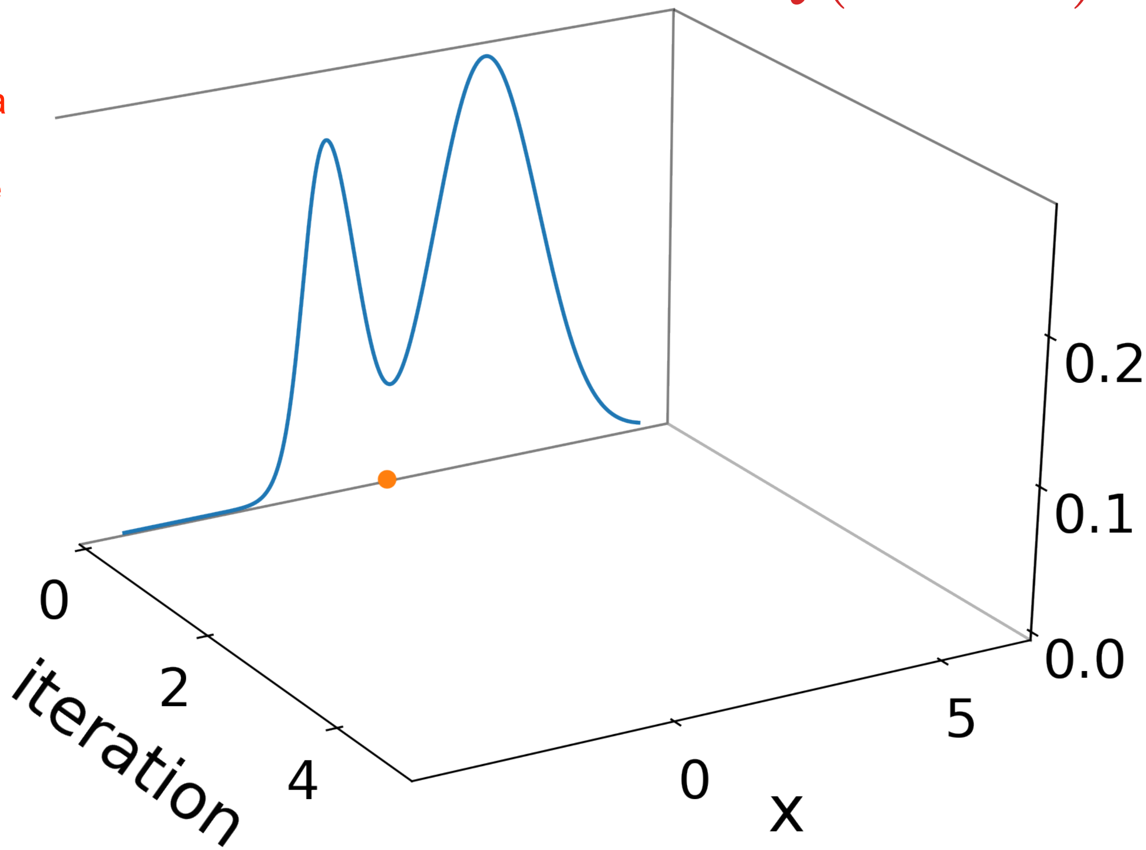
# Demo

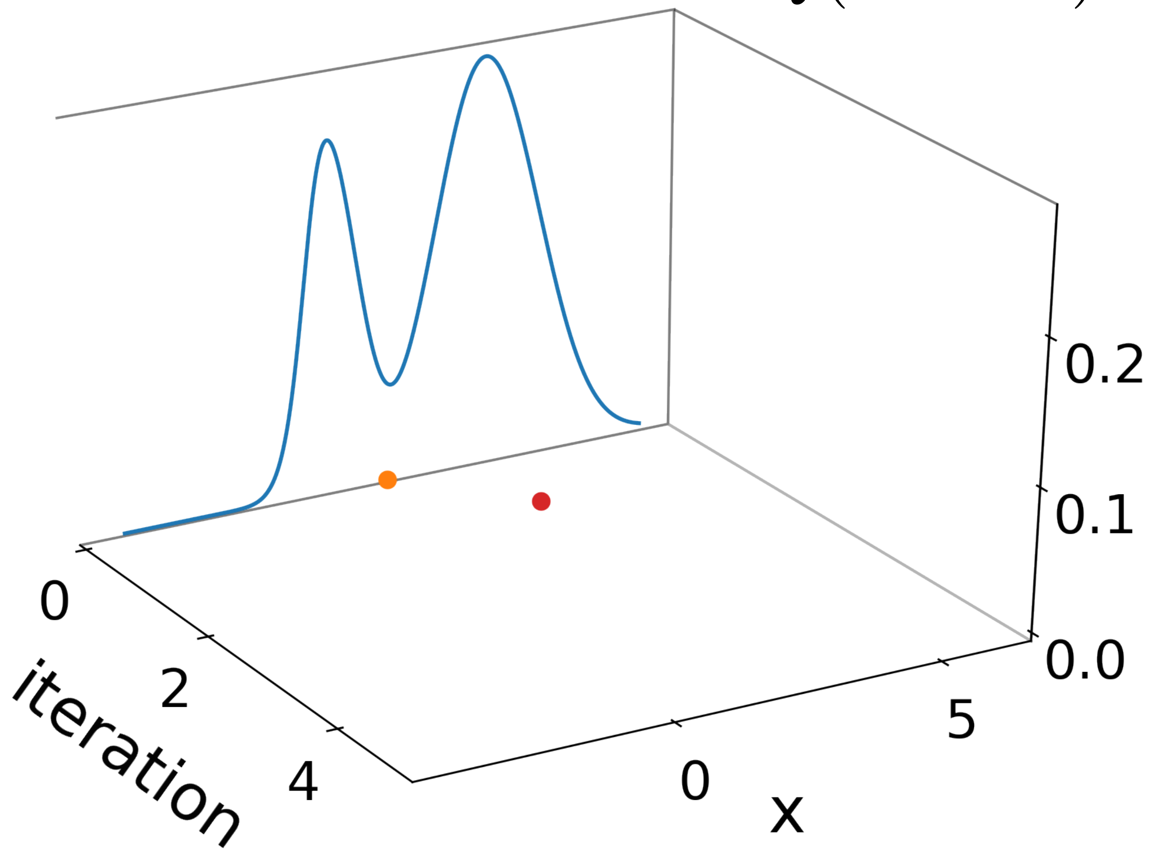
$$Q(x \to x') = \mathcal{N}(x, 1)$$

Now let's do this for a 1D case.
The blue curve is the true pdf. We want to model it using the Q distribution above.



