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1 Basic

1.1 Default [055771]

```
#include <bits/stdc++.h>
using namespace std;
#define pb emplace_back
#define iter(x) (x).begin(), (x).end()
#define size(x) (int)(x).size()
#define INF 0x3f3f3f3f
#define lowbit(x) ((x) & -(x))
#define tmin(a, b) (a) = min((a), (b))
#define tmax(a, b) (a) = max((a), (b))
typedef long long ll;
typedef pair<int, int> pii;

void db() { cerr << endl; }
template <class T, class... U>
void db(T a, U... b) { cerr << a << " ", db(b...); }

signed main() {
    cin.tie(0) -> sync_with_stdio(0);
}

setxkbmap -option caps:swapescape #caps to esc
cpp file.cpp -dD -P -fpreprocessed |
tr -d '[:space:]' | md5sum | cut -c-6 #hash command
```

```
#!/usr/bin/bash
for ((i=0;;i++))
do
python gen.py > case.in
./A < case.in > aout
./B < case.in > bout

if ! (cmp -s aout bout);
then
cat case.in
fi

done
```

1.2 Pragma [0ca2dd]

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,arch=skylake")
__builtin_ia32_ldmxcsr(__builtin_ia32_stmxcsr() | 0x8040)
```

1.3 FastIO [11a75d]

```
inline char gc() {
    const static int SZ = 1 << 16;
    static char buf[SZ], *p1, *p2;
```

```
    if (p1 == p2 && (p2 =
        buf + fread(p1 = buf, 1, SZ, stdin), p1 == p2))
        return -1;
    return *p1++;
}

void rd() {}
template <typename T, typename... U>
void rd(T &x, U &...y) {
    x = 0;
    bool f = 0;
    char c = gc();
    while (!isdigit(c))
        f ^= !(c ^ 45), c = gc();
    while (isdigit(c))
        x = (x << 1) + (x << 3) + (c ^ 48), c = gc();
    f && (x = -x), rd(y...);
}

template <typename T>
void prt(T x) {
    if (x < 0)
        putchar('-'), x = -x;
    if (x > 9)
        prt(x / 10);
    putchar((x % 10) ^ 48);
}
```

2 Geometry

2.1 Point [4a7f8a]

```
struct P {
#define eps 1e-9
    double x, y;
    P() { x = y = 0; }
    P(double x, double y) : x(x), y(y) {}
    friend bool operator<(const P &a, const P &b)
        { return a.x == b.x ? a.y < b.y : a.x < b.x; }
    friend bool operator<=(const P &a, const P &b) {
        return a.x == b.x ? a.y <= b.y : a.x <= b.x; }
    friend bool operator==(const P &a, const
        P &b) { return a.x == b.x && a.y == b.y; }
    friend bool operator!=(const P &a, const
        P &b) { return a.x != b.x || a.y != b.y; }
    P operator+(const
        P &b) const { return P(x + b.x, y + b.y); }
    void operator+=(const P &b) { x += b.x, y += b.y; }
    P operator-(const
        P &b) const { return P(x - b.x, y - b.y); }
    void operator-=(const P &b) { x -= b.x, y -= b.y; }
    P operator
        *(double b) const { return P(x * b, y * b); }
    void operator*=(double b) { x *= b, y *= b; }
    P operator
        /(double b) const { return P(x / b, y / b); }
    void operator/=(double b) { x /= b, y /= b; }
    double operator*(
        const P &b) const { return x * b.x + y * b.y; }
    double operator^(
        const P &b) const { return x * b.y - y * b.x; }
    double lth() const { return sqrt(x * x + y * y); }
    double lth2() const { return x * x + y * y; }
    inline void print
        () { cout << '(' << x << ' ' << y << ')'; }
    friend istream &operator>>(
        istream &is, P &a) { return is >> a.x >> a.y; }
    friend ostream &operator<<(ostream &os,
        const P &a) { return os << a.x << ' ' << a.y; }
};

int ori(const P &a, const P &b, const P &c) {
    double k = (b - a) ^ (c - a);
    if (fabs(k) < eps)
        return 0;
    return k > 0 ? 1 : -1;
}
```

2.2 Polar Angle Vector [56401e]

```

P BASE(0, 0), BASEV(1, 0);
inline bool updown(const P &a) {
    int tmp = ori(BASE, BASE + BASEV, a);
    if (!tmp)
        return BASEV * (a - BASE) > 0;
    return tmp > 0;
}
bool cmp(const P &a, const P &b) {
    bool ba = updown(a), bb = updown(b);
    if (ba ^ bb)
        return ba;
    return ori(BASE, a, b) > 0;
}

```

2.3 Polar Angle Sort [05fa80]

```

P BASE(0, 0);
inline bool updown(const P &a) {
    if (fabs(a.y) < eps)
        return a.x > eps;
    return a.y > eps;
}
bool cmp(const P &a, const P &b) {
    bool ba = updown(a - BASE), bb = updown(b - BASE);
    if (ba ^ bb)
        return ba;
    return ori(BASE, a, b) > 0;
}

```

2.4 Intersections [c5adb9]

```

inline
    bool within(const P &a, const P &b, const P &c) {
        return (b - a) * (c - a) < eps;
    }
bool intersects
    (const P &a, const P &b, const P &c, const P &d) {
    int abc = ori(a, b, c);
    int abd = ori(a, b, d);
    int cda = ori(c, d, a);
    int cdb = ori(c, d, b);
    if (!abc && !abd)
        return within(a, c, d) || within(b, c, d) ||
            within(c, a, b) || within(d, a, b);
    return abc * abd <= 0 && cda * cdb <= 0;
}
P intersection
    (const P &a, const P &b, const P &c, const P &d) {
    double abc = (b - a) ^ (c - a);
    double abd = (b - a) ^ (d - a);
    return (d * abc - c * abd) / (abc - abd);
}

```

2.5 Convex Hull [f07084]

```

void fdhl(vector<P> &ar, vector<P> &hl) {
    for (int i = 0; i < 2; i++) {
        int prln = hl.size();
        for (int j = 0; j < size(ar); j++) {
            while
                (size(hl) - prln > 1 && ori(hl[size(hl) - 1], hl[size(hl) - 2], ar[j]) >= 0)
                hl.pop_back();
            hl.push_back(ar[j]);
        }
        if (hl.size() > 1)
            hl.pop_back();
        reverse(ar.begin(), ar.end());
    }
    if (hl.size() > 1 && hl.front() == hl.back())
        hl.pop_back();
}

```

2.6 In Convex Hull [e6722a]

```

bool in(const P &a, vector<P> &hl) {
    int ln = hl.size();
    if (ln == 1)
        return a == hl[0];
}

```

```

if (ln == 2)
    return within(a, hl[0], hl[1]);
int l = 1, r = ln - 1, m;
while (l < r - 1) {
    m = (l + r) >> 1;
    if (ori(hl[l], a, hl[m]) < 0)
        l = m;
    else
        r = m;
}
return ori(hl[l], hl[l], a) >= 0 && ori(hl[l], hl[r], a) >= 0 && ori(hl[r], hl[0], a) >= 0;
}

```

2.7 Rotation Sweep Line [9fd768]

```

void sweep(int n, vector<P> ar) {
    static const int N = 2005;
    static int id[N], po[N];
    static pii lr[N * N];
    int m = 0;
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            if (i != j)
                lr[m++] = pii(i, j);
    sort(lr, lr + m, [&](const pii &a, const pii &b) { return cmp(ar[a.first] - ar[a.second], ar[b.first] - ar[b.second]); });
    iota(id, id + n, 0);
    sort(id, id + n, [&](int a, int b) { return ar[a].y == ar[b].y ? ar[a].x < ar[b].x : ar[a].y < ar[b].y; });
    for (int i = 0; i < n; i++)
        po[id[i]] = i;
    for (int i = 0; i < m; i++) {
        swap(id[po[lr[i].first]], id[po[lr[i].second]]);
        swap(po[lr[i].first], po[lr[i].second]);
    }
}

```

2.8 Minkowski [542fed]

```

//need Geometry template
void reorder_polygon(vector<P> &pt) {
    int pos = 0;
    for (int i = 1; i < size(pt); i++) {
        if (pt[i].y < pt[pos].y || (pt[i].y == pt[pos].y && pt[i].x < pt[pos].x))
            pos = i;
    }
    rotate(pt.begin(), pt.begin() + pos, pt.end());
}
vector<P> minkowski(vector<P> A, vector<P> B) {
    reorder_polygon(A);
    reorder_polygon(B);
    A.push_back(A[0]); A.push_back(A[1]);
    B.push_back(B[0]); B.push_back(B[1]);
    vector<P> result;
    int i = 0, j = 0;
    while (i < size(A) - 2 || j < size(B) - 2) {
        result.push_back(A[i] + B[j]);
        auto cross
            = (A[i + 1] - A[i]) ^ (B[j + 1] - B[j]);
        if (cross >= 0 && i < size(A) - 2)
            ++i;
        if (cross <= 0 && j < size(B) - 2)
            ++j;
    }
    return result;
}

```

2.9 Minimum Enclosing Circle [1f681e]

