(63, 51) BCH soft-decision decoding

1. Parameters

$m = 6,\ n = 63,\ k = 51,\ t = 2$. Primitive polynomial $p(X) = 1 + X + X^6$.

2. Received codeword

Suppose that $r_{43},\ r_{44}$, and r_{53} are the error bits.

(1) Finding two least reliable bits

$\boldsymbol{r} = (r_0, r_1, \dots, r_{62})$

Least reliable bits are r_1 and r_{44}.

	LLR Data		Coefficients
LLR0	0	Don't care	x
LLR1	-127	r62	1
LLR2	92	r61	0
LLR3	-105	r60	1
LLR4	-127	r59	1
LLR5	72	r58	0
LLR6	82	r57	0
LLR7	86	r56	0
LLR8	-109	r55	1
LLR9	113	r54	0
LLR10	57	r53	0
LLR11	63	r52	0
LLR12	78	r51	0
LLR13	-54	r50	1
LLR14	-90	r49	1
LLR15	76	r48	0
LLR16	63	r47	0
LLR17	66	r46	0
LLR18	120	r45	0
LLR19	24	r44	0
LLR20	-86	r43	1
LLR21	121	r42	0
LLR22	58	r41	0
LLR23	-108	r40	1

LLR24	-107	r39	1
LLR25	93	r38	0
LLR26	64	r37	0
LLR27	-96	r36	1
LLR28	73	r35	0
LLR29	-60	r34	1
LLR30	-66	r33	1
LLR31	119	r32	0
LLR32	55	r31	0
LLR33	68	r30	0
LLR34	-54	r29	1
LLR35	84	r28	0
LLR36	-51	r27	1
LLR37	119	r26	0
LLR38	65	r25	0
LLR39	57	r24	0
LLR40	73	r23	0
LLR41	-85	r22	1
LLR42	57	r21	0
LLR43	-127	r20	1
LLR44	75	r19	0
LLR45	-73	r18	1
LLR46	-54	r17	1
LLR47	73	r16	0
LLR48	-53	r15	1
LLR49	-89	r14	1
LLR50	109	r13	0
LLR51	-99	r12	1
LLR52	-57	r11	1
LLR53	56	r10	0
LLR54	-110	r9	1
LLR55	120	r8	0
LLR56	-91	r7	1
LLR57	58	r6	0
LLR58	114	r5	0
LLR59	76	r4	0
LLR60	-72	r3	1

LLR61	108	r2	0
LLR62	50	r1	0
LLR63	-53	r0	1

(2) Test pattern generation

Given $\boldsymbol{r}$ and the least reliable bits r_1 and r_{44}, we generate four test patterns:

$\boldsymbol{r}_{p1} = \boldsymbol{r}$ except for $r_{p1,1} = 0$ and $r_{p1,44} = 0$.

$\boldsymbol{r}_{p2} = \boldsymbol{r}$ except for $r_{p2,1} = 1$ and $r_{p2,44} = 0$.

$\boldsymbol{r}_{p3} = \boldsymbol{r}$ except for $r_{p3,1} = 0$ and $r_{p3,44} = 1$.

$\boldsymbol{r}_{p4} = \boldsymbol{r}$ except for $r_{p4,1} = 1$ and $r_{p4,44} = 1$.

(3) Test pattern decoding

We decode each test pattern by using the hard-decision decoding algorithm.

Decode $\boldsymbol{r}_{p1}$: Failed.

Decode $\boldsymbol{r}_{p2}$: Successful, $e_{p2}(X) = X^{14} + X^{17}$. $\hat{\boldsymbol{c}}_{p2} = \boldsymbol{r}_{p2} + \boldsymbol{e}_{p2}$.

Decode $\boldsymbol{r}_{p3}$: Successful, $e_{p3}(X) = X^{43} + X^{53}$. $\hat{\boldsymbol{c}}_{p3} = \boldsymbol{r}_{p2} + \boldsymbol{e}_{p3}$.

Decode $\boldsymbol{r}_{p4}$: Successful, $e_{p4}(X) = X^{4} + X^{18}$. $\hat{\boldsymbol{c}}_{p4} = \boldsymbol{r}_{p4} + \boldsymbol{e}_{p4}$.

(4) Decoding result evaluation

Fail to decode test pattern 1, so we don’t need to calculate the correlation value.

For test pattern 2, correlation value = 4754.

For test pattern 3, correlation value = 4806. It is maximal.

For test pattern 4, correlation value = 4694.

We choose the decoded result for test pattern 3 as the answer. The corresponding $e(X) = X^{43} + X^{44} + X^{53}$.

So, the output error location numbers are 43, 44, 53, in ascending order.