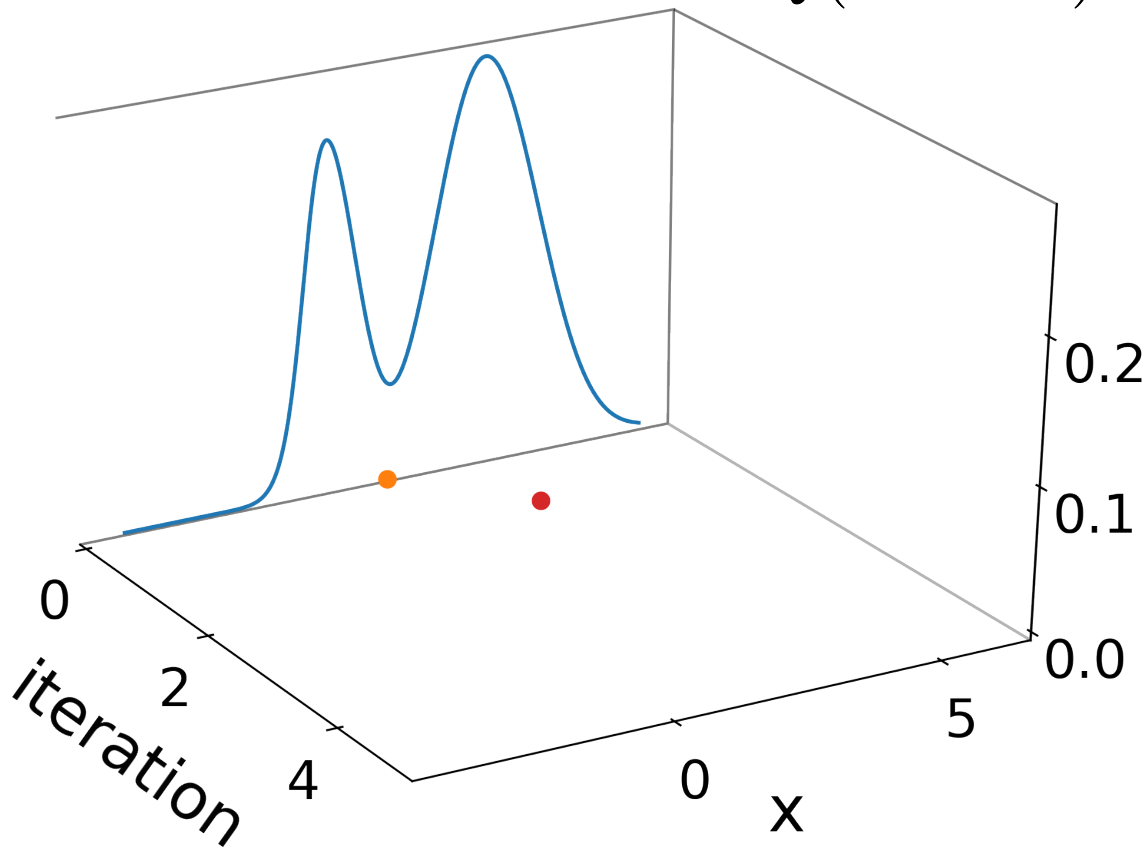
# Demo

$$Q(x \to x') = \mathcal{N}(x, 1)$$



# Demo

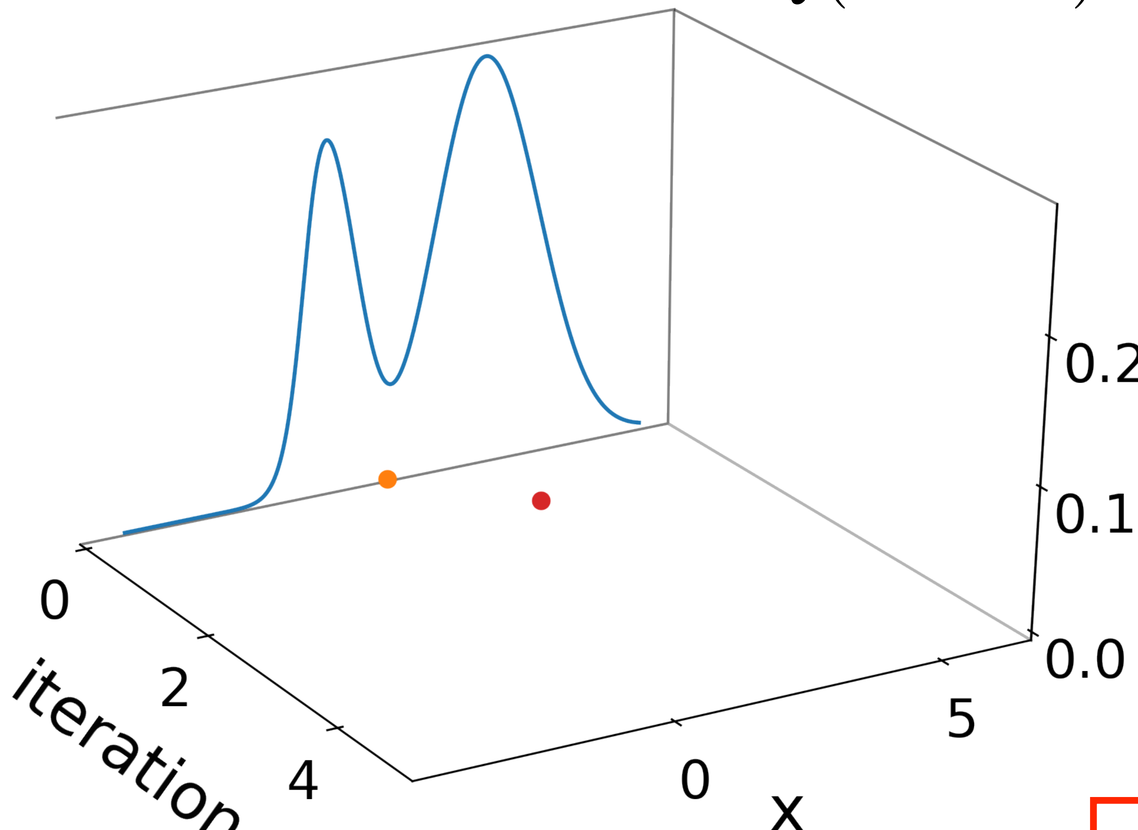
$$Q(x \to x') = \mathcal{N}(x, 1)$$




$$A(x \to x') = \min\left(1, \frac{\pi(x')Q(x' \to x)}{\pi(x)Q(x \to x')}\right)$$

