get

$$\sum_{j=1}^N \epsilon_{ij}^H = 0$$

This says that the sum of all of good  $i$ 's cross-price elasticities is 0. But symmetry also gives us some intuition into elasticities. We can multiply and divide by 1 a few times to get from  $\frac{\partial X_i^H}{\partial P_j} = \frac{\partial X_j^H}{\partial P_i}$  to  $\frac{X_i P_i}{M} \frac{P_j}{X_i} \frac{\partial X_i^H}{\partial P_j} = \frac{\partial X_j^H}{\partial P_i} \frac{P_i}{X_j} \frac{X_i P_i}{M}$ . Using the income shares for each good (recall that  $s_i = \frac{X_i P_i}{M}$ ) we get  $s_i \epsilon_{ij}^H = s_j \epsilon_{ji}^H$ . Then the ratio of the cross-elasticities is equivalent to the ratio of the shares. In general, this says that the  $\epsilon_{ij}^H$  is not the same as  $\epsilon_{ji}^H$ . But it says more than that. Suppose  $s_j > s_i$ . Then it's clear that the big good  $j$  matters more than the small good  $i$  – the elasticity of demand for good  $i$  with respect to the price of good  $j$  is larger than the elasticity of demand for good  $j$  with respect to the price of good  $i$ .

Now we can think about utility again. Note, by construction, we have

$$U(X_1^H(P_1, \dots, P_N, \bar{U}), \dots, X_N^H(P_1, \dots, P_N, \bar{U})) = \bar{U}$$

But this holds for any  $P$ , so we can differentiate with respect to  $P$  to get

$$\sum_{j=1}^N \frac{\partial U}{\partial X_j^H} \frac{\partial X_j^H}{\partial P_i} = 0 \xrightarrow{\text{F.O.C.}} \sum_{j=1}^N P_j \frac{\partial X_j^H}{\partial P_i} = 0$$

That is, we can once again use that marginal utilities are proportional to prices. But this gives us a corollary in elasticity form, namely that

$$\sum_{j=1}^N s_j \epsilon_{ji}^H = 0$$

This sum over goods is called “adding up” (for the Hicksian demand system). It adds the effects of a single price change ( $P_i$ ) on all demands ( $j = 1 \dots N$ ). Homogeneity, on the other hand, was about a single demand equation: it adds all of the price effects in that equation. If we change all prices by the same percentage, we get no change in consumption. Adding up says that, looking across all equations, whenever a single price changes, all the goods have to change in such a way that, weighted by their shares, the total change is 0. So three restrictions come out: homogeneity, symmetry, and adding up. Note (convince yourself of this!) that symmetry and homogeneity imply adding up, and adding up and symmetry imply homogeneity. In the two good case, adding up and homogeneity imply symmetry, but this does not hold in general.

Chapter [3] links the Hicksian and Marshallian systems using the Slutsky equation.

## Chapter 3 Relating the Marshallian and Hicksian Systems

Chapters [1] and [2] covered two different demand systems, the Marshallian approach—maximizing utility subject to a budget constraint—and the Hicksian approach—minimizing cost subject to a utility constraint. Because they’re two ways of looking at the same problem, we are not limited to using the tool that corresponds to the problem the consumer is solving. That is, if we know a consumer is solving the utility maximization problem subject to a budget constraint, then, as analysts, we can still use the tools of the Hicksian approach. For many problems, the Hicksian approach will prove very useful. Throughout the book we both methods, picking the one better suited to solving the problem at hand.

### The Slutsky Equation

It’s convenient to know how to go back and forth between the Hicksian and Marshallian systems. See Figure 3-1. Suppose a cost-minimizing consumer picks the optimal point  $(X_1^*, X_2^*)$ . Then  $X_1^* = X_1^H(P_1, P_2, \bar{U})$ . But now suppose we give the consumer a level of income equal to  $C(P_1, P_2, \bar{U})$  and he solves the utility maximization problem rather than the cost minimization problem. The consumer will choose the same optimal point  $(X_1^*, X_2^*)$ . That is,  $X_1^* = X_1^H(P_1, P_2, \bar{U}) = X_1^M(P_1, P_2, C(P_1, P_2, \bar{U}))$ . We’ll call this the Slutsky correspondence. This is one level above the Slutsky equation, which is about derivatives.

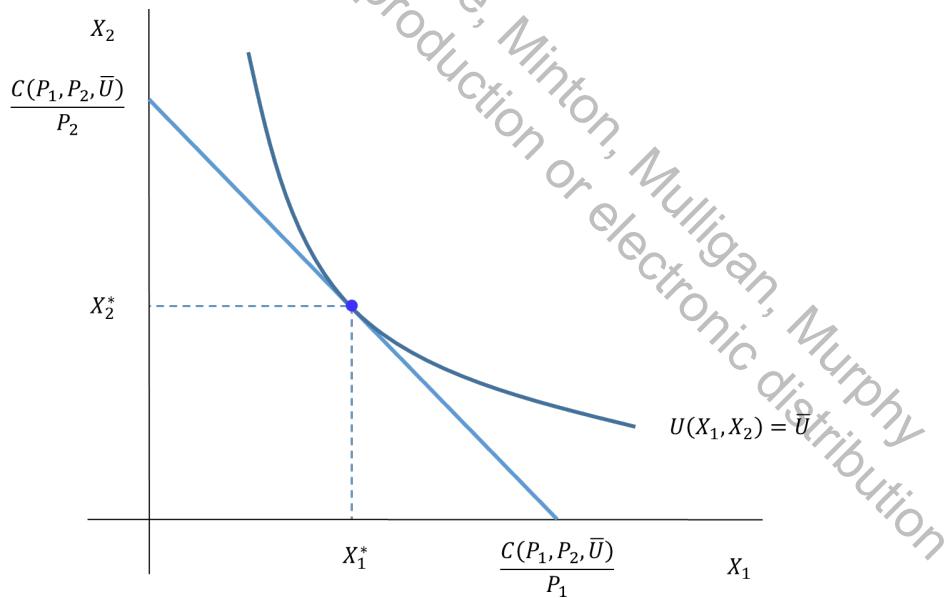


Figure 3-1: A graphical depiction of the idea that  $X_1^* = X_1^H(P_1, P_2, \bar{U}) = X_1^M(P_1, P_2, C(P_1, P_2, \bar{U}))$ .

But note that the Slutsky correspondence is *not* an equilibrium condition determining a price or utility level. Rather, the Hicksian and Marshallian functions are the same when the latter has income replaced by the cost function: the equation holds for every price and every utility level.

More generally, the Slutsky correspondence for good  $i$  is  $X_i^H(P_1, \dots, P_N, \bar{U}) = X_i^M(P_1, \dots, P_N, C(P_1, \dots, P_N, \bar{U}))$ . Then we can take the derivative with respect to the

$j$ th price and use the fact that each price derivative of the cost function is equal to the corresponding quantity demanded ( $\frac{\partial C}{\partial P_j} = X_j$ ), to get

$$\frac{\partial X_i^H}{\partial P_j} = \frac{\partial X_i^M}{\partial P_j} + \frac{\partial X_i^M}{\partial M} X_j$$

This is the Slutsky equation. It lets us go back and forth between the derivatives of the Hicksian and Marshallian systems, and it tells us how income-constant changes in price are related to utility-constant changes in price. The difference between them comes from the rightmost term, called the income effect.

Let's rearrange this expression to be  $\frac{\partial X_i^M}{\partial P_j} = \frac{\partial X_i^H}{\partial P_j} - \frac{\partial X_i^M}{\partial M} X_j$ . The left-hand side is the Marshallian effect, the first term on the right-hand side is the substitution or Hicksian effect, and the last term is the income effect.

What does the Slutsky equation say intuitively? Suppose  $P_j$  increases by a dollar. There will be a response holding utility constant, which is the substitution effect. Even though we call this the “substitution effect,” it could be either a substitution relationship or a complementarity relationship. But there is also an effect coming from the fact that the change in price has changed our income. If we were consuming ten of good  $j$ , then we have, in some sense, ten fewer dollars of income. That is, to buy what I bought before would cost ten more dollars after the price hike. The income effect tells us how responsive good  $i$  is to income. That is, if my income changes by ten dollars, how much does my demand for good  $i$  change?

The Marshallian and Hicksian price effects differ by the income-effect term. The income-effect term tends to be small for most goods because most goods are a small share of the overall household budget. If I make \$100,000/year but only become \$10 poorer as a result of the good  $j$ 's price change, the income effect on my demand for good  $i$  will not be large.

To see all of this more clearly, let's translate the Slutsky equation into elasticity form. Multiply by  $\frac{P_j}{X_i}$  on both sides and by  $\frac{M}{M}$  on the term designating the income effect to get

$$\epsilon_{ij}^M = \epsilon_{ij}^H - s_j \eta_i$$

This is the elasticity version of the Slutsky equation. It says the Marshallian percentage response of a percent increase in the price of good  $j$  on good  $i$  is the Hicksian response to the price change minus the income share of the good whose price has changed times the income elasticity of the good we're looking at. Thus, the size of the income effect depends on the share of good  $j$  and how responsive to income good  $i$  is.

This is where the law of demand can run into some problems. Let's consider the own-price case, where  $j = i$ . Then

$$\epsilon_{ii}^M = \epsilon_{ii}^H - s_i \eta_i$$

Theory tells us that  $\epsilon_{ii}^H \leq 0$ . Then, if the good is normal,  $\eta_i > 0$ , and the share is positive, we're guaranteed that  $\epsilon_{ii}^M < 0$  so that the law of demand holds. Moreover, for normal goods, the Hicksian demand curve is downward sloping, but the Marshallian demand curve is even more responsive to price because the income effect reinforces the substitution effect. The negativity of  $\epsilon_{ii}^M$  could be violated,

however, if the good has a large share and is inferior, so that  $\eta_i < 0$ . This case is called a *Giffen good*, where typically we have a weak substitution effect and a large share inferior good. These are very difficult to find because most high share goods are normal. See Figure 3-2 for a depiction of the Giffen good case.

