

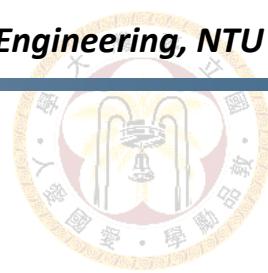
Computer-Aided VLSI System Design

**Final Project:
BCH Decoder
Lecturer: Po-Jen Chen**

Graduate Institute of Electronics Engineering, National Taiwan University



NTU GIEE

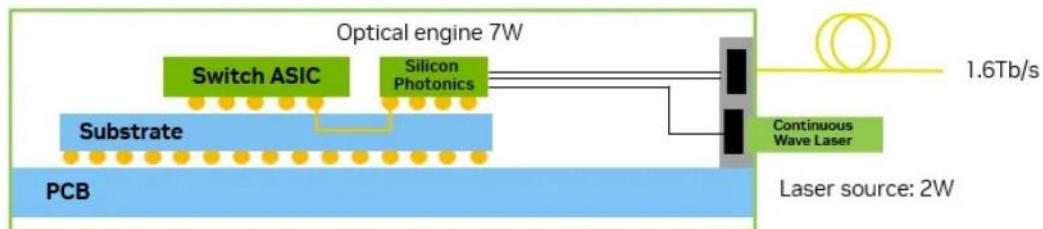


Introduction

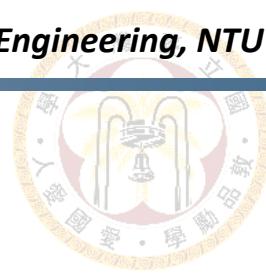
- Advanced AI programs rely on high computing capability and a large network bandwidth
- Over the past two decades [1] M. F. Chen, *IEEE ECTC*, 2025
 - Computing capability: 60,000 times ↑
 - I/O bandwidth: 30 times ↑
- Optical-based interconnects between GPU servers have been regarded as a key to address the I/O bandwidth issue



GPU server

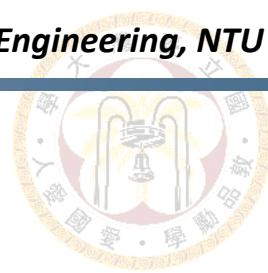


Optical interconnect



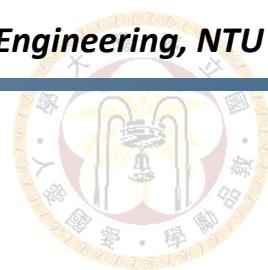
Project Overview

- What is the importance of the BCH code?
 - Optical communications still need error correction codes
 - The BCH code is a building block of more advanced codes
- Project goal
 - Implement a BCH decoder, which can decode BCH codes of (63, 51), (255, 239), and (1023, 983)
 - The BCH decoder supports hard-decision decoding and soft-decision decoding
- In this project, you will learn:
 - Basic algebra concepts: ring, field, and polynomial
 - BCH codes: encoding and decoding
 - Algebraic decoding algorithm: Berlekamp algorithm
 - Soft-decision decoding algorithm: Chase algorithm



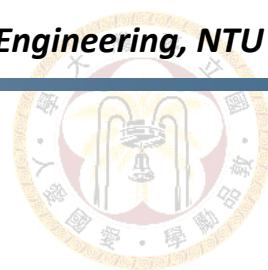
Outline

- Algebra Preliminaries
- BCH Code: Encoding and Decoding
- Project Specifications
- Project Submission
- Grading Policy



Finite Field

- A **finite field**, or called **Galois field** (GF), is a field that contains a finite number of elements
- A **field** is an algebraic structure. It is a set F with addition ‘+’ and multiplication ‘·’
 - The following properties hold for all elements $a, b, c \in F$:
 - (closure) $a + b \in F, a \cdot b \in F$
 - (associativity) $a + (b + c) = (a + b) + c, a \cdot (b \cdot c) = (a \cdot b) \cdot c$
 - (identity element) $a + 0 = 0 + a = a, a \cdot 1 = 1 \cdot a = a$
 - (inverse) there exists $(-a) \in F$ such that $a + (-a) = (-a) + a = 0$,
 - there exists $a^{-1} \in F$ such that $a \cdot a^{-1} = a^{-1} \cdot a = 1$ for $a \neq 0$
 - (commutativity) $a + b = b + a, a \cdot b = b \cdot a$
 - (distributivity) $a \cdot (b + c) = a \cdot b + a \cdot c$
- Finite fields are widely used in coding theory



GF(2)

- A finite field of two elements is F_2 or $GF(2)$
- $GF(2) = \{0, 1\}$ with the following operations
- Addition modulo 2

$$0 + 0 = 0,$$

$$0 + 1 = 1,$$

$$1 + 0 = 1,$$

$$1 + 1 = 0$$

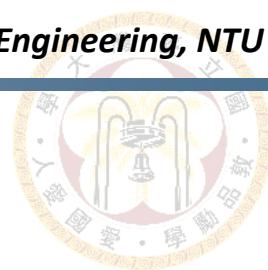
- Multiplication modulo 2

$$0 \cdot 0 = 0,$$

$$0 \cdot 1 = 0,$$

$$1 \cdot 0 = 0,$$

$$1 \cdot 1 = 1$$



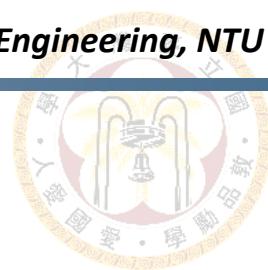
Polynomial (1/2)

- Only prime number p can form F_p or $GF(p)$ that has elements $\{0, 1, \dots, p - 1\}$
- To have more practical elements of the set, we construct finite fields by field extension
- We consider the set $F_p[X]$ of all polynomials

$$f(X) = a_0 + a_1X + \dots + a_{n-1}X^{n-1},$$

where $a_0, \dots, a_{n-1} \in F_p$.

- $F_p[X]$ is called the **polynomial ring** over F_p
- It fulfills the **ring** properties:
 - For addition: associativity, identity, inverse, commutativity
 - For multiplication: associativity
 - Distributivity



Polynomial (2/2)

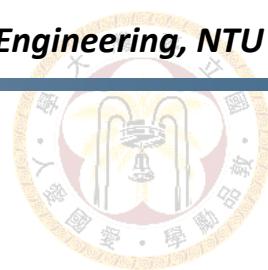
- Two polynomials $a(X)$ and $b(X)$ can be divided as

$$a(X) = b(X)q(X) + r(X)$$

with quotient $q(X)$ and remainder $r(X)$

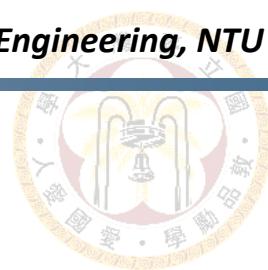
- The degree of $r(X)$ is less than the degree of $b(X)$
- It can also be written as $a(X) \equiv r(X) \bmod b(X)$

- A **factor ring** $F_p[X]/g(X)$ is the set of remainders calculated from all polynomials in $F_p[X]$ divided by $g(X)$
- This factor ring has other properties:
 - Its multiplication is commutative
 - It has multiplicative identity (unity), i.e., $f(X) = 1$



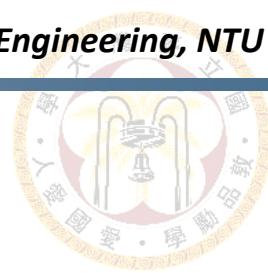
$GF(2^m)$

- **Irreducible polynomial:** a polynomial that cannot be factored into the product of two non-constant polynomials
- For the commutative factor ring $F_p[X]/g(X)$ with unity, this ring is a field if and only if $g(X)$ is irreducible
- Now we can construct an **extension field** $GF(2^m)$ with an irreducible polynomial $p(X) = p_0 + p_1X + \cdots + p_mX^m$ over $GF(2)$
- Such $p(X)$ is called a **primitive polynomial**
- Any primitive polynomial divides $X^{2^m-1} + 1$
- The factor ring $F_2[X]/p(X)$ is a field called $GF(2^m)$, which is also finite



