Excerpts from

Chicago Price Theory

Comments are welcome: please email to

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Part III: Technological Progress and Markets for Durable Goods

Chapter 15 Durable Production Factors

Stocks and Flows for Factor Prices and Quantities

When we think about production, we can think about 3 types of inputs. There are inputs like labor, which are purchased in a service market, inputs like capital, which are purchased in a capital goods market, and materials, which are "used up" in the production process. Materials are not like capital assets because they do not have durability. Capital yields a flow of services over an extended period of time.

Land is a standard example of capital. It was, in fact, the most important form of capital for much of history. Initially, however, there was not a notion of "investment" in land. In more recent times, we have created land, investing on the quantity side. More frequently, however, we invest in the quality aspect of land. We make land useable by draining swamps, eradicating diseases, or terracing mountains.

A home is also a good example of capital. It provides shelter, privacy, storage, etc., year after year.

So there is a quantity side of capital. $K_t = K_{t-1} + I_t - DEP$, where I denotes investment and DEP denotes depreciation. This type of investment is building new houses, draining a swamp to create new land, etc. Most commonly in economics, we let $DEP = \delta K_{t-1}$. That is, a constant fraction of capital yesterday depreciates in every period. We also do not care whether the capital from yesterday was built last year or ten years ago. It all depreciates at the same rate.

$$K_t = I_t + (1-\delta)I_{t-1} + (1-\delta)^2 I_{t-2} + \cdots = I_t + (1-\delta)K_{t-1}$$

The entire investment history is summarized in K_{t-1}. Exponential depreciation embodies the idea that capital of various ages is perfectly substitutable (just not at 1 for 1 rates) and also the idea that capital depreciates at the same rate regardless of how old it is.

Now that we are considering durable goods, we need to consider both the stock of durable assets and the flow of new investment. When the assets are very durable, these numbers can differ significantly. Consider the housing market: the stock of housing is much larger than the number of new houses built per year. The American Community Survey estimates that there are about 75 million single-family homes in the United States whereas about 1 million single-family homes are built each year (U.S. Census Bureau).

There are also two notions of price, which we have mentioned before. There is the rental price, R_t, which is what we pay for capital for a moment in time, and a capital price, P_t, which is what we pay to purchase the asset and own the right to its use for the rest of its life. Note that this does not refer to renters vs. buyers in the housing market, for example. Even if a house is purchased, the owner is still paying the rental price in each period because the owner could be renting the house and charging a tenant the rental price. Just as we could write K_t in terms of last period's capital *or* the perpetual inventory method, we can write

$$R_t = P_t - \frac{P_{t+1}(1-\delta)}{1+r} = \frac{rP_t + \delta P_{t+1} + P_t - P_{t+1}}{1+r}$$

So the rental price is the interest cost plus the depreciation cost plus the capital loss (a negative number if price goes up), expressed in next year's dollars. Or we could rearrange to put tomorrow's purchase price on the left, which would yield an estimate of the expected price tomorrow as a function of today's rental and purchase prices.

Yet another way to look at it: the relation between rental price and capital gains is a difference equation that can be solved for the initial purchase price:

$$P_t = R_t + \frac{1-\delta}{1+r} R_{t+1} + \left(\frac{1-\delta}{1+r}\right)^2 R_{t+2} + \cdots$$

So the amount K of capital we have is a backward-looking concept, but the capital price P of an asset is a forward-looking concept.

The Use and Investment Markets for Capital Goods

A simple model involves thinking about a use market. In the use market, there is a demand schedule for capital. At any point in time, there is a capital stock K_t. Because we assume the use market clears, the capital stock, together with the demand for capital today, determines the rental price R_t. We can say all of this without thinking about the supply side at all.

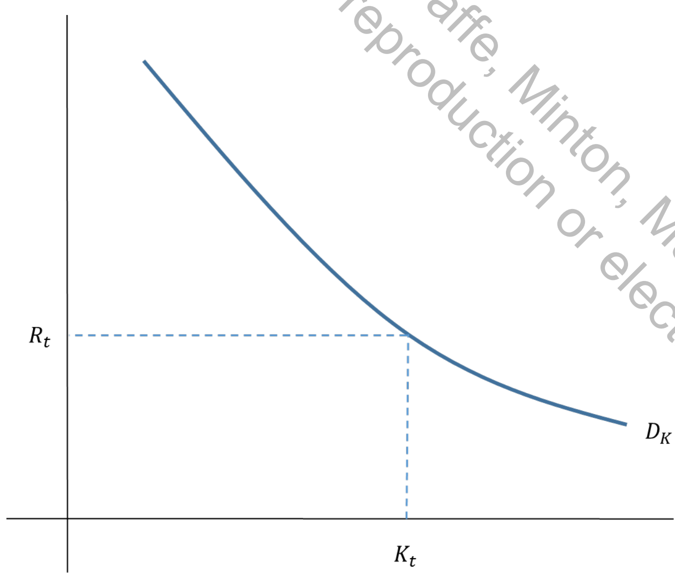

Figure 15-1: The use market. In equilibrium, the quantity of capital demanded today K_t along with its associated price R_t lie on the demand curve.

Note this is the *use* market, so we're talking about demand for use of housing, automobiles, durable manufacturing inputs, etc. Now we can think about the supply side, which will deal with the production of capital. There are many models we could use for this, but we will use a simple model that assumes a supply curve $I(P)$. Suppliers of capital care about the price, P_t, not the rental price, R_t, because producing a unit of capital will return P_t over its life.

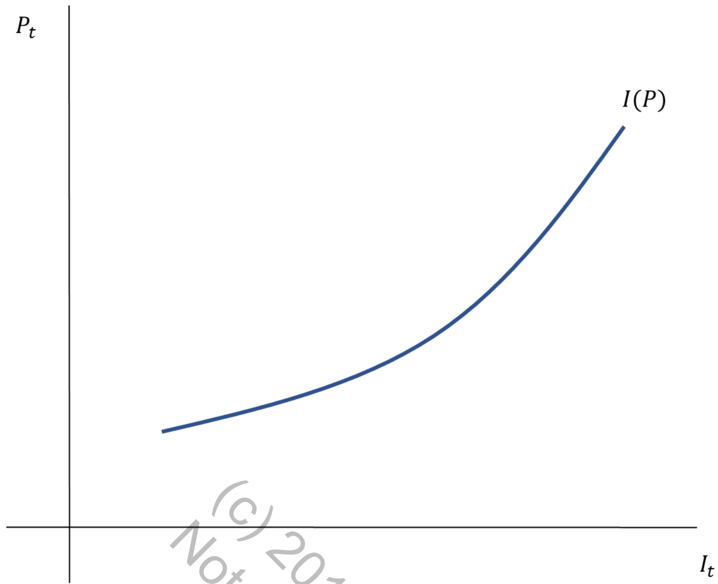

Figure 15-2: Investment market. Investment today, I_t, is chosen based on the capital price P_t.

Four Equilibrium Conditions

We have assembled four equations for each point in time.

1. Rental market equilibrium: $K_t = D(R_t)$. This can be explicit, if transactions are rental transactions (i.e. apartments), or implicit, if purchases are made, since the owner still pays the rental cost every period by *not* renting out the capital.
2. Purchase-price equation: $P_t = R_t + \frac{1-\delta}{1+r}R_{t+1} + \frac{(1-\delta)^2}{(1+r)^2}R_{t+2} + \cdots$. This is the forward-looking part of the model.
3. Investment-good market equilibrium: $I_t = I(P_t)$. This is the simplest possible supply model for investment.
4. Law of motion for capital: $K_t = (1-\delta)K_{t-1} + I_t$. The stock equation is the backward-looking equation. Knowing the capital we had yesterday is important for knowing the capital we have today.

The four unknowns at each point in time are the two prices and the two quantities.

We could, for example, endogenize δ. Consider a simple model of maintenance, where one can choose an optimal value of δ given a cost for reducing it:

$$\max_{\delta} \frac{P_{(t+1)}(1-\delta)}{1+r} - c(\delta)$$

This is an alternate form of investment. Expanding our conception of the model a bit – note that to get more capital tomorrow, one can produce more capital in addition to slowing depreciation by investing more into maintenance. For now, maintenance has simply been absorbed into the investment supply function $I(P)$.

Steady State

Let's write these four equations for the steady state, where an upper bar denotes steady state:

1. $\bar{K} = D(\bar{R})$. Capital in steady state must equal the amount of capital demanded at the steady state rental rate.
2. $\bar{P} = \frac{\bar{R}(1+r)}{r+\delta}$, just by condensing the geometric series.
3. $\bar{I} = I(\bar{P})$
4. $\bar{K} = \frac{\bar{I}}{\delta}$

We can use equation 1 and combine equations 2, 3, and 4 to write down steady state supply and demand equations that pin down the steady state rental rate and steady state capital.

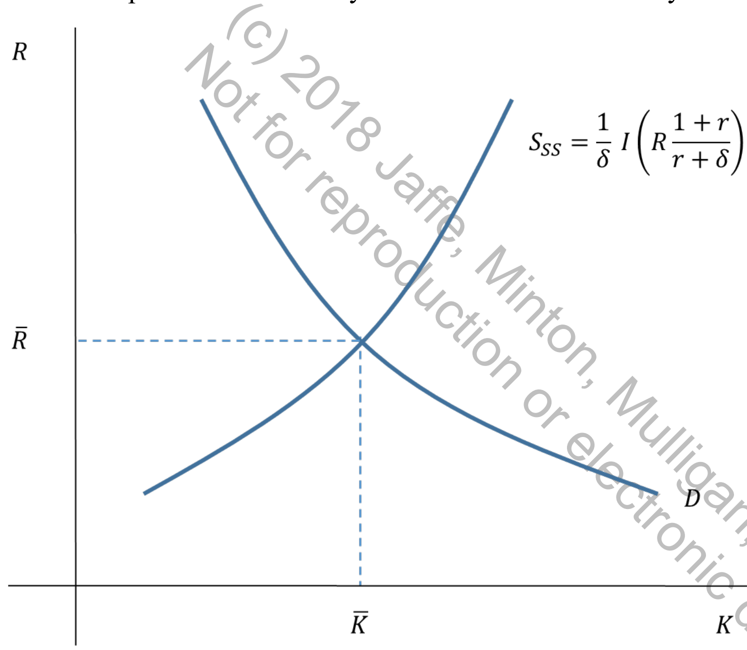

Figure 15-3: Steady state rental rate and capital.

Perturbing the Steady State

But if all we think about is the steady state, the concept is not very interesting. More interesting is the fact that the steady state tells us about where the system is headed. If changes put the system on a path towards a new steady state, what does that path look like? Suppose we were in a steady state, but a hurricane suddenly destroys some capital. Now we have some capital $K_0 < \bar{K}$. Then rents must be higher today. But then the price also suddenly rises today, because the price is just the present value of rents. So investment also increases today, which will lead K to begin to rebound. These dynamics will continue working, and we will asymptotically return to the steady state. See Figure 15-4.