```

P perp(const P &a) {
    return P(-a.y, a.x);
}
struct Circle {
    P o;
    double r;
}

```

```

    inline bool in(const P
        &a) const { return (a - o).lth() <= r + eps; }
};
Circle getcircle(const P &a, const P &b) {
    return Circle{(a + b) / 2, (a - b).lth() / 2};
}
Circle getcircle(const P &a, const P &b, const P &c) {
    const P p1 = (a + b) / 2, p2 = (a + c) / 2;
    Circle res;
    res.o = intersection
        (p1, p1 + perp(a - b), p2, p2 + perp(a - c));
    res.r = (res.o - a).lth();
    return res;
}
Circle findcircle(vector<P> &ar) {
    int n = size(ar);
    shuffle(iter(ar), mt19937(time(NULL)));
    Circle res = {ar[0], 0};
    for (int i = 0; i < n; i++) {
        if (res.in(ar[i]))
            continue;
        res = {ar[i], 0};
        for (int j = 0; j < i; j++) {
            if (res.in(ar[j]))
                continue;
            res = getcircle(ar[i], ar[j]);
            for (int k = 0; k < j; k++) {
                if (res.in(ar[k]))
                    continue;
                res = getcircle(ar[i], ar[j], ar[k]);
            }
        }
    }
    return res;
}

```

3 Graph

3.1 Block Cut [ac5ea2]

```

struct BCT {
    static const int N = 1e6 + 5; // change
    vector<int> e1[N], e2[N], sk; // e2 is the new tree
    int low[N], dfn[N], ctd = 0, ctn = 0;
    bool vs[N];
    void dfs(int u, int fa) {
        dfn[u] = low[u] = ++ctd;
        sk.pb(u), vs[u] = 1;
        for (int v : e1[u]) {
            if (v == fa)
                continue;
            if (dfn[v]) {
                if (vs[v])
                    tmin(low[u], dfn[v]);
                continue;
            }
            dfs(v, u), tmin(low[u], low[v]);
            if (low[v] >= dfn[u]) {
                e2[u].pb(++ctn);
                int x;
                do {
                    x = sk.back();
                    sk.pop_back();
                    e2[ctn].pb(x);
                } while (x != v);
            }
        }
        vs[u] = 0;
    }
    inline void addedge
        (int x, int y) { e1[x].pb(y), e1[y].pb(x); }
    inline void ini(int n, int rt) {
        for (int i = 1; i <= ctn; i++)
            e1[i].clear(), e2[i].clear();
        ctn = n, ctd = 0;
        memset(dfn + 1, 0, n << 2);
        memset(vs + 1, 0, n);
    }
}

```

```

        sk.clear();
        dfs(rt, -1);
    }
};

```

3.2 Centroid Decomp [314317]

```

struct CentroidDecomposition {
    vector<vector<int>> > g;
    vector<int> sub;
    vector<bool> v;
    vector<vector<int>> tree;
    int root;
    void add_edge(int a, int b) {
        g[a].push_back(b);
        g[b].push_back(a);
    }
    CentroidDecomposition(const vector<vector
        <int>> &g_, int isbuild = true) : g(g_) {
        sub.resize(size(g), 0);
        v.resize(size(g), false);
        if (isbuild) build();
    }
    void build() {
        tree.resize(size(g));
        root = build_dfs(0);
    }
    int get_size(int cur, int par) {
        sub[cur] = 1;
        for (auto &dst : g[cur]) {
            if (dst == par || v[dst]) continue;
            sub[cur] += get_size(dst, cur);
        }
        return sub[cur];
    }
    int get_centroid(int cur, int par, int mid) {
        for (auto &dst : g[cur]) {
            if (dst == par || v[dst]) continue;
            if (sub[dst] > mid
                ) return get_centroid(dst, cur, mid);
        }
        return cur;
    }
    int build_dfs(int cur) {
        int centroid = get_centroid
            (cur, -1, get_size(cur, -1) / 2);
        v[centroid] = true;
        for (auto &dst : g[centroid]) {
            if (!v[dst]) {
                int nxt = build_dfs(dst);
                if (centroid != nxt
                    ) tree[centroid].emplace_back(nxt);
            }
        }
        v[centroid] = false;
        return centroid;
    }
};

```

3.3 Dominater Tree [602345]

```

struct DOT {
    static const int N = 2e5 + 5; // change
    int dfn
        [N], id[N], dfc, fa[N], idm[N], sdm[N], bst[N];
    vector<int> G[N], rG[N];
    void ini(int n) { // remember to initialize
        for (int i = 1; i <= n; i++)
            G[i].clear(), rG[i].clear();
        fill(dfn, dfn + n + 1, 0);
    }
    inline void addedge
        (int u, int v) { G[u].pb(v), rG[v].pb(u); }
    int f(int x, int lm) {

```

```

    if (x <= lm)
        return x;
    int cr = f(fa[x], lm);
    if (sdm[bst[fa[x]]] < sdm[bst[x]])
        bst[x] = bst[fa[x]];
    return fa[x] = cr;
}
void dfs(int u) {
    id[dfn[u] = ++dfc] = u;
    for (int v : G[u])
        if (!dfn[v])
            dfs(v), fa[dfn[v]] = dfn[u];
}
void tar(vector<int> *eg, int rt) {
    dfc = 0, dfs(rt);
    for (int i = 1; i <= dfc; i++)
        sdm[i] = bst[i] = i;
    for (int i = dfc; i > 1; i--) {
        int u = id[i];
        for (int v : rG[u])
            if ((v = dfn[v]))
                f(v, i), tmin(sdm[i], sdm[bst[v]]);
        eg[sdm[i]].pb(i), u = fa[i];
        for (int v : eg[u])
            f(v, u), idm[v]
                = (sdm[bst[v]] == u ? u : bst[v]);
        eg[u].clear();
    }
    for (int i = 2; i <= dfc; i++) {
        if (sdm[i] != idm[i])
            idm[i] = idm[idm[i]];
        eg[id[idm[i]]].pb(id[i]);
    }
}
};

```

3.4 Matching [becd87]

```

struct Matching {
    static const int maxn
        = 505, p = (int)1e9 + 7; // change this, 1-base
    int size = 0;
    int sub_n = 0;
    int id[maxn], vertices[maxn], matches[maxn];
    bool row_marked
        [maxn] = {false}, col_marked[maxn] = {false};
    int A[maxn][maxn], B[maxn][maxn], t[maxn][maxn];
    vector<pair<int, int>> sidearr;
    void init(int _n) {
        size = _n;
        sub_n = 0;
        fill(id, id + _n + 1, 0);
        fill(vertices, vertices + _n + 1, 0);
        fill(matches, matches + _n + 1, 0);
        fill(row_marked, row_marked + _n + 1, 0);
        fill(col_marked, col_marked + _n + 1, 0);
        for (int i = 0; i <= _n; i++) {
            fill(A[i], A[i] + _n + 1, 0);
            fill(B[i], B[i] + _n + 1, 0);
            fill(t[i], t[i] + _n + 1, 0);
        }
        sidearr.clear();
    }
    Matching(int _n) {
        init(_n);
    }
    int qpow(int a, int b) {
        int ans = 1;
        while (b) {
            if (b & 1) ans = (long long)ans * a % p;
            a = (long long)a * a % p;
            b >>= 1;
        }
        return ans;
    }
    void Gauss(int A[][maxn], int B[][maxn], int n) {
        if (B) {

```

