Think Bayesian

A man is running. Why?

Principle 1: Use prior knowledge

1 He is in a hurry

2 He is doing sports

3 He always runs

4 ~~He saw a dragon~~

Low prior probability

A man is running. Why?

Principle 2: Chose answer that explains observations the most

Suppose, for the sake of argument, that he is not wearing a sports suit. Then, 'he is doing sports' doesn't really explain our observation.

1

He is in a hurry

2

~~He is doing sports~~

Contradicts the data

3

He always runs

4

~~He saw a dragon~~

A man is running. Why?

Principle 3: Avoid making extra assumptions

1

He is in a hurry

2

~~He is doing sports~~

3

~~He always runs~~

Too many assumptions

4

~~He saw a dragon~~

Main principles

Principle 1:

Use prior knowledge

Principle 2:

Chose answer that explains observations the most

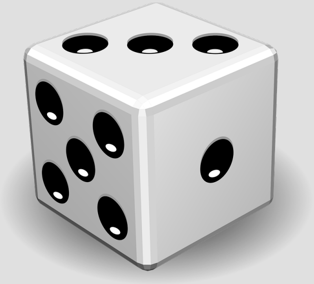
Principle 3:

Avoid making extra assumptions

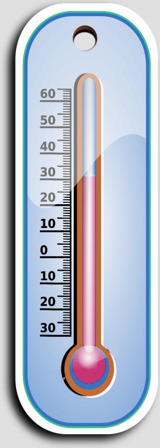
Principle 3 is also known as Occam's razor.

