



电阻 R_5 中的电流等于 R_1R_2 的电流差，也等于 R_3R_4 的电流差：等式（一）

$$I_{r5} = \frac{G - U_1}{R_1} - \frac{U_1}{R_2} = \frac{U_2}{R_4} - \frac{G - U_2}{R_3} = \frac{U_1 - U_2}{R_5}$$

求解得：

$$I_{r5} = \frac{R_2R_3 - R_1R_4}{R_1R_2R_3 + R_1R_2R_4 + R_1R_3R_4 + R_1R_3R_5 + R_1R_4R_5 + R_2R_3R_4 + R_2R_3R_5 + R_2R_4R_5}$$

$$U_1 = \frac{R_1R_2R_4 + R_2R_3R_4 + R_2R_4R_5 + R_2R_3R_5}{R_1R_2R_3 + R_1R_2R_4 + R_1R_3R_4 + R_1R_3R_5 + R_1R_4R_5 + R_2R_3R_4 + R_2R_3R_5 + R_2R_4R_5}$$

$$U_2 = \frac{R_1R_2R_4 + R_2R_3R_4 + R_2R_4R_5 + R_1R_4R_5}{R_1R_2R_3 + R_1R_2R_4 + R_1R_3R_4 + R_1R_3R_5 + R_1R_4R_5 + R_2R_3R_4 + R_2R_3R_5 + R_2R_4R_5}$$

等式（一）变形：等式（二）

$$I_{r5} = \frac{G}{R_1} - U_1 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = -\frac{G}{R_3} + U_2 \left(\frac{1}{R_3} + \frac{1}{R_4} \right) = \frac{U_1 - U_2}{R_5}$$

$$G \left(\frac{1}{R_1} + \frac{1}{R_3} \right) = U_1 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) + U_2 \left(\frac{1}{R_3} + \frac{1}{R_4} \right)$$

电阻 R_5 不接入电桥时的电压为：

$$U'_1 = \frac{GR_2}{R_1 + R_2} \quad U'_2 = \frac{GR_4}{R_3 + R_4} \quad G = \frac{U'_1(R_1 + R_2)}{R_2} = \frac{U'_2(R_3 + R_4)}{R_4}$$

代入等式（一）：

$$I_{r5} = (U'_1 - U_1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = -(U'_2 - U_2) \left(\frac{1}{R_3} + \frac{1}{R_4} \right) = \frac{U_1 - U_2}{R_5}$$

有电压差值的比例关系：

$$\frac{(U'_2 - U_2)}{(U'_1 - U_1)} = -\frac{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}{\left(\frac{1}{R_3} + \frac{1}{R_4}\right)} = -\frac{\frac{1}{\left(\frac{1}{R_3} + \frac{1}{R_4}\right)}}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

再看 U_1 通过电容接入交流信号 ΔU_1 时的情况：

$$\begin{aligned} I''_{r5} &= \frac{G - (U_1 + \Delta U_1)}{R_1} - \frac{(U_1 + \Delta U_1)}{R_2} = \frac{(U_2 + \Delta U_2)}{R_4} - \frac{G - (U_2 + \Delta U_2)}{R_3} \\ &= \frac{(U_1 + \Delta U_1) - (U_2 + \Delta U_2)}{R_5} \end{aligned}$$

$$G\left(\frac{1}{R_1} + \frac{1}{R_3}\right) = U_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + U_2\left(\frac{1}{R_3} + \frac{1}{R_4}\right) + \Delta U_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \Delta U_2\left(\frac{1}{R_3} + \frac{1}{R_4}\right)$$

$$\frac{\Delta U_2}{\Delta U_1} = -\frac{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}{\left(\frac{1}{R_3} + \frac{1}{R_4}\right)} = -\frac{\frac{1}{\left(\frac{1}{R_3} + \frac{1}{R_4}\right)}}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

ΔU_2 与 ΔU_1 方向相反，它们的比值为 R_3R_4 并联电阻与 R_1R_2 并联电阻的阻值之比。如果 $R_1 = 1M\Omega$ ， $R_2 = 2M\Omega$ ， $R_3 = 20M\Omega$ ， $R_4 = 10M\Omega$ ， R_3R_4 端对 R_1R_2 端有 10 倍的电压放大。