```

            memset(B, 0, sizeof(t));
            for (int i = 1; i <= n; i++) B[i][i] = 1;
        }
        for (int i = 1; i <= n; i++) {
            if (!A[i][i]) {
                for (int j = i + 1; j <= n; j++) {
                    if (A[j][i]) {
                        swap(id[i], id[j]);
                        for (int k = i; k <= n; k++)
                            swap(A[i][k], A[j][k]);
                        if (B
                            ) for (int k = 1; k <= n; k++)
                                swap(B[i][k], B[j][k]);
                        break;
                    }
                }
                if (!A[i][i]) continue;
            }
            int inv = qpow(A[i][i], p - 2);
            for (int j = 1; j <= n; j++) {
                if (i != j && A[j][i]) {
                    int t
                        = (long long)A[j][i] * inv % p;
                    for (int k = i; k <= n; k++)
                        if (A[i][k]) A[j][k] = (
                            A[j][k] - (ll)t * A[i][k]) % p;
                    if (B) {
                        for (int k = 1; k <= n; k++) if
                            (B[i][k]) B[j][k] = (B[j][
                                k] - (ll)t * B[i][k]) % p;
                    }
                }
            }
        }
        if (B) {
            for (int i = 1; i <= n; i++) {
                int inv = qpow(A[i][i], p - 2);
                for (int j = 1; j <= n; j++) {
                    if (B[i][j]) B[i][j]
                        = (long long)B[i][j] * inv % p;
                }
            }
        }
        void eliminate(int r, int c) {
            row_marked[r] = col_marked[c] = true;
            int inv = qpow(B[r][c], p - 2);
            for (int i = 1; i <= sub_n; i++) {
                if (!row_marked[i] && B[i][c]) {
                    int t = (long long)B[i][c] * inv % p;
                    for (int j = 1; j <= sub_n; j++)
                        if (!col_marked[j] && B[r][j])
                            B[i][j] = (B[i][j] - (
                                long long)t * B[r][j]) % p;
                }
            }
        }
        void add_edge(int a, int b) {
            sidearr.pb(min(a, b), max(a, b));
        }
        void build_matching() {
            auto rng = mt19937(chrono::steady_clock
                ::now().time_since_epoch().count());
            for (auto e : sidearr) {
                int x = e.first;
                int y = e.second;
                A[x][y] = rng() % p;
                A[y][x] = -A[x][y];
            }
            for (int i = 1; i <= size; i++) id[i] = i;
            memcpy(t, A, sizeof(t));
            Gauss(A, nullptr, size);
            for (int i = 1; i <= size; i++) {
                if (A[id[i]][id[i]]) vertices[++sub_n] = i;

```

```

    }
    for (int i = 1; i <= sub_n; i++) {
        for (int j = 1; j <= sub_n; j++)
            A[i][j] = t[vertices[i]][vertices[j]];
    }
    Gauss(A, B, sub_n);
    for (int i = 1; i <= sub_n; i++) {
        if (!matches[vertices[i]]) {
            for (int j = i + 1; j <= sub_n; j++) {
                if (!matches
                    [vertices[j]] && t[vertices
                    [i]][vertices[j]] && B[j][i]) {
                    matches[
                        vertices[i]] = vertices[j];
                    matches[
                        vertices[j]] = vertices[i];
                    eliminate(i, j);
                    eliminate(j, i);
                    break;
                }
            }
        }
    }
}

int matched(int x) {
    return matches[x];
}
};

```

3.5 Max Clique [2b1496]

```

struct MaxClique { // fast when N <= 100
    static const int N = 105;
    bitset<N> G[N], cs[N];
    int ans, sol[N], q, cur[N], d[N], n;
    MaxClique(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].reset();
    }
    void add_edge(int u, int v) {
        G[u][v] = G[v][u] = 1;
    }
    void pre_dfs(vector<int> &r, int l, bitset<N> mask) {
        if (l < 4) {
            for (int i : r) d[i] = (G[i] & mask).count();
            sort(r.begin(), r.end(), [&](int x, int y) { return d[x] > d[y]; });
        }
        vector<int> c(size(r));
        int lft = max(ans - q + 1, 1), rgt = 1, tp = 0;
        cs[1].reset(), cs[2].reset();
        for (int p : r) {
            int k = 1;
            while ((cs[k] & G[p]).any()) ++k;
            if (k > rgt) cs[++rgt + 1].reset();
            cs[k][p] = 1;
            if (k < lft) r[tp++] = p;
        }
        for (int k = lft; k <= rgt; ++k)
            for (int p = cs[k]._Find_first(); p < N; p = cs[k]._Find_next(p))
                r[tp] = p, c[tp] = k, ++tp;
        dfs(r, c, l + 1, mask);
    }
    void dfs(vector<
        int> &r, vector<int> &c, int l, bitset<N> mask) {
        while (!r.empty()) {
            int p = r.back();
            r.pop_back(), mask[p] = 0;
            if (q + c.back() <= ans) return;
            cur[q++] = p;
            vector<int> nr;
            for (int i : r) if (G[p][i]) nr.pb(i);
            if (!nr.empty()) pre_dfs(nr, l, mask & G[p]);
            else if (q > ans) ans = q, copy_n(cur, q, sol);
            c.pop_back(), --q;
        }
    }
};

```

```

    }
    int solve() {
        vector<int> r(n);
        ans = q = 0, iota(r.begin(), r.end(), 0);
        pre_dfs(r, 0, bitset<N>(string(n, '1')));
        return ans;
    }
};

```

3.6 Max Flow [f1e191]

```

struct FLOW {
    static const int N = 1e3 + 5, M = N * 10; // change
    int dp[N], cr[N], hd[N], ct = 2, s = 0, t = 1, n, flow;
    inline void ini(int _n) { n = _n; }
    struct E {
        int to, cap, nxt;
    } eg[M];
    inline void addedge(int u, int v, int w, int d=0) {
        eg[ct] = {v, w, hd[u]};
        hd[u] = ct++;
        eg[ct] = {u, d, hd[v]};
        hd[v] = ct++;
    }
    bool bfs() {
        memset(dp, INF, (n + 1) << 2);
        queue<int> q;
        q.push(s), dp[s] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int i = hd[u]; i; i = eg[i].nxt) {
                const int v = eg[i].to;
                if (!eg[i].cap || dp[u] + 1 >= dp[v])
                    continue;
                dp[v] = dp[u] + 1, q.push(v);
            }
        }
        return dp[t] != INF;
    }
    int dfs(int u, int fl) {
        if (u == t)
            return fl;
        int sm = 0;
        for (int &i = cr[u]; i; i = eg[i].nxt) {
            int v = eg[i].to, &w = eg[i].cap;
            if (!w || dp[u] + 1 != dp[v])
                continue;
            int tp = dfs(v, min(w, fl - sm));
            w -= tp, sm += tp, eg[i ^ 1].cap += tp;
            if (sm == fl)
                return fl;
        }
        return sm;
    }
    int getflow() {
        flow = 0;
        while (bfs())
            memcopy(cr,
                hd, (n + 1) << 2), flow += dfs(s, INF);
        return flow;
    }
};

```

3.7 MCMF [5190af]

```

struct MCMF {
    static const int N = 5005, M = N * 20;
    struct E {
        int to, cap, co, nxt;
    } eg[M];
    int dp[N], hd[
        N], cr[N], ct = 2, s = 0, t = 1, n, flow, cost;
    bool vd[N];
    inline void ini(int _n) { n = _n; }
    inline void addedge(int u, int v, int w, int c) {

```

```

    eg[ct] = {v, w, c, hd[u]};
    hd[u] = ct++;
    eg[ct] = {u, 0, -c, hd[v]};
    hd[v] = ct++;
}
bool spfa() {
    queue<int> q;
    memset(dp, INF, (n + 1) << 2);
    q.push(s), vd[s] = 1, dp[s] = 0;
    while (!q.empty()) {
        int u = q.front();
        q.pop(), vd[u] = 0;
        for (int i = hd[u]; i; i = eg[i].nxt) {
            const int v = eg[i].to, c = eg[i].co;
            if (!eg[i].cap || dp[u] + c >= dp[v])
                continue;
            dp[v] = dp[u] + c;
            if (!vd[v])
                vd[v] = 1, q.push(v);
        }
    }
    return dp[t] != INF;
}
int dfs(int u, int fl) {
    if (u == t)
        return fl;
    int sm = 0;
    vd[u] = 1;
    for (int &i = cr[u]; i; i = eg[i].nxt) {
        int &w =
            eg[i].cap, v = eg[i].to, c = eg[i].co;
        if (!w || vd[v] || dp[u] + c != dp[v])
            continue;
        int tp = dfs(v, min(fl - sm, w));
        w -= tp, eg[i] ^
            1].cap += tp, sm += tp, cost += tp * c;
    }
    vd[u] = 0;
    return sm;
}
pii getflow() {
    flow = cost = 0;
    while (spfa())
        memcpy(cr,
            hd, (n + 1) << 2), flow += dfs(s, INF);
    return pii(flow, cost);
}
};

```

3.8 Gomory-Hu Tree [ba0b07]

```

struct edge{
    int u,v,w;
};
vector<edge> result;
vector<edge> ed;
void GomoryHu(int N){
    vector<int> par(N,0);
    for(int i=1;i<N;i++){
        FLOW din;
        for(const auto &[u,v,c]:ed){
            din.addedge(u,v,c,c);
        }
        din.s = i;
        din.t = par[i];
        result.push_back({i,par[i],din.getflow()});
        for(int j=i+1;j<N;j++){
            if(par[j]==par[i] && din.dp[j]<INF) par[j]=i;
        }
    }
}

```

4 Math

4.1 Mod Int [41cafb]