# Demo

$$Q(x \to x') = \mathcal{N}(x, 1)$$

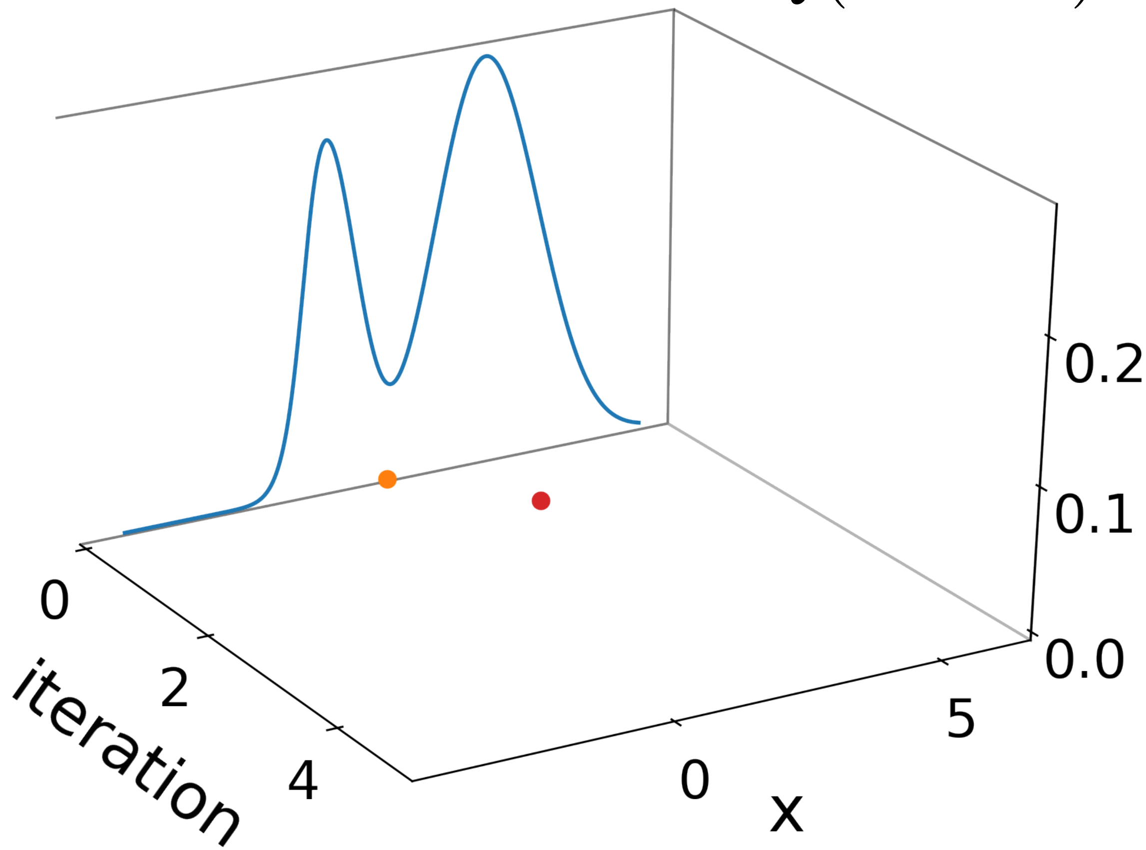



$$A(x \to x') = \min \boxed{\left(1, \frac{\pi(x')Q(x' \to x)}{\pi(x)Q(x \to x')}\right)} = \min \boxed{\left(1, \frac{\pi(x')}{\pi(x)}\right)}$$

Q(x' -> x) and Q(x -> x') is the same because
we have normal at N(x,1). This depends on x alone.
So this ration becomes 1

