ADSP HW4

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(1) 程式已隨這份作業文件繳交到 NTU COOL

(2)

(2) (a) $\text{entropy} = \sum_{j=1}^{J} P(S_j) \ln \frac{1}{P(S_j)} = \sum_{n=1}^{80000} (e^{0.002}-1)e^{-0.002n} \cdot \ln\left(\frac{1}{(e^{0.002}-1)e^{-0.002n}}\right)$

$= -\sum_{n=1}^{80000} (e^{0.002}-1)e^{-0.002n} \cdot \ln\left((e^{0.002}-1)e^{-0.002n}\right) = -\sum_{n=1}^{80000} (e^{0.002}-1)e^{-0.002n} \cdot \left(\ln(e^{0.002}-1) - 0.002n\right)$

Simplify and appoximate $\Rightarrow$ entropy ≈ 7.21

(b) Assume total coding length is b, we know that $\left\lceil N \frac{\text{entropy}}{\ln k} \right\rceil \le b \le \left\lfloor N \frac{\text{entropy}}{\ln k} + N \right\rfloor$

$\Rightarrow \left\lceil 10^5 \cdot \frac{7.21}{\ln 2} \right\rceil \le b \le \left\lfloor 10^5 \cdot \frac{7.21}{\ln 2} + 10^5 \right\rfloor \Rightarrow 1040183 \le b \le 1140183$

(c) We know that $\left\lceil N \frac{\text{entropy}}{\ln k} \right\rceil \le b \le \left\lfloor N \frac{\text{entropy}}{\ln k} + \log_k 2 + 1 \right\rfloor$

$\Rightarrow \left\lceil 10^5 \cdot \frac{7.21}{\ln 2} \right\rceil \le b \le \left\lfloor 10^5 \cdot \frac{7.21}{\ln 2} + 2 \right\rfloor \Rightarrow 1040183 \le b \le 1040185$

(3)

(3) $e^{j\theta} = \cos\theta + j\sin\theta$

令 $x = a + jb$

$\Rightarrow x \cdot e^{j\theta} = (a+jb)(\cos\theta + j\sin\theta) = (a\cos\theta - b\sin\theta) + j(b\cos\theta + a\sin\theta) = c + jd$

$\Rightarrow \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & \cos\theta \\ \cos\theta & \cos\theta \end{bmatrix}}_{前} \begin{bmatrix} a \\ b \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & -\sin\theta - \cos\theta \\ \sin\theta - \cos\theta & 0 \end{bmatrix}}_{後} \begin{bmatrix} a \\ b \end{bmatrix}$

① If $\sin\theta = \pm\cos\theta \Leftrightarrow \theta = \pm\left(\frac{\pi}{4} + 2k\pi\right)$ or $\pm\left(\frac{3\pi}{4} + 2k\pi\right)$, $k = 0, 1, 2, \ldots$

前後各只要 1 MUL $\Rightarrow$ 共 2 MUL

② If $\cos\theta = 0 \Leftrightarrow \theta = \pm\left(\frac{\pi}{2} + 2k\pi\right)$, $k = 0, 1, 2, \ldots$

前 0 MUL, 後 2 MUL $\Rightarrow$ 共 2 MUL

③ If $\sin\theta = 0 \Leftrightarrow \theta = \pm 2k\pi$, $k = 0, 1, 2, \ldots$

$\begin{cases} c = a \cdot \cos\theta \\ d = b \cdot \cos\theta \end{cases} \Rightarrow$ 共 2 MUL

④ If $\cos\theta = \pm 2^{-k} \Leftrightarrow \theta = \cos^{-1}(\pm 2^{-k})$, $k = 0, 1, 2, \ldots$

前 0 MUL, 後 2 MUL $\Rightarrow$ 共 2 MUL

⑤ If $\sin\theta = \pm 2^{-k} \Leftrightarrow \theta = \sin^{-1}(\pm 2^{-k})$, $k = 0, 1, 2, \ldots$

$\begin{cases} c = a \cdot \cos\theta + b \cdot (\pm 2^{-k}) \\ d = a \cdot (\pm 2^{-k}) + b \cdot \cos\theta \end{cases} \Rightarrow$ 共 2 MUL

