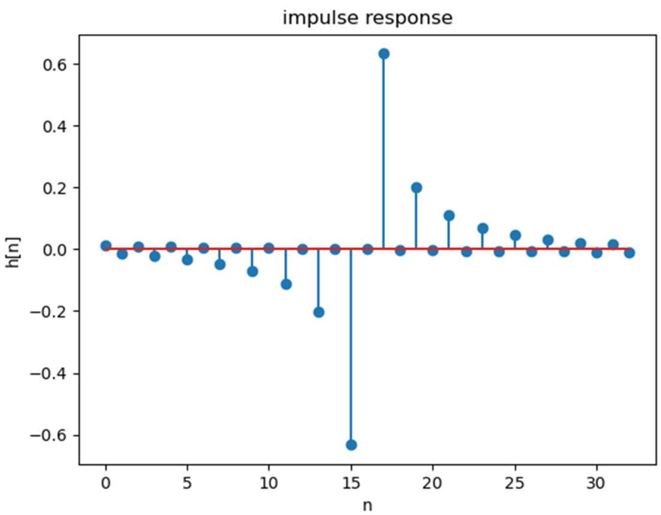
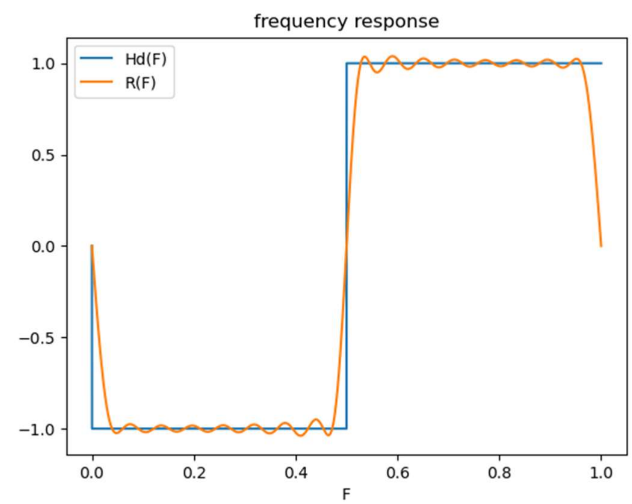
ADSP HW2

B10505047 電機三 邱郁喆

(1) 設 k = 16





(2)

(2)

$$f_s = \frac{1}{\Delta t} = 20000\ Hz$$

Set $\delta_1 = \delta_2 = 0.01$

$$\Delta F = \frac{6000-5000}{f_s} = \frac{1000}{20000} = 0.05$$

$$\Rightarrow \text{length } N = \frac{2}{3}\frac{1}{\Delta F}\log_{10}\left(\frac{1}{10\delta_1\delta_2}\right)$$

$$= \frac{2}{3}\times\frac{1}{0.05}\times\log_{10}\left(\frac{1}{0.001}\right)$$

$$= 40 \ \#$$

(3) 由於 DFT 的運算複雜度為$\theta(NlogN)$，隨著 N 越大，運算量也會變得相當可觀，因此這不是一個適合用來設計 FIR filter 的方法。

(4)

(4) Type 4： $R(F)=\sum_{n=1}^{k+\frac{1}{2}} s[n]\sin\left(2\pi(n-\frac{1}{2})F\right)$

由於 n^{th} term 和 $(n+1)th$ term 兩項相減得

$$\sin\left(2\pi(n+\tfrac{1}{2})F\right)-\sin\left(2\pi(n-\tfrac{1}{2})F\right)=2\sin(\pi F)\cos(2\pi nF)$$

∴判斷何將 $R(F)$ 改寫為 $R(F)=\sin(\pi F)\sum_{n=0}^{k_1} s_1[n]\cos(2\pi nF)$

$$\Rightarrow R(F)=\sum_{n=0}^{k_1} s_1[n]\sin(\pi F)\cos(2\pi nF)=\frac{1}{2}\sum_{n=0}^{k_1} s_1[n]\sin\left(2\pi(n+\tfrac{1}{2})F\right)-\frac{1}{2}\sum_{n=0}^{k_1} s_1[n]\sin\left(2\pi(n-\tfrac{1}{2})F\right)$$

$$=\frac{1}{2}\sum_{n=1}^{k_1+1} s_1[n-1]\sin\left(2\pi(n-\tfrac{1}{2})F\right)-\frac{1}{2}\sum_{n=0}^{k_1} s_1[n]\sin\left(2\pi(n-\tfrac{1}{2})F\right).$$

$$=-\frac{1}{2}s_1[0]\sin(\pi F)+\frac{1}{2}\sum_{n=1}^{k_1}\left(s_1[n-1]-s_1[n]\right)\cdot\sin\left(2\pi(n-\tfrac{1}{2})F\right)+\frac{1}{2}s_1[k_1]\sin\left(2\pi(k_1+\tfrac{1}{2})F\right)$$