Steps 3 and 4 are summarized in the following table.

In the following table, blue bits are assigned by Step 2 (test pattern generation) and green bits are corrected by Step 3 (test pattern decoding).

Besides, l_i is the LLR value of r_i. The Partial sum 2 column contains $l_i(1-2\hat{c}_{p2,i})$, the Partial sum 3 column contains $l_i(1-2\hat{c}_{p3,i})$, and the Partial sum 4 column contains $l_i(1-2\hat{c}_{p4,i})$.

i	l_i	$\hat{c}_{p2,i}$	$1-2\hat{c}_{p2,i}$	Partial sum 2	$\hat{c}_{p3,i}$	$1-2\hat{c}_{p3,i}$	Partial sum 3	$\hat{c}_{p4,i}$	$1-2\hat{c}_{p4,i}$	Partial sum 4
62	-127	1	-1	127	1	-1	127	1	-1	127
61	92	0	1	92	0	1	92	0	1	92
60	-105	1	-1	105	1	-1	105	1	-1	105
59	-127	1	-1	127	1	-1	127	1	-1	127
58	72	0	1	72	0	1	72	0	1	72
57	82	0	1	82	0	1	82	0	1	82
56	86	0	1	86	0	1	86	0	1	86
55	-109	1	-1	109	1	-1	109	1	-1	109
54	113	0	1	113	0	1	113	0	1	113
53	57	0	1	57	1	-1	-57	0	1	57
52	63	0	1	63	0	1	63	0	1	63
51	78	0	1	78	0	1	78	0	1	78
50	-54	1	-1	54	1	-1	54	1	-1	54
49	-90	1	-1	90	1	-1	90	1	-1	90
48	76	0	1	76	0	1	76	0	1	76
47	63	0	1	63	0	1	63	0	1	63
46	66	0	1	66	0	1	66	0	1	66
45	120	0	1	120	0	1	120	0	1	120
44	24	0	1	24	1	-1	-24	1	-1	-24
43	-86	1	-1	86	0	1	-86	1	-1	86
42	121	0	1	121	0	1	121	0	1	121
41	58	0	1	58	0	1	58	0	1	58
40	-108	1	-1	108	1	-1	108	1	-1	108
39	-107	1	-1	107	1	-1	107	1	-1	107
38	93	0	1	93	0	1	93	0	1	93
37	64	0	1	64	0	1	64	0	1	64
36	-96	1	-1	96	1	-1	96	1	-1	96
35	73	0	1	73	0	1	73	0	1	73
34	-60	1	-1	60	1	-1	60	1	-1	60

33	-66	1	-1	66	1	-1	66	1	-1	66
32	119	0	1	119	0	1	119	0	1	119
31	55	0	1	55	0	1	55	0	1	55
30	68	0	1	68	0	1	68	0	1	68
29	-54	1	-1	54	1	-1	54	1	-1	54
28	84	0	1	84	0	1	84	0	1	84
27	-51	1	-1	51	1	-1	51	1	-1	51
26	119	0	1	119	0	1	119	0	1	119
25	65	0	1	65	0	1	65	0	1	65
24	57	0	1	57	0	1	57	0	1	57
23	73	0	1	73	0	1	73	0	1	73
22	-85	1	-1	85	1	-1	85	1	-1	85
21	57	0	1	57	0	1	57	0	1	57
20	-127	1	-1	127	1	-1	127	1	-1	127
19	75	0	1	75	0	1	75	0	1	75
18	-73	1	-1	73	1	-1	73	0	1	-73
17	-54	0	1	-54	1	-1	54	1	-1	54
16	73	0	1	73	0	1	73	0	1	73
15	-53	1	-1	53	1	-1	53	1	-1	53
14	-89	0	1	-89	1	-1	89	1	-1	89
13	109	0	1	109	0	1	109	0	1	109
12	-99	1	-1	99	1	-1	99	1	-1	99
11	-57	1	-1	57	1	-1	57	1	-1	57
10	56	0	1	56	0	1	56	0	1	56
9	-110	1	-1	110	1	-1	110	1	-1	110
8	120	0	1	120	0	1	120	0	1	120
7	-91	1	-1	91	1	-1	91	1	-1	91
6	58	0	1	58	0	1	58	0	1	58
5	114	0	1	114	0	1	114	0	1	114
4	76	0	1	76	0	1	76	1	-1	-76
3	-72	1	-1	72	1	-1	72	1	-1	72
2	108	0	1	108	0	1	108	0	1	108
1	50	1	-1	-50	0	1	50	1	-1	-50
0	-53	1	-1	53	1	-1	53	1	-1	53
				Sum 2 = 4754			Sum 3 = 4806			Sum 4 = 4694