$GF(2^m)$ Elements

- We define a **primitive element** α , for $GF(2^m)$ with primitive polynomial $p(X)$
 - Let α be a root of $p(X)$
 - Since we have $X^{2^m-1} + 1 = q(X)p(X)$,
replacing X by α , $\alpha^{2^m-1} + 1 = q(\alpha)p(\alpha) = q(\alpha) \cdot 0 = 0$
 - So, $\alpha^{2^m-1} = 1$
- $GF(2^m)$ has the elements $\{0, 1, \alpha, \alpha^2, \dots, \alpha^{2^m-2}\}$
- These elements can also be expressed as polynomials $r_i(\alpha)$
 - Recall the definition of factor ring
 - $\alpha^i \equiv r_i(\alpha) \bmod p(\alpha), i = 0, \dots, 2^m - 2$



$GF(2^m)$ Arithmetics

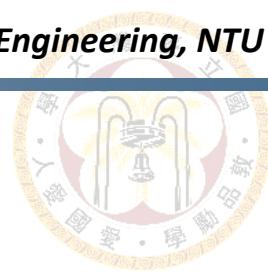
- $GF(2^m)$ elements can be expressed as polynomials over $GF(2)$
- Polynomials over $GF(2)$ can be added, subtracted, multiplied, and divided in the usual way
- The coefficients are modulo-2
- Multiplication example

$$(1 + X + X^2)(1 + X) = 1 + (X + X) + (X^2 + X^2) + X^3 = 1 + X^3$$

- Division example

$$X^3 + X + 1 = (X + 1)(X^2 + X) + 1$$

$$\begin{array}{r} x^2 + x \\ \hline x + 1 \left[\begin{array}{r} x^3 + x + 1 \\ -x^3 - x^2 \\ \hline x^2 + x + 1 \\ -x^2 - x \\ \hline 1 \end{array} \right] \end{array}$$



Example: $GF(2^4)$ Elements

- $m = 4, p(X) = 1 + X + X^4$

- Calculation examples:

$$\alpha^4 \equiv 1 + \alpha \pmod{p(\alpha)}$$

$$\begin{aligned}\alpha^7 &= \alpha^6 \cdot \alpha = (\alpha^2 + \alpha^3) \cdot \alpha \\ &= \alpha^3 + \alpha^4 \equiv 1 + \alpha + \alpha^3 \pmod{p(\alpha)}\end{aligned}$$

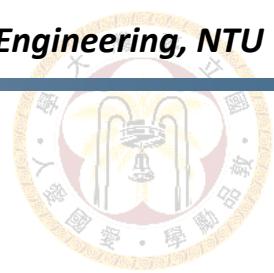
You can check $\alpha^{15} = 1$

$$\begin{aligned}\alpha^{15} &= \alpha^{14} \cdot \alpha = (1 + \alpha^3) \cdot \alpha \\ &= \alpha + \alpha^4 \equiv 1 \pmod{p(\alpha)}\end{aligned}$$

Inverse

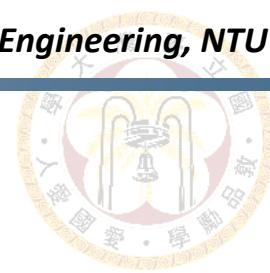
$$\begin{aligned}\alpha^{-i} \alpha^i &= 1 = \alpha^{15} \\ \Rightarrow \alpha^{-i} &= \alpha^{15-i}\end{aligned}$$

Power representation	Polynomial representation	4-tuple representation
0	0	(0 0 0 0)
1	1	(1 0 0 0)
α	α	(0 1 0 0)
α^2	α^2	(0 0 1 0)
α^3	α^3	(0 0 0 1)
α^4	$1 + \alpha$	(1 1 0 0)
α^5	$\alpha + \alpha^2$	(0 1 1 0)
α^6	$\alpha^2 + \alpha^3$	(0 0 1 1)
α^7	$1 + \alpha + \alpha^3$	(1 1 0 1)
α^8	$1 + \alpha^2$	(1 0 1 0)
α^9	$\alpha + \alpha^3$	(0 1 0 1)
α^{10}	$1 + \alpha + \alpha^2$	(1 1 1 0)
α^{11}	$\alpha + \alpha^2 + \alpha^3$	(0 1 1 1)
α^{12}	$1 + \alpha + \alpha^2 + \alpha^3$	(1 1 1 1)
α^{13}	$1 + \alpha^2 + \alpha^3$	(1 0 1 1)
α^{14}	$1 + \alpha^3$	(1 0 0 1)



Outline

- Algebra Preliminaries
- BCH Code: Encoding and Decoding
- Project Specifications
- Project Submission
- Grading Policy

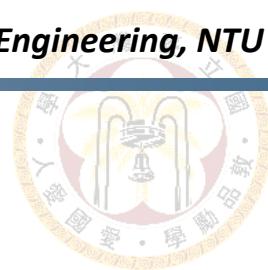


BCH Code

- The Bose, Chaudhuri, and Hocquenghem (BCH) codes form a large class of error-correcting cyclic codes
- Binary BCH codes
 - For positive integers $m \geq 3$ and $t < 2^{m-1}$, there exists a binary BCH code with the following parameters:

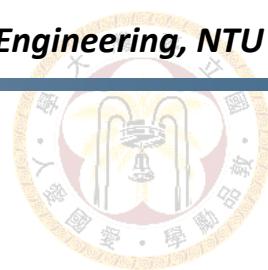
Block length	$n = 2^m - 1$
Number of parity-check bits	$n - k \leq mt$
Minimum distance	$d_{min} \geq 2t + 1$

- It can correct any combination of **t or fewer errors** in a codeword
- It is denoted by **(n, k) BCH code**



BCH Encoding (1/3)

- The $(n = 2^m - 1, k)$ BCH code has codewords in $GF(2^m)$
- Let α be a primitive element of $GF(2^m)$
- Encoding is to generate a codeword polynomial that is a multiple of the generator polynomial $g(X)$
- $g(X)$ of the t -error-correcting BCH code is the lowest-degree polynomial over $GF(2)$ that has $\alpha, \alpha^2, \alpha^3, \dots, \alpha^{2t}$ as its roots
- The degree of $g(X)$ is $n - k$
- Let $\phi_i(X)$ be the minimal polynomial of α^i , i.e., $\phi_i(\alpha^i) = 0$
- Then, $g(X) = \text{LCM}\{\phi_1(X), \phi_2(X), \dots, \phi_{2t}(X)\}$, where LCM is the least common multiple



BCH Encoding (2/3)

- Suppose that the message to be encoded is $u = (u_0, u_1, \dots, u_{k-1})$
- The corresponding message polynomial is

$$u(X) = u_0 + u_1X + \dots + u_{k-1}X^{k-1}$$

- Multiply $u(X)$ by X^{n-k}

$$X^{n-k}u(X) = u_0X^{n-k} + u_1X^{n-k+1} + \dots + u_{k-1}X^{n-1}$$

- Divide $X^{n-k}u(X)$ by the generator polynomial $g(X)$

$$X^{n-k}u(X) = g(X)a(X) + b(X),$$

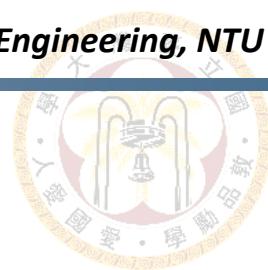
where $b(X)$ is the remainder and $\deg(b(X)) < n - k$

- Add $b(X)$ and $X^{n-k}u(X)$ to obtain codeword polynomial

$$b(X) + X^{n-k}u(X) = b_0 + b_1X + \dots + b_{n-k-1}X^{n-k-1}$$

$$+ u_0X^{n-k} + u_1X^{n-k+1} + \dots + u_{k-1}X^{n-1}$$

- The codeword is $(b_0, b_1, \dots, b_{n-k-1}, u_0, u_1, \dots, u_{k-1})$



BCH Encoding (3/3)