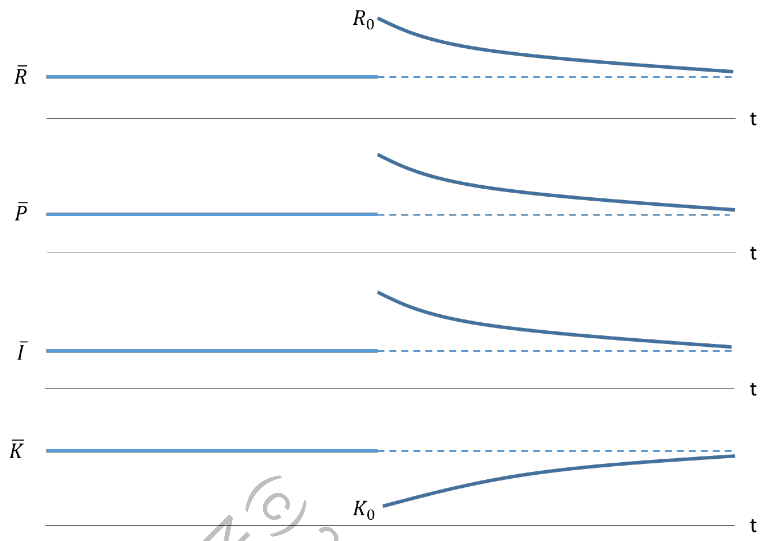

Figure 15-4: The system returning back to steady state. An initial negative shock to capital, which falls from $\bar{K}$ to K_0, causes rents to rise to R_0. Rents are related to the price, however, so higher rents in the future cause the price to rise today. Since the price is higher today, investment also jumps today.

Now, think about how one would solve this system. Given K_0, R_0 is also known. But what would P_0 be? Think about guessing a value and call it $\hat{P}_0$. If $\hat{P}_0$ is too high, investment will initially rise a lot, driving the capital stock up quickly, which will lower rents too quickly to justify the high value for $\hat{P}_0$. If $\hat{P}_0$ is too low, we have the opposite problem. Investment will rise only slightly, which means the capital stock will not rise much. Then rents will stay high, and these high rents will imply that $\hat{P}_0$ should have been higher.

The immediate jump from $\bar{P}$ to P_0 is the efficient markets hypothesis. We are assuming that all the information about the new future is incorporated into the price today. The jump from $\bar{P}$ to P_0 is the capitalization of the information that became available.

What if people have naïve forecasts? For example, suppose people think the higher rent will stay high forever. Then P would rise more than P_0, and I would jump above I_0. But then R would fall quickly, and P would fall quickly, so we would get much faster convergence.

Now we will consider a rise in demand. We will move to a new steady state with a new level of output and price, as shown in Figure 15-5. When demand rises, we know capital in the future will be higher, as will rents. The figure shows us this. Further, we know that the price will be higher, as rents are higher, and we know that investment has to have risen as well. Depending on the elasticity of supply, more of effects of higher demand will show up in capital or the rental price. If supply is very inelastic, most of the higher demand will appear in the new rental price, whereas elastic supply results in most of the higher demand appearing in the new steady state level of capital.

The dynamics of an increase in demand are depicted in Figure 15-6. Note that the dynamics here are very similar to when we destroyed capital. While the new steady states are higher, unlike before, we are still engaged in a process of building new capital to meet demand.

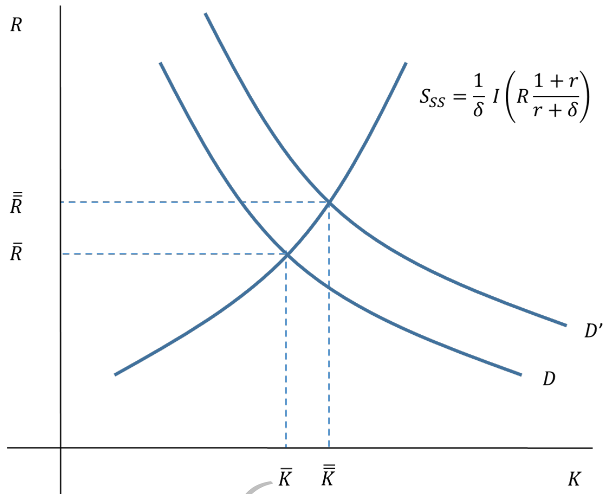

Figure 15-5: A rise in demand. The day demand jumps, rents will rise sharply, to the intersection of $\bar{K}$ with the new demand curve, D'. This results in the price rising sharply, causing investment to rise, and further causing capital to begin to move to the right, from $\bar{K}$ to $\bar{\bar{K}}$.

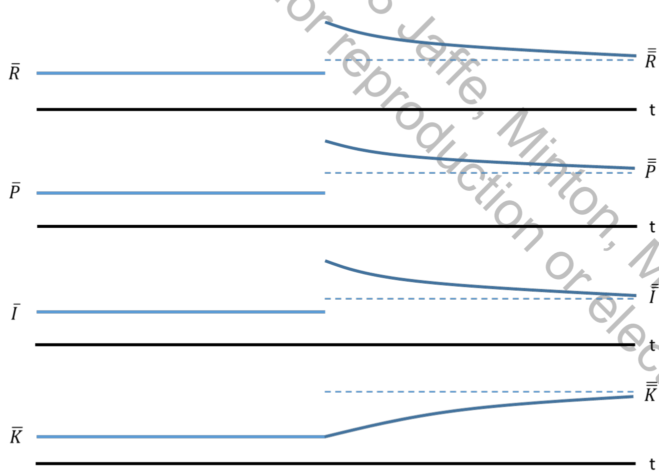

Figure 15-6: Transition dynamics for a rise in demand. Since capital is fixed in the short run, rents rise sharply. So prices rise sharply today, causing investment to rise drastically. This causes capital to begin to converge to the new steady state. As the capital stock converges, rents fall, causing prices to fall. Investment slows over time, and we converge to a new steady state.

Note that pinning down the higher price that results from higher rent involves firms making good forecasts about the future. If the pinned down price is too high, investment will be too high, the stock of capital will rise quickly, and rents will fall quickly. Similarly, if the pinned down price is too low, investment will be too low, the stock will rise slowly, and rents will fall only gradually. This problem of pinning down capital becomes very difficult when the capital is long-lived and discount rates are low – forecasts must be accurate far into the future.

Consider the case of non-durable goods in this model; that is, consider capital that lasts for only one period. If demand jumps, then all variables move right to the steady state immediately. The difference for durable goods is that rents, prices, and investment will all jump more than they would in the non-durable case. And they jump more the more durable the asset is. Consider a very long-lived asset with $\delta = .02$. This asset will survive for 50 years. In steady state, we are only producing 2% of the stock per year. If demand increases suddenly by 10%, investment will rise dramatically. This is one reason why investment is so much more volatile than consumption. Despite the fact that demand rises in a given period, the stock has not moved significantly, so consumption cannot move significantly. Investment, on the other hand, is free to adjust much more in a short period. Suppose, even further, that a durable asset A' is used to produce a durable asset A. If demand for A rises, and investment jumps significantly, investment in durable asset A' will have to jump even more to meet the significantly higher demand resulting from increased production of A.

Now suppose we have the same demand increase, but instead of happening today, we learn that the demand increase will happen in the future. Rents will not change today, but because rents will be higher in the future, the price rises today. As a result, investment increases today, and the capital stock begins to increase. Thus, rents begin to fall. Though rents are falling, prices continue to rise because the higher demand in the future is less discounted. Thus, in the short run, rental rates will fall. See Figure 15-7 for a depiction of these dynamics.

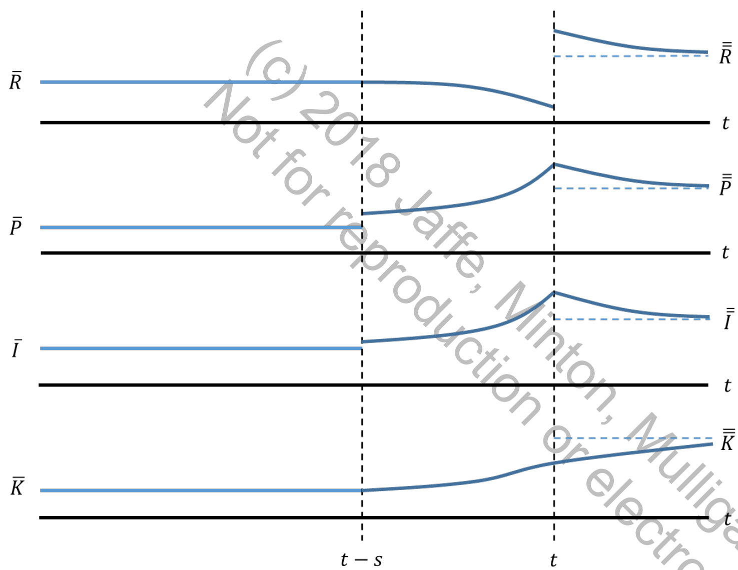

Figure 15-7: At time $t-s$, information about higher demand in the future is announced. Prices and investment jump today, causing rents to fall and capital to rise. When the higher demand is realized at period t, rents jump.

Note that we cannot build up all of the required capital for the new steady state in advance. Suppose we did—we will derive a contradiction. If $K_t = \bar{\bar{K}}$, then it must be that $R_t = \bar{\bar{R}}$ and that R continues to equal $\bar{\bar{R}}$ after period t. Prior to period t, R must be below $\bar{R} < \bar{\bar{R}}$ because demand has not yet increased and the stock is above $\bar{K}$. But then we must have that prices before period t were less than $\bar{\bar{P}}$, which implies investment before period t must be less than $\bar{\bar{I}}$. If we are investing less than $\bar{\bar{I}}$ prior to period t, however, there is no way we could have reached $\bar{\bar{K}}$.

These dynamics reflect the ideas of rational expectations and market efficiency. We expect that upon learning new information, the prices today jump to reflect this new information. This is why we see change in period $t-s$; prices today are changing to reflect the new information. These dynamics reflect boom periods we see in the real economy. In the housing market, for instance, when people think demand is going to be high in the future, rents are low but prices are high and rising. This process describes a fully rational housing boom. An observation of low rents together with high and rising prices is no proof of irrational pricing.

Now let's suppose that at time $t - s$, we learn there is a chance that demand increases at date t. At date t, we actually learn that this hunch was wrong; demand remains unchanged. These dynamics are depicted in Figure 15-8. Now, when we learn demand has remained unchanged, we have too much stock. Instead of jumping, rents remain low. Prices fall drastically, and as a result investment falls as well. Over time, the capital stock reverts to the steady state, causing rents to rise, prices to rise, and investments to rise back to the previous steady state level.

