ADSP HW5

B10505047 電機三 邱郁喆

(1) 程式已隨這份作業文件繳交到 NTU COOL

(2) 注：在使用 sectioned convolution 方法計算時的 L_0 是用 MATLAB 得到的，而最後選的 P 則是取附近適合的幾個點計算比較而得到的最好選擇。

(2)(a) $N=1200,\ M=300$

① Direct method: $3MN = 1080000$

② non-sectioned convolution method(DFT): $P \geq N+M-1 = 1499 \Rightarrow$ Use 1680-point DFT

$\Rightarrow 2\cdot MUL_{1680} + 3\cdot 1680 = 2\cdot 10420 + 3\cdot 1680 = 25880$

③ sectioned convolution method: $L_0 = 600,\ P_0 = L_0+M-1 = 899$

$\Rightarrow$ Set $P = 1152,\ L = P-M+1 = 853,\ S = \lceil \frac{N}{L} \rceil = 2 \Rightarrow S(2\cdot MUL_P + 3\cdot P) = 2\cdot(27088 + 3\cdot 1152) = 35264$

∴ The best method is non-sectioned conv. method(DFT), total # of real mul. = 25880 #

(b) $N=1200,\ M=30$

① Direct: $3MN = 108000$

② DFT: $P \geq N+M-1 = 1229 \Rightarrow$ Use 1260-point DFT

$\Rightarrow 2\cdot MUL_{1260} + 3\cdot 1260 = 2\cdot 7640 + 3\cdot 1260 = 19060$

③ Sectioned conv.: $L_0 = 174,\ P_0 = L_0+M-1 = 203$

$\Rightarrow$ Set $P=144,\ L=P-M+1=115,\ S=\lceil \frac{N}{L} \rceil = 11 \Rightarrow S(2MUL_P+3P) = 11\cdot(2\cdot 436+3\cdot 144) = 14344$

∴ The best method is sectioned conv. method, total # of real mul. = 14344 #

(2)(c) $N=1200,\ M=8$

① Direct: $3MN = 28800$

② DFT: $P \geq N+M-1 = 1207 \Rightarrow$ Use 1260-point DFT

$\Rightarrow 2\cdot MUL_{1260} + 3\cdot 1260 = 2\cdot 7640 + 3\cdot 1260 = 19060$

③ sectioned conv.: $L_0 = 30,\ P_0 = L_0+M-1 = 37$

$\Rightarrow$ Set $P=24,\ L=P-M+1=17,\ S=\lceil \frac{N}{L} \rceil = 71 \Rightarrow S(2MUL_P+3P) = 71\cdot(2\cdot 28+3\cdot 24) = 9088$

∴ The best method is sectioned conv. method, total # of real mul. = 9088 #

(d) $N=1200,\ M=2$

① Direct: $3MN = 7200$

② DFT: $P \geq N+M-1 = 1201 \Rightarrow$ Use 1260-point DFT

$\Rightarrow 2\cdot MUL_{1260} + 3\cdot 1260 = 2\cdot 7640 + 3\cdot 1260 = 19060$

③ Sectioned conv.: $L_0 = 2,\ P_0 = L_0+M-1 = 3$

$\Rightarrow$ Set $P=4,\ L=P-M+1=3,\ S=\lceil \frac{N}{L} \rceil = 400 \Rightarrow S(2MUL_P+3P) = 400\cdot(2\cdot 0+3\cdot 4) = 4800$

∴ The best method is sectioned conv. method, total # of real mul. = 4800 #

(3) (a) 我們可以知道 Walsh transform 除了第一個 row 都是 1 之外，其他每個 row 中都是有一半數量為 1，一半數量為 -1。因此，等於 1 的 entries 總共有 $2^k + \frac{2^k}{2} \times (2^k - 1) = 2^k + 2^{2k-1} - 2^{k-1} = 2^{k-1} + 2^{2k-1}$ 個，而等於 -1 的 entries 則總共有$\frac{2^k}{2} \times (2^k - 1) = 2^{2k-1} - 2^{k-1}$ 個。

(b) 按照上課所講，2^k- point Haar transform 共可分成 k+1 個 groups

等於 1 的 entries：除了第一個 group (其實就是第一個 row)都是 1 之外，其他每個 group 中各有 2^{k-1} 個 1，因此共有 $2^k + k \cdot 2^{k-1}$ 個。

等於 -1 的 entries：除了第一個 group (其實就是第一個 row) 沒有 -1 之外，其他每個 group 中各有 2^{k-1} 個 -1，因此共有$k \cdot 2^{k-1}$ 個。

等於 0 的 entries：總 entries 數減去等於 1 或 -1 的 entries，即共有 $2^k \times 2^k - (2^k + k \cdot 2^{k-1}) - (k \cdot 2^{k-1}) = 2^{2k} - (k+1) \cdot 2^k$ 個。

(c) The most important application of the Walsh transform nowadays is modulation, which is using some man-made waveform to represent a data. 像是用在 CDMA 這個無線通訊技術上, which is using the basis (rows) of the Walsh transform to perform modulation.

(d) The most important advantage of the Haar transform nowadays is analysis of the local high frequency component (edges of different locations and scales).