- (63, 51) BCH: $m = 6, n = 63, k = 51, t = 2$

$$p(X) = 1 + X + X^6$$

$$\phi_1(X) = \phi_2(X) = \phi_4(X) = 1 + X + X^6$$

$$\phi_3(X) = 1 + X + X^2 + X^4 + X^6$$

$$g(X) = 1 + X^3 + X^4 + X^5 + X^8 + X^{10} + X^{12}$$

- (255, 239) BCH: $m = 8, n = 255, k = 239, t = 2$

$$p(X) = 1 + X^2 + X^3 + X^4 + X^8$$

$$\phi_1(X) = \phi_2(X) = \phi_4(X) = 1 + X^2 + X^3 + X^4 + X^8$$

$$\phi_3(X) = 1 + X + X^2 + X^4 + X^5 + X^6 + X^8$$

$$g(X) = 1 + X + X^5 + X^6 + X^8 + X^9 + X^{10} + X^{11} + X^{13} + X^{14} + X^{16}$$

- (1023, 983) BCH: $m = 10, n = 1023, k = 983, t = 4$

$$p(X) = 1 + X^3 + X^{10}$$

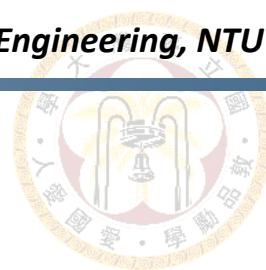
$$\phi_1(X) = \phi_2(X) = \phi_4(X) = \phi_8(X) = 1 + X^3 + X^{10}$$

$$\phi_3(X) = \phi_6(X) = 1 + X + X^2 + X^3 + X^{10}$$

$$\phi_5(X) = 1 + X^2 + X^3 + X^8 + X^{10}$$

$$\phi_7(X) = 1 + X^3 + X^4 + X^5 + X^6 + X^7 + X^8 + X^9 + X^{10}$$

$$g(X) = 1 + X + X^3 + X^4 + X^7 + X^9 + X^{10} + X^{11} + X^{12} + X^{16} + X^{19} + X^{21} \\ + X^{22} + X^{23} + X^{24} + X^{25} + X^{27} + X^{29} + X^{30} + X^{31} + X^{33} + X^{39} + X^{40}$$

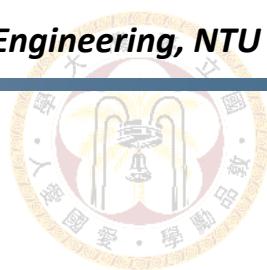


BCH Decoding (1/8)

- Suppose that a codeword polynomial $c(X) = c_0 + c_1X + \cdots + c_{n-1}X^{n-1}$ is transmitted
- The received polynomial is $r(X) = r_0 + r_1X + \cdots + r_{n-1}X^{n-1}$
- Let $e(X)$ be the error pattern. Then,

$$r(X) = c(X) + e(X)$$

- The algebraic (hard-decision) decoding steps:
 1. Compute the syndromes from the received polynomial $r(X)$
 2. Determine the error-location polynomial $\sigma(X)$ from the syndromes
 3. Determine the error locations by finding the roots of $\sigma(X)$, and correct the errors in $r(X)$



BCH Decoding (2/8)

- **Step 1: Syndrome calculation, $S = (S_1, S_2, \dots, S_{2t})$**

- For $1 \leq i \leq 2t$,

$$S_i = r(\alpha^i) = r_0 + r_1\alpha^i + r_2\alpha^{2i} + \dots + r_{n-1}\alpha^{(n-1)i}$$

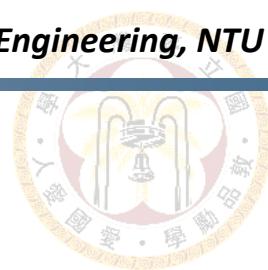
- We can also compute S_i as follows

- Since $r(X) = a_i(X)\phi_i(X) + b_i(X)$,

$$S_i = r(\alpha^i) = b_i(\alpha^i),$$

where $\phi_i(X)$ is the minimal polynomial of α^i

- Error pattern $e(X)$ is not divisible by $\phi_i(X)$
- Therefore, $e(\alpha^i) = S_i$, which is the relationship between the syndrome and the error pattern



BCH Decoding (3/8)

- The syndromes depend on the error pattern $e(X)$ only
- Suppose that $e(X)$ has v errors at locations $X^{j_1}, X^{j_2}, \dots, X^{j_v}$, that is,

$$e(X) = X^{j_1} + X^{j_2} + \dots + X^{j_v}$$

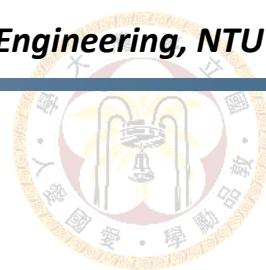
- From $e(\alpha^i) = S_i$, we obtain the following set of equations:

$$S_1 = \alpha^{j_1} + \alpha^{j_2} + \dots + \alpha^{j_v}$$

$$S_2 = (\alpha^{j_1})^2 + (\alpha^{j_2})^2 + \dots + (\alpha^{j_v})^2$$
$$\vdots$$

$$S_{2t} = (\alpha^{j_1})^{2t} + (\alpha^{j_2})^{2t} + \dots + (\alpha^{j_v})^{2t}$$

- We need to solve for the unknown $\alpha^{j_1}, \alpha^{j_2}, \dots, \alpha^{j_v}$
- These j_1, j_2, \dots, j_v are called the error location numbers



BCH Decoding (4/8)

- Define the error-location polynomial

$$\begin{aligned}\sigma(X) &= (1 + \alpha^{j_1}X)(1 + \alpha^{j_2}X) \dots (1 + \alpha^{j_v}X) \\ &= \sigma_0 + \sigma_1X + \sigma_2X^2 + \dots + \sigma_vX^v\end{aligned}$$

- Compare the coefficients of $\sigma(X)$ with previous set of equations
- We find that σ_i 's are related to S_i 's by the following:

$$S_1 + \sigma_1 = 0$$

$$S_2 + \sigma_1S_1 + 2\sigma_2 = 0$$

$$S_3 + \sigma_1S_2 + \sigma_2S_1 + 3\sigma_3 = 0$$

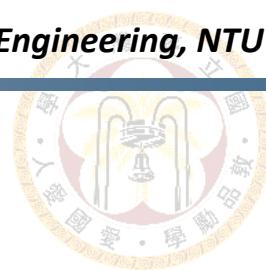
$$\vdots$$

$$S_v + \sigma_1S_{v-1} + \dots + \sigma_{v-1}S_1 + v\sigma_v = 0$$

$$S_{v+1} + \sigma_1S_v + \dots + \sigma_{v-1}S_2 + \sigma_vS_1 = 0$$

$$\vdots$$

- The Berlekamp's iterative algorithm is used to calculate all σ_i 's



BCH Decoding (5/8)

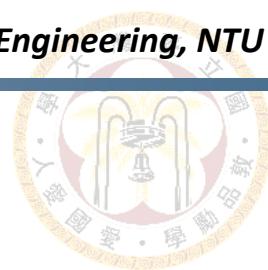
- Step 2: Berlekamp's algorithm
- Initialization

μ	$\sigma^{(\mu)}(X)$	d_μ	l_μ	$\mu - l_\mu$
-1	1	1	0	-1
0	1	S_1	0	0

- Loop: Assuming that we have filled out $-1, 0, 1, \dots, \mu$ th rows, we fill out the $(\mu + 1)$ th row as follows
 - If $d_\mu = 0$, then $\sigma^{(\mu+1)}(X) = \sigma^{(\mu)}(X)$, and $l_{\mu+1} = l_\mu$
 - If $d_\mu \neq 0$, then we find another row $\rho < \mu$ such that $d_\rho \neq 0$ and the number $\rho - l_\rho$ has the largest value. Then,

$$\sigma^{(\mu+1)}(X) = \sigma^{(\mu)}(X) + d_\mu d_\rho^{-1} X^{(\mu-\rho)} \sigma^{(\rho)}(X), \text{ and}$$

$$l_{\mu+1} = \max(l_\mu, l_\rho + \mu - \rho)$$



BCH Decoding (6/8)