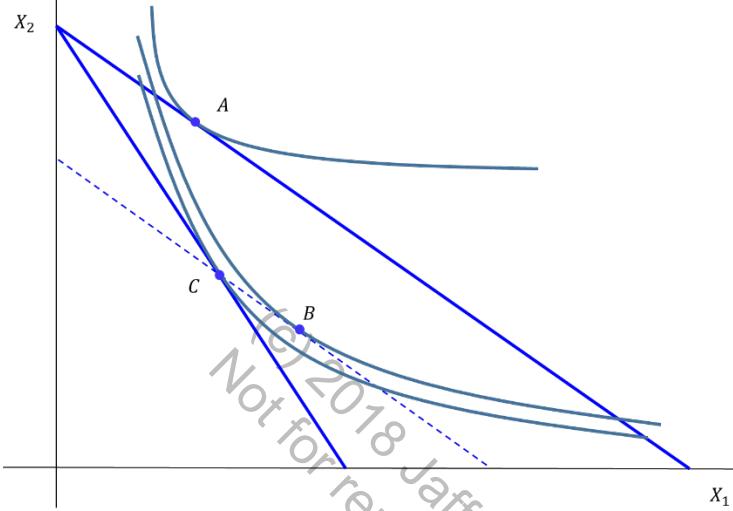


Figure 3-2: After an increase in  $P_1$ , consumption moves from A to C. The move from A to B denotes the income effect part of the change in consumption. As income falls, much more of  $X_1$  is consumed: good 1 is inferior. The move from B to C is the substitution effect: consuming less of the good that got more expensive. The figure shows the Giffen case because an increase in  $P_1$  has an income effect on good 1 that more than offsets its substitution effect.

### Adding Up and Symmetry for the Marshallian System

The Slutsky equation also allows us to go between the conditions on Marshallian and Hicksian demand. In the Marshallian system, we have homogeneity in prices and in income; in the Hicksian system, we had homogeneity just in prices. “Adding up” for the Marshallian system comes right off the budget constraint; That is, because  $\sum_{i=1}^N X_i P_i = M$ , this must hold at the optimum, so that  $\sum_{i=1}^N X_i^M(P_1, \dots, P_N, M) P_i = M$ . This doesn’t have a whole lot to do with rationality; we’ve just said that people spend all their income. This holds for all prices, so we can differentiate with respect to the price of good  $j$  to get (see Chapter 1 for the full derivation):

$$\sum_{i=1}^N s_i \epsilon_{ij}^M + s_j = 0$$

Let’s suppose  $s_j = 0.1$ . Then a 10% increase in the price of good  $j$  will be a 1% reduction in my real income. Since I have a 1% reduction in my real income, the formula says that, on average, I have to reduce my consumption of other goods by 1%. Gary Becker (1962) used to love this point: a lot of demand comes right off the budget constraint. Even the law of demand is not that far from the budget constraint taken from this perspective. If a good becomes more expensive, you’re poorer; you have to consume less, on average. But where will you consume less? It makes sense to think you will consume less of the good that has become more expensive, which is the law of demand.

Chapter 1 does not look at symmetry for Marshallian demand, but we can get it from Hicksian symmetry (Chapter 2) and the Slutsky equation. Symmetry for Hicksian demand is  $\frac{\partial x_i^H}{\partial P_j} = \frac{\partial x_j^H}{\partial P_i}$ . We can use the Slutsky equation to write:

$$\begin{aligned}\frac{\partial X_i^M}{\partial P_j} &= \frac{\partial X_i^H}{\partial P_j} - \frac{\partial X_i^M}{\partial M} X_j = \frac{\partial X_i^H}{\partial P_i} - \frac{\partial X_i^M}{\partial M} X_j = \frac{\partial X_i^M}{\partial P_i} + \frac{\partial X_j^M}{\partial M} X_i - \frac{\partial X_i^M}{\partial M} X_j \\ &= \frac{\partial X_j^M}{\partial P_i} + \frac{\partial X_j^M}{\partial M} \frac{M}{X_j} \frac{X_i X_j}{M} - \frac{\partial X_i^M}{\partial M} \frac{M}{X_i} \frac{X_j X_i}{M} = \frac{\partial X_j^M}{\partial P_i} + \frac{X_i X_j}{M} (\eta_j - \eta_i)\end{aligned}$$

In elasticity format, this Marshallian symmetry is:

$$s_i \epsilon_{ij}^M = s_j \epsilon_{ji}^M + s_i s_j (\eta_j - \eta_i)$$

Thus, symmetry holds for the Marshallian case when the two goods have equal income elasticities (i.e.  $\eta_i = \eta_j$ ). Recall from Chapter 1 that the shares and elasticities are evaluated at a particular set of prices and a particular income. So the adding up, symmetry, homogeneity, and Slutsky are describing the demand system at a particular point.

### Demand System Degrees of Freedom

To make progress on a lot of problems, it's very important to keep your model simple. Limiting the number of goods helps a lot because, at a particular income and set of prices, an  $N$ -good model has  $N$  expenditure shares,  $N$  income elasticities, and  $N^2$  Marshallian price elasticities. These  $N(N+2)$  parameters are interrelated because of symmetry, the budget constraint on shares, the budget constraint on income elasticities (i.e., Engel aggregation), and adding up, but those are only  $(N-1)N/2 + N + 2$  restrictions so the  $N$ -good model still has  $(N+4)(N-1)/2$  free parameters.<sup>14</sup> For  $N = 2, 3, 4, 5$ , that means 3, 7, 12, and 18 free parameters, respectively.

Take the two-good case. If we are given values for, say,  $s_1$ ,  $\eta_1$ , and  $\epsilon_{11}^M$ , then we can use the budget constraint, adding up, and symmetry to obtain the other five parameters ( $\epsilon_{12}^M$ ,  $\epsilon_{21}^M$ ,  $\epsilon_{22}^M$ ,  $s_2$ , and  $\eta_2$ ). Specifically, the budget constraint gives us the second share from the first:  $s_2 = 1 - s_1$ . With these shares, the budget constraint also gives us the second income elasticity from the first:

$$\eta_2 = \frac{1 - s_1 \eta_1}{s_2}$$

Homogeneity gives us  $\epsilon_{12}^M$  from  $\epsilon_{11}^M$  and the income elasticity:

$$\epsilon_{12}^M = -\epsilon_{11}^M - \eta_1$$

Symmetry gives us  $\epsilon_{21}^M$  from  $\epsilon_{12}^M$ , the income elasticities, and the shares:

$$\epsilon_{21}^M = \frac{s_1 \epsilon_{12}^M + s_2 s_1 (\eta_1 - \eta_2)}{s_2}$$

Homogeneity gives us  $\epsilon_{22}^M$  from  $\epsilon_{21}^M$  and the income elasticity:

$$\epsilon_{22}^M = -\epsilon_{21}^M - \eta_2$$

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<sup>14</sup> Recall that symmetry and adding up together imply homogeneity.

In the three good case, we need two shares and two income elasticities and from these and the budget constraint can determine the remaining share and the remaining income elasticity. We also need three cross-price elasticities, such as  $\varepsilon_{21}^M, \varepsilon_{31}^M, \varepsilon_{32}^M$ , from which symmetry and homogeneity determine the remaining six price elasticities.

The two good model is one way to keep the model simple. Other times we use the three-good model with additional restrictions. Chapter 8's rent gradient model is an example where two goods – work time and commuting time – are assumed to be perfect substitutes. Chapter 10 introduces production and thereafter the constant returns restriction is frequently referenced. A number of policy questions inherently refer to more than two goods, such as the effects of the corporate-income tax on noncorporate business activity (Chapter 18). Here, and for the analysis of optimization over time (Chapter 17), the idea of a composite commodity is helpful. All of these special cases of the three-good model are relatively easy to work with because there are fewer than seven free parameters.

## The Income Effect of a Price Change

Another point Becker (1962) made is about where the opportunities are when prices change.<sup>15</sup> Suppose the price of good 1 goes down, as in Figure 3-3. The new opportunities will mostly arise for the bundles where a lot of the cheaper good is consumed. After the price is reduced, the “center of gravity,” in some sense, moves towards good 1; that is, if I pick a bundle within the budget set at random, it will contain more of good 1 on average.

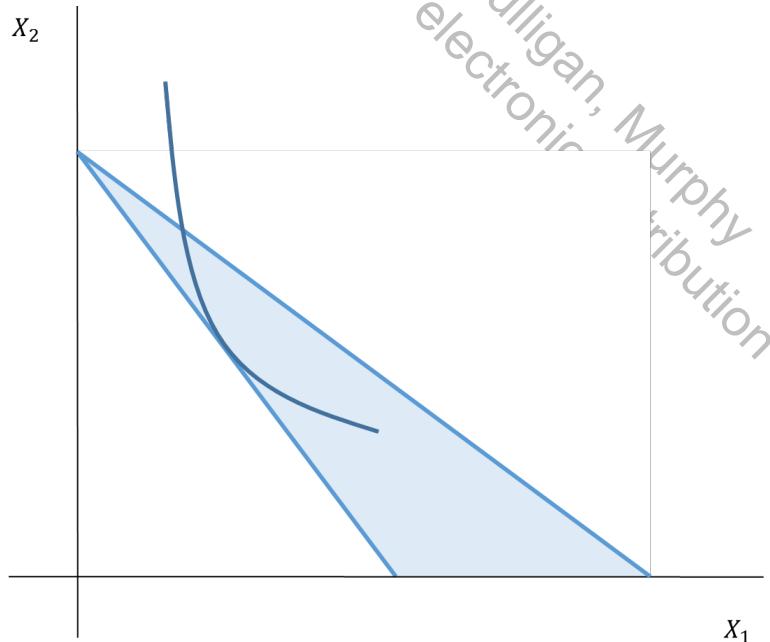


Figure 3-3: A decrease in the price of good 1 creates more opportunities for consumption of good 1 than consumption of good 2.

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<sup>15</sup> Becker (1962).

The income effects discussed so far are at the individual level, but they are informative about aggregate cases too. See Figure 3-4. Different people make different choices along a given budget constraint. Consider the average choice. Now suppose prices change in such a way that we pivot around the average point; one price goes up, and one goes down. Those who were consuming more of the good that is now more expensive may not like the new prices. More generally, even though incomes are constant on average, they're being redistributed from the group consuming more of the good whose price increased—in the figure, the redistribution transfers income from the heavy  $X_2$  consumers to the heavy  $X_1$  consumers. Now suppose further that indifference curves for individuals are perfect complements, so individuals do not substitute. On aggregate, we still get substitution because effectively we make the people who like  $X_1$  richer and the people who like  $X_2$  poorer.