(4)

4.(a) [1 0 1] → [1 -1 1] , [1 1 0] → [1 1 -1] , [0 1 1] → [-1 1 1]

1,-1,1 modulated by [1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1] = v_1 → [v_1 $-v_1$ v_1]

1,1,-1 modulated by [1,1,1,1,-1,-1,-1,-1,1,1,1,1,-1,-1,-1,-1] = v_2 → [v_2 v_2 $-v_2$]

-1,1,1 modulated by [1,-1,-1,1,1,-1,-1,1,-1,1,1,-1,-1,1,1,-1] = v_3 → [$-v_3$ v_3 v_3]

相合 ⇒ [1 3 3 1 -1 1 1 -1 3 1 1 3 1 -1 -1 1 1 1 -1 -1 1 1 -1 -3 -3 -1 -1 1 1 -1 -3 -1 -1 -3 1 -1 -1 1 3 1 1 3 -1 1 1 -1 1 3 3 1]

(b) 7th and 19th entries missed

⇒ [1 3 3 1 -1 1 0 -1 3 1 1 3 1 -1 -1 1 | 1 -1 0 1 -1 -3 -3 -1 -1 1 1 -1 -3 -1 -1 -3 | ...]

ch1 inner product = 15
$\frac{15}{N} = 0.9375 > 0 \Rightarrow 1$

inner product = -15
$\frac{-15}{N} = -0.9375 < 0 \Rightarrow -1$

⇒ ch1 success [1, -1, 1]

ch2 inner product = 17
$\frac{17}{N} = 1.0625 > 0 \Rightarrow 1$

inner product = 17
$\frac{17}{N} = 1.0625 > 0 \Rightarrow 1$

⇒ ch2 success [1, 1, -1]

ch3 inner product = -15
$\frac{-15}{N} = -0.9375 < 0 \Rightarrow -1$

inner product = 15
$\frac{15}{N} = 0.9375 > 0 \Rightarrow 1$

⇒ ch3 success [-1, 1, 1]

∴ Yes, we can recover the original data #

(5) 根據上課講義如下

最後結論，大家可以把 DFT 的地方換成 NTT ，結論還是一樣

$$\begin{tikzcd}
\text{gcd-delta} \arrow[r, shift left, "A_N"] \arrow[d, "\text{DFT}"'] & \text{impulse train} \arrow[l, shift left, "M_N"] \arrow[d, "\text{DFT}"] \\
\text{Ramanujan's sum} \arrow[r, shift left, "A_N"] & \text{weighted impulse train} \arrow[l, shift left, "M_N"]
\end{tikzcd}$$

現在我們知道 fft[x] = [4 0 2 0 -2 0 -4 0 -2 0 2 0]，也就是 Ramanujan's Sum。因此 CNT of x 的結果就是 fft[x] modulo M 的結果，即為[4 0 2 0 9 0 7 0 9 0 2 0]

(6)

6.(a)

By 費馬小定理, $3^{102} \equiv 1 \pmod{103}$

∴ $(3^{102})^k \equiv 1 \pmod{103}$, for every positive integer k

Thus, $3^{2049} = 3^{102\cdot 20+9} = (3^{102})^{20} \cdot 3^9 \equiv 1 \cdot 3^9 \equiv 10 \pmod{13}$

∴ $3^{2049} \bmod 103 = 10$ #

(b) We have $x \equiv 2 \pmod{43}$ and $x \equiv 13 \pmod{67}$

Let $m = 43 \cdot 67 = 2881$, $M_1 = \frac{m}{43} = 67$, $M_2 = \frac{m}{67} = 43$

We see that 9 is an inverse of $M_1 = 67$ modulo 43 since $67 \cdot 9 \equiv 1 \pmod{43}$

53 is an inverse of $M_2 = 43$ modulo 67 since $43 \cdot 53 \equiv 1 \pmod{67}$

∴ By Chinese Remainder Thm., $x = 2 \cdot 67 \cdot 9 + 13 \cdot 43 \cdot 53 = 30833 \equiv 2023 \bmod 2881$

$\Rightarrow x \bmod 2881 = 2023$ #

(c)

Wilson's Thm.: $(p-1)! \equiv -1 \pmod{p}$ for a prime p

For $p=43$, we have $42! \equiv -1 \pmod{43}$

We also know $42 \equiv -1 \pmod{43}$, $41 \equiv -2 \pmod{43}$, $40 \equiv -3 \pmod{43}$

∴ $42 \times 41 \times 40 \pmod{43} = \{42 \pmod{43} \times 41 \pmod{43} \times 40 \pmod{43}\} \pmod{43}$

$= [(-1)\times(-2)\times(-3)] \pmod{43} = -6 \pmod{43} = 37$

Thus, $42! \pmod{43} = \{42 \times 41 \times 40 \pmod{43} \times \underline{39! \pmod{43}}\} \pmod{43} = 37x \pmod{43}$

$\Rightarrow 42! \equiv 37x \equiv -1 \pmod{43}$

We can get $x = 36 \Rightarrow 39! \bmod 43 = 36$ #

學號尾數(2, 7)的 extra 問題：

If length(x) = N, x is real

$h[n] = [0.1, 0.2, 0.4, 0.2, 0.1]$ for $n = -2 \sim 2$

$y[n] = x[n] * h[n] = \sum_{m=-2}^{2} x[n-m]h[m] = 0.1x[n+2] + 0.2x[n+1] + 0.4x[n] + 0.2x[n-1] + 0.1x[n-2]$

$= 0.1(x[n+2] + x[n-2]) + 0.2(x[n+1] + x[n-1]) + 0.4x[n]$

$= 0.1[(x[n+2] + x[n-2]) + 2(x[n+1] + x[n-1]) + 2^{2} \cdot x[n]]$

$\Rightarrow N$ MULs ※