- In either case,

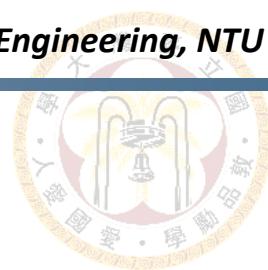
$$d_{\mu+1} = S_{\mu+2} + \sigma_1^{(\mu+1)} S_{\mu+1} + \cdots + \sigma_{l_{\mu+1}}^{(\mu+1)} S_{\mu+2-l_{\mu+1}}$$

where $\sigma_i^{(\mu+1)}$'s are the coefficients of $\sigma^{(\mu+1)}(X)$

- Termination

After we obtain $\sigma^{(2t)}(X)$, we can terminate the loop

- The error-location polynomial $\sigma(X) = \sigma^{(2t)}(X)$



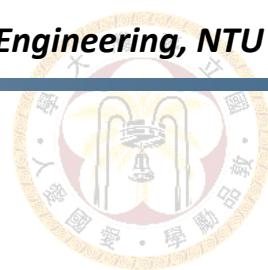
BCH Decoding (7/8)

- **Step 3: Finding the roots of $\sigma(X)$**

We find the roots by substituting α^{-j} into $\sigma(X)$

If $\sigma(\alpha^{-j}) = 0$ for $j = j_i$, then α^{-j_i} is a root

- All the error location numbers j_1, j_2, \dots, j_v can be found
- So, we obtain the error pattern $e(X) = X^{j_1} + X^{j_2} + \dots + X^{j_v}$
- If the number of roots found is less than the degree of $\sigma(X)$, the decoding is failed
- If the number of roots found is equal to the degree of $\sigma(X)$, the decoding is successful



BCH Decoding (8/8)

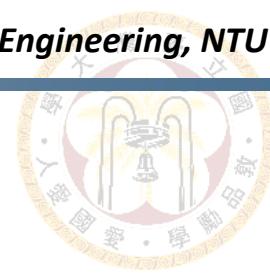
- Finally, the error correction is simply adding $e(X)$

$$\hat{c}(X) = r(X) + e(X)$$

$$= \hat{b}_0 + \hat{b}_1 X + \cdots + \hat{b}_{n-k-1} X^{n-k-1} + \hat{u}_0 X^{n-k} + \cdots + \hat{u}_{k-1} X^{n-1}$$

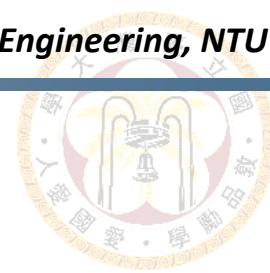
- The decoded message is the k highest-degree terms of $\hat{c}(X)$

$$\hat{u} = (\hat{u}_0, \hat{u}_1, \dots, \hat{u}_{k-1})$$



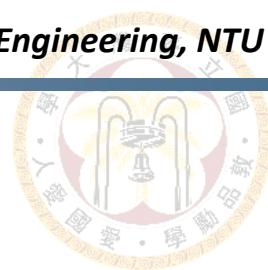
Soft-Decision Decoding (1/3)

- The soft-decision decoding algorithms utilize channel measurement information to enhance performance
- Chase proposed a soft-decision decoding algorithm, which tests multiple $r(X)$ candidates to find the most probable result
- Chase algorithm is outlined as follows:
 1. Determine the p least reliable bits of the received polynomial $r(X)$
 2. Generate 2^p test patterns by enumerating all combinations of the p least reliable bits
 3. Decode all test patterns by using the hard-decision decoding algorithm
 4. Evaluate each successfully decoded result, and choose the most probable result



Soft-Decision Decoding (2/3)

- **Step 1: Finding p least reliable bits**, $p = 2$ in this project
- Received polynomial $r(X) = r_0 + r_1X + \cdots + r_{n-1}X^{n-1}$
- The log-likelihood ratio (LLR) of each received bit r_i indicates the coefficient of $r(X)$ and the reliability of that bit
 - LLR ≥ 0 : the bit tends to be 0, i.e., $r_i = 0$
 - LLR < 0 : the bit tends to be 1, i.e., $r_i = 1$
 - When the absolute value of LLR is smaller, it means the bit is less reliable
- We find the bits that have p smallest absolute values of LLRs
- **Step 2: Test pattern generation**
- Each of the p least reliable bits can be 0 or 1
- We generate all 2^p test patterns from $r(X)$



Soft-Decision Decoding (3/3)

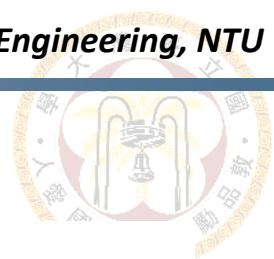
- **Step 3: Test pattern decoding**
- We decode all test patterns by using the hard-decision decoding algorithm
- **Step 4: Decoding result evaluation**
- We only consider successfully decoded results
- Let $\hat{c}(X) = \hat{c}_0 + \hat{c}_1 X + \cdots + \hat{c}_{n-1} X^{n-1}$ be a decoded result
- The correlation value is calculated as

$$\sum_{i=0}^{n-1} l_i(1 - 2\hat{c}_i),$$

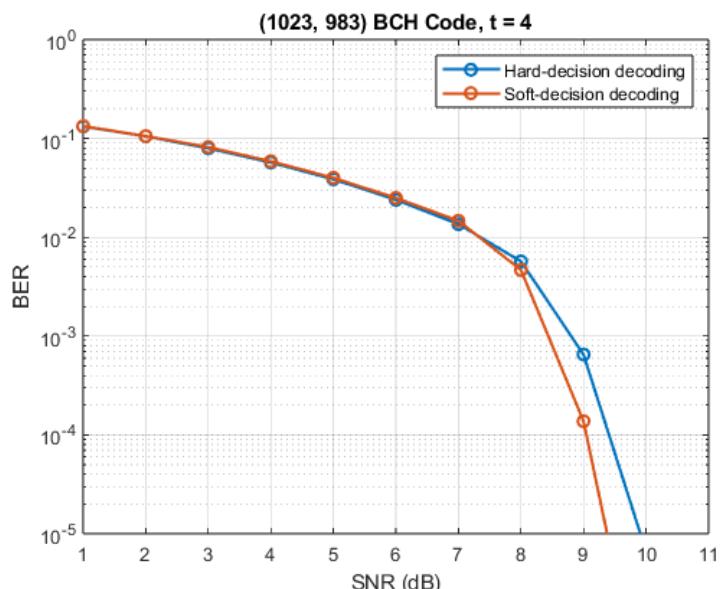
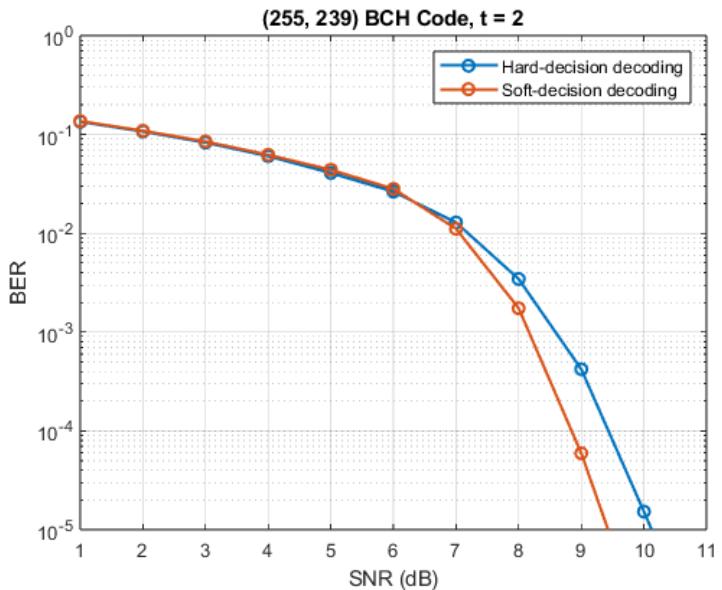
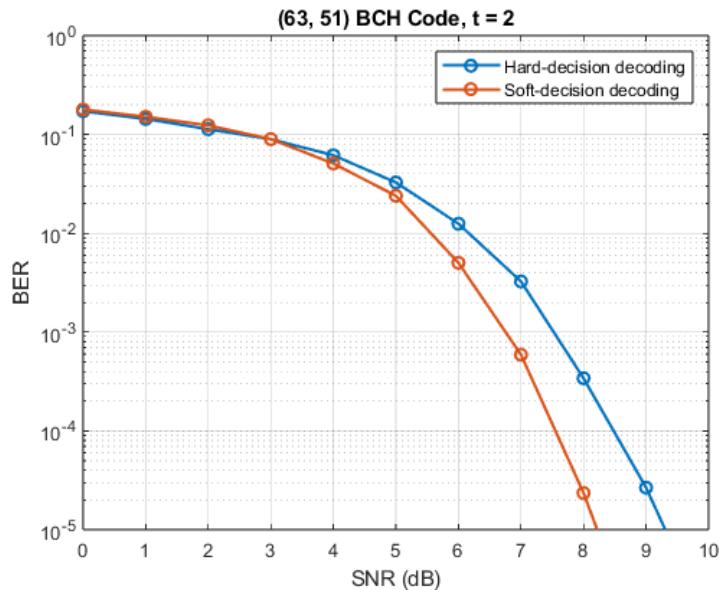
where l_i is the LLR of r_i