```

template <unsigned P>
struct mint { // P not prime break /=

```

```

    unsigned v;
    mint(ll v = 0) : v(v % P) {}
    mint &operator+=(mint const &o) {
        v = (v += o.v) >= P ? v - P : v;
        return *this;
    }
    mint &operator-=(mint const &o) {
        v = (v < o.v) ? v + P - o.v : v - o.v;
        return *this;
    }
    mint &operator*=(mint const &o) {
        ll ret = ll(v) * ll
            (o.v) - P * ll(1.L / P * ll(v) * ll(o.v));
        v = ret + P * (ret < 0) - P * (ret >= (ll)P);
        return *this;
    }
    friend
        mint operator+(mint const &a, mint const &b) {
            return mint(a) += b;
        }
    friend
        mint operator-(mint const &a, mint const &b) {
            return mint(a) -= b;
        }
    friend
        mint operator*(mint const &a, mint const &b) {
            return mint(a) *= b;
        }
    mint pow(ll n) const {
        mint r(1);
        mint a = v;
        for (; n; a *= a, n >>= 1)
            r *= (n & 1) ? (a) : (mint(1));
        return r;
    }
    mint &operator/=(mint const &o) {
        *this *= o.pow(P - 2);
        return *this;
    }
    friend
        mint operator/(mint const &a, mint const &b) {
            return mint(a) /= b;
        }
    friend ostream
        &operator<<(ostream &os, mint const &m) {
            return os << m.v;
        }
    friend istream &operator>>(istream &is, mint &m) {
            return is >> m.v;
        }
};

```

4.2 Fraction [9c92bf]

```

struct frac {
    ll n, d;
    frac(const
        ll &n = 0, const ll &d = 1) : n(_n), d(_d) {
        ll t = __gcd(n, d);
        n /= t, d /= t;
        if (d < 0)
            n = -n, d = -d;
    }
    frac operator-() const {
        return frac(-n, d);
    }
    frac operator+(const frac &b) const {
        return frac(n * b.d + b.n * d, d * b.d);
    }
    void operator+=(const frac &b) {
        *this = frac(n * b.d + b.n * d, d * b.d);
    }
    frac operator-(const frac &b) const {
        return frac(n * b.d - b.n * d, d * b.d);
    }
    void operator-=(const frac &b) {
        *this = frac(n * b.d - b.n * d, d * b.d);
    }

```



```

}
frac operator*(const frac &b) const {
    return frac(n * b.n, d * b.d);
}
void operator*=(const frac &b) {
    *this = frac(n * b.n, d * b.d);
}
frac operator/(const frac &b) const {
    return frac(n * b.d, d * b.n);
}
void operator/=(const frac &b) {
    *this = frac(n * b.d, d * b.n);
}
friend ostream
    &operator<<(ostream &os, frac const &f) {
    if (f.d == 1)
        return os << f.n;
    return os << f.n << '/' << f.d;
}
friend istream &operator>>(istream &is, frac &f) {
    istream &tp = is >> f.n >> f.d;
    f = frac(f.n, f.d);
    return tp;
}
};

```

4.3 FFT [cefbf5]

```

typedef complex<double> cd;
struct FFT {
    // #define
    // M_PI 3.14159265358979323846264338327950288
    static const int K = 21, N = 1 << K; // change
    cd pl[N];
    int rv[N];
    void fft(vector<cd> &ar) {
        int n = size(ar), k = log2(n);
        if (n <= 1)
            return;
        for (int i = 1; i < n; i++)
            if (i < rv[i] >> (K - k))
                swap(ar[i], ar[rv[i] >> (K - k)]);
        cd a, b;
        for (int l = 1, p
            = 1 << (K - 1); l < n; l <= 1, p >>= 1) {
            for (int i = 0; i < n; i += l << 1) {
                for (int j = 0; j < l; j++) {
                    a = ar[i + j], b = ar[i + j + l];
                    ar[i + j] = a + b * pl[j * p];
                    ar[i + j
                        + l] = a + b * pl[(j + l) * p];
                }
            }
        }
    }
    void pmul
        (vector<cd> &a, vector<cd> &b, vector<cd> &c) {
        int n = size(a) + size(b) - 1;
        int pn = n;
        while (n & (n - 1))
            n += lowbit(n);
        a.resize(n), b.resize(n);
        fft(a), fft(b);
        c.resize(n);
        for (int i = 0; i < n; i++)
            c[-i & (n - 1)] = a[i] * b[i] / (double)n;
        fft(c), c.resize(pn);
    }
    FFT() {
        for (int i = 0; i < N; i++)
            pl[i] = polar(1.0, 2 * M_PI * i / N);
        for (int i = 1, hb = -1; i < N; i++) {
            if (!(i & (i - 1)))
                hb++;
            rv[i] =
                rv[i ^ (1 << hb)] | 1 << (K - 1 - hb);
        }
    }
};

```

```

};
}
4.4 NTT [08d19d]
struct NTT {
    static const int
        K = 21, N = 1 << K, M = 998244353; // change
    typedef mint<M> mi;
    mi pl[N];
    int rv[N];
    void ntt(vector<mi> &ar) {
        int n = size(ar), k = log2(n);
        if (n <= 1)
            return;
        for (int i = 1; i < n; i++)
            if (i < rv[i] >> (K - k))
                swap(ar[i], ar[rv[i] >> (K - k)]);
        mi a, b;
        for (int l = 1, p
            = 1 << (K - 1); l < n; l <= 1, p >>= 1) {
            for (int i = 0; i < n; i += l << 1) {
                for (int j = 0; j < l; j++) {
                    a = ar[i + j], b = ar[i + j + l];
                    ar[i + j] = a + b * pl[j * p];
                    ar[i + j
                        + l] = a + b * pl[(j + l) * p];
                }
            }
        }
    }
    void pmul
        (vector<mi> &a, vector<mi> &b, vector<mi> &c) {
        int n = size(a) + size(b) - 1;
        int pn = n;
        while (n & (n - 1))
            n += lowbit(n);
        a.resize(n), b.resize(n);
        ntt(a), ntt(b);
        c.resize(n);
        for (int i = 0; i < n; i++)
            c[-i & (n - 1)] = a[i] * b[i] / n;
        ntt(c), c.resize(pn);
    }
    NTT() {
        pl[0] = 1, pl[1] = mi(3).pow((M - 1) / N);
        for (int i = 2; i < N; i++)
            pl[i] = pl[i - 1] * pl[1];
        for (int i = 1, hb = -1; i < N; i++) {
            if (!(i & (i - 1)))
                hb++;
            rv[i] =
                rv[i ^ (1 << hb)] | 1 << (K - 1 - hb);
        }
    }
};

```

4.5 Big Integer [4347d6]

```

#undef size
template<typename T>
inline string to_string(const T& x) {
    stringstream ss;
    return ss<<x, ss.str();
}
struct bigN:vector<ll> {
    const static int base=1000000000, width=log10(base);
    bool negative;
    bigN(const_iterator
        a, const_iterator b):vector<ll>(a, b) {}
    bigN(string s) {
        if (s.empty()) return;
        if (s[0] == '-') negative=1, s=s.substr(1);
        else negative=0;
        for (int i=int(s.size())-1; i>=0; i-=width) {
            ll t=0;
            for (int j=max(0, i-width+1); j<=i; ++j)
                t=t*10+s[j]-'0';
        }
    }
};

```