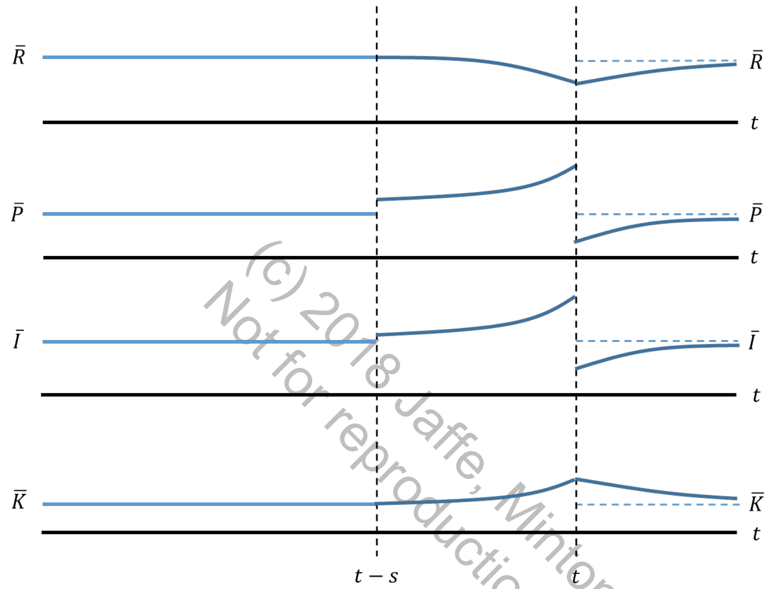

Figure 15-8: At date $t - s$, we learn that demand might rise on date t. At date t, we learn that guess was wrong.

Now, this story looks a lot like the housing bust. How do we tell the difference between this story and the "irrational exuberance" story? We need to think about whether or not it was reasonable to think demand was going to be high in the future. We can always write the following equation:

$$P_t = R_t + \frac{R_{t+1}(1-\delta)}{1+r} + \frac{R_{t+2}(1-\delta)^2}{(1+r)^2} + \cdots + \frac{R_{t+N-1}(1-\delta)^{N-1}}{(1+r)^{N-1}} + \frac{P_{t+N}(1-\delta)^N}{(1+r)^N}$$

Today's price must be justified by the rents we expect to receive in the future plus some terminal value. In the housing boom, we know that rents were low, but prices today were high. This means people thought prices were going to be high in the future. It's easy to say after the fact that what we saw in the housing boom was just an irrational bubble. But before the crash happened, if we say that people had expectations that future prices were going to be high, i.e. there was some high terminal value, determining whether the expectations were rational after the fact is very difficult. Put differently, in the housing crisis, it turned out people's expectations were wrong relative to what happened. The question remains, however: were people wrong about what *could* have happened?

Chapter 16 Capital Accumulation in Continuous Time

Perturbing the Steady State (Continued)

In the previous chapter we analyzed a model of capital investment and applied it to the housing market. There were several major points here. During the boom period, there were two key features: prices were high, so producers found it an attractive time to produce homes, but rents were low, so it was cheap (on net) to live in homes. Even purchasing an expensive home was "cheap" because people believed homes were appreciating in value.

During the bust period, after the housing market crashed, prices plummeted. In fact, they often fell below construction cost. It looked like buying houses was profitable during the bust. The problem was that it is hard to police depreciation, especially when renting out single-family homes. We can start to formalize some of these ideas.

Consider what occurs in the market when the interest rate is reduced. Because interest rates appear in the denominator for the present value of future rents, then for given rent values, a lower interest rate means a higher price. This shifts the steady state supply curve to the right. In the new equilibrium then, $\bar{\bar{R}} < \bar{R}$, $\bar{\bar{I}} > \bar{I}$, $\bar{\bar{K}} > \bar{K}$, and $\bar{\bar{P}} > \bar{P}$. This looks much like a boom period. This change is depicted in Figure 16-1, and the transitional dynamics are shown in Figure 16-2. During the housing boom, note that we also had decreasing rates, which contributes even more to the housing boom effects we see.

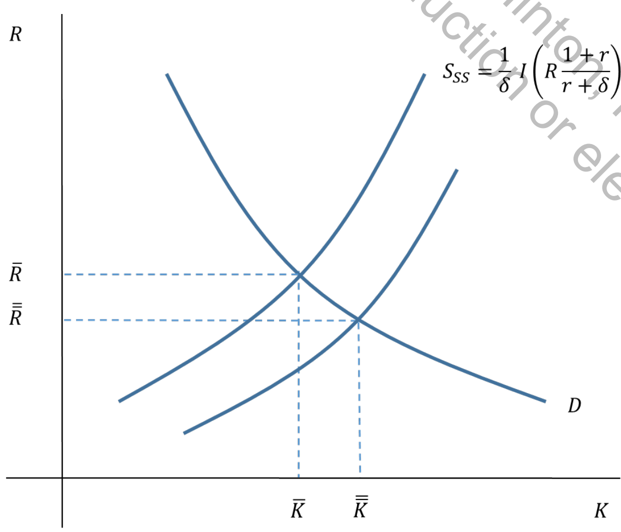

Figure 16-1: A decrease in the interest rate shifts the supply curve to the right. In the new steady state, there is more capital, lower rents, higher prices, and higher investment.

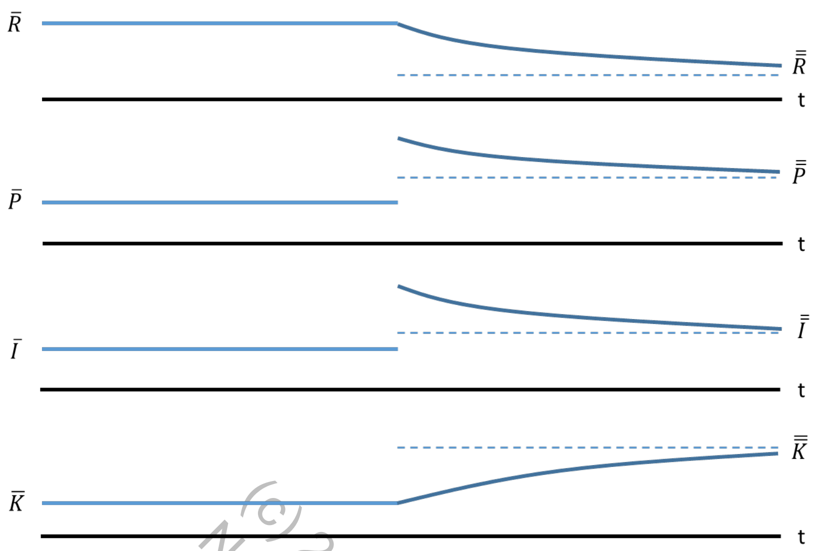

Figure 16-2: These transitional dynamics result from a decrease in the interest rate. Price increases immediately, driving up investment, which causes rents to begin to fall and the capital stock to begin to rise.

Now let's think about the speed of convergence to the steady state. Several parameters can be important here. The depreciation rate, for example, is very important. Suppose I, for example, is constant. Then the convergence rate is simply given by δ. In the continuous case, $\dot{K} = -\delta K + I$, where $\dot{K}$ gives the instantaneous rate of change of capital. This equation shows that convergence under these assumptions will occur at rate δ. Also, the elasticity of supply is important. A higher elasticity means faster convergence. Finally, demand elasticity has an effect, since less elastic demand implies faster convergence. When the stock is reduced, for example, rents jump up more when demand is more inelastic. This causes prices to jump higher and investment to rise higher, which implies the capital stock will rise faster to meet the new demand.

Note that learning about a demand change, for example, far in advance of it occurring does not mean that the stock will reach the new steady state in advance of the demand change occurring. Building homes prior to the demand increase means losing some money in the short run because the new houses are not yet demanded. Similarly, however, building houses in excess of steady state investment after the demand increase means more money could be made if those houses existed already. The costs of building houses early, however, prevent the new steady state from being achieved before the higher demand is realized.

The future is almost as important as the past when the interest rate is low. The past is much more important than the future, however, when interest rates are high.

Continuous-time versions of the four equilibrium conditions

Now, let's think about this entire model in continuous time. The equations become

1. Rental market equilibrium. $K(t) = D(R(t))$
2. Purchase-price equation. $P(t) = \int_t^\infty e^{-(r+\delta)(\tau-t)} R(\tau) d\tau$
3. Investment-goods market equilibrium. $I(t) = I(P(t))$
4. Law of motion for capital. $\dot{K}(t) = -\delta K(t) + I(t)$

2'. $R(t) = (r+\delta)P(t) - \dot{P}(t)$

Sometimes equation 2' is considered a primitive of the model. But equation 2 holds for all t, so one can differentiate it with respect to t. Equation 2 implies equation 2'. Note, importantly, that equation 2' does not imply equation 2. Equation 2' does not include the boundary condition that says prices must be set at the correct level. This is important, because we will now use equations 1, 3, 4, and 2' in Figure 16-3.

We can combine equations 1 and 2' to form Equation 5: $K(t) = D((r+\delta)P(t) - \dot{P}(t))$. We can further combine 3 and 4 to form Equation 6: $\dot{K}(t) = -\delta K(t) + I(P(t))$. Now everything is in terms of K, P, and their derivatives. We will use these equations to draw a phase diagram. It is useful to consider the steady state, when $\dot{K}(t) = 0$ and $\dot{P}(t) = 0$.

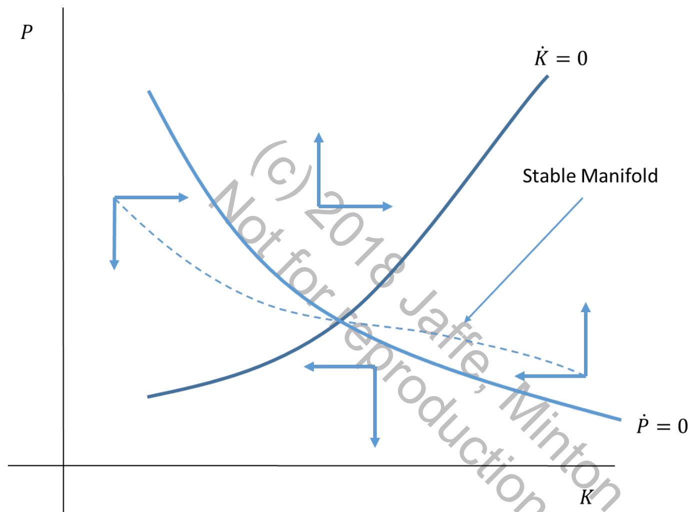

Figure 16-3: : A phase diagram. The lines $\dot{K} = 0$ and $\dot{P} = 0$ denote when capital and price are constant, respectively. The stable manifold is the line along which an economy converges to the steady state, where $\dot{P} = \dot{K} = 0$.