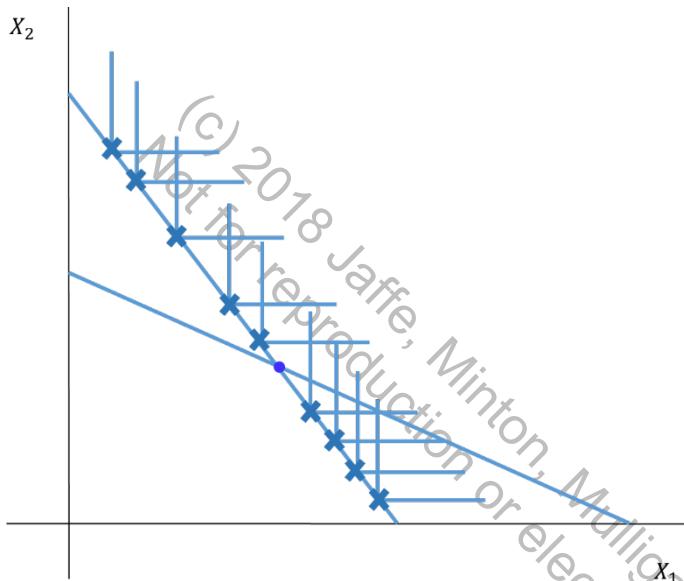


Figure 3-4: An aggregate compensated change (real incomes stay constant) redistributes income from the people who consume lots of  $X_2$  to the people who consume lots of  $X_1$ .

Conceivably people could have enough  $x_1$  so that when it becomes cheaper, they spend all their additional money on  $x_2$ . There's a famous theorem by Hugo Sonnenschein that says because of these types of income effects, you can get just about anything in the aggregate, i.e. "anything goes." Draw a crazy curve—then this curve could be a demand curve, in the aggregate. While that can happen, the most common way that income effects work is just to reinforce the story; that is, most frequently it works in the same way as substitution to reinforce the law of demand. See Figure 3-5 for an illustration of how one of these more nonstandard demand curves might result.

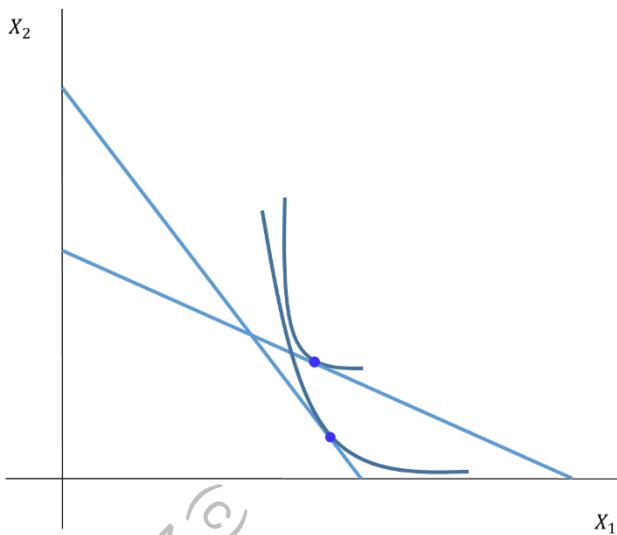


Figure 3-5: After the shift in relative prices, the heavy  $X_1$  consumers buy less of  $X_1$ .

To think about these nonstandard cases, consider aggregating the Slutsky equation and the multiplying the income effects term by  $\frac{\sum_{\text{people}} X_j}{\sum_{\text{people}} X_j}$

$$\begin{aligned} \sum_{\text{people}} \frac{\partial X_i^M}{\partial P_j} &= \sum_{\text{people}} \frac{\partial X_i^H}{\partial P_j} - \left( \sum_{\text{people}} X_j \frac{\partial X_i}{\partial M} \right) \left( \frac{\sum_{\text{people}} X_j}{\sum_{\text{people}} X_j} \right) \\ &= \sum_{\text{people}} \frac{\partial X_i^H}{\partial P_j} - \left( \sum_{\text{people}} X_j \right) \left( \frac{\sum_{\text{people}} X_j \frac{\partial X_i}{\partial M}}{\sum_{\text{people}} X_j} \right) \end{aligned}$$

The Marshallian and Hicksian price terms have a natural aggregate interpretation: what happens to aggregate purchases of good  $i$  when every person faces the same increase in the price of good  $j$ . The aggregate Slutsky equation's final term has aggregate good  $j$  consumption ( $\sum_{\text{people}} X_j$ ) exactly where the individual equation had individual consumption. However, consumption is multiplied by an effect that depends on which good's price changes, whereas the individual Slutsky equation would have  $\partial X_i / \partial M$ , which is properly called an income effect because it is independent of the source of the income (that is, good  $j$  does not appear). Unless everyone has the same income effect, in which case  $\partial X_i / \partial M$  can be factored out of the sum across people, it makes little sense to refer to an aggregate income effect because it depends on who gets the income.

From now on, we draw Marshallian demand curves as downward sloping. That is, we rule out the Giffen good case, which means either that the income effect is in the same direction as the substitution effect or that the income effect is sufficiently small.

## Chapter 4 Price Indices: Consumer Theory Guides Measurement

Consumer theory gives us a lot of guidance about how to measure things like real income and GDP; it suggests weighting changes in quantities by the prices.

### Laspeyres and Paasche Decompositions of Expenditure Growth

Expenditure is the cost of the chosen bundle of goods – the sum of prices times quantities. We can decompose expenditure growth  $E_{t+1}/E_t$  into price and quantity indices:

$$\frac{E_{t+1}}{E_t} = \frac{\sum_i X_{i,t+1} P_{i,t+1}}{\sum_i X_{i,t} P_{i,t}} = \frac{\sum_i X_{i,t} P_{i,t+1}}{\sum_i X_{i,t} P_{i,t}} \times \frac{\sum_i X_{i,t+1} P_{i,t+1}}{\sum_i X_{i,t} P_{i,t+1}} = \frac{P_{t+1}}{P_t} \times \frac{Q_{t+1}}{Q_t}$$

It's especially useful because often we only measure two of the three. Price is sometimes easier to measure than quantity because price can be seen from a sample; just go to one of the sellers in the market—say, go to one of the grocery store and look at their price for eggs. Quantity measurement can be more difficult you have to ask every seller what they sold; you'd need some kind of census of sellers. In these situations we often back out quantities by measuring expenditure and price and using the decomposition above.

To make the decomposition, we simply multiplied  $\frac{\sum_i X_{i,t+1} P_{i,t+1}}{\sum_i X_{i,t} P_{i,t}}$  by  $\frac{\sum_i X_{i,t} P_{i,t+1}}{\sum_i X_{i,t} P_{i,t+1}} = 1$ . But why does this make sense? So long as the numerator and denominator are the same, we could multiply by any number of things to decompose  $\frac{\sum_i X_{i,t+1} P_{i,t+1}}{\sum_i X_{i,t} P_{i,t}}$ . The first part of the answer is that  $\frac{\sum_i X_{i,t} P_{i,t+1}}{\sum_i X_{i,t} P_{i,t}}$  is a first-order approximation to the cost function. That is, this is saying how much the cost increased along the tangent line; it tells us how much it would cost us to buy today's bundle tomorrow relative to today. Consider Figure 4-1. Because the line is a first-order approximation, we overstate an increase in the cost of living and underestimate a decrease in the cost of living because of the concavity of the cost function.

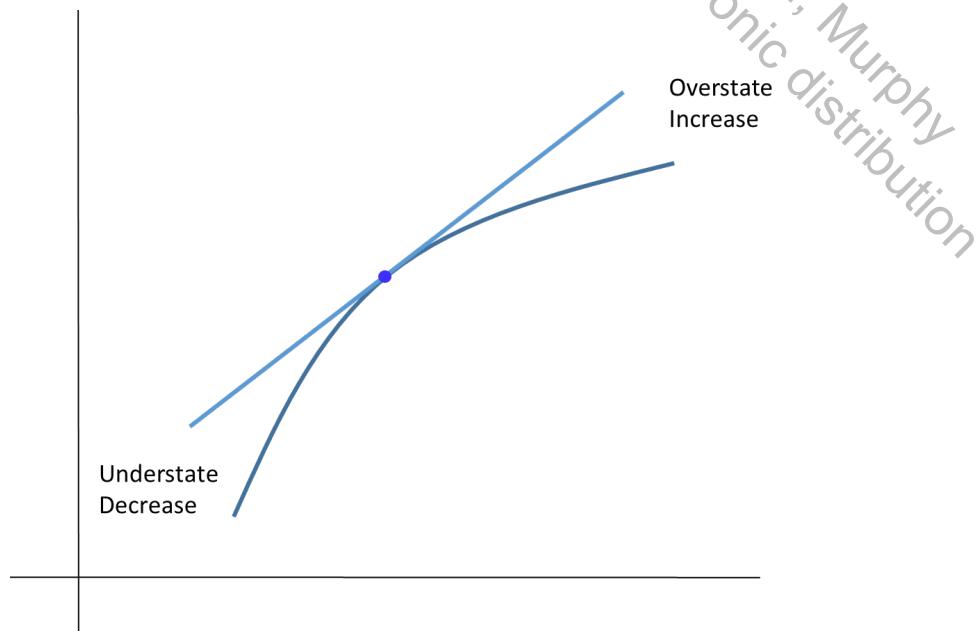


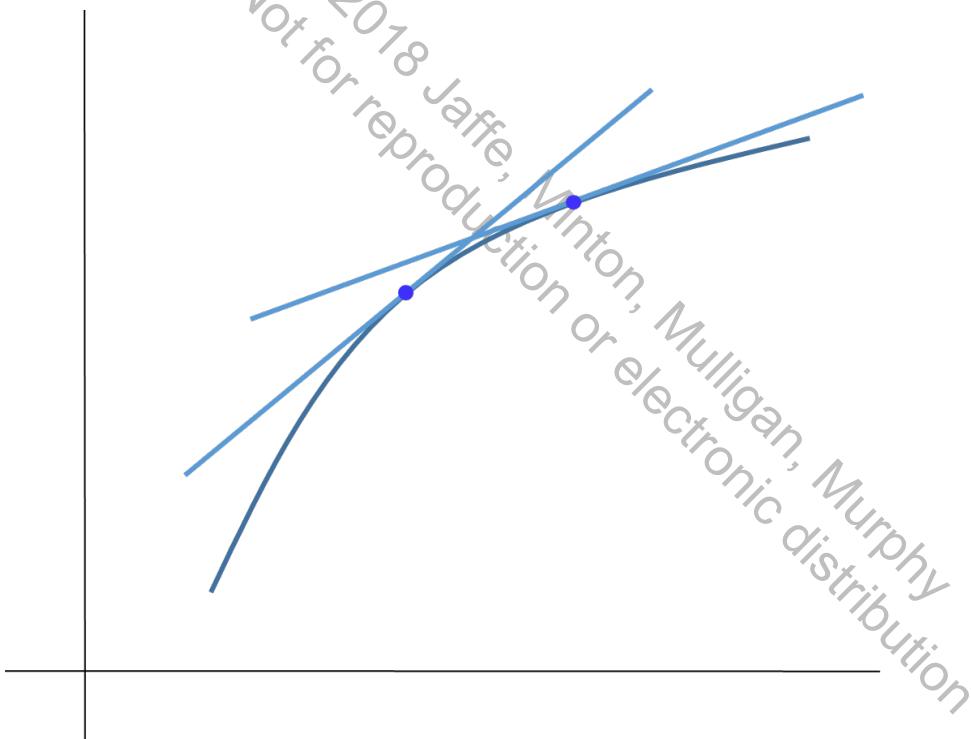
Figure 4-1: Because of the concavity of the cost function, an increase in the cost of living is overstated, and a decrease in the cost of living is understated.