```

    push_back(t);
}
trim();
}
template<typename T>
bigN(const T &x):bigN(to_string(x)){
bigN():negative(0){}
void trim(){
    while(size()&&!back())pop_back();
    if(empty())negative=0;
}
void carry(int _base=base){
    for(size_t i=0;i<size();++i){
        if(at(i)>=0&&at(i)<_base)continue;
        if(i+1u==size())push_back(0);
        int r=at(i)%_base;
        if(r<0)r+=_base;
        at(i+1)+=(at(i)-r)/_base,at(i)=r;
    }
}
int abscmp(const bigN &b)const{
    if(size()>b.size())return 1;
    if(size()<b.size())return -1;
    for(int i=int(size())-1;i>=0;--i){
        if(at(i)>b[i])return 1;
        if(at(i)<b[i])return -1;
    }
    return 0;
}
int cmp(const bigN &b)const{
    if(negative!=b.negative)return negative?-1:1;
    return negative?-abscmp(b):abscmp(b);
}
bool operator<(const bigN&b)const{return cmp(b)<0;}
bool operator>(const bigN&b)const{return cmp(b)>0;}
bool operator<=(const bigN&b)const{return cmp(b)<=0;}
bool operator>=(const bigN&b)const{return cmp(b)>=0;}
bool operator==(const bigN&b)const{return !cmp(b);}
bool operator!=(const bigN&b)const{return cmp(b)!=0;}
bigN abs()const{
    bigN res=*this;
    return res.negative=0, res;
}
bigN operator-()const{
    bigN res=*this;
    return res.negative=!negative,res.trim(),res;
}
bigN operator+(const bigN &b)const{
    if(negative)return -(-(*this)+(-b));
    if(b.negative)return *this-(-b);
    bigN res=*this;
    if(b.size()>size())res.resize(b.size());
    for(size_t i=0;i<b.size();++i)res[i]+=b[i];
    return res.carry(),res.trim(),res;
}
bigN operator-(const bigN &b)const{
    if(negative)return -(-(*this)-(-b));
    if(b.negative)return *this+(-b);
    if(abscmp(b)<0)return -(b-(*this));
    bigN res=*this;
    if(b.size()>size())res.resize(b.size());
    for(size_t i=0;i<b.size();++i)res[i]-=b[i];
    return res.carry(),res.trim(),res;
}
bigN operator*(const bigN &b)const{
    bigN res;
    res.negative=negative!=b.negative;
    res.resize(size()+b.size());
    for(size_t i=0;i<size();++i)
        for(size_t j=0;j<b.size();++j)
            if((res[i+j]+at(i)*b[j])>=base){
                res[i+j+1]+=res[i+j]/base;
                res[i+j]%=base;
            }
    return res.trim(),res;
}

```

```

}
bigN operator/(const bigN &b)const{
    int norm=base/(b.back()+1);
    bigN x=abs()*norm;
    bigN y=b.abs()*norm;
    bigN q,r;
    q.resize(x.size());
    for(int i=int(x.size())-1;i>=0;--i){
        r=r*base+x[i];
        int s1=r.size()<=y.size()?0:r[y.size()];
        int s2=r.size()<y.size()?0:r[y.size()-1];
        int d=(ll(base)*s1+s2)/y.back();
        r=r-y*d;
        while(r.negative)r=r+y,--d;
        q[i]=d;
    }
    q.negative=negative!=b.negative;
    return q.trim(),q;
}
bigN operator%(const bigN &b)const{
    return *this-(*/this/b)*b;
}
friend istream& operator>>(istream &ss, bigN &b){
    string s;
    return ss>>s, b=s, ss;
}
friend
    ostream& operator<<(ostream &ss, const bigN &b){
        if(b.negative)ss<<"-";
        ss<<(b.empty()?0:b.back());
        for(int i=int(b.size())-2;i>=0;--i)
            ss<<setw(width)<<setfill('0')<<b[i];
        return ss;
    }
template<typename T>
operator T(){
    stringstream ss;
    ss<<*this;
    T res;
    return ss>>res,res;
}
}
#define size(x) (int)(x).size()

```

4.6 Exgcd/CRT [f45f4d]

```

// find x,y such that ax+by=gcd(a,b)
ll exgcd(ll a, ll b, ll &x, ll &y) {
    if (!b) return x = 1, y = 0, a;
    ll d = exgcd(b, a % b, y, x);
    return y -= a/b * x, d;
}
ll CRT(int k, ll* a, ll* r) {
    ll n = 1, ans = 0;
    for (int i = 1; i <= k; i++) n = n * r[i];
    for (int i = 1; i <= k; i++) {
        ll m = n / r[i], b, y;
        exgcd(m, r[i], b, y); // b * m mod r[i] = 1
        ans = (ans + a[i] * m * b % n) % n;
    }
    return (ans % n + n) % n;
}
// not coprime
: x = m1p+a1 = m2q+a2 => m1p-m2q = a2-a1, use exgcd

```

4.7 Miller Rabin [67a711]

```

bool isPrime(const uint64_t n) {
    if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
    uint64_t A[] = {2,
        325, 9375, 28178, 450775, 9780504, 1795265022},
    s = __builtin_ctzll(n-1), D = n >> s;
    for (auto a : A) {
        uint64_t p = 1, g = a % n, i = s, d = 0;
        for (; d; g = __int128
            (g)*g%n, d/=2) if (d&1) p = __int128(p)*g%n;
        while (p != 1 && p
            != n - 1 && a % n && i--) p = __int128(p)*p%n;
    }
}

```



```

    if (p != n-1 && i != s) return 0;
}
return 1;
}

```

4.8 Pollard's Rho [8252e7]

```

ll PollardRho(
    ll x) { // get a factor of x(not prime) in O(x^0.25)
    ll s = 0, t = 0;
    ll c = (ll)rand() % (x - 1) + 1;
    int step = 0, g = 1;
    ll val = 1;
    for (g = 1;; g <= 1, s = t, val = 1) {
        for (step = 1; step <= g; ++step) {
            t = (__int128(t)*t+c)%x;
            val = __int128(val) * abs(t - s) % x;
            if ((step % 127) == 0) {
                ll d = __gcd(val, x);
                if (d > 1) return d;
            }
        }
        ll d = __gcd(val, x);
        if (d > 1) return d;
    }
}

```

4.9 FWT [02d887]