Above the $\dot{K} = 0$ line, the price is high, so the capital stock is increasing. Above $\dot{K} = 0$ on Figure 16-3, we therefore draw arrows pointing to the right. Similarity, below the $\dot{K} = 0$ on Figure 16-3, we denote by a left arrow the fact that the capital stock is shrinking over time because prices are too low. To the right of the $\dot{P} = 0$ line, the level of capital stock is too high, so the price is too low and is therefore rising over time. We denote this with an up arrow. The opposite is true when we are to the left of the $\dot{P} = 0$ line, so we denote the movement of price to the left of this line with a down arrow. All of these dynamics can be easily shown mathematically using equations 5 and 6.

Convergence to the steady state happens along the saddle path or stable manifold, denoted by a dotted line on the phase diagram in Figure 16-3. As an example, suppose demand unexpectedly rises today, and Figure 16-3 shows the new steady state in the world with higher demand. Then the capital stock, as we know, is too low today to meet the higher demand. Rents rise, causing prices to jump today. Prices cannot just jump simply an arbitrary amount, however. The saddle path gives the unique value to which the old price must jump in the presence of higher demand.

What happens if price jumps above or below the saddle path? This is shown in Figure 16-4.

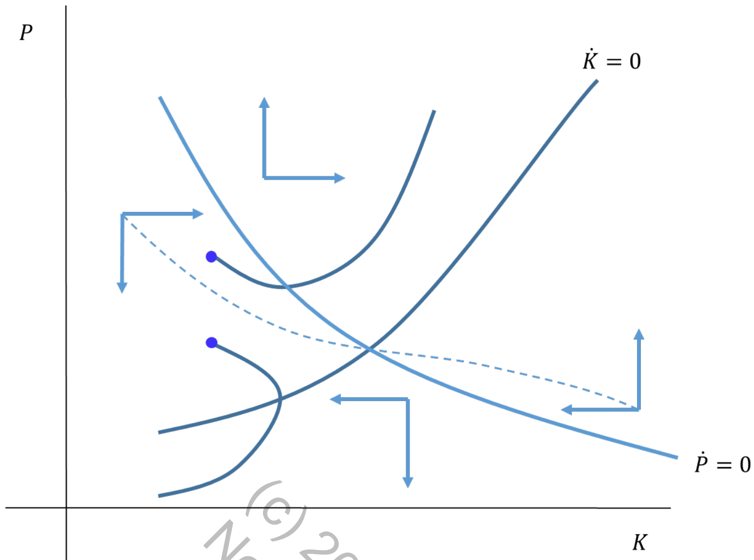

Figure 16-4: If price jumps to the wrong point, the mathematics do not lead the economy to converge to the steady state.

Finally, consider a myopic model in which we replace the rational expectations equation for $P(t)$ with

$$P(t) = \frac{R(t)}{r + \delta}$$

Then future anticipated shocks would have no effect. Further, we would adjust to the new steady state faster, since instead of jumping to the saddle path, P jumps to the demand curve. Note that the approached steady state will be the same, and that this will not look that different from the rational expectations case. Finally, note that under this model, holding capital is a bad deal if prices are falling, since the cost is actually higher than $R(t)$.

Lastly, observe that if investment cannot be negative, then uncertainty causes people to underinvest. Suppose there's a 50% chance of a 10% increase in demand and a 50% chance of a 10% decrease. Then the initial jumps in rental rate will be equal (in opposite directions), but in the case of the negative shock, the rent will return to steady state much more slowly because the stock cannot adjust as quickly downward because there is no negative investment. So on average rents are lower, and thus the price today must be lower.

In the next chapter, we look at a model that has an endogenous interest rate instead of a simple upward sloping supply curve as we have been assuming.

Chapter 17 Investment from a Planning Perspective

Last time, we considered a very simple investment model. The key underlying fact was that we had an upward sloping supply of investment. That's why the dynamics we discussed came about. It is cheaper to build the product more slowly than it is to build it all at once. There are a number of reasons for this in practice: heterogeneity of underlying resources, for example, or multiple capital goods required as factors of production. With upward sloping supply, production is smoothed over time to reduce the total cost of production. This is like a convexity in the cost structure. The upward sloping supply curve, or a rising marginal cost, corresponds to convexity in overall cost. As a result, the behavior we had previously represented as market equilibrium involving multiple agents (e.g., renters of capital, builders of new capital goods, etc.) can be restated as a maximization problem of a single producer.

Take the inverse demand function $D(x)$ and define the function $V(x) = \int_0^x D(z)dz$. Then, by the fundamental theorem of calculus, $\frac{dV(x)}{dx} = D(x)$. Similarly, take the supply function $S(x)$ and define $C(x) = \int_0^x s(z)dz$; then, by the same reasoning, $\frac{dC(x)}{dx} = S(x)$. Then consider the problem of maximizing total surplus:

$$\max \int_0^\infty e^{-rt}\left[V\big(K(t)\big) - C\big(I(t)\big)\right]dt$$

Because of how we have defined V and C, taking derivatives here gives the typical first-order conditions. Note that this is not maximizing the true surplus necessarily. The true equilibrium, however, is generated by taking derivatives, since we get back to the true demand and supply curves. To see this, we can rewrite this problem as

$$\max \int_0^\infty e^{-rt}\left[V\big(K(t)\big) - C\big(\delta K(t) + \dot{K}(t)\big)\right]dt$$

Since $\dot{K} = -\delta K + I$. Note that, when we take the derivative here, we are not choosing both K and $\dot{K}$. If we know the initial level of capital, for example, as well as $\dot{K}$ for every period, then we also know K in every period. Consider the experiment where we take a proposed path $K^*(t)$ for capital and from there consider changing $\dot{K}$ for one period and then maintaining that additional capital in future periods—that is, we change $\dot{K}$ in one period but fix $\dot{K}$ in future periods, as illustrated in Figure 17-1.

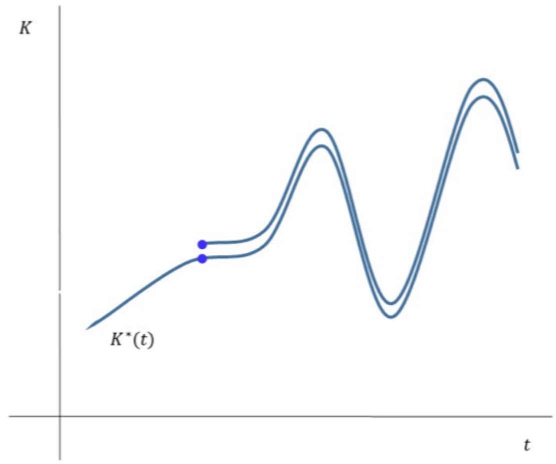

Figure 17-1: Increasing $\dot{K}$ for one period and maintaining that additional capital over the equilibrium path, $K^*(t)$, for the remaining periods.

In this experiment we are maintaining the same increment to capital after date t, which means that gross investment has to be incremented enough. If we have capital on an optimal path, then this experiment – for that matter, any deviation from the path – cannot add to our objective. In other words, the derivative of the objective is zero:

$$C'(t) = \int_t^{\infty} e^{-r(\tau-t)}[V'(\tau) - \delta C'(\tau)]d\tau$$

where we have condensed the notation and replaced each function's argument with just the time index for the argument.[41]

This is just the fact that marginal cost of investing must be the same as the present value, discounted at rate r, of having a bit more capital at all future dates. We typically refer to $[V'(\tau) - \delta C'(\tau)]$ as the net return on capital, so the right-hand side is the present discounted value of the net return on capital.

Alternatively, we could consider investing in one more unit of capital in a given period but then simply letting it depreciate. This means changing $\dot{K}$ for one period and then holding I fixed at all other points in time. This experiment also cannot add to the objective:

$$MC = C'(t) = \int_t^{\infty} e^{-(r+\delta)(\tau-t)}V'(\tau)d\tau$$

Where we are now discounting at rate $r + \delta$ because this experiment involves adding smaller bits to capital at the dates more distant in the future. An optimal path not only involves marginal cost equal to the (r-discounted) present value of the future net returns on capital, but it also involves marginal cost equal to the (r+δ-discounted) present value of the future gross returns on capital.

Both of these approaches are equally valid for modeling the world, but be careful to be consistent! Many people often confuse gross and net returns when working with these problems.

[41] For example, $C'(t)$ refers to $C'\big(\delta K(t) + \dot{K}(t)\big)$.

Adjustment Costs Applied to Net Investment

Prior to this, we have considered the approach using *gross* returns because we set marginal cost equal to the present value of rental payments discounted at $r + \delta$. Now, let's eliminate the assumption that the supply price of investment is rising—that is, before we had a rising short-run and long-run supply of capital. Let's consider an adjustment cost model. This can be combined with upward sloping supply, but for now let's leave supply perfectly elastic for simplicity. In other words,

$$C(I) = PI$$

In other words, the marginal cost of investment is P. In this world, our old model would give us a highly volatile capital stock. Here, however, we will impose a cost to changing the capital stock, $A(\dot{K}) = \frac{1}{2}A\dot{K}^2$. It is therefore costly to increase or to decrease capital. Convexity gives us that faster stock adjustments are more costly. Note that this may not fit reality well in many circumstances because it will give that many, tiny adjustments over time are preferred to few, large adjustments. One could consider incorporating a fixed cost of adjustment, but this does not make much sense in a continuous time model. Certainly, this assumption is often acceptable for macroeconomic situations, which aggregate micro-level facts in a way that creates a smooth result largely consistent with our continuous time analysis. As we will see, however, the long-run behavior of this model is very different than the one we just analyzed. Now, we solve

$$\max \int_0^\infty e^{-rt}\left[V\big(K(t)\big) - P\left(\delta K(t) + \dot{K}(t)\right) - \frac{A}{2}\left(\dot{K}(t)\right)^2\right]dt$$

As before, in Figure 17-1, consider perturbing $\dot{K}$ for one period and then holding it fixed in future periods. The first-order condition, differentiating with respect to $\dot{K}$ at date t, will be

$$-e^{-rt}\left(P + A\dot{K}(t)\right) + \int_t^\infty e^{-rt}(V'(\tau) - \delta P)d\tau = 0$$

So

$$MC = P + A\dot{K}(t) = \int_t^\infty e^{-r(\tau-t)}(V'(\tau) - \delta P)d\tau$$

Thus, investment is not immediate in this model because adjustment of the capital stock is costly. But now consider the steady state:

$$P = \frac{1}{r}(V' - \delta P)$$

Which implies

$$V' = rP + \delta P = (r + \delta)P$$

So in this model, though we have upward sloping supply of capital in the short run, there is perfectly elastic supply of capital in the long run. The structural difference from before is that, while we had a rising supply of gross investment, now we only have a rising supply of net investment. That is, the adjustment cost function is a function of $\dot{K}$, not $\dot{K} + \delta K$.