The second part of the answer is that  $\frac{\sum_i X_{i,t+1} P_{i,t+1}}{\sum_i X_{i,t} P_{i,t+1}}$  is a quantity index that approximates the indifference curve using the budget line. This makes sense because the consumer's indifference curve is tangent to the budget line. Thus, changes in utility can be approximated by movements along the budget line. This is again, of course, a first-order approximation, and it is measured in dollars; more specifically, it tells us the income equivalent required to give you the change in utility.

This case was for a price index based in period  $t$  and a quantity index based in  $t + 1$ . Since, again, our choice of numerator and denominator were arbitrary, we could do this the opposite way. Consider a price index based in  $t + 1$  and a quantity index based in  $t$ . Then we equivalently get

$$\frac{E_{t+1}}{E_t} = \frac{\sum_i X_{i,t+1} P_{i,t+1}}{\sum_i X_{i,t} P_{i,t}} = \frac{\sum_i X_{i,t+1} P_{i,t+1}}{\sum_i X_{i,t} P_{i,t}} \times \frac{\sum_i X_{i,t+1} P_{i,t+1}}{\sum_i X_{i,t+1} P_{i,t+1}} = \frac{P_{t+1}}{P_t} \times \frac{Q_{t+1}}{Q_t}$$

In general, one decomposition is not better than another. See Figure 4-2. The different methods simply change where the first-order approximations occur—either in period  $t$  or period  $t + 1$ .



*Figure 4-2: A cost function approximated at two different points. Because the cost function is concave, depending on where we move from  $t$  to  $t + 1$ , one estimation is too big and one is too small. These correspond to the Paasche and Laspeyres price indices.*

The first price index we considered, where the price index is based in period  $t$  (that is, prices are weighted by period  $t$  quantities), is called the Laspeyres price index. The second case we considered, where the price index is based in period  $t + 1$ , is called the Paasche price index.

These measures are sometimes combined, as a geometric average, in an index called the Fisher ideal index. For prices, the Fisher ideal index is:

$$\left( \frac{\sum_i X_{i,t} P_{i,t+1}}{\sum_i X_{i,t} P_{i,t}} \right)^{\frac{1}{2}} \left( \frac{\sum_i X_{i,t+1} P_{i,t+1}}{\sum_i X_{i,t+1} P_{i,t}} \right)^{\frac{1}{2}}$$

Unlike the Laspeyres and Paasche indices, we now no longer know the direction of the bias (as a measure of the movement along the cost function from  $t$  prices to  $t+1$  prices). On the other hand, we know that this measure will be better than at least one of the Laspeyres and Paasche indices (i.e. less biased).

### **Chained Price Indices**

Suppose we want to perform this calculation over a long period of time. We can consider the two Laspeyres and Paasche ratios for looking at this

$$\frac{\sum_i X_{i,1950} P_{i,2015}}{\sum_i X_{i,1950} P_{i,1950}} \text{ vs. } \frac{\sum_i X_{i,2015} P_{i,2015}}{\sum_i X_{i,2015} P_{i,1950}}$$

These will give such radically different answers that they are essentially useless. Consider the denominator on the right term, and take  $i$  to be cell phones. What is the price of a cell phone in 1950? It would be absurdly high. A single modern phone probably does more computing than the aggregation of all computing prior to 1950. You'd never consider buying your 2015 bundle at 1950 prices. Similarly, consider the numerator on the left side. We'd leave out all the new goods that weren't around in 1950. Cell phones, for instance, would get far too little weight. In short, ignoring the substitution effect introduces large errors over long time horizons. The way we address this is with a chained price index. Consider the following:

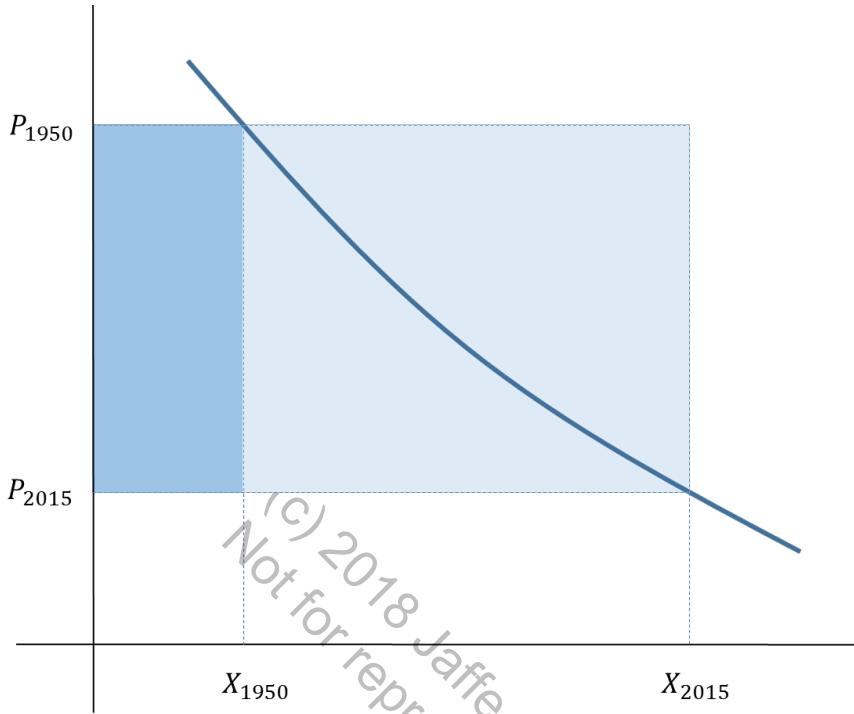
$$\frac{P_{2015}}{P_{1950}} = \left( \frac{P_{1951}}{P_{1950}} \right) \left( \frac{P_{1952}}{P_{1951}} \right) \dots \left( \frac{P_{2015}}{P_{2014}} \right)$$

That is, the change in the cost of living between 1950 and 2015 was really a series of changes going from one year to the next. Then we can write all of these in terms of price indices:

$$\left( \frac{\sum_i X_{i,1950} P_{i,1951}}{\sum_i X_{i,1950} P_{i,1950}} \right) \left( \frac{\sum_i X_{i,1951} P_{i,1952}}{\sum_i X_{i,1951} P_{i,1951}} \right) \dots \left( \frac{\sum_i X_{i,2014} P_{i,2015}}{\sum_i X_{i,2014} P_{i,2014}} \right)$$

In this formulation, new goods will be added into the price indices as they arrive. Cell phones will be added in when they become cheap enough that people will actually buy them.

Now let's visualize our naïve approaches to measuring the change in the cost of living between 1950 and 2015. Expressed in dollars rather than ratios, the Laspeyres and Paasche approaches are  $\sum_i X_{i,1950} (P_{i,2015} - P_{i,1950})$  and  $\sum_i X_{i,2015} (P_{i,2015} - P_{i,1950})$ . Let's assume that only one price changed, and that income is the same in the two years, so that we can draw the changes in the usual Marshallian demand picture, as in Figure 4-3.



*Figure 4-3: The change in the cost of living is vastly underapproximated or overapproximated by the two naïve attempts to measure it.*

The smaller rectangle estimates the change in the cost of living the same way that the Laspeyres index does: using the initial consumption bundle to weight the price changes. The larger rectangle uses the final consumption bundle, as the Paasche index does.

The true change in the cost of living that the two indices are designed to approximate is

$$C(P_{1,t+1}, \dots, P_{N,t+1}, \bar{U}) - C(P_{1,t}, \dots, P_{N,t}, \bar{U})$$

We still have the same issue of a reference point. The ambiguity this time is about which  $\bar{U}$  to use (period  $t$  utility or period  $t + 1$  utility?). Now, however, the issue is not about what approximation to use; changing which  $\bar{U}$  we use corresponds to different questions. We may want to know how much it costs to get 2015 utility in 1950, or we may want to know how much it costs to get 1950 utility in 2015.

Now, for simplicity, as in Figure 4-3, assume only the price of good 1 is changing. Then we can write

$$C(P_{1,t+1}, \bar{P}_2, \dots, \bar{P}_N, \bar{U}) - C(P_{1,t}, \bar{P}_2, \dots, \bar{P}_N, \bar{U}) = \int_{P_{1,t}}^{P_{1,t+1}} \frac{\partial C}{\partial P_1} dP_1 = \int_{P_{1t}}^{P_{1,t+1}} x^H(P_1, \bar{P}_2, \dots, \bar{P}_N, \bar{U}) dP_1$$

Because the change in cost is an area to the left of a Hicksian demand curve, Figure 4-4 adds some Hicksian demand curves to Figure 4-3, assuming that good 1 is a normal good, which makes the Hicksian curves less price sensitive than the Marshallian curve.