Review of probability

Random variables

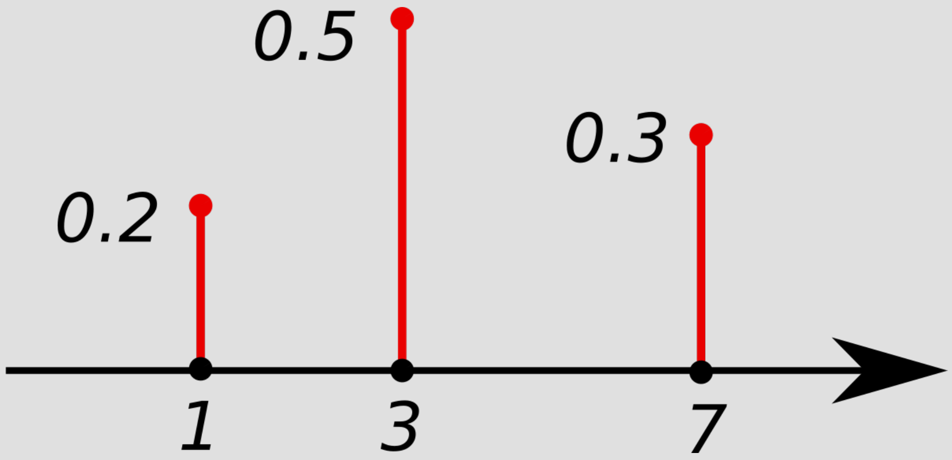


Discrete



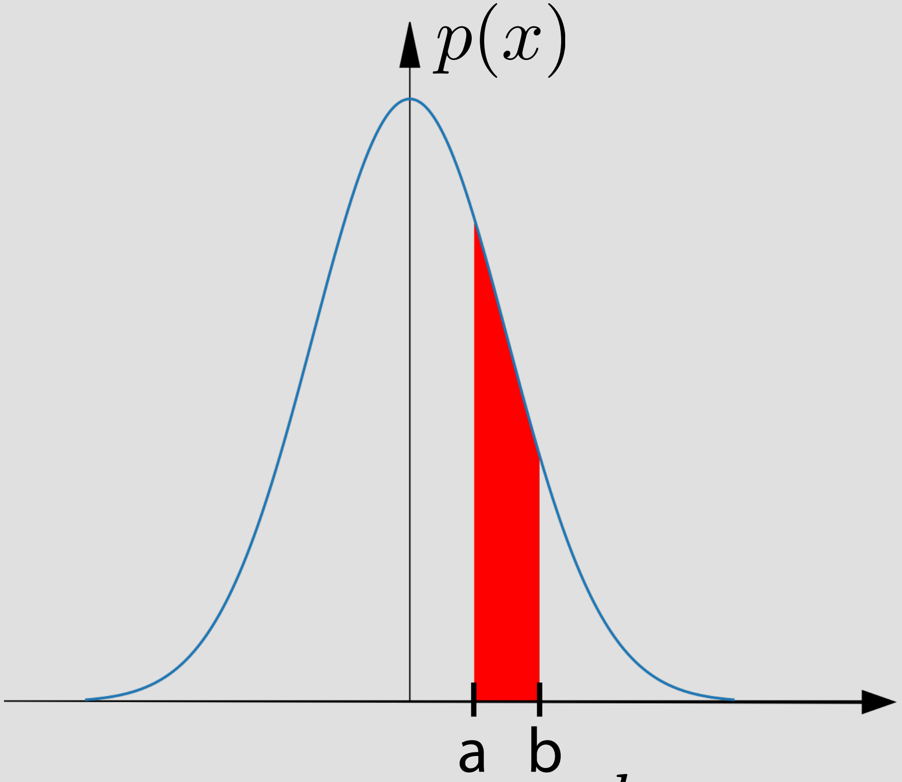
Continuous

Discrete: Probability Mass Function (PMF)



$$P(X) = \begin{cases} 0.2 & X = 1 \\ 0.5 & X = 3 \\ 0.3 & X = 7 \\ 0 & otherwise \end{cases}$$

Continuous: Probability Density Function (PDF)



$$P(x \in [a,b]) = \int_a^b p(x)dx$$

Independence

X and Y are independent if:

Marginals

$$P(X,Y) = P(X)P(Y)$$

Joint

Conditional probability

Probability of **X** given that **Y** happened:

Joint

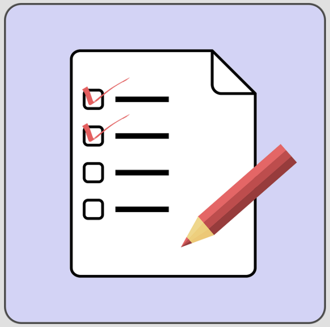
$$P(X|Y) = \frac{P(X,Y)}{P(Y)}$$

Conditional

Marginal

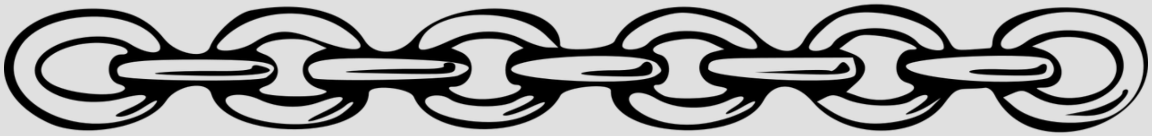
Conditional probability

$$P(M) = 0.4$$
$$P(M \,\&\, F) = 0.25$$



$$P(F|M) = \frac{P(M \,\&\, F)}{P(M)} = \frac{0.25}{0.4} = 0.625$$

Chain rule



$$P(X,Y) = P(X|Y)P(Y)$$

$$P(X,Y,Z) = P(X|Y,Z)P(Y|Z)P(Z)$$

$$P(X_1,\ldots,X_N) = \prod_{i=1}^{N} P(X_i|X_1,\ldots,X_{i-1})$$

Sum rule

Marginalization

$$p(X) = \int_{-\infty}^{\infty} p(X, Y) dY$$

Bayes theorem

This is important!

θ — parameters

X — observations

Posterior Likelihood Prior

$$P(\theta|X) = \frac{P(X,\theta)}{P(X)} = \frac{P(X|\theta)P(\theta)}{P(X)}$$

Evidence