Endogenous Interest Rates: the Neoclassical Growth Model

Now, we will consider a third model. We will have a single good economy, where that good can be consumed or invested:

$$Y = C + I$$

$$Y = F(K)$$

So Y is produced using K, and Y is either consumed or invested. This formulation also means that C and I are perfect substitutes. There is no rising supply curve or adjustment costs here.

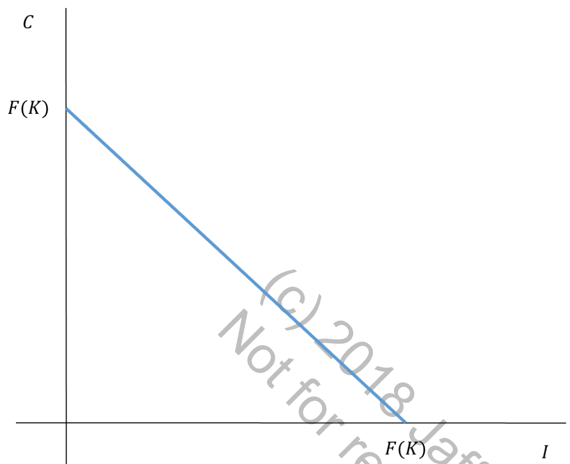

Figure 17-2: The production possibility frontier. C and I are perfect substitutes, but their combination must equal the total amount produced $F(K)$.

Now, the cost of increasing investment will be sacrificing consumption. Since we want to smooth consumption over time, we will in effect have a rising supply price. The consumer will solve

$$\max \int_0^\infty e^{-\rho t} U(C(t))dt$$

Where $C(t) = F\big(K(t)\big) - I(t)$ and ρ is the rate of time preference. Thus, $C(t) = F\big(K(t)\big) - \delta K(t) - \dot{K}(t)$. Rewriting the problem, we have

$$\max \int_0^\infty e^{-\rho t} U(F\big(K(t)\big) - \delta K(t) - \dot{K}(t))dt$$

Holding future $\dot{K}$ constant and increasing it today, we get the first-order condition

$$-e^{-\rho t}U'(t) + \int_t^\infty e^{-\rho \tau} U'(\tau)[F'K(\tau) - \delta]d\tau = 0$$

Or

$$1 = \int_t^\infty e^{-\rho(\tau - t)} \frac{U'(\tau)}{U'(t)} [F'K(\tau) - \delta]dt = \int_t^\infty e^{-\int_t^\tau r(s)ds}[F'K(\tau) - \delta]d\tau$$

The right-hand side is the market value of the future net returns on capital. The left-hand side is simply the number 1 because the cost of capital is 1 unit of consumption. This is a simple, consumption-based valuation of capital.

The dynamics will come from the fact that people do not want to starve themselves to build up capital quickly. Let's consider the steady state, where $U'(\tau) = U'(t)$:

$$\frac{F'(K) - \delta}{\rho} = 1$$

Which implies

$$F'(K) = \rho + \delta$$

Thus, in steady state, the marginal product of capital only depends on two numbers, ρ and δ. Now suppose, in steady state, we change technology by doubling the output we get per unit of capital. That is, we double F. The results of this experiment are depicted in Figure 17-3. The marginal product of capital will increase today, but we know it must remain the same in steady state, since ρ and δ are constant. Thus it falls toward the new steady state, which means capital is rising over time. Since F doubles, we also have that Y jumps. It pays to invest, since $F'(K)$ is above the steady state, so investment jumps. But we do not know what will happen to C in the short run. On the one hand, the agent is richer and will want to increase consumption so as to smooth it over time. On the other, it really pays to invest, since the marginal product of capital is so high, so consuming today is expensive. Consumption may increase or decrease in the short run, but we know it will be higher in the long run. To see this, differentiate the first-order condition with respect to time to get

$$\dot{C}(t) = -\frac{U'\big(C(t)\big)}{U''\big(C(t)\big)}\big(F'(K(t)) - (\rho + \delta)\big)$$

Thus, since $U' > 0$, $U'' < 0$, and $F'(K) > \rho + \delta$, we get that $\dot{C}(t) > 0$. Whether consumption increases or decreases in the short run, however, depends on the curvature of the utility function.

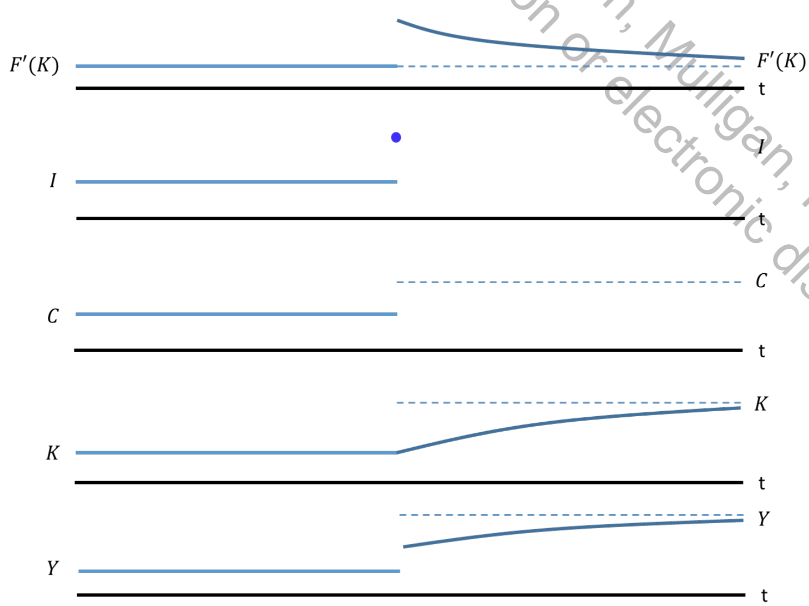

Figure 17-3: The effect of doubling F.

This model captures the essence of the neoclassical growth model. In this case, we have perfectly elastic long run supply because the rate of time preference is constant. What happens, however, if the rate of time preference falls as people become richer? Now, long run supply will actually be downward sloping. In the experiment depicted in Figure 17-3, if, as we become richer, we become more patient, the steady state level of capital will be above the original steady state level after we double F.

This model is very much like the adjustment cost model. The adjustment cost here is the fact that adjusting capital quickly may mean drastically altering consumption. To see this another way, note that the interest rate in this model is endogenous. The interest rate r between t and τ is

$$e^{-r(\tau-t)} = e^{-\rho(\tau-t)} \frac{U'(\tau)}{U'(t)}$$

The more one invests, the higher future consumption is relative to present consumption. Thus, the marginal utility from additional consumption in period τ is reduced relative to the marginal utility received today, driving the interest rate up. This dynamic prevents the economy from reaching the new equilibrium right away.

If ρ is a constant, then the long-run supply of capital curve is horizontal. Otherwise the supply curve could slope upward or downward depending on whether people become more patient or less patient as they become richer (i.e, that ρ depends on the level of consumption or income).

Chapter 18 Applied Factor Supply and Demand 1: Technological Progress and Capital-income Tax Incidence

Definitions of Labor Productivity

Now we want to think about technological progress. Most generally, we think about technological progress in terms of productivity. We measure this commonly through labor productivity,

$$\frac{Y}{L} = \text{Real Output per Man Hour}$$

We also have the measure

$$\frac{W}{P} = MP_L = \text{Marginal Product of Labor}$$

These are the average and marginal products of labor, respectively. Finally, we can consider the ratio of these two numbers,

$$\frac{W/P}{Y/L} = \frac{WL}{PY} = S_L$$

The share of labor stays roughly constant over time if and only if both measures of productivity grow roughly at the same rate.

Explaining Economic Growth in the Presence of Complementarity

But what lies behind the growth in labor productivity and rising real wages? We have a few hypotheses here.

1. Capital deepening (K/L is rising).
2. Technological change (more output for the same inputs).
3. Human capital accumulation (perhaps people have more skills on average than they did before).
4. Better allocations (we use our resources better).

This fourth point tends to be considered on the macro level, whereas the first three can be considered on a micro level. Note further that it is difficult to consider these as independent explanatory factors. If I invent a new technology, for example, I am going to need capital to get my project going. Similarly, when we see $Y = F(K, L)$, we do not try to ask how much of the output came from capital vs. labor. We do ask, however, how much each gets rewarded in the market.

In short, there's complementarity between all of these factors. All of the decompositions of the growth that we will do depend on the path we take to get there. For example, suppose to get more output we had to build a road and a vehicle. If the vehicle arrives after the road is already there, it seems like the vehicle is the key factor for getting the output. If the opposite occurs, however, the road looks like the key factor.

Take an output level $F(K, L)$. How we account for that output in terms of capital and labor depends on how we acquired those two factors. See Figure 18-1, which shows two paths. On the upper (lower) path, capital (labor) tended to be accumulated first, respectively. Due to the complementarity in production, the marginal product of labor is high along the upper path, so we would attribute a lot of the additions to output to additions to labor. The opposite happens on the lower path.[42]

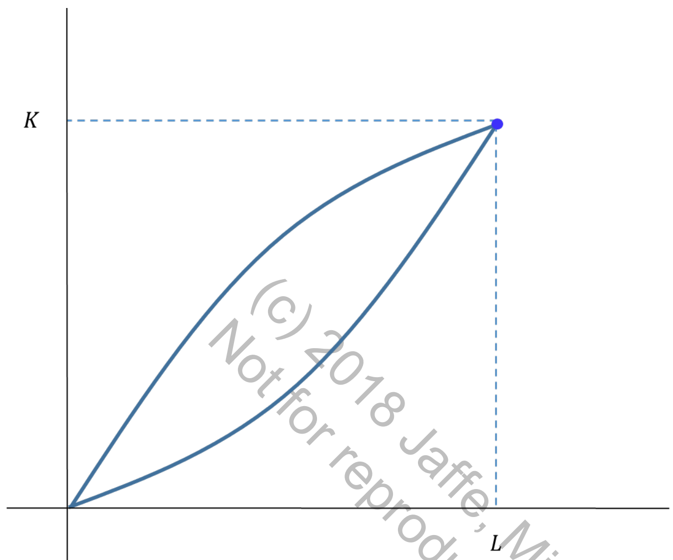

Figure 18-1: If I allocate along the margin according to the bottom path, there will be relatively abundant labor, and much of the output will be attributed to capital. If I allocate along the margin according to the top path, the opposite is true.

A cake, for example, can be changed by altering the amount of egg, flour, etc. on the margin. But in general, once the cake as a whole exists, we cannot decompose what "portion" of the cake comes from the egg, the flour, etc.