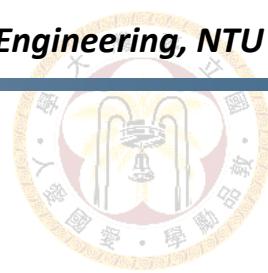
$$\begin{aligned}\hat{c}_i = 0 &\Rightarrow 1 - 2\hat{c}_i = 1 \\ \hat{c}_i = 1 &\Rightarrow 1 - 2\hat{c}_i = -1\end{aligned}$$

- Treat the coefficients of $\hat{c}(X)$ as real-valued numbers
- The decoded result of maximal correlation is the most probable result



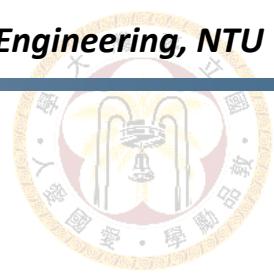
Bit Error Rate of BCH Codes



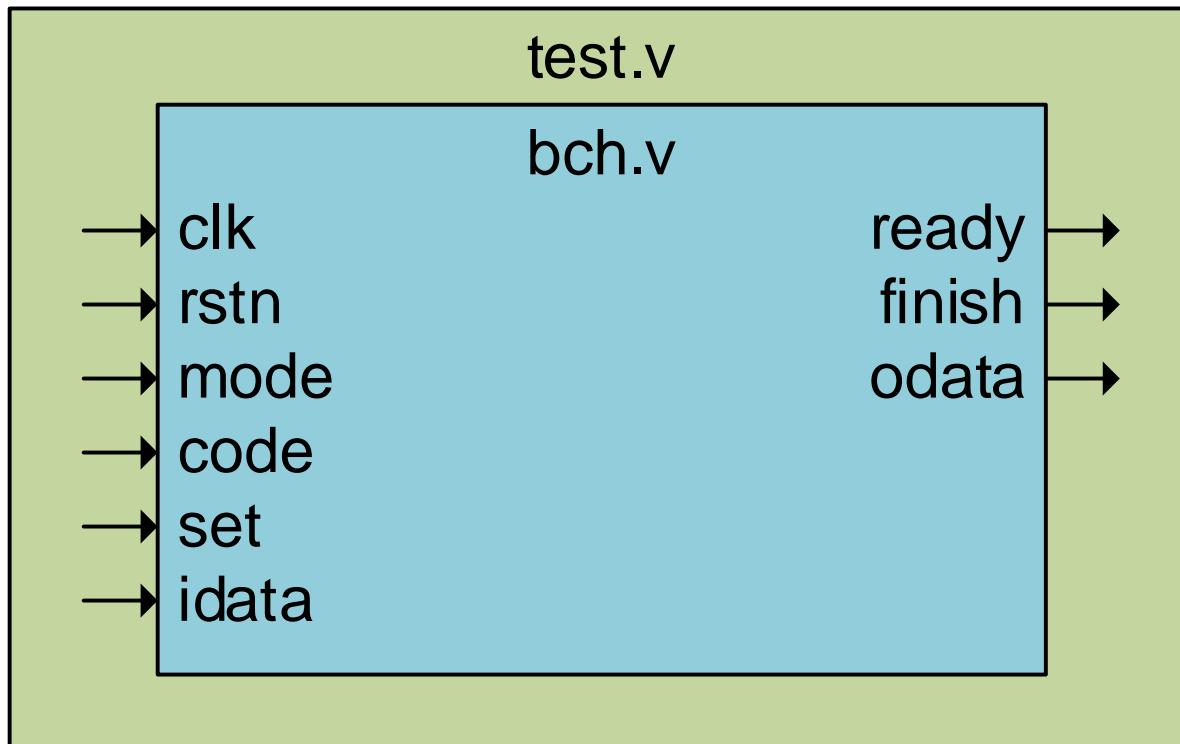


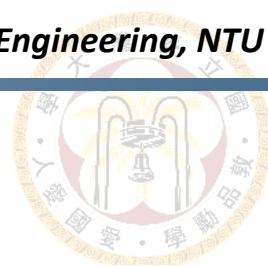
Outline

- Algebra Preliminaries
- BCH Code: Encoding and Decoding
- Project Specifications
- Project Submission
- Grading Policy