```

struct Fast_Walsh_Transform { // Modint needed
    string op; // and, or, xor
    void fwt(vector<mint> &v, bool ifwt) {
        int n = __lg(size(v));
        mint iv2 = mint(1) / 2;
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < 1 << n; ++j)
                if (op == "and" && (~j >> i & 1) || op == "or" && (j >> i & 1)) {
                    if (!ifwt)
                        v[j] += v[j ^ (1 << i)];
                    else
                        v[j] -= v[j ^ (1 << i)];
                } else
                    if (op == "xor" && (j >> i & 1)) {
                        mint x = v[j ^ (1 << i)], y = v[j];
                        if (!ifwt)
                            v[j ^ (1 << i)] = x + y, v[j] = x - y;
                        else
                            v[j ^ (1 << i)] = (x + y) * iv2, v[j] = (x - y) * iv2;
                    }
            }
        vector<mint> v1, v2; // size(v1) = size(v2) = 2^k
        Fast_Walsh_Transform(const vector<mint> &_v1, const vector<mint> &_v2, const string &_op) : v1(_v1), v2(_v2), op(_op) {}
        vector<mint> solve() { // ans_k = \sum_{i op j = k} a_i * b_j
            fwt(v1, 0), fwt(v2, 0);
            for (int i = 0; i < size(v1); ++i)
                v1[i] *= v2[i];
            fwt(v1, 1);
            return v1;
        }
    };
}

```

4.10 Theorem

- LTE Lemma $\nu_p(x^n - y^n) = \nu_p(x - y) + \nu_p(n)$, if $p|x - y$. $\nu_2(x^n - y^n) = \nu_2(x - y) + \nu_2(n)$, if $4|x - y$. $\nu_2(x^n - y^n) = \nu_2(x - y) + \nu_2(x + y) + \nu_2(n) - 1$, if $2|x - y$ and n is even. $\nu_p(x^n + y^n) = \nu_p(x + y) + \nu_p(n)$, if $p|x + y$ and n is odd. $\nu_2(x^n + y^n) = 1$, if $2|x + y$ and n is even. $\nu_2(x^n + y^n) = \nu_2(x + y)$ if $2|x + y$ and n is odd.
- Cramer's rule

$$\begin{aligned}
 ax + by &= e & x &= \frac{ed - bf}{ad - bc} \\
 cx + dy &= f & y &= \frac{af - ec}{ad - bc}
 \end{aligned}$$

- Vandermonde's Identity

$$C(n+m, k) = \sum_{i=0}^k C(n, i) C(m, k-i)$$

- Kirchhoff's Theorem
Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G , where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G .
- The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
- The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.
- Tutte's Matrix
Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniformly at random) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{\text{rank}(D)}{2}$ is the maximum matching on G .
- Cayley's Formula
- Given a degree sequence d_1, d_2, \dots, d_n for each labeled vertices, there are $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\dots(d_n-1)!}$ spanning trees.
- Let $T_{n,k}$ be the number of labeled forests on n vertices with k components, such that vertex $1, 2, \dots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.

- Erdős-Gallai theorem

A sequence of nonnegative integers $d_1 \geq \dots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only

if $d_1 + \dots + d_n$ is even and $\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$ holds for

every $1 \leq k \leq n$.

- Gale-Ryser theorem

A pair of sequences of nonnegative integers $a_1 \geq \dots \geq a_n$ and b_1, \dots, b_n

is bigraphic if and only if $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$ and $\sum_{i=1}^k a_i \leq \sum_{i=1}^n \min(b_i, k)$ holds

for every $1 \leq k \leq n$.

- Fulkerson-Chen-Anstee theorem

A sequence $(a_1, b_1), \dots, (a_n, b_n)$ of nonnegative integer pairs with $a_1 \geq \dots \geq a_n$ is digraphic if and only if $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$ and

$\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k-1) + \sum_{i=k+1}^n \min(b_i, k)$ holds for every $1 \leq k \leq n$.

- Pick's theorem

For simple polygon, when points are all integer, we have $A = \#(\text{lattice points in the interior}) + \frac{\#(\text{lattice points on the boundary})}{2} - 1$.

- Möbius inversion formula

$$\begin{aligned}
 f(n) &= \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f\left(\frac{n}{d}\right) \\
 f(n) &= \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) f(d)
 \end{aligned}$$

- Spherical cap

- A portion of a sphere cut off by a plane.
- r : sphere radius, a : radius of the base of the cap, h : height of the cap, θ : $\arcsin(a/r)$.
- Volume $= \pi h^2(3r - h)/3 = \pi h(3a^2 + h^2)/6 = \pi r^3(2 + \cos\theta)(1 - \cos\theta)^2/3$.
- Area $= 2\pi r h = \pi(a^2 + h^2) = 2\pi r^2(1 - \cos\theta)$.

- Lagrange multiplier

- Optimize $f(x_1, \dots, x_n)$ when k constraints $g_i(x_1, \dots, x_n) = 0$.
- Lagrangian function $\mathcal{L}(x_1, \dots, x_n, \lambda_1, \dots, \lambda_k) = f(x_1, \dots, x_n) - \sum_{i=1}^k \lambda_i g_i(x_1, \dots, x_n)$.
- The solution corresponding to the original constrained optimization is always a saddle point of the Lagrangian function.

- Nearest points of two skew lines

- Line 1: $\mathbf{v}_1 = \mathbf{p}_1 + t_1 \mathbf{d}_1$
- Line 2: $\mathbf{v}_2 = \mathbf{p}_2 + t_2 \mathbf{d}_2$
- $\mathbf{n} = \mathbf{d}_1 \times \mathbf{d}_2$
- $\mathbf{n}_1 = \mathbf{d}_1 \times \mathbf{n}$
- $\mathbf{n}_2 = \mathbf{d}_2 \times \mathbf{n}$
- $\mathbf{c}_1 = \mathbf{p}_1 + \frac{(\mathbf{p}_2 - \mathbf{p}_1) \cdot \mathbf{n}_2}{\mathbf{d}_1 \cdot \mathbf{n}_2} \mathbf{d}_1$
- $\mathbf{c}_2 = \mathbf{p}_2 + \frac{(\mathbf{p}_1 - \mathbf{p}_2) \cdot \mathbf{n}_1}{\mathbf{d}_2 \cdot \mathbf{n}_1} \mathbf{d}_2$

- Derivatives/Integrals

$$\begin{aligned}
 \text{Integration by parts: } \int_a^b f(x)g(x)dx &= [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx \\
 \left| \frac{d}{dx} \sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \right| & \left| \frac{d}{dx} \cos^{-1}x = -\frac{1}{\sqrt{1-x^2}} \right| & \left| \frac{d}{dx} \tan^{-1}x = \frac{1}{1+x^2} \right| \\
 \left| \frac{d}{dx} \tan x = 1 + \tan^2 x \right| & \left| \int \tan ax = -\frac{\ln|\cos ax|}{a} \right| \\
 \left| \int e^{-x^2} = \frac{\sqrt{\pi}}{2} \text{erf}(x) \right| & \left| \int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1) \right| \\
 \int \sqrt{a^2 + x^2} &= \frac{1}{2} \left(x \sqrt{a^2 + x^2} + a^2 \text{asinh}(x/a) \right)
 \end{aligned}$$

- Spherical Coordinate

$$(x, y, z) = (r \sin \theta \cos \phi, r \sin \theta \sin \phi, r \cos \theta)$$

$$(r, \theta, \phi) = (\sqrt{x^2 + y^2 + z^2}, \arccos(z / \sqrt{x^2 + y^2 + z^2}), \operatorname{atan2}(y, x))$$

- Rotation Matrix

$$M(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}, R_x(\theta_x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_x & -\sin \theta_x \\ 0 & \sin \theta_x & \cos \theta_x \end{bmatrix}$$

- Pell's equation $x^2 - ny^2 = 1 \Rightarrow$ let x_1, y_1 be the smallest solution: the other solution has the form $x_i + y_i \sqrt{n} = (x_1 + y_1 \sqrt{n})^i$ (x_1, y_1 can be found by continuous fraction expansion of \sqrt{n})
- Lucas' Theorem

$$\binom{n}{i} \equiv \prod_{j=0}^m \binom{n_j}{i_j} \pmod{p}$$

where a_j is a 's representation in p -base

5 String

5.1 AC auto [6e5a60]

```
struct ACauto {
    const static int N = 2e5 + 5; // change
    int tr
        [26][N], fail[N], ctn = 0, cnt[N], endat[N], n;
    vector<int> top; // fail tree topological order
    inline void clr(int p) {
        fail[p] = cnt[p] = 0;
        for (int i = 0; i < 26; i++)
            tr[i][p] = 0;
    }
    inline int add(const string &s) {
        int cr = 1;
        string tmp;
        for (int c : s) {
            tmp += c;
            c -= 'a';
            if (!tr[c][cr])
                clr(tr[c][cr] = ++ctn);
            cr = tr[c][cr];
        }
        return cr;
    }
    void blt(const vector<string> &ar) {
        for (int i = 0; i < 26; i++)
            tr[i][0] = 1;
        clr(ctn = 1), n = size(ar);
        for (int i = 0; i < n; i++)
            endat[i] = add(ar[i]);
        queue<int> q;
        q.push(1);
        while (!q.empty()) {
            int pr = q.front();
            q.pop();
            top.pb(pr);
            for (int i = 0; i < 26; i++) {
                int &cr = tr[i][pr];
                if (cr)
                    fail[cr] = tr[i][fail[pr]], q.push(cr);
                else
                    cr = tr[i][fail[pr]];
            }
        }
        reverse(iter(top));
    }
    void qry(const string &s) {
        int cr = 1;
        for (char c : s) // ways to walk
            cr = tr[c - 'a'][cr], cnt[cr]++;
        for (int i : top)
            cnt[fail[i]] += cnt[i];
    }
};
```

5.2 SA [58db4a]

```
struct SA {
    static const int N = 5e5 + 5; // change
    int sa[N], rk[N], cnt[N], lcp[N], tmp[N], n;
    void blt(const string &s) {
        n = s.length();
        int m = 128;
        memset(cnt + 1, 0, m << 2);
        for (int i = 0; i < n; i++)
            cnt[rk[i] = s[i]]++;
        for (int i = 1; i <= m; i++)
            cnt[i] += cnt[i - 1];
        for (int i = n - 1; i >= 0; i--)
            sa[--cnt[rk[i]]] = i;
        for (int k = 1; k <= 1) {
            int ln = 0;
            for (int i = n - k; i < n; i++)
                tmp[ln++] = i;
            for (int i = 0; i < n; i++)
                if (sa[i] >= k)
                    tmp[ln++] = sa[i] - k;
            memset(cnt + 1, 0, m << 2);
            for (int i = 0; i < n; i++)
                cnt[rk[i]]++;
            for (int i = 1; i <= m; i++)
                cnt[i] += cnt[i - 1];
            for (int i = n - 1; i >= 0; i--)
                sa[--cnt[rk[tmp[i]]]] = tmp[i];
            memcpy(tmp, rk, n << 2), rk[sa[0]] = m = 1;
            for (int i = 1; i < n; i++) {
                if (tmp[sa[i]] != tmp[sa[i - 1]] ||
                    tmp[sa[i] + k] != tmp[sa[i - 1] + k])
                    m++;
                rk[sa[i]] = m;
            }
            if (m == n)
                break;
        }
        for (int i = 0, k = 0; i < n; i++, k -= !!k) {
            if (rk[i] == n)
                continue;
            int j = sa[rk[i]];
            while (i + k <
                n && j + k < n && s[i + k] == s[j + k])
                k++;
            lcp[rk[i]] = k;
        } // lcp[1~n-1], sa[0~n-1]
    }
};
```

5.3 KMP [2b5dca]

```
vector<int> kmp(string &s) {
    int n = size(s);
    vector<int> pi(n, 0);
    for (int i = 1; i < n; i++) {
        int j = pi[i - 1];
        while (j > 0 && s[i] != s[j])
            j = pi[j - 1];
        if (s[i] == s[j])
            j++;
        pi[i] = j;
    }
    return pi;
}
```

5.4 Z [29dad4]