The Consequences of Unbiased Technological Change

Returning to our question of technological progress, for now we will focus on capital deepening and technological change. Note that $\Delta(Y/L) = \Delta Y - \Delta L$, where we are thinking about Δ as a percent change and make use of the log properties. The change in total factor productivity, TFP, will then be given by

$$\Delta TFP = \Delta Y - \Delta Total\ Input$$

$$\Delta TFP = \Delta Y - (S_L \Delta L + S_K \Delta K)$$

In the constant returns to scale case, $S_L + S_K = 1$, but CRS is not needed to formulate the general form of this equation given above. We can also measure TFP on the price side, just as we did with the real wage for labor productivity:

$$\Delta TFP = (S_L \Delta W + S_K \Delta R) - \Delta P$$

[42] The same issue arises in the utility context, which is why the Laspayres and Paasche quantity indices give different answers as to the change in utility.

This measure will include technological change, human capital change, and better allocations. Together we have that TFP growth reflects both output growing faster than inputs and factor prices growing faster than output prices.

For now, we are simply assuming growth is driven just by capital deepening and technological change. Then

$$\Delta Y - \Delta L = S_K(\Delta K - \Delta L) + \Delta TFP$$

Thus, growth in labor productivity can be decomposed into capital deepening, $S_K(\Delta K - \Delta L)$, and technological progress, ΔTFP. These are both related, however. In the neoclassical growth model, technological growth from increasing the productivity of capital would induce capital deepening.

The price-based measure of TFP can also be rewritten

$$\Delta TFP = S_L \Delta \frac{W}{P} + S_K \Delta \frac{R}{P}$$

If both real wages and the real rental rate on capital are growing, then we must have technological progress.

What would the neoclassical growth model say if we have a large sudden improvement in technology, such as $\Delta TFP = \Delta \frac{W}{P} = \Delta \frac{R}{P} = 20\%$? The economy will respond with capital deepening. Recall the elasticity of substitution:

$$\Delta \frac{W}{P} - \Delta \frac{R}{P} = \Delta W - \Delta R = \frac{1}{\sigma}(\Delta K - \Delta L)$$

In the short run, capital does not deepen (by definition) and the two real rental rates increase by the same percentage. As capital is accumulated, $\Delta(W/P)$ increases further and $\Delta(R/P)$ decreases. In the long run, we will have $\Delta \frac{W}{P} = \frac{\Delta TFP}{S_L}$. That is, all of the benefit of the TFP growth will accrue to the real wage. We also have that $\frac{S_L}{S_K} = \frac{WL}{RK} = \frac{\frac{W}{R}}{\frac{K}{L}}$, but we don't know the way that this is moving, because both $\frac{W}{R}$ and $\frac{K}{L}$ are moving upwards.

We arrived that these long-run results with a supply and demand framework. On the factor-supply side, we said capital is perfectly elastically supplied ($\Delta R = 0$), and that labor is not ($\Delta W \neq 0$). From there the price-based TFP measure immediately gives us that all of TFP goes to wages. When we add the factor demand curves – specifically their differences in log-linear form – we additionally find that TFP increases the capital-labor ratio, with a magnitude that increases with the elasticity of substitution σ.

The Incidence of a Capital-income Tax

We can augment this framework to address a number of other growth questions, but already it tells us about the incidence of a capital-income tax, which is a fraction $\tau \in (0,1)$ of capital income RK that is paid to the government with the revenue distributed equally to all of the owners of labor. With such a tax in place, perfectly elastic capital supply means that $\Delta R + \Delta(1-\tau) = 0$ because, by definition, no finite amount of capital is supplied unless the after-tax rental rate is the same as it would be without the tax.

Let's hold TFP constant while we focus on the tax. From our price-based definition of TFP, we have that $0 = S_L \Delta W + S_K \Delta R$. This and the supply condition give us

$$\Delta W = \frac{S_K}{S_L} \Delta(1-\tau) < 0$$

where we are considering a change from a low tax rate to a higher one. In other words, the capital-income tax reduces wages in the long run. The magnitude of this effect is decreasing in labor's share, which is interesting because labor's share has fallen over the past two decades. The long-run wage-impact of capital taxation may be more negative than it used to be.

Bringing in the factor-demand equations, we have the amount that the tax reduces capital intensity, and therefore average labor productivity:

$$\Delta K - \Delta L = \sigma(\Delta W - \Delta R) = -\Delta R \sigma \left(\frac{S_K}{S_L} + 1\right) = \sigma \frac{\Delta(1-\tau)}{S_L} < 0$$

$$\Delta Y - \Delta L = S_K(\Delta K - \Delta L) = \frac{S_K}{S_L} \sigma \Delta(1-\tau) < 0$$

The workers are not only getting their wage, but also the revenue from the tax. Holding labor fixed, what we have above is enough to prove that workers lose more in wages than they gain in tax revenue from a marginal increase in the tax rate. As a share of aggregate income, the increase in workers' post-fisc income is:[43]

$$S_L \Delta(WL) + \tau S_K \Delta(\tau RK)$$

Recall that Δ denotes log changes: the first (second) Δ term is multiplied by labor income (tax revenue τS_K) so that it is an absolute change, respectively. Using $\Delta L = 0$ and the results above for capital and wages, we have that workers' post-fisc income falls with the tax; the first term is more negative that the second term is positive (if at all).[44]

These results are easy to see in the capital demand diagram Figure 18-2. For any given capital stock K, aggregate output, and therefore aggregate income, is the area under the demand curve to the left of that amount of capital.[45] The equilibrium capital-rental rate R/P vertically divides that area between labor income (above) and capital income (below), both before taxes and transfers. The capital income tax increases R/P and therefore reduces labor income.[46]

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[43] Post-fisc income refers to own income minus taxes plus subsidies. In this case, workers pay no taxes but receive the revenue from the capital income tax.
[44] The increase in workers' post-fisc income can also be written as sum of five terms: $S_L(\Delta W + \Delta L) + \tau S_K(\Delta\tau + \Delta R + \Delta K)$. The second term is assumed to be zero. Substituting with the above equations for ΔW, ΔR and ΔK, we have $S_K \Delta(1-\tau) + \tau S_K \left(\Delta\tau - \Delta(1-\tau) + \sigma \frac{\Delta(1-\tau)}{S_L}\right) = \tau S_K \sigma \frac{\Delta(1-\tau)}{S_L} < 0$.
[45] This assumes $F(0, L) = 0$. The results are the same if we allow $F(0, L) \neq 0$, but then we would have to track that amount. Also note that the capital demand curve here holds labor and technology constant, and not output.
[46] R/P reduces both the base and the height of the labor-income triangle shown in the figure.

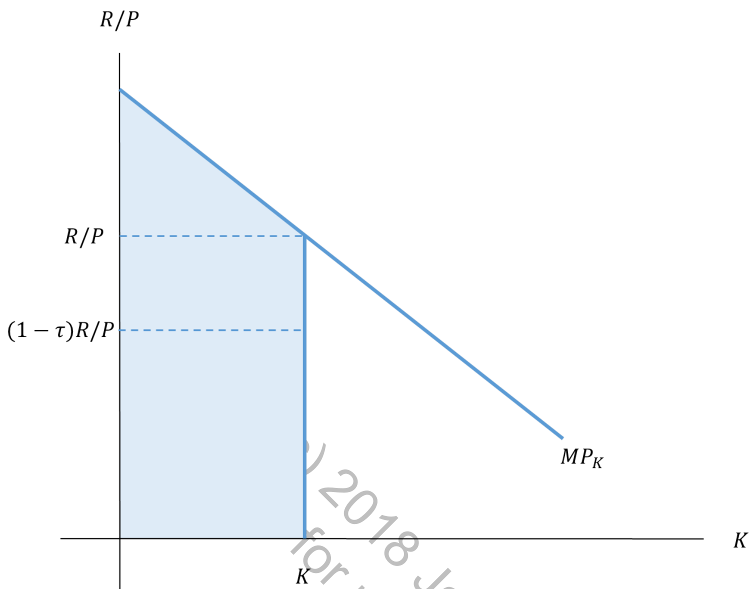

Figure 18-2: Output and factor incomes as areas under the marginal product schedule.

The pre-tax capital income rectangle is further divided in two at the height of the after-tax equilibrium rental rate $(1-\tau)R/P$, with tax payments above and after-tax capital income below. With perfectly elastic capital supply, the after-tax rental rate is independent of the tax rate, so that the tax serves to reduce the sum of labor income and tax revenue by raising R/P and moving the economy up the marginal product schedule. Even when labor receives all of the revenue, they still pay the deadweight loss of the tax: the extra output that additional capital would add beyond what it costs to supply that capital.

Why Capital is Elastically Supplied in the Long Run

The supply of capital reflects its marginal cost, which itself reflects (i) the willingness of people to delay consumption and (ii) the ability of producers to make investment goods rather than consumption goods. In the long run, consumption and investment are constant; whether they are constant at a high level or a low level depends on technology, taxes, etc. But the level of consumption can affect (i) and (ii) in either direction. For example, people might be more patient when they are richer (higher consumption levels), which by definition means that capital is less costly to supply. In other words, this example has the supply of capital sloping down in its rental rate. On point (ii), the production of investment goods could be more capital intensive than consumption goods, which means that the economy with more capital in the long run supplies capital at a lower marginal cost in terms of foregone consumption.

The neoclassical growth model is neutral on points (i) and (ii). Its long-run capital-supply curve therefore neither slopes down or up: it is horizontal. A horizontal long-run supply curve was the basis for this chapter's long-run analysis of the incidence of productivity and capital taxes.

The Incidence of a Corporate-income Tax

A corporate-income tax is different from a capital-income tax because the non-corporate capital does not pay corporate-income taxes. We address this by modifying the production function a bit so that it is $F(K, L)$ where K is now a homogeneous aggregate of two capital inputs: $G(K_1, K_2)$. This two-capital-input approach allows us to recognize that the effects of the corporate tax is different on corporate and non-corporate capital. Assume also that output can be used for consumption or either type of investment: $Y = C + I_1 + I_2$. Investment can go into either sector but the types of investment are perfect substitutes in terms of foregone consumption.

Because the aggregate is homogenous, we can focus on the unit isoquant, which are the combinations of K_1 and K_2 that result in exactly one unit of aggregate capital K. See Figure 18-3. The cost of any one of those possibilities in terms of foregone consumption is where the line with slope -1 intersects the vertical axis.

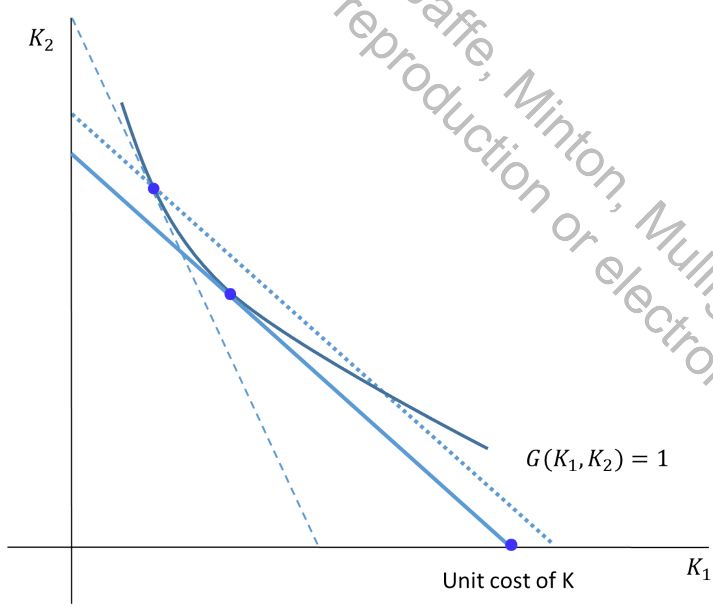

Figure 18-3: Using the unit isoquant to measure the cost of sectoral distortions.