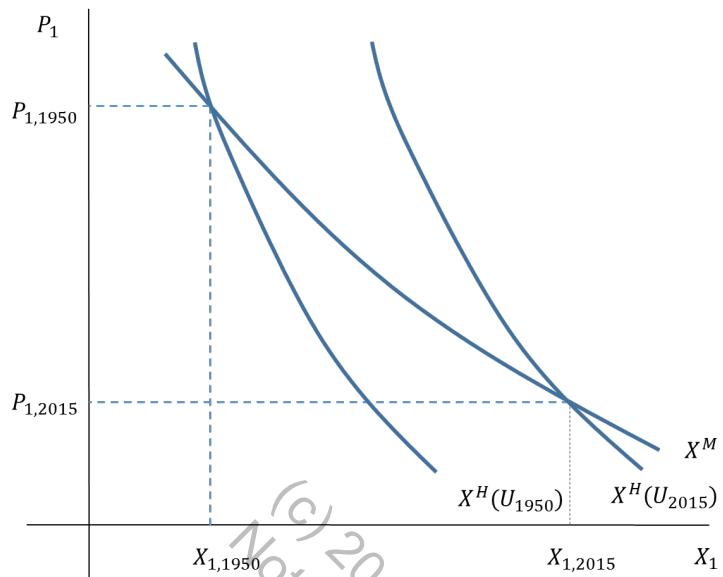


Figure 4-4: Hicksian demand curves tell the answers to two different questions: how much cheaper is it to get the 1950 level of utility, or how much more expensive is it to get the 2015 level of utility?

In Figure 4-4, we drew the Hicksian demand curves much steeper than the Marshallian demand curves. This is useful for visualization, but a more realistic picture would show less contrast between the slopes. Recall the Slutsky equation. If the income elasticity is 1, and the share of the good is 1%, then Marshallian and Hicksian elasticities will differ only by .01. Keep this in mind when thinking about how the Marshallian and Hicksian demand curves are related.

But what about using the Marshallian demand curve to measure the cost of living? Looking at Figure 4-4, it doesn't seem so crazy, because the area under the Marshallian curve gives us something in between the areas under the two Hicksian curves.

It should be noted, however, that the two Hicksian demand curves in Figure 4-4 are answering two slightly different questions. The Hicksian demand curve on the left answers how much cheaper it is to attain the 1950 level of utility while the Hicksian demand curve on the right indicates how much more expensive it is to attain the 2015 level of utility.

A chained price index estimates a cost change from 1950 to 1951, adds a cost change from 1951 to 1952, etc. Each link in the chain is weighting price changes by quantities that people were buying at that time: namely the quantity on the Marshallian demand curve. The chained price index can therefore be visualized in the same picture: see Figure 4-5.

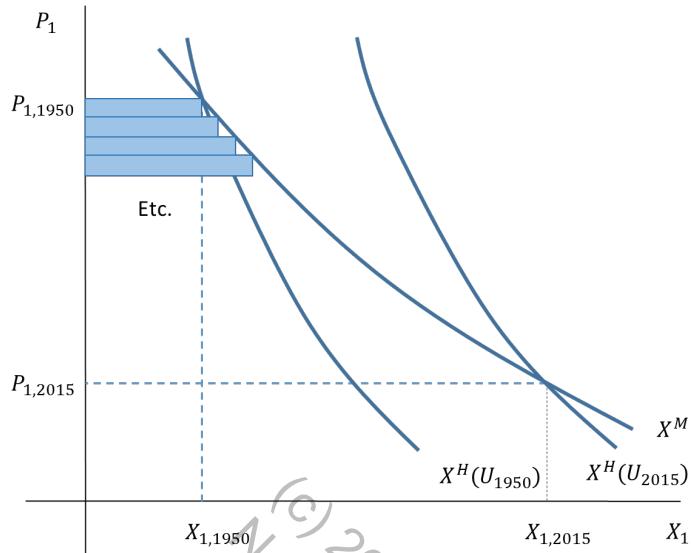


Figure 4-5: The chained price index roughly gives us the area under the Marshallian demand curve.

The Fisher index, on the other hand, will simply average the Laspeyres and the Paasche indices that we've already depicted in Figure 4-3. Consider Figure 4-6.

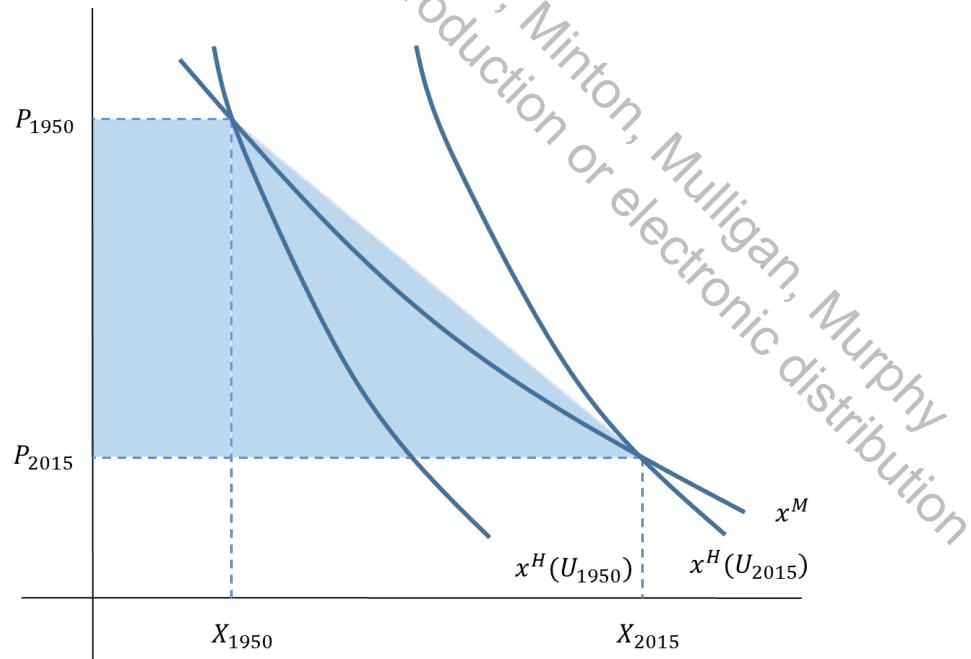


Figure 4-6: The Fisher index averages the Laspeyres and Paasche indices.

Remember the problem motivating our use of price indices. We want to figure out how important price changes are over time. The demand curve gives us a lot of information about this. Think again about the one good case and consider Figure 4-7. The marginal value of an additional unit has to be equal to the price I'm paying. So we don't ask people how important a good is—people reveal how important it is by how much they're willing to consume when the price is higher than it is today.

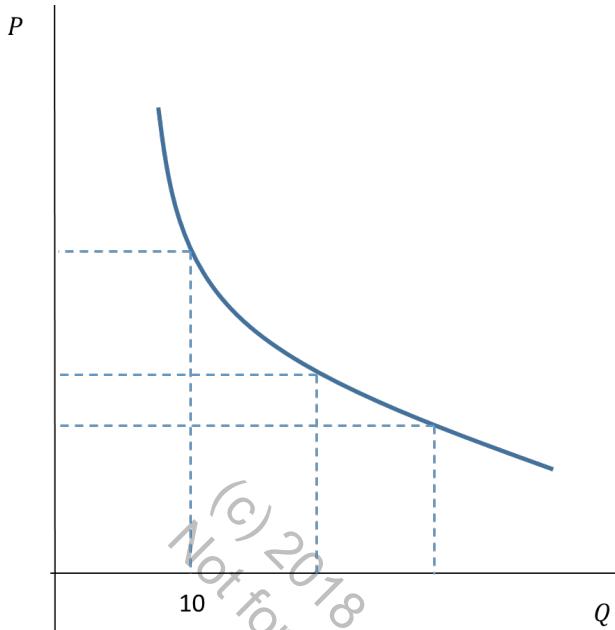


Figure 4-7: The marginal value of the 10th unit must be equal to the price. So people reveal how much the 10th unit is worth to them.

Even with the chained index, we only account for new goods with some lag. That is, there is a period where people are buying them but we haven't incorporated them into our index yet. Then we miss the initial benefit when the price was high. See Figure 4-8.

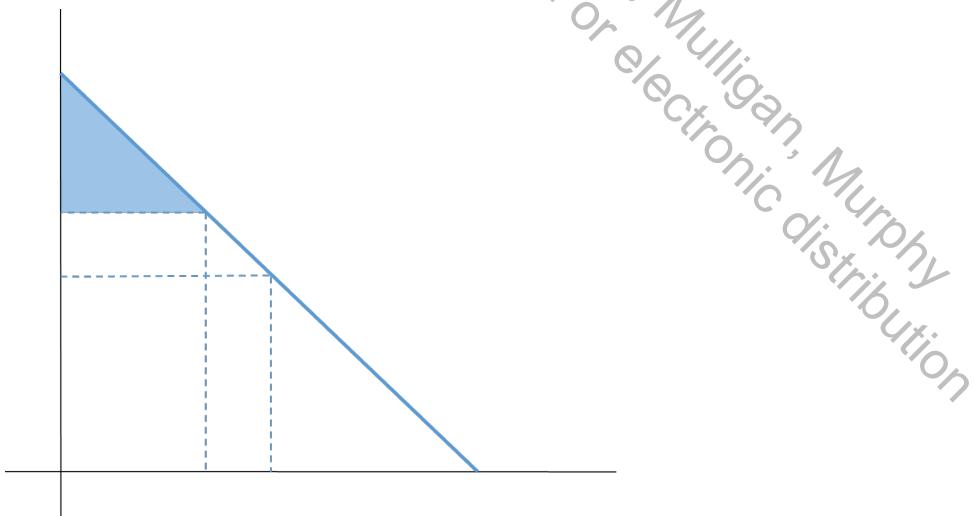


Figure 4-8: When we incorporate new goods with some lag, we miss the shaded region in the graph.

### Using the Cost Function to Value Quality Change

The other issue with these price indices is that goods are not the same goods over time; quality changes. One method to account for quantity is using components. With a car, for instance, we might keep track of miles per gallon over time as one measure of the car's quality. However, a simple model that many economists use is just that higher quality means a lower effective price. Then one unit of today's good equals  $K$  units of the old good. Suppose  $X_i$  is a good that got better, so that there is an old  $X_i$  and a new  $X_i$ . Then

$$X_i^{new}(P_1, \dots, P_{i-1}, P_i, P_{i+1}, \dots, P_N, M) = \frac{1}{K} X_i^{old}(P_1, \dots, P_{i-1}, \frac{P_i}{K}, P_{i+1}, \dots, P_N, M)$$

Because the per-unit price of good  $i$  is scaled by  $1/K$ , and the old units only require  $1/K$  new units. Think about it this way. Candy bars used to cost \$0.50 for 1 oz. Now, they're \$0.50 for 2 oz. Then the price per oz has moved from \$0.50 to \$0.25. It's obvious that we want more ounces after the bars got bigger, but do we want more bars? Suppose the elasticity of demand is -1. Then we demand twice as many ounces after the price is cut in half, and this means the same number of bars. Thus, if demand is elastic or inelastic, we demand more or fewer bars, respectively. Thus, an increase in quality might reduce demand (for total quantity of units). This is realistic. If tire quality increases, for instance, we replace our tires less frequently.