# Demo

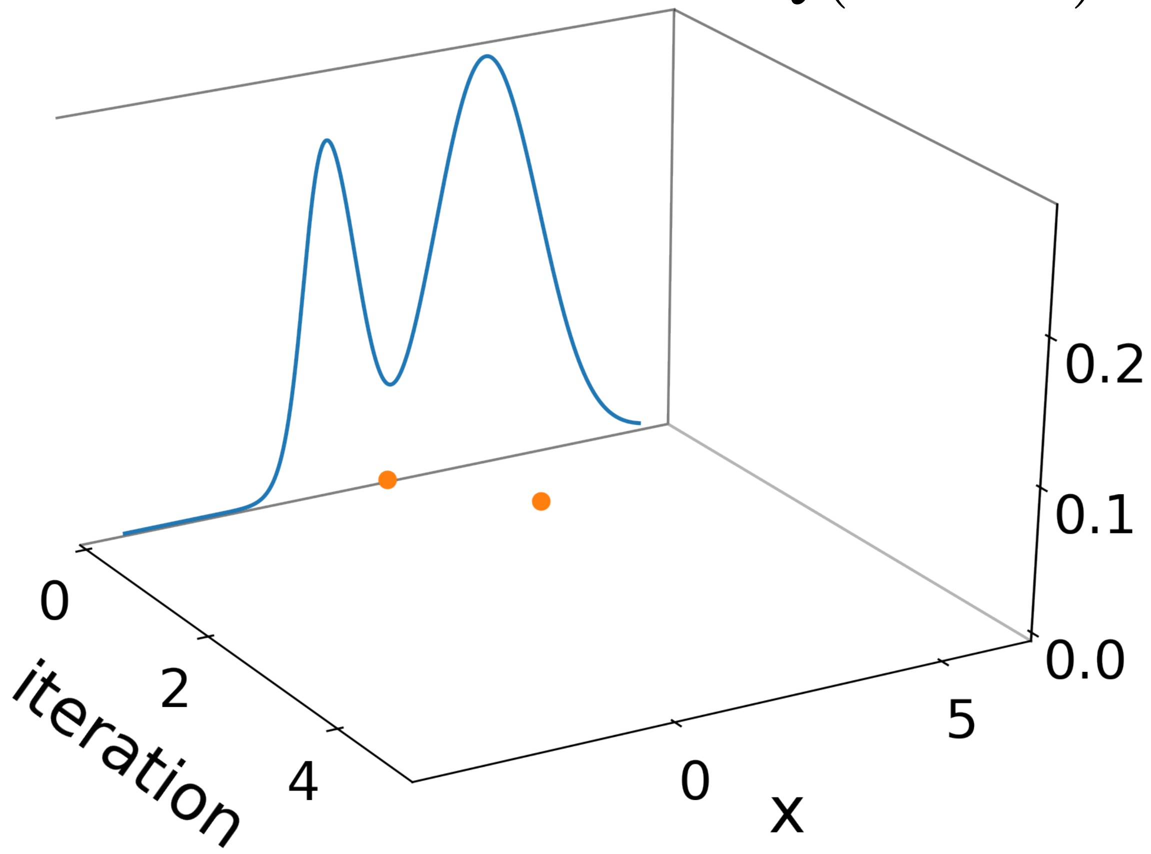
$$Q(x \to x') = \mathcal{N}(x, 1)$$




$$A(x \to x') = \min\left(1, \frac{0.27}{0.07}\right) = \min(1, 3.87)$$

# Demo

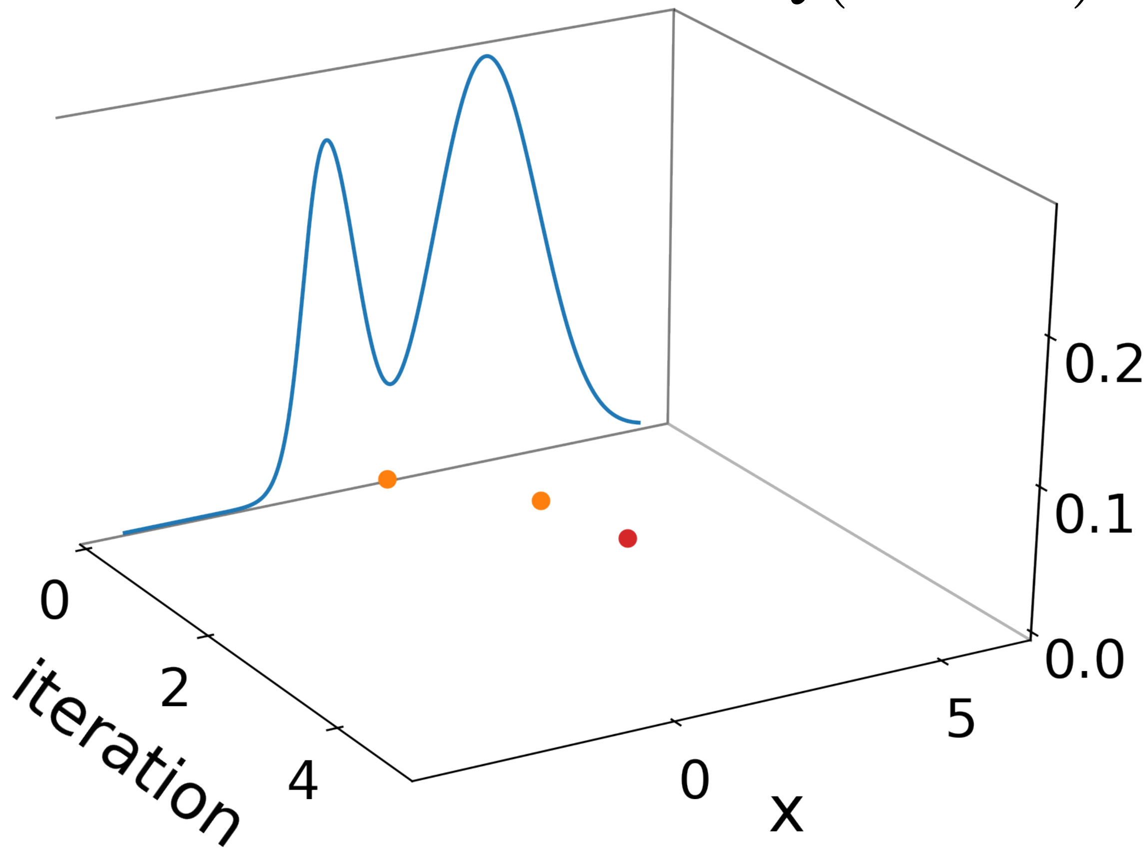