The social cost is minimized where the isoquant also has slope -1, as shown in the figure. But when K_1 is taxed more than K_2, the investors will not minimize social cost because they want to avoid taxes too. Instead they have too little K_1. This raises the cost of each unit of K output by the vertical distance between the solid line and the dotted line.

Now let's go back to Figure 18-2. Before we said that the after-tax return on K is fixed, so that each unit of taxation increased the pretax rental rate R/P one-for-one. But now the after-tax return on K must also increase because it takes more investment to get each unit of K. See Figure 18-4. The loss to workers is not only the trapezoid *abcd* between R/P and $(1-\tau)R/P$, but also the trapezoid *cdef* between the new after-tax return and the old one.

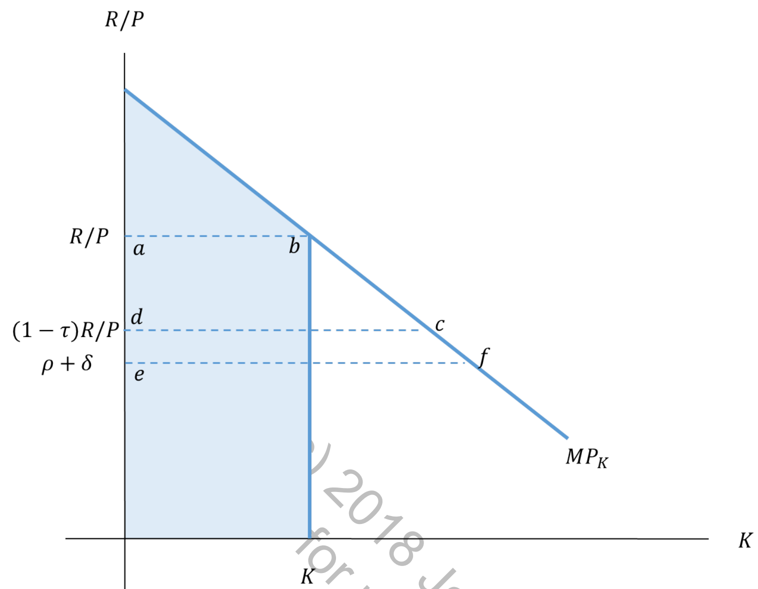

Figure 18-4: Labor's extra loss from distorting the composition of investment.

In other words, holding tax revenue constant, labor loses more when that revenue is raised by taxing just some of the capital rather than all of it.

Chapter 19 Applied Factor Supply and Demand 2: Factor-biased Technological Progress, Factor Shares and the Malthusian Economy

The Definition of Technological Bias

To summarize: the neoclassical growth model has the following view. In the long-run, $\Delta\frac{W}{P} = \frac{\Delta TFP}{S_L}$ and $\Delta\frac{R}{P} = 0$. But now, consider what $\Delta\frac{K}{L}$ looks like in the long run. We have to think about a point we have neglected thus far: technical bias. Consider Figure 19-1. As technology improves, the isoquant shifts inward, since we can produce the same amount as before using fewer inputs. If the isoquant gets steeper as it shifts inwards, this is a bias towards labor. There are three ways to think about this:

1. At a fixed $\frac{L}{K}$, $\frac{W}{R}$ is rising.
2. At a fixed $\frac{W}{R}$, $\frac{L}{K}$ is rising.
3. More TFP at higher L/K.

All of these say the same thing: we are twisting the isoquant clockwise as we shift it inward.

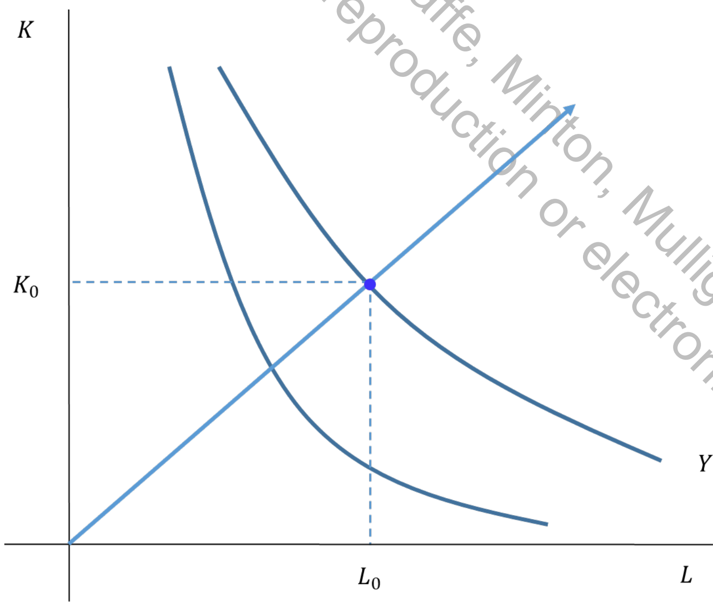

Figure 19-1: Technological bias towards labor.

How do we measure technical bias? $\Delta B = \Delta W - \Delta R - \frac{1}{\sigma}(\Delta K - \Delta L)$. It is again a residual. This is the difference between $\Delta\frac{W}{R}$ and what we predict this change will be using the elasticity of substitution (that is, if there is no technological bias). If we want to measure it using quantities, we can write $\Delta B = \Delta L - \Delta K - \sigma(\Delta R - \Delta W)$.

From now on, we assume all the variables are measured in real terms, so we will just write the change in the (log) real wage, for example, as ΔW. From last time, we related the change in total factor productivity to the (log) factor-price changes:

$$\Delta TFP = S_L \Delta W + S_K \Delta R$$

To interpret this, remember that $P = MC = \frac{Factor\ Price}{Marginal\ Product}$. If marginal productivity rises 10%, this allows factor prices to rise 10% and maintain the same level of output price. We do not know which factor prices will rise, however; so we cannot say whether higher TFP will be seen in wages or capital prices. Let us assume, in the short-run, that labor and capital are fixed. Then in the short-run, the increase in output will just be the increase in productivity.

In the previous chapter, we also related factor-price changes to factor-quantity changes using the two factor-demand curves. We now consider the more general case where the bias is not zero:

$$\Delta W - \Delta R = \frac{1}{\sigma}(\Delta K - \Delta L) + \Delta B$$

Some people will also talk about factor-augmenting technological progress, which is different from technological bias. In factor augmenting technological progress, $Y = F(A_L(t)L(t), A_K(t)K(t))$, where A_L and A_K are labor-augmenting and capital-augmenting technological progress, respectively. This doesn't even tell us the direction of technological bias. In Cobb-Douglas, for example, A and B are really the same thing, because we have

$$F\big(A_L(t)L(t), A_K(t)K(t)\big) = \big(A_L(t)L(t)\big)^{\alpha}\big(A_K(t)K(t)\big)^{1-\alpha} = A_L(t)^{\alpha}A_K(t)^{1-\alpha}L(t)^{\alpha}K(t)^{1-\alpha}$$

In general, the elasticity of substitution will determine which way A_L and A_K bias technological growth. If the elasticity of substitution is 1, they're both neutral.

Remember we can also measure technological progress from the quantities:

$$\Delta TFP = \Delta Y - (S_L\Delta L + S_K\Delta K)$$

In order to measure technological progress, we don't need to know much about the production function other than shares. Measuring technological bias is a different story, because it requires us to say something about σ, the elasticity of factor substitution in production. The bias can be signed, for example, if we know $\Delta W - \Delta R < 0$ and $\Delta K - \Delta L > 0$. In general, however, we will have to know the elasticity of substitution to measure technological bias.

Relating Labor's Share to Economic Growth

We have worked out three equations relating the percentage changes in different quantities and prices. These tell us the theoretically possible relationships between quantity changes, factor price changes, and technology changes. We apply them by taking special cases for factor supply, such as constant factor quantities, or a constant capital-rental rate, or Malthusian supply (a constant wage rate). This is the same supply and demand framework from the previous chapter, except now technological change does not have to increase the capital demand curve by the same percentage that it increases the labor demand curve.

Returning to our short run analysis (constant factor quantities), the factor-demand curves and TFP definition are:

$$\Delta W - \Delta R = \Delta B \rightarrow \Delta R = \Delta W - \Delta B$$

$$\Delta TFP = S_L\Delta W + S_K\Delta R$$

These imply that

$$\Delta TFP = S_L \Delta W + S_K(\Delta W - \Delta B)$$

Which simplifies to

$$\Delta W = \Delta TFP + S_K \Delta B$$

So the change in the real wage, in the short-run, will be affected by the technological progress as well as by the bias of the progress. Remember ΔB is defined as bias in favor of labor, so positive bias towards labor will mean the change in the wage is higher than it would be without the bias. What about the relative shares of labor and capital, $\frac{S_L}{S_K}$? By definition of shares,

$$\Delta \frac{S_L}{S_K} = (\Delta W + \Delta L) - (\Delta K + \Delta R)$$

Where we continue to use Δ to denote change in logs. In the short run, we have assumed $\Delta L = \Delta K = 0$, so

$$\Delta \frac{S_L}{S_K} = \Delta \frac{W}{R}$$

In general, the shares evolve according to

$$\Delta \frac{S_L}{S_K} = (\Delta W - \Delta R) - (\Delta K - \Delta L) = \left(\frac{1}{\sigma} - 1\right)(\Delta K - \Delta L) + \Delta B$$

where the second equality comes from the factor-demand curves. Factor shares were constant for a hundred years or so. During that "traditional" time frame, our model would be $\Delta S_L = 0, \Delta R = 0,$ which imply that $\Delta K = \Delta Y$ and $\Delta W = \frac{\Delta TFP}{S_L}$.

To give more detail, consider the following example. We start in period 0, where $\frac{Y}{L} = 1,\ L = \alpha$, $K = 1 - \alpha$, $W = 1$, and $R = 1$. Suppose, in period 1, we know that $\frac{Y}{L}$ will be 10. Then the traditional model tells us that $W_1 = 10W_0$, $Y_1 = 10Y_0$, $R_1 = R_0$, $K_1 = 10K_0$, and $L_1 = L_0$. Labor share remains constant because the wage has gone up ten times while the labor quantity has remained constant, and capital share has remained constant because the quantity of capital has gone up ten times while the rental rate has remained constant.