令 $k_1=k-\frac{1}{2}$

$$=\left(s_1[0]-\frac{1}{2}s_1[1]\right)\sin(\pi F)+\frac{1}{2}\sum_{n=2}^{k-\frac{1}{2}}\left(s_1[n-1]-s_1[n]\right)\sin\left(2\pi(n-\tfrac{1}{2})F\right)+\frac{1}{2}s_1[k-\tfrac{1}{2}]\sin(2\pi kF)$$

比較係數可得

$$\begin{cases} s[1]=\frac{1}{2}s_1[1]-s_1[0] \\ s[n]=\frac{1}{2}\left(s_1[n-1]-s_1[n]\right) \text{ for } n=2,3,\dots,k-\frac{1}{2} \\ s[k+\frac{1}{2}]=\frac{1}{2}s_1[k-\frac{1}{2}] \end{cases}$$

$$err(F)=\left[R(F)-H_d(F)\right]W(F)=\left[\sin(\pi F)\sum_{n=0}^{k-\frac{1}{2}} s_1[n]\cos(2\pi nF)-H_d(F)\right]W(F)$$

$$=\left[\sum_{n=0}^{k-\frac{1}{2}} s_1[n]\cos(2\pi nF)-\csc(\pi F)H_d(F)\right]\sin(\pi F)\cdot W(F)$$

∴只需要將講義 p.58-61 方法中，$H_d(F)$ 換成 $\csc(\pi F)H_d(F)$

$W(F)$ 換成 $\sin(\pi F)W(F)$

k 換成 $k-\frac{1}{2}=\frac{N}{2}-1$

注意 $s_1[n]$ 和 $s[n]$ 之間的關係即可

(5)

(5) (a) $x[n]=1+\sin(n)\xrightarrow{DTFT}X(F)=\delta(F)+\frac{1}{2j}\delta(F-\frac{1}{2\pi})-\frac{1}{2j}\delta(F+\frac{1}{2\pi})$

$X_H(F)=X(F)\cdot H(F)=\frac{-1}{2}\delta(F-\frac{1}{2\pi})-\frac{1}{2}\delta(F+\frac{1}{2\pi})$

$\therefore x_H[n]=\int_0^1 X(F)e^{j2\pi Fn}dF=\frac{-1}{2}e^{jn}-\frac{1}{2}e^{-jn}=-\cos(n)$

(b) $x_a[n]=x[n]+jx_H[n]=1+\sin(n)-j\cos(n)$

(6) (a) (ii) the Hilbert transform, (iii) the matched filter, and (iv) the difference
(b) (v) the Kalman filter and (vi) the particle filter

(7) (a) One advantage is that it is stable since all the poles and all the zeros are within the unit circle. Another advantage is that it can let the energy concentrate on the region near to n = 0.
(b) 若使用 equalizer 解決 multipath 問題必須要先知道參數 α 跟 τ ，而且會有因為取倒數而造成不穩定的問題，而使用 ceptrum 的兩個主要的 advantages 就在於可以避免這兩個問題。

(8)

(8) (a) $$H(z)=\frac{1+z^{-1}-1.5z^{-2}+z^{-3}}{1-0.3z^{-1}-0.4z^{-2}}=\frac{(0.5+z^{-1})(1+j-z^{-1})(1-j-z^{-1})}{0.1(5-4z^{-1})(2+z^{-1})}=\frac{2z^{-1}(1-(\frac{-z}{2}))(1-\frac{1-j}{2}z^{-1})(1-\frac{1+j}{2}z^{-1})}{(1-\frac{4}{5}z^{-1})(1-(-\frac{1}{2}z^{-1}))}$$

By 講義 P.185,
$$\hat{x}[n]=\begin{cases}\log 2, & n=0\\ -\frac{(\frac{1-j}{2})^n}{n}-\frac{(\frac{1+j}{2})^n}{n}+\frac{(\frac{4}{5})^n}{n}+\frac{(\frac{-1}{2})^n}{n}, & n>0\\ \frac{(\frac{-1}{2})^n}{n}, & n<0\end{cases}$$
#

(b)
$$H(z)=\frac{(z+2)(z-\frac{1-j}{2})(z-\frac{1+j}{2})}{z(z-\frac{4}{5})(z+\frac{1}{2})}$$

-2 is the zero that is not within the unit circle

$$\Rightarrow H_1(z)=\frac{(z+2)(z-\frac{1-j}{2})(z-\frac{1+j}{2})}{z(z-\frac{4}{5})(z+\frac{1}{2})}\cdot(-2)\cdot\frac{z-\overline{(-2)^{-1}}}{z+2}$$
$$=\frac{-2(z+\frac{1}{2})(z-\frac{1-j}{2})(z-\frac{1+j}{2})}{z(z-\frac{4}{5})(z+\frac{1}{2})}=\frac{-2(z-\frac{1-j}{2})(z-\frac{1+j}{2})}{z(z-\frac{4}{5})}$$
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學號尾數 7 的 bonus 問題：為什麼 notch filter 比起一般的 pass-stop filter 來得難以設計？
答：因為 notch filter 的 transition band 不能設太寬(若設太寬會沒有 stopband)，而在 transition band 很窄的情形下，若是要達到同樣的誤差則要將 N 設得比較大，然而濾波器 N 太大又會比較耗費運算資源。