Gibbs Sampling

It would be nice to build a markov chain to sample from a distribution. But how to do it?

We use gibbs sampling.

$$p(x_1, x_2, x_3) = \frac{\widehat{p}(x_1, x_2, x_3)}{Z}$$

Suppose we know the above probability distribution. Then:

Start with (x_1^0, x_2^0, x_3^0), e. g. $(0, 0, 0)$

Start from an initial point.

$$x_1^1 \sim p(x_1 \mid x_2 = x_2^0, x_3 = x_3^0)$$

$$= \frac{\widehat{p}(x_1, x_2^0, x_3^0)}{Z_1}$$

Then, build our next iteration point.
Build this ONE dimension at a time.
So in this case, we use dimension x_1.

But how do we sample from this conditional distribution? 1d distributions are normally easier.

Gibbs Sampling

$$p(x_1, x_2, x_3) = \frac{\widehat{p}(x_1, x_2, x_3)}{Z}$$

Start with (x_1^0, x_2^0, x_3^0), e. g. $(0, 0, 0)$

$$x_1^1 \sim p(x_1 \mid x_2 = x_2^0, x_3 = x_3^0)$$

$$x_2^1 \sim p(x_2 \mid x_1 = x_1^1, x_3 = x_3^0)$$

$$x_3^1 \sim p(x_3 \mid x_1 = x_1^1, x_2 = x_2^1)$$

Then the other dimensions follow suit!

Gibbs Sampling

$$p(x_1, x_2, x_3) = \frac{\widehat{p}(x_1, x_2, x_3)}{Z}$$

Start with (x_1^0, x_2^0, x_3^0), e. g. $(0, 0, 0)$

For k = 0, 1, ...

$$x_1^{k+1} \sim p(x_1 \mid x_2 = x_2^k, x_3 = x_3^k)$$

We perform several iterations.

$$x_2^{k+1} \sim p(x_2 \mid x_1 = x_1^{k+1}, x_3 = x_3^k)$$

$$x_3^{k+1} \sim p(x_3 \mid x_1 = x_1^{k+1}, x_2 = x_2^{k+1})$$

NOTE: there is a proof of this. See typora notes.