We can also use this example about tires to think again about demand elasticity. Why do we think that the demand for tires is inelastic, for instance? Suppose the price of tire-miles falls so that it is half of what it used to be. How much has the price of driving fallen? It's clear that the amount is much less. So even if demand for driving is elastic, the fall in the price of tires does not reduce the total price of driving by much, since tires are a small share of driving costs. And thus the increase in demand for driving will not increase the demand for tires by very much. Now, what about a single manufacturer of tires? Suppose a single manufacturer reduces the price of his tires. We know the demand will be elastic here. Why? If demand were inelastic, the manufacturer would *raise* prices to make more money.

Suppose our econometrics skills are bad, so that we can't estimate the elasticity of demand for  $X_i$  as in the equation above. An increase in quality still has a very predictable effect on complements and substitutes:

$$X_j = X_j(P_1, \dots, P_{i-1}, \frac{P_i}{K}, P_{i+1}, \dots, P_N, M)$$

That is, if good  $i$  gets better, people will buy fewer of the substitutes for  $i$  and more of the complements to  $i$ .

Now suppose we know the demand function for a good. Take cigarettes, for instance. Assume people learn that cigarettes are bad for you and, in response, decrease consumption from  $Q_0$  to  $Q_1$ . How much did people update their assessment of the health cost of smoking? Well, how much would we have had to raise the price of cigarettes to get people to reduce their consumption to  $Q_1$ ? The shaded region in Figure 4-9 tells us the answer to this: on a per-unit-quantity basis, people increased their assessment of the health costs of smoking by  $P_1 - P_0$ .

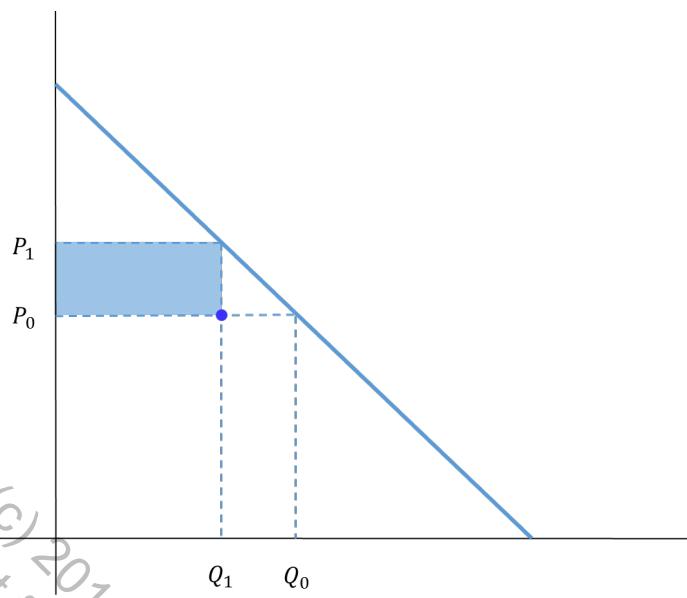


Figure 4-9: We can value people's change in views about the health effects of cigarettes in response to new information by the shaded region above.

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## Chapter 5 Nudges in Consumer Theory

### Indifference curves for buyers

Recall that an individual's demand curve tells us, for a given price, what quantity she chooses to buy. Then, for a given price, any point to the right or left of this point leaves the consumer worse off. Further, for a given quantity, lower prices are always preferred. Using this intuition, we can draw an indifference curve together with a (Marshallian) demand curve in demand space. See Figure 5-1.

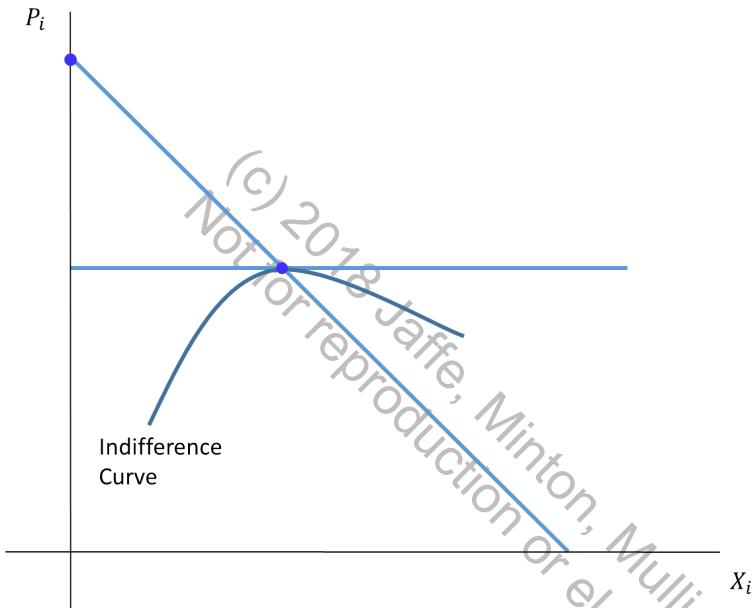


Figure 5-1: In  $(X_i, P_i)$  space, we can draw indifference curves. Each indifference curve is tangent to a price line where it crosses the demand curve, which indicates the optimal quantity to purchase at a given price.

By including the indifference curve, which is tangent to a horizontal price line where it crosses the demand curve, Figure 5-1 makes it more obvious that any point on demand curve represents the consumer's optimal quantity to purchase given the price being charged.

In general, what do we think about someone making a purchase to the right of their demand curve? Is this a puzzle for consumer theory? No, because on the margin it is not often worth the effort to get it exactly right. Notice, in Figure 5-1, that the slope of the indifference curve is zero at the demand curve, which means that a consumer is essentially indifferent between the point on his demand curve and points slightly to the right of his demand curve. There is only the second-order effect coming from the curvature of the indifference curve that makes him worse off. So we're not normally worried if people purchase a few more units at a given price than is optimal for them.

What if, however, it's a matter of someone being *above* their demand curve, in the sense of Figure 5-2 below. This is a consumer paying more for a given bundle than that bundle would warrant on the demand curve. Here, consumer theory is contradicted. The cost of paying a higher price for a given bundle on the demand curve is much higher than the cost of buying more units than demand would suggest at a given price. What the consumer loses from paying a higher price is depicted by the shaded region in Figure 5-2.

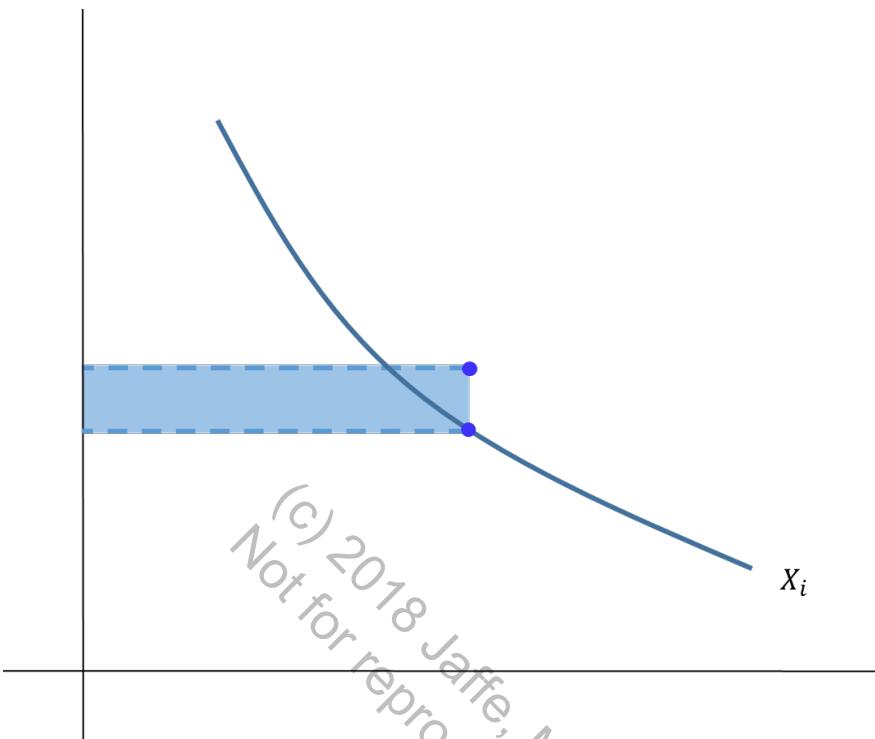


Figure 5-2: We care about going to a supermarket that charges more than another, but we're not too concerned about Coke vs. Pepsi.

People shop this way at grocery stores. They shop around intensively on price, but once they're in the store, they frequently buy more than their demand curve would suggest at given prices.

### Consumer misinformation and “nudgeability” is a prediction of consumer theory

Consumers have a strong incentive to pay less for a given quantity, but little incentive to purchase exactly the right quantity at a given price. Consumer theory is telling us that people should be pretty open to suggestions as to how much to buy, as long as it is not too far to the right or left of their demand curve. It is also telling us that, to the extent that information is costly, people will be somewhat misinformed about the quantity they should be purchasing. Such misinformation is causing them to buy too much or too little, but those purchase mistakes are hardly affecting consumer welfare as long as they are not too large. Why pay for information that has little value?

A growing literature in economics asserts that consumers are sometimes misinformed about what they buy and will make different choices if nudged to do so.<sup>16</sup> The observation is not surprising, but we are surprised when the authors of such studies assert that consumer theory has been refuted. Consumer misinformation and responsiveness to suggestion are predictions of consumer theory as long as the acquisition of information is not free.

Chapter 13 revisits this in the market context, showing that the “nudgeability” of consumers has many times been a force promoting competition and thereby enhancing efficiency and consumer welfare.

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<sup>16</sup> Thaler and Sunstein (2008).