(4)

(4) We know that 1D N-point DFT complexity $= O(N \log N)$

$M \times N \times P$-point DFT $=$ MN times of 1D P-point DFT

$+$ NP times of 1D M-point DFT

$+$ MP times of 1D N-point DFT

$\therefore$ complexity $= MN \times O(P \log P) + NP \times O(M \log M) + MP \times O(N \log N)$

$= O(MNP(\log M + \log N + \log P)) = O(MNP \log(MNP))$ ※

(5)、(6)、(7)

(5)

$$\begin{bmatrix} X[1] \\ X[3] \end{bmatrix} = \begin{bmatrix} a & b \\ b & -a \end{bmatrix} \begin{bmatrix} x[1]+x[4] \\ x[2]+x[3] \end{bmatrix} \Rightarrow 3\ \text{MUL (case(4) in page 346)}$$

$$\begin{bmatrix} X[2] \\ X[4] \end{bmatrix} = \begin{bmatrix} b & a \\ a & -b \end{bmatrix} \begin{bmatrix} x[1]-x[4] \\ x[2]-x[3] \end{bmatrix} \Rightarrow 3\ \text{MUL (case(4) in page 346)}$$

∴ need 6 nontrivial multiplications #

(6) (a) $143 = 13 \times 11$

$\Rightarrow MUL_{143} = 13 MUL_{11} + 11 MUL_{13} = 13 \times 40 + 11 \times 52 = 1092$

(b) $195 = 3 \times 5 \times 13 = 15 \times 13$

$\Rightarrow MUL_{195} = 15 MUL_{13} + 13 MUL_{15} = 15 \times 52 + 13 \times 40 = 1300$

(c) $196 = 4 \times 49$

By $P_2B_1 + P_1B_2 + 3D_1 + 2D_2$

We know $MUL_{49} = 7MUL_7 + 7MUL_7 + 3 \times 6 \times 6 = 7 \times 16 \times 2 + 108 = 332$

$\therefore MUL_{196} = 4MUL_{49} + 49MUL_4 = 4 \times 332 + 0 = 1328$

(7) (a) $\alpha^5 = 1 \pmod{11} \Rightarrow$ can find smallest $\alpha = 3$

By $F(k) = \sum_{n=0}^{N-1} f(n)\alpha^{nk} \pmod M$, $k = 0, 1, 2, \ldots, N-1$ ($N=5$ and $M=11$ in this case)

We have

$$\begin{bmatrix} F[0] \\ F[1] \\ F[2] \\ F[3] \\ F[4] \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 5 & 4 \\ 1 & 9 & 4 & 3 & 5 \\ 1 & 5 & 3 & 4 & 9 \\ 1 & 4 & 5 & 9 & 3 \end{bmatrix}}_{\text{transform matrix}} \begin{bmatrix} f(0) \\ f(1) \\ f(2) \\ f(3) \\ f(4) \end{bmatrix}$$

(b) INTT:

By $f(n) = N^{-1} \sum_{k=0}^{N-1} F(k)\alpha^{-nk} \pmod M$, ($N^{-1}$ is 9 since $(9 \cdot \overset{N}{5}) \bmod 11 = 1$)

we have

$$\begin{bmatrix} f(0) \\ f(1) \\ f(2) \\ f(3) \\ f(4) \end{bmatrix} = \underbrace{\begin{bmatrix} 9 & 9 & 9 & 9 & 9 \\ 9 & 3 & 1 & 4 & 5 \\ 9 & 1 & 5 & 3 & 4 \\ 9 & 4 & 3 & 5 & 1 \\ 9 & 5 & 4 & 1 & 3 \end{bmatrix}}_{\text{transform matrix}} \begin{bmatrix} F[0] \\ F[1] \\ F[2] \\ F[3] \\ F[4] \end{bmatrix}$$

學號尾數(2, 7)的 extra 問題：h 在什麼情形之下，x 和 h 的 convolution 可以用 recursive 的方法來實現(就算 h 的長度是無限長)？

答：當 h[n]是 $\alpha^n u[n]$ 的形式時($u[n] = 1\ for\ n \geq 0, u[n] = 0\ for\ n < 0$)