```
vector<int> z_algo(string &s) { // 0-base
    int n = size(s);
    vector<int> z(n, 0);
    for (int i = 1, l = 0, r = 0; i < n; i++) {
        if (i <= r)
            z[i] = min(z[i - l], r - i + 1);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]])
            z[i]++;
    }
```

```

    if (i + z[i] - 1 > r)
        l = i, r = i + z[i] - 1;
}
z[0] = n;
return z;
}

```

5.5 Manacher [c08260]

```

vector<int> manacher(string &ss) {
    // biggest k such [i-k+1,i]=[i~i+k-1], padded with $
    string s;
    s.resize(size(ss) * 2 + 1, '$');
    for (int i = 0; i < size(ss); i++) {
        s[i * 2 + 1] = ss[i];
    }
    vector<int> p(size(s), 1);
    for (int i = 0, l = 0, r = 0; i < size(s); i++) {
        p[i] = max(min(p[l * 2 - i], r - i), 1);
        while (0 <= i - p[i] && i + p[i] < size(s) && s[i - p[i]] == s[i + p[i]]) {
            l = i, r = i + p[i], p[i]++;
        }
    }
    return p;
}

```

6 Data Structure

6.1 Li-Chao [f2885c]

```

struct LiChaoMin {
    struct line {
        ll m, k, id;
        line(ll _m = 0, ll _k
            = 0, ll _id = 0) : m(_m), k(_k), id(_id) {}
        ll at(ll x) { return m * x + k; }
    };
    struct node {
        node *l, *r;
        line f;
        node(line v) : f(v), l(NULL), r(NULL) {}
    };
    node *root;
    int sz;
    void insert(node *x, int l, int r, line &ln) {
        if (!x) {
            x = new node(ln);
            return;
        }
        ll trl = x->f.at(l), trr = x->f.at(r);
        ll vl = ln.at(l), vr = ln.at(r);
        if (trl <= vl && trr <= vr)
            return;
        if (trl > vl && trr > vr) {
            x->f = ln;
            return;
        }
        if (trl > vl)
            swap(x->f, ln);
        int mid = (l + r) >> 1;
        if (x->f.at(mid) < ln.at(mid))
            insert(x->r, mid + 1, r, ln);
        else
            swap(x->f, ln), insert(x->l, l, mid, ln);
    }
    ll query(node *x, int l, int r, ll idx) {
        if (!x)
            return LONG_LONG_MAX;
        if (l == r)
            return x->f.at(idx);
        int mid = (l + r) >> 1;
        if (mid >= idx)
            return min(x
                ->f.at(idx), query(x->l, l, mid, idx));
        return min(x
            ->f.at(idx), query(x->r, mid + 1, r, idx));
    }
    LiChaoMin(int _sz) : sz(_sz + 1), root(NULL) {}
}

```

```

void add_line(ll m, ll k, ll id = 0) {
    auto ln = line(m, k, id);
    insert(root, -sz, sz, ln);
} // -sz <= query_x <= sz
ll query
    (ll idx) { return query(root, -sz, sz, idx); }
};

```

6.2 Treap [5ab1a1]

```

struct node {
    int data, sz;
    node *l, *r;
    node(int k) : data(k), sz(1), l(0), r(0) {}
    void up() {
        sz = 1;
        if (l) sz += l->sz;
        if (r) sz += r->sz;
    }
    void down() {}
};
int sz(node *a) { return a ? a->sz : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (rand() % (sz(a) + sz(b)) < sz(a))
        return a->down(), a->r = merge(a->r, b), a->up(),
            a;
    return b->down(), b->l = merge(a, b->l), b->up(), b;
}
void split(node *o, node *&a, node *&b, int k) {
    if (!o) return a = b = 0, void();
    o->down();
    if (o->data <= k)
        a = o, split(o->r, a->r, b, k), a->up();
    else b = o, split(o->l, a, b->l, k), b->up();
}
void split2(node *o, node *&a, node *&b, int k) {
    if (sz(o) <= k) return a = o, b = 0, void();
    o->down();
    if (sz(o->l) + 1 <= k)
        a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
    else b = o, split2(o->l, a, b->l, k);
    o->up();
}
node *kth(node *o, int k) {
    if (k <= sz(o->l)) return kth(o->l, k);
    if (k == sz(o->l) + 1) return o;
    return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, int key) {
    if (!o) return 0;
    if (o->data < key)
        return sz(o->l) + 1 + Rank(o->r, key);
    else return Rank(o->l, key);
}
bool erase(node *&o, int k) {
    if (!o) return 0;
    if (o->data == k) {
        node *t = o;
        o->down(), o = merge(o->l, o->r);
        delete t;
        return 1;
    }
    node *t = k < o->data ? o->l : o->r;
    return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *&o, int k) {
    node *a, *b;
    split(o, a, b, k),
        o = merge(a, merge(new node(k), b));
}
void interval(node *&o, int l, int r) {
    node *a, *b, *c;
    split2(o, a, b, l - 1), split2(b, b, c, r);
    // operate
    o = merge(a, merge(b, c));
}

```

6.3 Link Cut Tree [49d7e6]

```

struct LCT {
    static const int N = 4e5 + 5; // change
    int fa[N], ch[2][N], sz[N], sv[N];
    #define gch(x) ((x) == ch[1][fa[x]])
    #define nrt(x) ((x) == ch[gch(x)][fa[x]])
    #define up
    (x) sz[x] = sz[ch[0][x]] + sz[ch[1][x]] + sv[x] + 1
    inline void rota(int x) {
        int f = fa[x], ff = fa[f], k = gch(x);
        if (nrt(f))
            ch[gch(f)][ff] = x;
        fa[x] = ff;
        if (ch[!k][x])
            fa[ch[!k][x]] = f;
        ch[k][f] = ch[!k][x];
        ch[!k][x] = f, fa[f] = x;
        up(f), up(x);
    }
    inline void splay(int x) {
        for (int f = fa[x]; nrt(x); rota(x), f = fa[x])
            if (nrt(f))
                rota(gch(x) ^ gch(f) ? x : f);
    }
    inline int acc(int x) {
        int p;
        for (p = 0; x; p = x, x = fa[x])
            splay(x), sv[x] += sz[
                ch[1][x]] - sz[p], ch[1][x] = p, up(x);
        return p;
    }
    inline int findroot(int x) {
        int cr = acc(x);
        while (ch[0][cr])
            cr = ch[0][cr];
        splay(cr);
        return cr;
    }
    void link(int x, int y) {
        int rt = findroot(y);
        acc(x), acc(rt);
        acc(y), splay(y);
        sv[y] += sz[x];
        fa[x] = y, up(y);
        acc(rt);
    }
    void cut(int x, int y) {
        int rt = findroot(y);
        acc(rt);
        acc(y), splay(y), splay(x);
        sv[y] -= sz[x];
        fa[x] = 0, up(y);
        acc(x), acc(rt);
    }
};

```

6.4 Ultimate Segment Tree [6e7e86]

```

struct SegBeat {
    int n, n0;
    vector<ll> max_v, smax_v,
        max_c, min_v, smin_v, min_c, sum, len, ladd, lval;
    void update_node_max(int k, ll x) {
        sum[k] += (x - max_v[k]) * max_c[k];
        if (max_v
            [k] == min_v[k]) max_v[k] = min_v[k] = x;
        else if (max_v
            [k] == smin_v[k]) max_v[k] = smin_v[k] = x;
        else max_v[k] = x;
        if (lval[k] != 1e18 && x < lval[k]) lval[k] = x;
    }
    void update_node_min(int k, ll x) {
        sum[k] += (x - min_v[k]) * min_c[k];
        if (max_v
            [k] == min_v[k]) max_v[k] = min_v[k] = x;

```