## Chapter 6 Short- and Long-run Demand, with an Application to Addiction

### An example: the demand for cars and gasoline

To illustrate the distinction between long-run and short-run demand, take the demand for gasoline. Let's begin with Figure 6-1

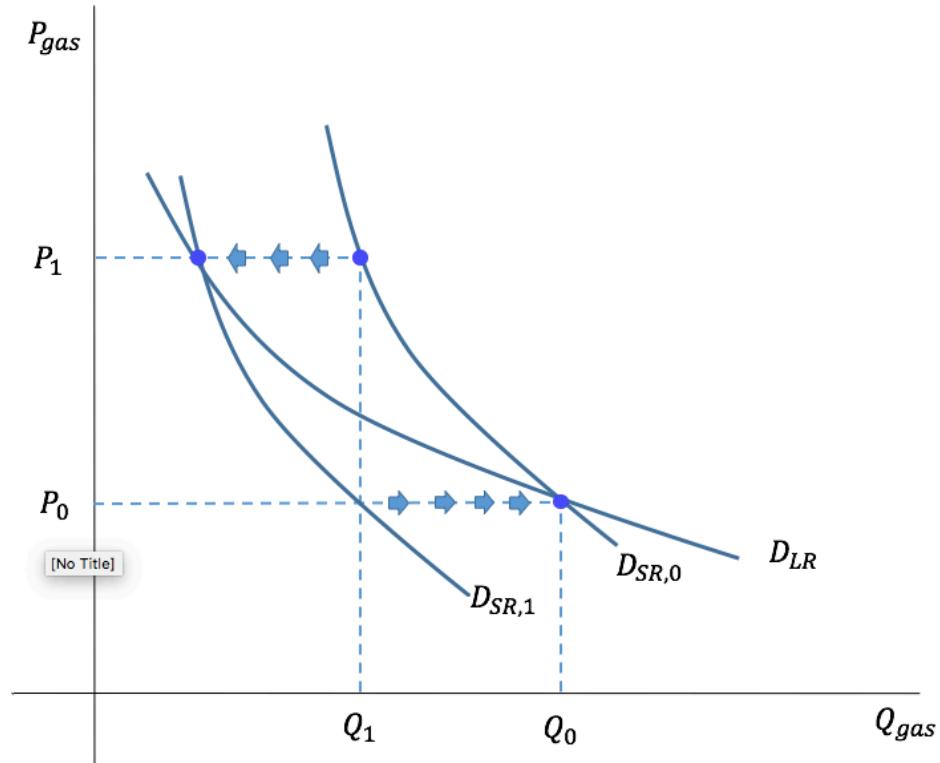


Figure 6-1: The long- and short-run demand for gasoline. Along long-run demand, all other prices are held constant, and all other quantities adjust, especially the quantity of cars. Each short-run demand curve corresponds to a specific quantity of cars.

The market starts with price and quantity  $P_0$ ,  $Q_0$ . What would happen if the price of gasoline instead moved up to  $P_1$  (a dramatic increase, as we have seen numerous times in history)? The law of demand tells us that people buy less gas. But we think that the demand for gasoline at least in the short run is going to be relatively inelastic. If I double the price of gasoline I might see the quantity of gas go down maybe 10 percent something like that, such as the quantity  $Q_1$  in the figure above.

Now a lot of people say that it is completely inelastic which is incorrect, people do save gas when the price of gas goes up.? What are the major ways in which people can conserve on gas? People take fewer or shorter road trips: they go on a shorter vacation or they don't drive as far on a vacation as they would have otherwise. People might also take the bus or the subway, walk, do something other than drive. Both

of these are ways of ‘driving less.’ Like when we talked about cars and car maintenance the activity that is being adjusted is to drive less. People would drive fewer miles. If we were going to achieve a 10 percent decrease in gas consumption, do we have to drive 10 percent fewer miles? Is that the only way we could consume less gas is drive fewer miles? Many households have more than one car, that’s one thing that’s changed over time. People can drive a smaller car and leave the minivan at home. They can still take the same trip, but they take the smaller, more fuel-efficient car. People won’t change the composition of cars they own overnight, but they can still change the composition of cars on the road. It may take time to adjust the car fleet that is in the garage but the car fleet on the road can change instantly. This short run effect of changing which cars are driven has gotten much more important as we have more and more multi-car households. Nonetheless, in the short run there’s a limited number of possible adjustments. That’s why the short-run demand curve  $D_{SR,0}$  is relatively steep.

Now what would happen if the price of gas stayed up at  $P_1$ ? People would buy smaller or more fuel efficient cars. When the price of gas goes up, sales of pickup trucks go down and sales of more fuel efficient cars go up. There are other margins for adjustment. Cities could build public transportation; people could move closer to work; there are many more possible responses than in the short run. So over time the consumption point moves to the left of  $Q_1$ . The response to the price increase is more elastic in the long run because we have more substitution. In the picture, the long-run demand curve  $D_{LR}$  is flatter than the short-run demand curve  $D_{SR,0}$ .

Again, this is where a production function effect is very useful to think about it because one of the major things that’s going on here is that the car fleet is changing, that’s the production function approach. I can see that the car fleet looks different at the right point than at the left point. You can think about the 1970s in the US, the price of gas went way up, and if you look at the fleet of cars we have dramatically changed. Basically the 8 cylinder car went out of business by the 80’s.

What would happen, once we got to the second point on the long run demand curve  $D_{LR}$ , if the price fell back to where it used to be? We would move along another short run demand curve  $D_{SR,1}$ . This curve would be again less elastic because the fleet of cars is being held constant along that curve. Then what would happen? It was amazing how far back we actually came. When prices went up cars became much more fuel efficient; then gas got cheap and what happened? The 8-cylinder car came back; even 10 cylinder cars came back. People have a strong taste for these cars. Technology did change; engines are much more efficient than they used to be, but what do we use that efficiency for? More power and more performance. Performance of cars has gone up enormously: a Honda civic is a high-performing car by prior standards. We have an elastic demand for performance, and as price of gas came down we used a lot of the more fuel-efficient technology for better performance rather than for saving gas.

The key notion is that there’s a lot more adjustment that happens in the long run that can not happen in the short run. This is important for many goods, since people can respond more as they have more and more things they can change, and cars and gas is a good example of that. We look at another example later in this chapter.

## Relating the short-run demand curve to the overall demand system

In the short-run, the quantity of cars is held constant; let's think about the demand system:

$$X_G = X_{Gas}(P_{Gas}, P_{Cars}, \dots)$$

$$X_C = X_{Cars}(P_{Gas}, P_{Cars}, \dots)$$

where I am denoting quantities with X, prices with P, and specific goods with subscripts. Each quantity depends on all of the prices.

The demand system says that the demand for gas depends on, among other things, the price of gas and the price of cars. Obviously, we expect the first effect to be negative, but the second effect is also negative because cars and gas are complements. When cars are cheaper, I will want more cars and more gas.

To get gasoline's long-run demand curve, we take the demand system and vary the price of gas, holding all other prices constant:

$$\frac{dX_G^{LR}}{dP_G} = \frac{\partial X_{Gas}}{\partial P_G}$$

The long run response says- hold other prices constant, how does the quantity of gas respond to its price when people can freely adjust their other goods and importantly freely adjust their cars.

Now let's use the same demand system to get the short-run demand curve. What is different about the short run is that people "cannot" change the quantity of cars. This is not a law or something, just that the supply of cars is fixed in the short run so prices, especially the price of cars, must adjust in the short run so people are willing to have those cars.

$$dX_G^{SR} = \frac{\partial X_{Gas}}{\partial P_G} dP_G + \frac{\partial X_{Gas}}{\partial P_C} dP_C$$

The first term is the long run effect and the second term reflects the short-run change in the price of cars. Moreover, the demand system tells us the amount  $dP_C$  that cars have to get cheaper when gas gets expensive so that people are still willing to have their cars:

$$0 = dX_C^{SR} = \frac{\partial X_{Cars}}{\partial P_G} dP_G + \frac{\partial X_{Cars}}{\partial P_C} dP_C$$

The zero is the change in supply, which has to equal the change in demand. We solve this for the car-price change that has to occur with each unit gas-price change in order for the quantity of cars to be unchanged:

$$\frac{dP_C}{dP_G} = -\frac{\frac{\partial X_{Cars}}{\partial P_G}}{\frac{\partial X_{Cars}}{\partial P_C}}$$

What does that say? Well if cars are complements for gas, the cross-price term (the numerator) is negative: as the price of gas goes up, I want fewer cars. The entire term is therefore negative which implies that the price of cars is going to have to go down to hold the stock of cars the people have. So you can substitute it into the gas-demand equation and get

$$\frac{dX_G^{SR}}{dP_G} = \frac{\partial X_{Gas}}{\partial P_G} \frac{\frac{\partial X_{Cars}}{\partial P_C} \frac{\partial X_{Cars}}{\partial P_G}}{\frac{\partial X_{Cars}}{\partial P_C}}$$

Remember that the first term is negative because it is the own price long-run effect. And the sign of the second term including the minus sign will be positive. Which means that the short run effect will be smaller than the long run effect, because it will be muted by that second term. As the price of gas goes up people will want to buy fewer cars, but the price of cars will go down which will cause them to hold more cars in the short run than they will in the long run, which makes them buy more gas in the short run than they will in the long run. This is the basic mechanism by which the short run demand will be less elastic than the long run.

Here cars and gas are complements but in other applications we are interested in substitutes. With substitutes, both of the two cross-price terms in the numerator are negative. But their product is still positive, so regardless of whether we have substitutes or complements, the long-run response is greater than the short-run response. The difference is in the mechanism: make gas more expensive and its substitutes get more expensive in the short run, because by definition their quantities are fixed, whereas its complements, such as cars, get cheaper.

## Using consumption stocks to understand addiction

What is the key aspect of addiction? That is, what is the key feature of demand for addictive goods that differs from what we have discussed in the context of the neoclassical model? We start by noting that past consumption will matter for consumption decisions today. The model in Becker and Murphy's (1988) paper on addiction includes the standard perpetual-inventory formula used for production stocks, except now it is used for a "consumption stock:"

$$S_t = (1 - \delta)S_{t-1} + C_t$$

with  $\delta \in (0,1]$ . The stock-evolution equation can be solved backwards to get the stock as a function of the entire consumption history.

$$S_t = \sum_{j=0}^T (1 - \delta)^j C_{t-j}$$

where date  $t-T$  is the first time that the consumer consumed this good. Here we have no algebraic difference from production stocks. The practical differences are that this stock  $S$  is not tangible the way that a stock of houses or vehicles would be, and that additions to the stock are referred to as "consumption" rather than "investment."  $S$  represents the consumer's historical experiences. Those are important for habits and addictions because your willingness to pay today depends on how much you have consumed in the past.

