Template Models

Overview

Sharing of both structure and the parameters across different template variables.

- **Template variable** $X(U_1...U_k)$ is instantiated multiple times. For example, the Grade variable for different class
- **Template Model** is languages that specify how (ground) variables inherit dependency model from template
- Dynamic Bayesian networks temporal data
- Object-relational models, like people, courses

Temporal Models

A system evolved over time.

Distributions over Trajectories

Since continued parameters are hard to deal with, we usually discretize time continued variable

- Pick time granularity Δ
- $X^{(t)}$ variable X at time $t\Delta$
- $X^{(t:t')} = \{X^{(t)}...X^{(t')}\}, (t \leq t')$

- Want to represent $P(X^{(t:t')})$ for any t and t'

Markov Assumption

For temporal case, we can draw joint distribution as $P(X^{(0:T)}) = P(X^{(0)}) \prod_{i=0}^{T-1} P(X^{(t+1)}|X^{(0:t)})$. This formula has no assumption. To calculate this we need to know all the state from 0 to $t-1$, which is computation-consuming.

Markov Assumption

Then we assume that once we know the present, we forget about past, and next state is only dependents on the current. Formally, $(X^{(t+1)} \perp X^{(0:t-1)}|X^t)$

Apply this assumption to the above equation, we have $P(X^{(0:T)}) = P(X^{(0)}) \prod_{i=0}^{T-1} P(X^{(t+1)}|X^{(t)})$.

Sometimes this assumption is too strong to the case, in order to fix it, we usually have two main methods:

- Enrich the state description

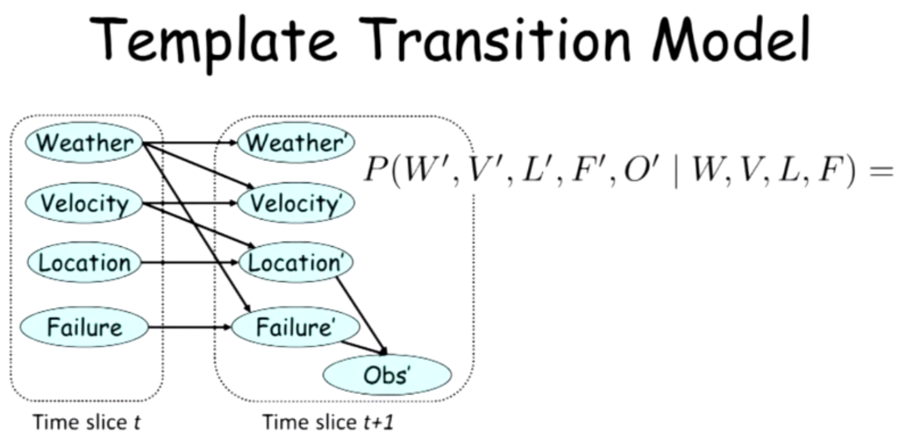
- Go further back in time, not just one time slice. This also called semi-Markov

Time Invariance

From Markov assumption, we can get a model template that assume model is duplicated for every single time point. $P(X'|X)$, where X' is the next time state and X is current state. The model won't be change along time.

Then for all t: $P(X^{(t+1)}|X^{(t)}) = P(X'|X)$

Template Transition Model



This graph describe a conditional distribution of time t given $t-1$. There are two types of edges in this graph:

- The edge across the time slice. This called **inter-time-slice**. And the edge from one variable to next state of it is called **persistence edge**, like the edge from Velocity to Velocity' in graph.
- Edge from RV to RV in same time slice. This called **intra-time-slice**. Which often refers to the events that happens rapidly.

2-time-slice Bayesian Network

A transition model(2TBN) over $X_1...X_n$ is specified as a BN fragment such that:

- The nodes include $X_1'...X_n'$ and a subset of $X_1...X_n$. Its subset since it only store the RVs that can directly affect the next state.
- Only the nodes $X_1'...X_n'$ have parents and a PCD

The 2TBN defines a conditional distribution $P(X'|X) = \prod_{i=1}^{n} P(X_i'|Pa_{X_i'})$

Dynamic Bayesian Network

A dynamic Bayesian network (DBN) over $X_1 ... X_n$ is defined by a

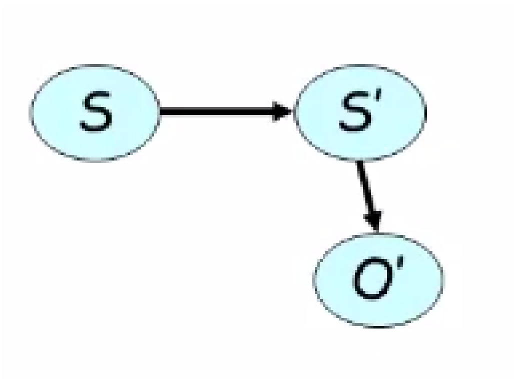
- 2TBN $BN_{\rightarrow}$ over $X'_1 ... X'_n$
- a Bayesian network $BN^{(0)}$ over $X_1^{(0)} ... X_n^{(0)}$. This is initial state

For a trajectory over $0, ..., T$ we define a ground (unrolled network) such that

- The dependency model for $X_1^{(0)} ... X_n^{(0)}$ is copied from $BN^{(0)}$.
- The dependency model for $X_1^{(t)} ... X_n^{(t)}$ for all $t >0 $ is copied from $BN_{\rightarrow}$

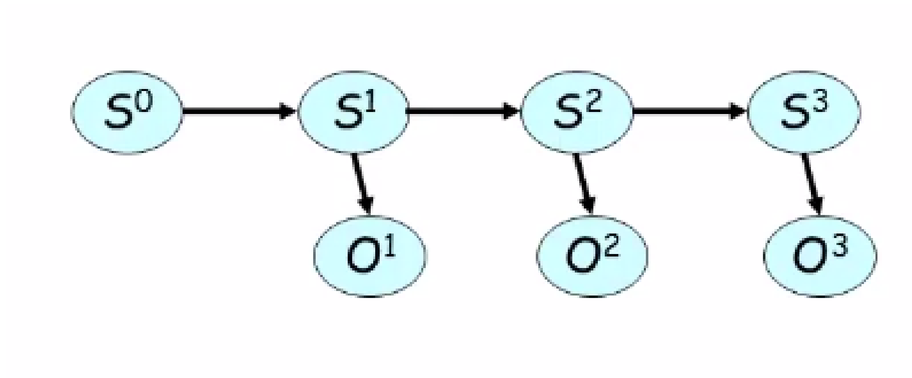
Hidden Markov Model

Hidden Markov Model can be simply viewed as probabilistic model that has a state variable S and a observation variable O. As following graph:



Therefore, we have a transition model that tell us how states change to next state and a observation model that can get observation we want from specific state.

Then we can unroll above 2TBN. We can get structure like following graph. It should be noticed that S_1 here is not random variable. It is assignment of S. The assignment of S might be change over time.



Summary

- HMMs can be viewd as a subclass of DBNs
- HMMs seems unstructured at the level of random variables.
- HMM structure typically manifests in sparsity and repeated elements within the transition matrix