```

        else if (smax_v
            [k] == min_v[k]) min_v[k] = smax_v[k] = x;
        else min_v[k] = x;
        if (lval[k] != 1e18 && lval[k] < x) lval[k] = x;
    }
    void push(int k) {
        if (n0-1 <= k) return;
        if (lval[k] != 1e18) {
            updateall(2*k+1, lval[k]);
            updateall(2*k+2, lval[k]);
            lval[k] = 1e18;
            return;
        }
        if (ladd[k] != 0) {
            addall(2*k+1, ladd[k]);
            addall(2*k+2, ladd[k]);
            ladd[k] = 0;
        }
        if (max_v[k] < max_v
            [2*k+1]) update_node_max(2*k+1, max_v[k]);
        if (min_v[2*k+1] <
            min_v[k]) update_node_min(2*k+1, min_v[k]);
        if (max_v[k] < max_v
            [2*k+2]) update_node_max(2*k+2, max_v[k]);
        if (min_v[2*k+2] <
            min_v[k]) update_node_min(2*k+2, min_v[k]);
    }
    void update(int k) {
        sum[k] = sum[2*k+1] + sum[2*k+2];
        if (max_v[2*k+1] < max_v[2*k+2]) {
            max_v[k] = max_v[2*k+2];
            max_c[k] = max_c[2*k+2];
            smax_v
                [k] = max(max_v[2*k+1], smax_v[2*k+2]);
        } else if (max_v[2*k+1] > max_v[2*k+2]) {
            max_v[k] = max_v[2*k+1];
            max_c[k] = max_c[2*k+1];
            smax_v
                [k] = max(smax_v[2*k+1], max_v[2*k+2]);
        } else {
            max_v[k] = max_v[2*k+1];
            max_c[k] = max_c[2*k+1] + max_c[2*k+2];
            smax_v
                [k] = max(smax_v[2*k+1], smax_v[2*k+2]);
        }
        if (min_v[2*k+1] < min_v[2*k+2]) {
            min_v[k] = min_v[2*k+1];
            min_c[k] = min_c[2*k+1];
            smin_v
                [k] = min(smin_v[2*k+1], min_v[2*k+2]);
        } else if (min_v[2*k+1] > min_v[2*k+2]) {
            min_v[k] = min_v[2*k+2];
            min_c[k] = min_c[2*k+2];
            smin_v
                [k] = min(min_v[2*k+1], smin_v[2*k+2]);
        } else {
            min_v[k] = min_v[2*k+1];
            min_c[k] = min_c[2*k+1] + min_c[2*k+2];
            smin_v
                [k] = min(smin_v[2*k+1], smin_v[2*k+2]);
        }
    }
    void _chmin
        (ll x, int a, int b, int k, int l, int r) {
        if (b <= l || r <= a || max_v[k] <= x) return;
        if (a <= l && r <= b && smax_v[k] < x) {
            update_node_max(k, x);
            return;
        }
        push(k);
        _chmin(x, a, b, 2*k+1, l, (l+r)/2);
        _chmin(x, a, b, 2*k+2, (l+r)/2, r);
        update(k);
    }
};

```

```

void _chmax
(ll x, int a, int b, int k, int l, int r) {
    if(b <= l || r <= a || x <= min_v[k]) return;
    if(a <= l && r <= b && x < smin_v[k]) {
        update_node_min(k, x);
        return;
    }
    push(k);
    _chmax(x, a, b, 2*k+1, l, (l+r)/2);
    _chmax(x, a, b, 2*k+2, (l+r)/2, r);
    update(k);
}

void addall(int k, ll x) {
    max_v[k] += x;
    if(smax_v[k] != -1e18) smax_v[k] += x;
    min_v[k] += x;
    if(smin_v[k] != 1e18) smin_v[k] += x;
    sum[k] += len[k] * x;
    if(lval[k] != 1e18) lval[k] += x;
    else ladd[k] += x;
}

void updateall(int k, ll x) {
    max_v[k] = x;
    smax_v[k] = -1e18;
    min_v[k] = x;
    smin_v[k] = 1e18;
    max_c[k] = min_c[k] = len[k];
    sum[k] = x * len[k];
    lval[k] = x;
    ladd[k] = 0;
}

void _add_val
(ll x, int a, int b, int k, int l, int r) {
    if(b <= l || r <= a) return;
    if(a <= l && r <= b) {
        addall(k, x);
        return;
    }
    push(k);
    _add_val(x, a, b, 2*k+1, l, (l+r)/2);
    _add_val(x, a, b, 2*k+2, (l+r)/2, r);
    update(k);
}

void _update_val
(ll x, int a, int b, int k, int l, int r) {
    if(b <= l || r <= a) return;
    if(a <= l && r <= b) {
        updateall(k, x);
        return;
    }
    push(k);
    _update_val(x, a, b, 2*k+1, l, (l+r)/2);
    _update_val(x, a, b, 2*k+2, (l+r)/2, r);
    update(k);
}

ll _query_max(int a, int b, int k, int l, int r) {
    if(b <= l || r <= a) return -1e18;
    if(a <= l && r <= b) return max_v[k];
    push(k);
    ll lv = _query_max(a, b, 2*k+1, l, (l+r)/2);
    ll rv = _query_max(a, b, 2*k+2, (l+r)/2, r);
    return max(lv, rv);
}

ll _query_min(int a, int b, int k, int l, int r) {
    if(b <= l || r <= a) return 1e18;
    if(a <= l && r <= b) return min_v[k];
    push(k);
    ll lv = _query_min(a, b, 2*k+1, l, (l+r)/2);
    ll rv = _query_min(a, b, 2*k+2, (l+r)/2, r);
    return min(lv, rv);
}

ll _query_sum(int a, int b, int k, int l, int r) {
    if(b <= l || r <= a) return 0;
    if(a <= l && r <= b) return sum[k];
    push(k);

```

```

    ll lv = _query_sum(a, b, 2*k+1, l, (l+r)/2);
    ll rv = _query_sum(a, b, 2*k+2, (l+r)/2, r);
    return lv + rv;
}

SegBeat(int _n) : n(_n) {
    max_v.resize(4*_n+4, 0); smax_v.resize
        (4*_n+4, -1e18); max_c.resize(4*_n+4, 1);
    min_v.resize(4*_n+4, 0); smin_v.
        resize(4*_n+4, 1e18); min_c.resize(4*_n+4, 1);
    sum.resize(4*_n+4, 0); len.resize(4*_n+4, 0);
    ladd.resize(4*_n+4, 0); lval.resize(4*_n+4, 1e18);
    n0 = 1;
    while(n0 < n) n0 <= 1;
    len[0] = n0;
    for(int i=0; i<n0-1; ++
        i) len[2*i+1] = len[2*i+2] = (len[i] >> 1);
    for(int i=n; i<n0; ++i) {
        max_v[n0-1+i] = smax_v[n0-1+i] = -1e18;
        min_v[n0-1+i] = smin_v[n0-1+i] = 1e18;
        max_c[n0-1+i] = min_c[n0-1+i] = 0;
    }
    for(int i=n0-2; i>=0; i--) update(i);
}

void chmin(int
    a, int b, ll x) {_chmin(x, a-1, b, 0, 0, n0);}
void chmax(int
    a, int b, ll x) {_chmax(x, a-1, b, 0, 0, n0);}
void add_val(int a
    , int b, ll x) {_add_val(x, a-1, b, 0, 0, n0);}
void update_val(int a, int
    b, ll x) {_update_val(x, a-1, b, 0, 0, n0);}
ll query_max(int a
    , int b) {return _query_max(a-1, b, 0, 0, n0);}
ll query_min(int a
    , int b) {return _query_min(a-1, b, 0, 0, n0);}
ll query_sum(int a
    , int b) {return _query_sum(a-1, b, 0, 0, n0);}
};

```

7 Misc

7.1 Total Binary Search [ac23f3]

```

struct TotalBS {
    int Q;
    vector<int> ans;
    TotalBS(int _Q) {
        Q=_Q;
        ans.resize(Q);
    }
    void split(vector<
        int> &qrys, vecotr<int> &ok, vector<int> &fail) {
        for(auto i : qrys) {
        }
        vector<int>.swap(qrys);
        return;
    }
    void do_things(int l, int mid) {
        return;
    }
    void undo_things(int l, int mid) {
        return;
    }
    void total_BS(int l, int r, vector<int> &qrys) {
        if (l == r) {
            for(auto i : qrys) {
                ans[i] = l;
            }
        }
        int mid = (l + r) / 2;
        do_things(l, mid);
        vector<int> lft, rgt;
        split(qrys, lft, rgt);
        total_BS(mid + 1, r, rgt);
        undo_things(l, mid);
        total_BS(l, mid, lft);
    }
};

```

```
|};
```

7.2 CDQ [11f96f]

```
|void CDQ(int l, int r) { // 三維偏序
|    if (l == r)
|        return;
|    int mid = (l + r) / 2;
|    CDQ(l, mid);
|    CDQ(mid + 1, r);
|    sort(arr + l, arr + mid + 1, cmpB);
|    sort(arr + mid + 1, arr + r + 1, cmpB);
|    int i = l;
|    int j = mid + 1;
|    while (j <= r) {
|        while (i <= mid && ue[i].b <= ue[j].b) {
|            BIT.Add(arr[i].c, arr[i].cnt);
|            i++;
|        }
|        ue[j].res += BIT.Ask(arr[j].c);
|        j++;
|    }
|    for (int k = l; k < i; k++)
|        BIT.Add(arr[k].c, -arr[k].cnt);
|    return;
|}
```