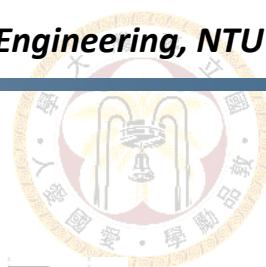
Block Diagram



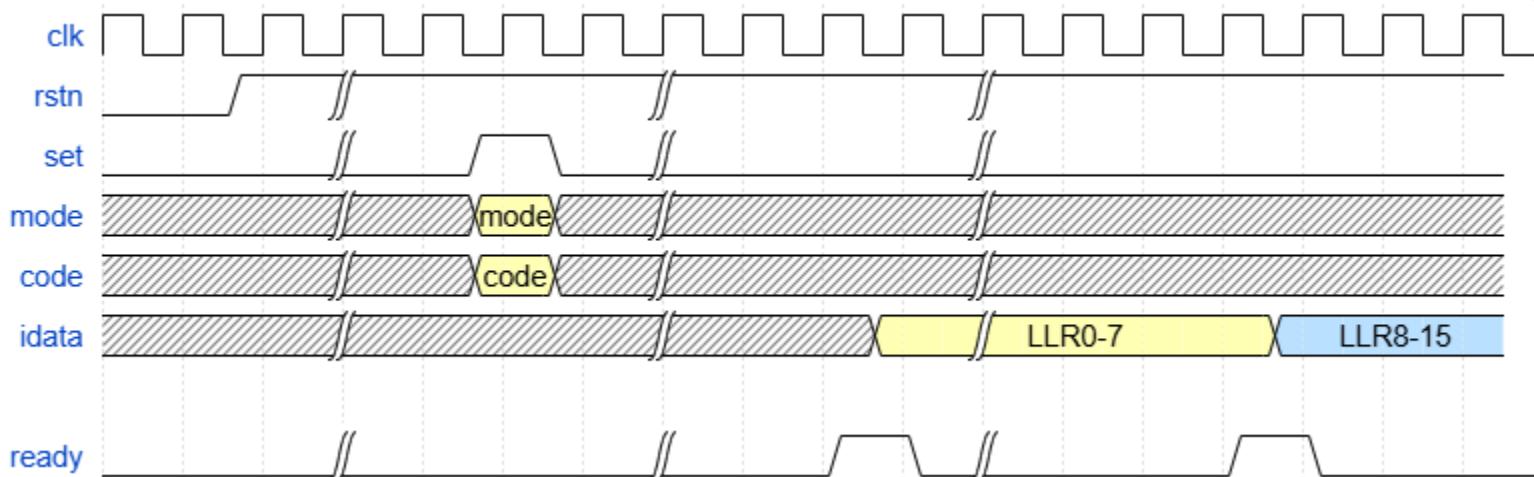


Input/Output

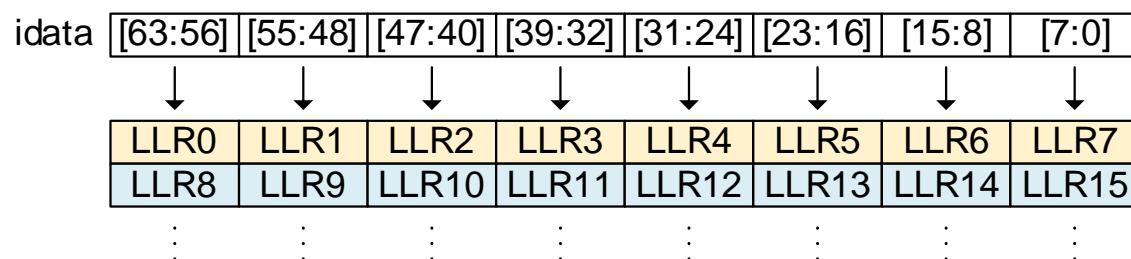
Signal name	I/O	Width	Description
clk	I	1	System clock signal
rstn	I	1	Synchronous active low reset signal
mode	I	1	0: hard-decision decoding 1: soft-decision decoding
code	I	2	1: (63, 51) BCH code 2: (255, 239) BCH code 3: (1023, 983) BCH code
set	I	1	Set the decoding scheme
idata	I	64	Eight LLR signals, each of which is 8 bits
ready	O	1	Indicates the state ready to get input LLR data
finish	O	1	Indicates the end of decoding
odata	O	10	Output error location number

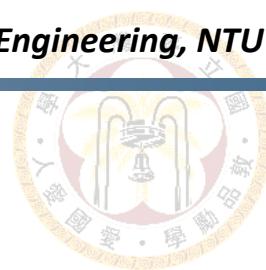


Input Waveform (1/2)



- **mode** and **code** are valid when **set** is HIGH
 - Your design should decode codewords under these settings
- After the design is set, LLR data are given through **idata** port
 - **idata** is valid right after **ready** is HIGH

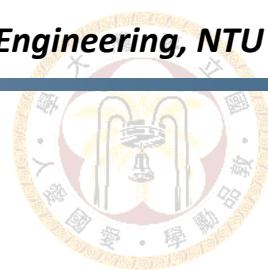




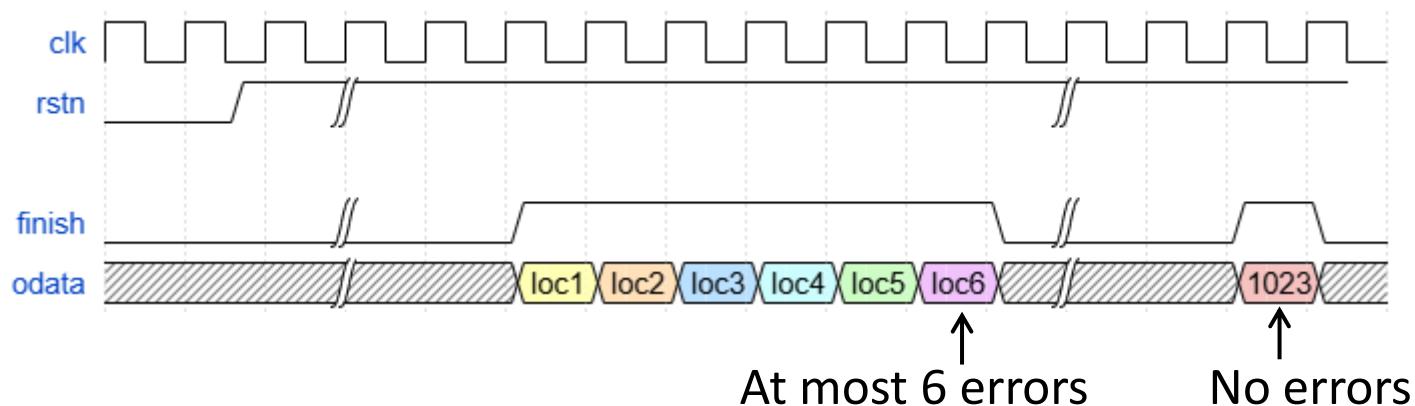
Input Waveform (2/2)

- The testbench streams out LLR data
- LLR data arrangement
 - Each LLR value is 8-bit signed integer
 - The order is in **descending power**:

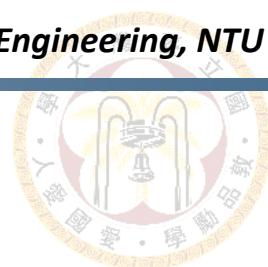
LLR0 = don't care,
LLR1 corresponds to X^{n-1} term of $r(X)$,
LLR2 corresponds to X^{n-2} term of $r(X)$, ...
- The testbench generates all signals at the **negative edge** of the clock



Output Waveform (1/2)

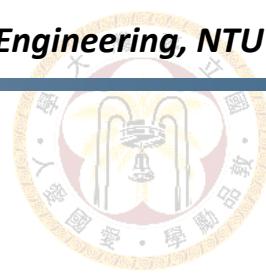


- Your design should continuously output error location numbers through **odata** port, along with **finish** raised to HIGH
 - Ascending error location numbers: $\text{loc1} < \text{loc2} < \dots$
 - The number of output cycles is the number of errors
 - If a codeword has no errors, then **odata** = 'd1023 for a cycle
- Note: $e(X) = X^{j_1} + \dots + X^{j_v}$. Then j_1, \dots, j_v are the error location numbers



Output Waveform (2/2)

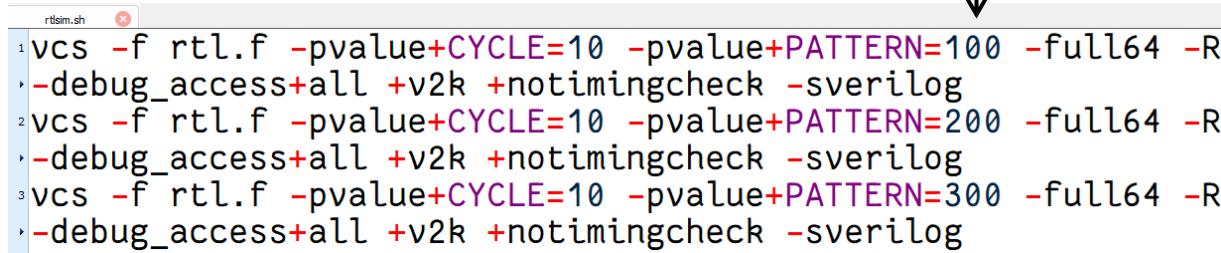
- The testbench will not raise **set** to HIGH until your design completes its output
 - Example sequence:
Testbench raises **set** → your design raises **finish** → ... →
your design raises **finish** → testbench raises **set** → ...
- The testbench checks all signals at the **negative edge** of the clock