But what if we saw an economy, still with $\Delta R = 0$, where labor's share was falling? If $\sigma = 1$, then it must be that $\Delta B < 0$, or that we have bias towards capital. But if $\sigma > 1$, there is another way to explain a reduction in labor's share even without biased technical change: the increase the capital stock.

Consider Figure 19-2. Before any shift, capital gets paid its marginal product, and labor gets paid the remaining surplus. These are depicted by K and L in the figure. In the short run, the demand curve shifts up and the rental rate rises; that is, capital gets the increase in the height of the curve. In the long run, however, the rental rate will remain the same, and all of the benefits from additional capital and increased productivity will accrue to labor.

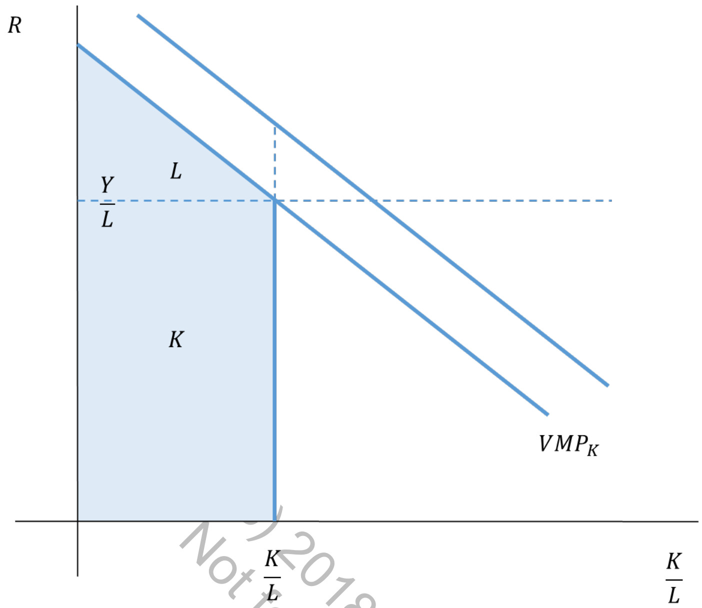

Figure 19-2: Short run vs. long run. The total shaded area denotes $\frac{Y}{L}$. The part K is paid to capital, and the part L is paid to labor. In the short run, capital receives the gains of the additional productivity through a higher rental rate. In the long run, however, the rental rate will remain constant, and all of the additional surplus will accrue to labor.

When $\sigma > 1$, capital accumulation does not so easily drive the capital rental rate back to where it was. We get more capital accumulation than we would with $\sigma = 1$. Labor benefits from this extra capital. In other words, in the long run labor gets all of the productivity growth (the area of the quadrilateral bounded by the two vertical lines and the two capital-demand curves) plus some of the additional output created by the extra capital (this part of labor's benefit is the triangle to the right of the vertical line and above the dashed horizontal line).

The irony here is that $\sigma > 1$ both confers extra benefit on labor while it reduces labor' share. Many people conclude that falling labor's share indicates harm to labor, but that conclusion is premature. It could be reflective of a capital deepening process that redistributes from capital to labor.

Combining some of our equations, we have

$$\Delta W = \Delta TFP + S_K[\frac{1}{\sigma}(\Delta K - \Delta L) + \Delta B]$$

This decomposes the wage change into a TFP component, a capital-deepening component, and a bias component, regardless of whether we are talking about short or long run.

Consider a world $\Delta B < 0$ and with $\sigma > 1$, so that technological progress is biased in favor of capital and capital substitutes more than one-for-one with labor. Looking back at the elasticity of substitution equation,

$$\Delta W - \Delta R = \frac{1}{\sigma}(\Delta K - \Delta L) + \Delta B$$

So if $\sigma > 1$, in order for wages to go up more than capital-rental rates – as they must in the long run – we will need capital deepening or a bias toward labor. If there's a bias in favor of capital to begin with, then we'll need even more capital deepening. There will be an elevated capital share in the short-run and an even larger increase in the capital share in the long-run. But is this bad for workers? No. The same process driving up capital share is driving up wages and driving down rental rates.

The Malthusian Special Case

Note that Malthus's model is a special case of our change equations. Instead of the long run fact that $\Delta R = 0$, however, he believed that $\Delta W = 0$. Because during his time, capital – largely land – was relatively inelastic, and labor supply, i.e. population, was very responsive to wage changes, he believed people would have more children in response to higher wages until the wage was driven back down to subsistence. In the Neoclassical growth model, we have the exact opposite fact that capital is perfectly elastically supplied in the long run, so that $\Delta R = 0$.

Two things changed after Malthus. Capital changed in that land is no longer as important a component of capital. The accumulation of people also changed; now, when people have more money they generally invest more in each child rather than having more children.

Capital-biased Technical Change Also Benefits Labor

Let's look at an extreme case where technical progress is biased rather extremely in favor of capital. In the worst case scenario, where a worker owns no capital at all, will that worker still be better off? The answer is no, in the short run, but yes, in the long run. Consider Figure 19-3. In the short run, the rise in productivity yields much higher rents for capital owners, and the amount paid to workers is decreased (the triangle for labor has the same base in the short run, but, due to the bias, a shorter height). In the long run, however, workers gain significantly more surplus than they used to have.

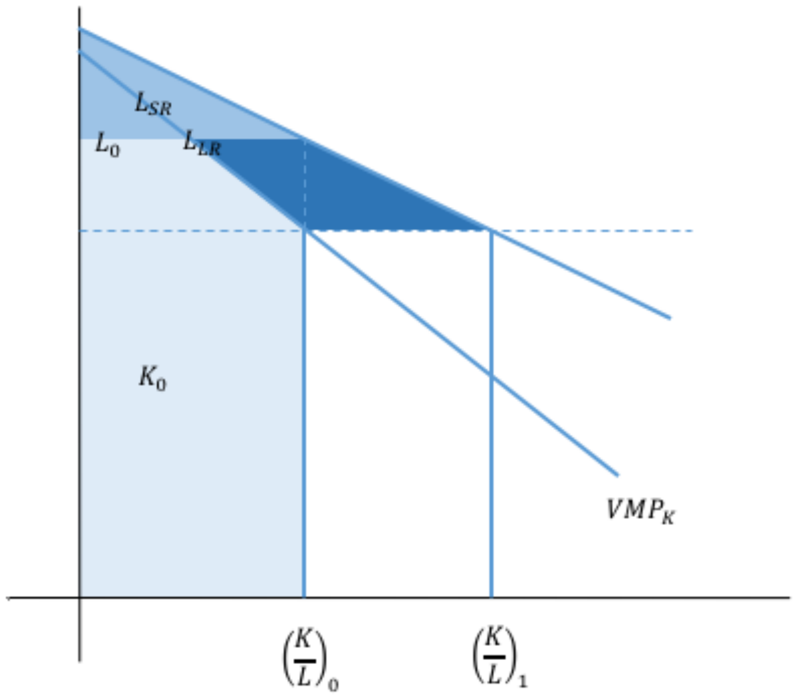

Figure 19-3: In period 0, labor earns the triangle labeled L_0 formed by the dotted horizontal line, the VMP_K line, and the y-axis. When productivity is increased, the surplus accruing to labor shrinks in the short run and is given by the area colored by the middle shade of blue and labeled L_{SR}. In the long run, the full triangle labeled by L_{LR} accrues to labor.

As with a lot of share phenomena in economics – consumer expenditure shares are another example – we need to know whether prices are increasing the share or some quantity change is doing it.

Now we'll suppose a case more favorable to workers. Suppose we have extreme technological bias toward workers. Consider Figure 19-4. Now, workers gain all of the surplus in the short run, but there is no capital deepening response that gives them additional surplus in the long run.

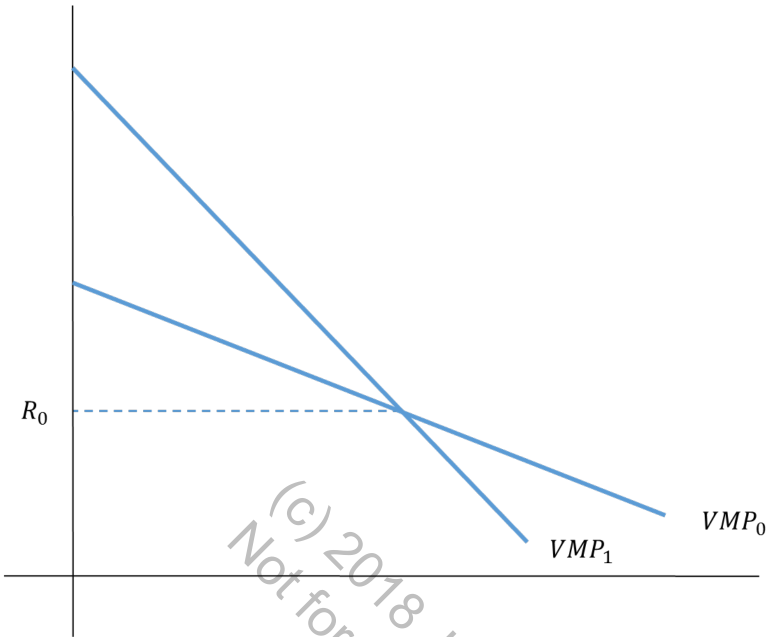

Figure 19-4: Technological bias in favor of workers. Here, there will be no capital deepening.

Adding Human Capital

Now, we are going to add human capital to this model. We'll have three inputs: L_H, high skilled labor, L_L, low skilled labor, and K, physical capital. We will have 3 associated prices, W_H, W_L, and R, respectively. We want to consider the process of long run growth. For this, we will want to consider technological growth (TFP increasing), capital deepening ($\frac{K}{L}$ increasing) and human capital deepening ($\frac{L_H}{L_L}$ increasing).

Note there are several dimensions to L_H. It is useful to think about $L_H = N_H H_H t_H$, or the idea that the amount of high skilled labor is the number of high skilled workers multiplied by their human capital and the amount of time they spend working. Defining L_L similarly, we get that

$$\frac{L_H}{L_L} = \left(\frac{N_H}{N_L}\right)\left(\frac{H_H}{H_L}\right)\left(\frac{t_H}{t_L}\right)$$

The first term is the extensive margin of human capital investment; that is, we look at entry into high skilled labor vs. low skilled labor. The second term is the intensive margin of human capital investment. We ask: within categories, how much human capital do the workers have?

We will assume technology is biased toward L_H relative to L_L, and that K is a substitute for L_L and a complement for L_H. As capital grows, it raises the relative demand for high skilled labor and lowers the relative demand for low skilled labor.

	Growth $\left(\frac{Y}{L}\right)$	$\frac{W_H}{W_L}$
(1) Technological Growth	+	+
(2) $\frac{K}{L}$	+	+
(3) $\frac{L_H}{L_L}$	+	-