Then consumers will solve

$$\max v(y) + \sum_{t=0}^{\infty} \frac{1}{(1 + \rho)^t} U(C_t, S_{t-1})$$

where  $y$  summarizes the consumption of all other goods that are not habit forming. At this point we could replace each stock term with its backward-looking consumption expression and note that the objective being maximized here is "just" a function of consumption at each point in time, fitting into the general consumer framework introduced in Chapter 1. But, as we always do when applying consumer theory, we take the simplest case that represents the phenomenon at hand, which here is habitual behavior. So we have added only one complexity to our previous setup, which is that utility depends on the consumption-stock variable.

We will assume  $U_{CS} > 0$ , so that I get more utility on the margin from consumption as I increase my stock of consumption of that good. As I'll explain in a minute, we will assume that  $U_{CS}$  is positive enough that consumption at one point in time reinforces consumption in the future (especially the near future).

We know  $U_C > 0$ . But what about  $U_S$ ? We will denote  $U_S < 0$  a harmful addiction and  $U_S > 0$  a beneficial addiction. Exercise might be considered a beneficial addiction, for example. But the distinction between harmful and beneficial is not as important as our assumption that current consumption reinforces future consumption. Current and future consumption are complements. The complementarity is a common element between exercising, listening to classical music, or doing price theory (!), on one hand, and smoking cigarettes or taking cocaine on the other.

For the same reason, the fact of complementarity (requiring that  $U_{CS} > 0$ ) does not tell us whether the complementary activity is a good ( $U_S > 0$ ) or bad ( $U_S < 0$ ). This is sometimes a source of confusion in social interactions research. When my neighbor buys a faster car, I respond by buying a faster car. That tells us that his consumption of fast cars is complementary with my consumption of fast cars. That does *not* tell us that his purchase harmed me, even though the result of his purchase is that I spend more money on cars. A better way to assess whether one person's fast car harms or helps his friends is to look at how people choose their friends. Do they choose friends with slow cars so that they can feel better?

Now think about the marginal utility received from an additional unit of consumption at time  $t$ :

$$\begin{aligned} & \frac{1}{(1+\rho)^t} U_C(C_t, S_{t-1}) + \frac{1}{(1+\rho)^{t+1}} U_S(C_{t+1}, S_t) + \frac{1-\delta}{(1+\rho)^{t+2}} U_S(C_{t+2}, S_{t+1}) + \dots \\ &= \frac{1}{(1+\rho)^t} \left[ U_C(C_t, S_{t-1}) + \frac{1}{1+\rho} \sum_{j=1}^{\infty} \left(\frac{1-\delta}{1+\rho}\right)^{j-1} U_S(C_{t+j}, S_{t+j-1}) \right] \end{aligned}$$

The first term is the usual term – more consumption today is valuable from today's perspective. The rest of the terms reflect that period  $t$ 's consumption adds to the habit stock, especially in the near future. A rational consumer takes the future terms into account in making his decision about period  $t$ 's consumption. For a harmful addiction, all of the future terms are negative.

We say that consumption is complementary over time when the marginal utility expression above is increasing in the stock  $S_{t-1}$ . This is stronger than  $U_{CS} > 0$  because  $S_{t-1}$  increases the future stocks, which reduces the marginal utility (or increases the marginal disutility) of future stocks.

### Short- and Long-run Price Effects on Addictive Behaviors

As in Chapters 15 and 16, we can think about a steady state and how the stock  $S$  approaches the steady state along an optimal time path. The steady state has  $C = \delta S$ . We expect the law of demand to hold: steady state consumption is less when the steady state price of  $C$  is higher.

The history of consumption matters only through  $S_{t-1}$ . That is,  $S$  is a state variable much like  $k$  was a state variable in Chapters 15 and 16. Let the policy function  $C(S)$  denote the consumer's optimal choice for current consumption when his past is summarized by the stock  $S$ . The slope of this function tells us whether the good is addictive – whether consumption is complementary over time. It slopes up in the addictive case.

Figure 6-2 is a diagram that uses these ideas to understand the optimal consumption dynamics. A steady state has to be on a ray from the origin. The policy function slopes up and may, as in the figure, cross the ray from above (ignore the upper policy function for the moment).

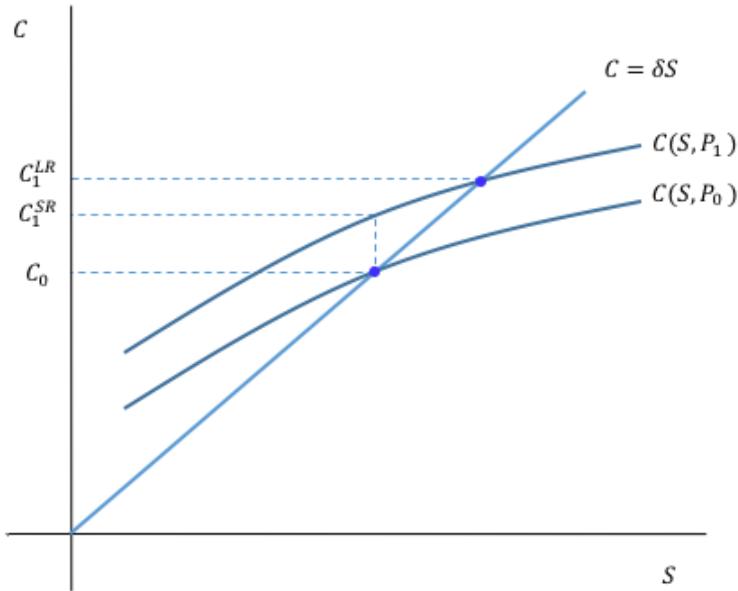


Figure 6-2: When prices are decreased, the  $C(S)$  schedule shifts upward and long run consumption of the addictive good increases. The 45 degree line is the steady state, where consumption simply replaces decreased consumption stock.

The policy function shows us that a consumer beginning with less than steady-state stock consumes above the ray, which is equivalent to saying that he consumes more than the stock's depreciation. His stock is therefore increasing over time and he approaches the steady state from the left. A similar argument says that a consumer beginning with more than steady-state stock approaches the steady state over time from the right. In other words, when the policy function crosses the ray from above, the crossing point is a stable steady state.

Now suppose that the consumer is in his steady state, and is surprised by a sudden and permanent decrease in the price of the addictive good. The new policy function is above the old one, as shown in Figure 6-3. Consumption increases in the short run – before the stock can change – to the point above on the new policy function. But that new consumption level is still below the new steady state consumption.

In other words, the sudden and permanent price decrease increases consumption more in the long run than in the short run. If you want to know how much price ultimately depresses the consumption of an addictive good, looking at the short-run response is going to be quite an underestimate.

This is similar to the short-run and long-run effects of a change in the price of gas on consumption of cars and gasoline. A sudden and permanent change in gas prices immediately changes gasoline purchases, without changing the number or types of cars that people own. Over time, cars change too so that the long run response of gasoline purchases is greater. Cars are not literally a stock of historical gasoline consumption, but the complementarity between gasoline and cars looks a lot like the complementarity between  $S$  and  $C$ .

The  $C(S)$  schedule might be sufficiently non-linear, however, that in some range it crosses the ray from below. It may also cross it again from above, as in Figure 6-3. In this case, there is more than one steady state and one of them is unstable. If you start to the right of  $S^*$  (the unstable steady state stock), one will move to the right, and if you start to the left, you move down and eventually quit.

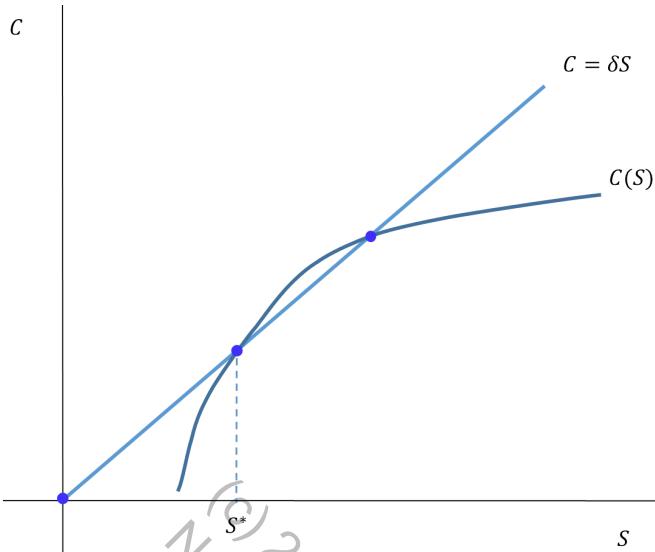


Figure 6-3: Case with an unstable steady state. A perturbation from  $S^*$  either leads someone to quit or to the higher level of steady state consumption.

We can think about rehabilitation in this model. Consider shifting down the  $C(S)$  schedule in Figure 6-3. Then someone beginning at  $S^*$ , for instance, will begin to move towards quitting along the policy function where it is below the ray. When rehab ends, the policy function returns to the original  $C(S)$  curve depicted, but by this time he may be at another  $C = 0$  steady state that is stable.

What does this tell us about potential problems caused when addictive substances are declared illegal? When consuming addictive substances is illegal, access to good ways to quit can be reduced. Someone who is addicted to an illegal drug may not be able to avail himself of quitting alternatives without exposing himself to the risk of prosecution, for example. However, with rational consumers, this is a double-edged sword. The easier it is to quit, the more people start. Depending on the elasticity, making it easier to quit could actually make *more* people addicted, if more people start than quit.

There are some policies we should also be careful about. Lying to people, for example, to overinflate how bad a drug is could actually get people to quit. In the long-term, however, it detracts from people's trust of those who lied. The liars, in the future, will have less credibility even when they tell the truth. The costs of this are difficult to measure. Furthermore, what about the policy of trying to inform people exactly how bad drugs are? Some studies have shown that people *overestimate* how likely smoking is to cause lung cancer, for instance. We have to take these studies with a grain of salt, however, because we are concerned with choices people make, not choices they say they would make. Fewer studies have been conducted along these lines.

There are also information problems with addictive goods, at least when they are new. It is often difficult to know how addictive a particular good is immediately. There is room for a company putting out a new product to misinform consumers about how addictive it is because their claims are not verifiable for an extended period of time. This is not necessarily unique to addictive goods, however. A dietary supplement could claim to have cancer prevention properties that are realized after twenty years of taking it. This would be very difficult to verify.