Simulation Settings

- RTL simulation: In **01_RTL/**

```
$ bash rtlsim.sh
```

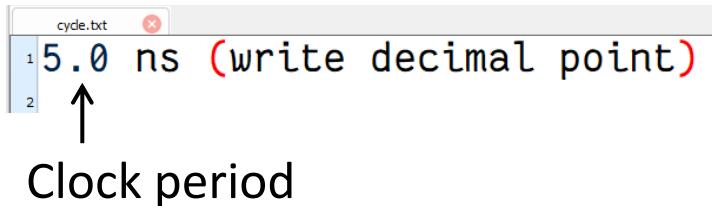


```
rtsim.sh
1 vcs -f rtl.f -pvalue+CYCLE=10 -pvalue+PATTERN=100 -full64 -R
> -debug_access+all +v2k +notimingcheck -sverilog
2 vcs -f rtl.f -pvalue+CYCLE=10 -pvalue+PATTERN=200 -full64 -R
> -debug_access+all +v2k +notimingcheck -sverilog
3 vcs -f rtl.f -pvalue+CYCLE=10 -pvalue+PATTERN=300 -full64 -R
> -debug_access+all +v2k +notimingcheck -sverilog
```

- Gate-level simulation: In **03_GATE/**

Write your cycle time in **cycle.txt**

```
$ bash gatesim.sh
```



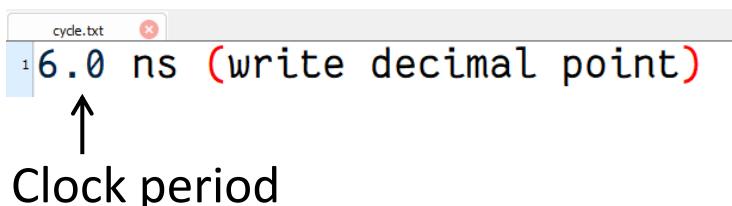
```
cycle.txt
1 5.0 ns (write decimal point)
2
```

↑
Clock period

- Post-layout simulation: In **05_POST/**

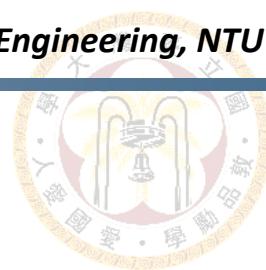
Write your cycle time in **cycle.txt**

```
$ bash postsim.sh
```



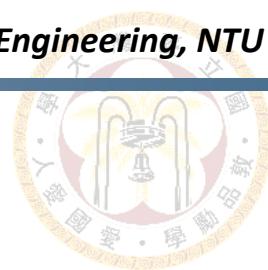
```
cycle.txt
1 6.0 ns (write decimal point)
2
```

↑
Clock period



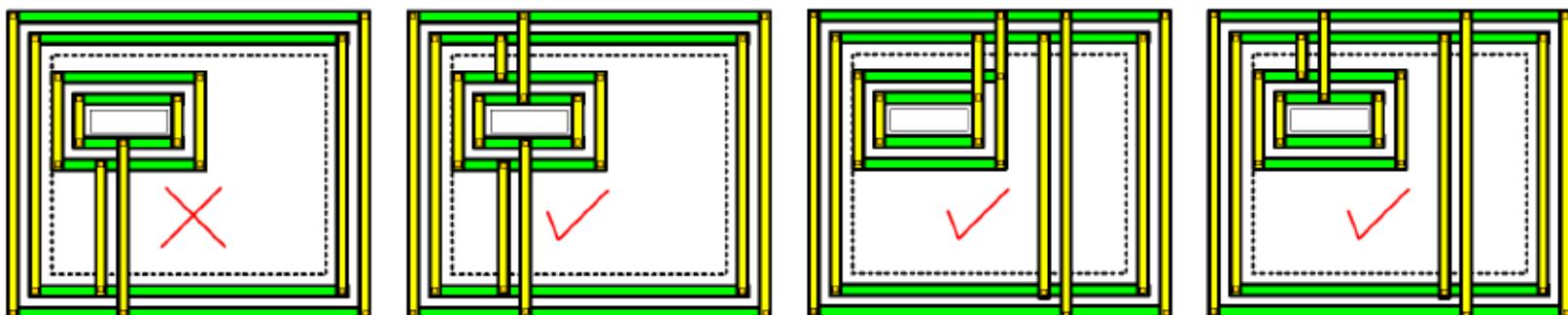
Timing Specifications

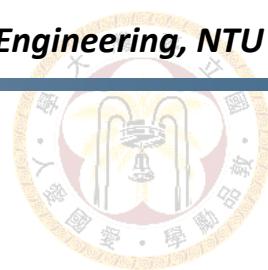
- Only the worst-case library is used for synthesis
- The slack for setup time should be nonnegative for synthesis
- The slack for setup time and hold time should be nonnegative for APR
- **No timing violation** for the gate-level simulation and post-layout simulation
 - The timing violation due to reset during the first 10 cycles can be ignored
- Your design should not exceed the maximum cycle of 1000000 for each pattern



APR Specifications (1/2)

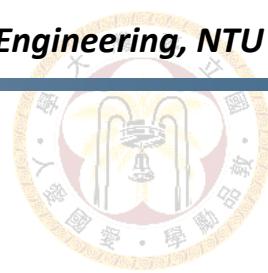
- Only the **macro layout** is needed
 - IO pads and bonding pads are not required
 - The core area is used for grading
- VDD and VSS power rings should each be $2 \mu\text{m}$ wide, with only one ring required for each
- Power stripes
 - At least one set, with VDD and VSS stripes each $2 \mu\text{m}$ wide
 - Vertical power stripes require at least one set





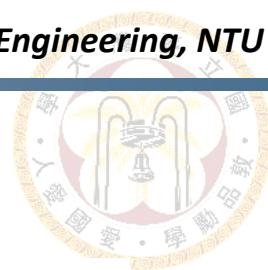
APR Specifications (2/2)

- Remember to add the power rail (follow pin)
- Dummy metal layers are not needed
- Core filler must be added
- The GDSII file after APR must be generated
- Ensure that the APR DRC/LVS is completely **error-free**



Outline

- Algebra Preliminaries
- BCH Code: Encoding and Decoding
- Project Specifications
- Project Submission
- Grading Policy



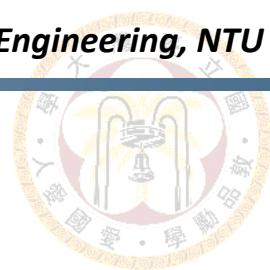
Submission (1/3)

Create a folder named **teamID_final**,

e.g., **team001_final**

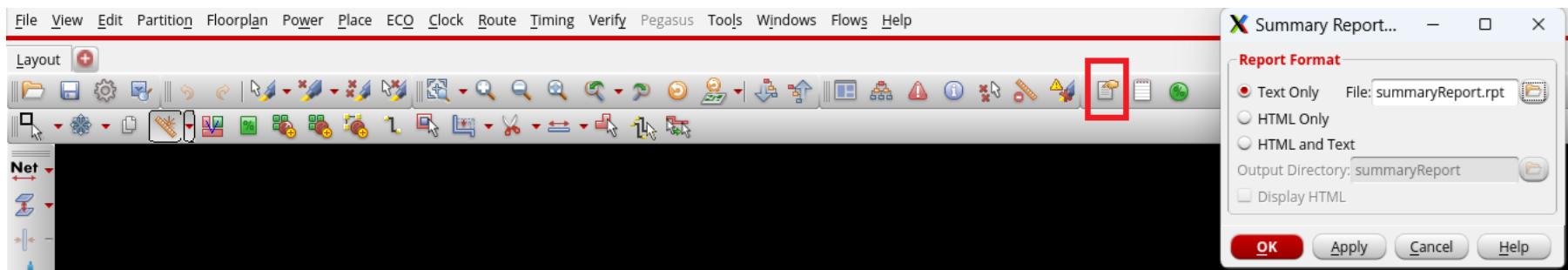
 Three digits

```
team00_final/
├── 01_RTL/
│   ├── bch.v (and other RTL files)
│   └── rtl.f (RTL filelist)
├── 02_SYN/
│   ├── Netlist/
│   │   ├── bch_syn.v
│   │   └── bch_syn.sdf
│   └── Report/
│       ├── bch_syn.area (area report)
│       └── bch_syn.timing_max (setup time report)
├── 03_GATE/
│   └── cycle.txt (your clock period)
├── 04(APR)/
│   ├── DBS/
│   │   ├── final (saved APR design)
│   │   └── final.dat (APR database)
│   ├── Netlist/
│   │   ├── bch_apr.v
│   │   └── bch_apr.sdf
│   └── Report/
│       ├── summaryReport.rpt (area report)
│       ├── bch_postRoute.summary (setup time report)
│       ├── bch_postRoute_hold.summary (hold time report)
│       ├── bch.geom.rpt (DRC report)
│       ├── bch.conn.rpt (LVS report)
│       └── bch.antenna.rpt (DRC report)
└── bch.gds
└── 05_POST/
    └── cycle.txt (your clock period)
└── team00_report.pdf
```

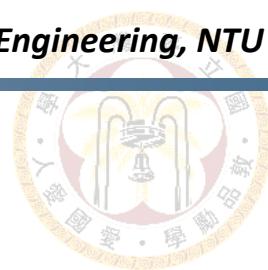


Submission (2/3)

- APR area report

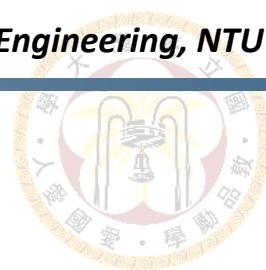


- APR timing reports can be found in **04_APR/timingReports/**
- Other reports can be found in **04_APR/**



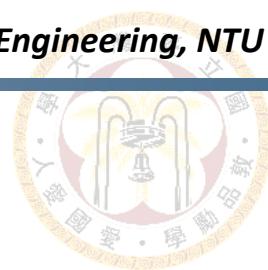
Submission (3/3)

- **Deadline:** 2025/12/16 13:59:59 (UTC+8)
- Compress the folder **teamID_final** (all lowercase) in a tar file named **teamID_final_vk.tar** (k: version number, e.g., 1, 2, ...)
- Ensure all required files are submitted
- Submit to NTU Cool



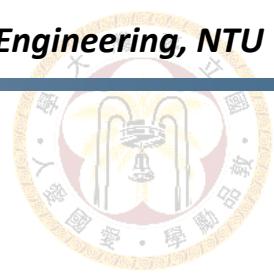
Report Requirements

- Format: written in English, single column
- Contents:
 1. Algorithm design
 - Describe the analysis and improvements you have done
 2. Hardware implementation
 - Provide the hardware architecture, key module implementation, and hardware optimization techniques
 3. APR results
 - Show the layout photo
 4. Performance evaluation
 - Assess the efficiency of your design, including metrics such as speed, power, and area



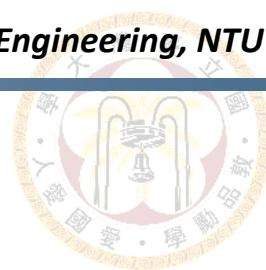
Discussion

- NTU Cool Discussion Forum
 - For any questions not related to assignment answers or privacy concerns, please use the NTU Cool discussion forum
 - TAs will prioritize answering questions on the NTU Cool discussion forum
- Email: d13943013@ntu.edu.tw
 - Title should start with [CVSD 2025 Fall Final Project]
 - Email with wrong title will be moved to trash automatically



Outline

- Algebra Preliminaries
- BCH Code: Encoding and Decoding
- Project Specifications
- Project Submission
- Grading Policy



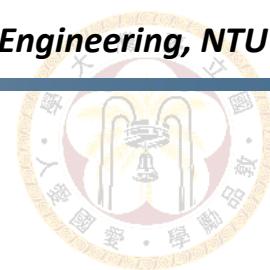
Grading Policy Overview

- Baseline 50% + Performance 40% + Report 10%
- Baseline

Item	Percentage	Description
RTL	10	RTL simulation using open patterns
Synthesis	10	Gate-level simulation using open patterns
APR	10	Post-layout simulation using open patterns
APR	20	Post-layout simulation using hidden patterns

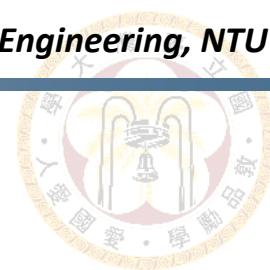
- Performance

Item	Percentage	Description
Latency per bit	30	Post-layout simulation using hidden patterns $\text{Area} \times \text{Time} / n$ (excluding data input time)
Energy per bit	10	Post-layout simulation using hidden patterns $\text{Power} \times \text{Time} / n$



Grading Policy Details (1/3)

- The points of each item can be partially given
 - It is according to the number of patterns you pass
- Open patterns
 - Three patterns ($n=63, 255, 1023$) for hard-decision decoding
 - Each pattern has two codewords
- Hidden patterns
 - Mixed combinations of $\{n=63, 255, 1023\} \times \{\text{hard-decision, soft-decision}\}$
 - Each pattern has **one** codeword for the simplicity
- A test pattern is regarded as passed if **all codewords** in that pattern are correctly decoded



Grading Policy Details (2/3)

- Performance ranking
 - Sorting order:
 - (1) Project completeness (no violation of specifications),
APR > Synthesis
 - (2) The number of passed patterns
 - (3) Performance metric $A \times T$ or $P \times T$ (sum of all patterns)
 - Example

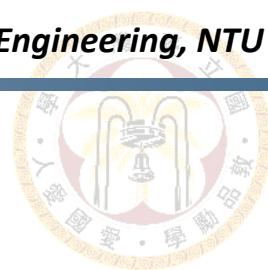
Student A: APR done, pass 10 patterns, $A \times T = 10000$

Student B: APR done, pass 8 patterns, $A \times T = 6000$

Student C: APR done, pass 8 patterns, $A \times T = 8000$

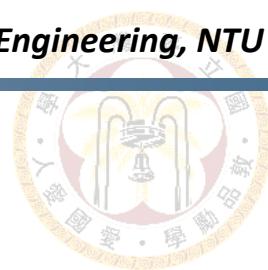
Student D: Synthesis done, pass 10 patterns, $A \times T = 8000$

Ranking: #1 is A, #2 is B, #3 is C, and #4 is D



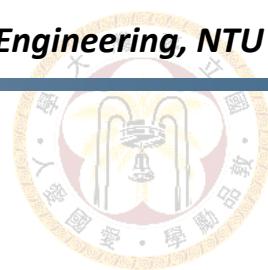
Grading Policy Details (3/3)

- **Late submissions are not accepted**
 - Any submission after the deadline will receive 0 points
- File corrections after the deadline should be avoided
 - Corrections for the folder name, file name, file hierarchy, missing file cause 5-point deduction
- The TA will grade your submissions by using scripts
- **No plagiarism**
 - Plagiarism in any form, including copying from online sources, is strictly prohibited



Final Project Presentation

- Date: December 23
 - Time: 14:20 – 17:20
 - Online
-
- Top-performing teams will be invited to present their design optimization strategies
 - Bonus points will be awarded for presentations



References

- Algebra textbook

John B. Fraleigh, “*A First Course in Abstract Algebra*,” Pearson, 2013.

- Coding theory textbook

Shu Lin and Daniel J. Costello, “*Error Control Coding: Fundamentals and Applications*,” Pearson, 2004.

- Soft-decision decoding

D. Chase, “A class of algorithms for decoding block codes with channel measurement information,” *IEEE Trans. Inf. Theory*, vol. 18, no. 1, pp. 170–182, Jan. 1972.