$$Q(x \to x') = \mathcal{N}(x, 1)$$




$$A(x \to x') = \min\left(1, \frac{0.27}{0.07}\right) = \min(1, 3.87)$$

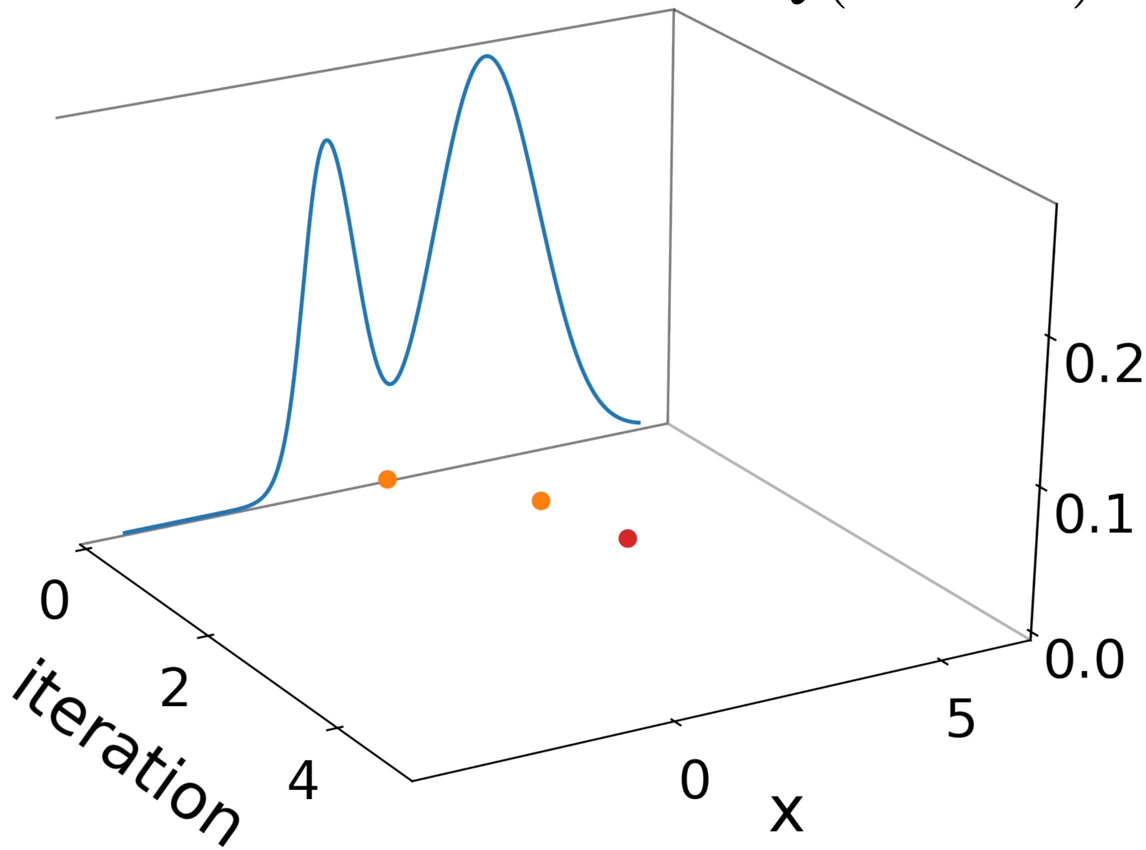
# Demo

$$Q(x \rightarrow x') = \mathcal{N}(x, 1)$$



# Demo

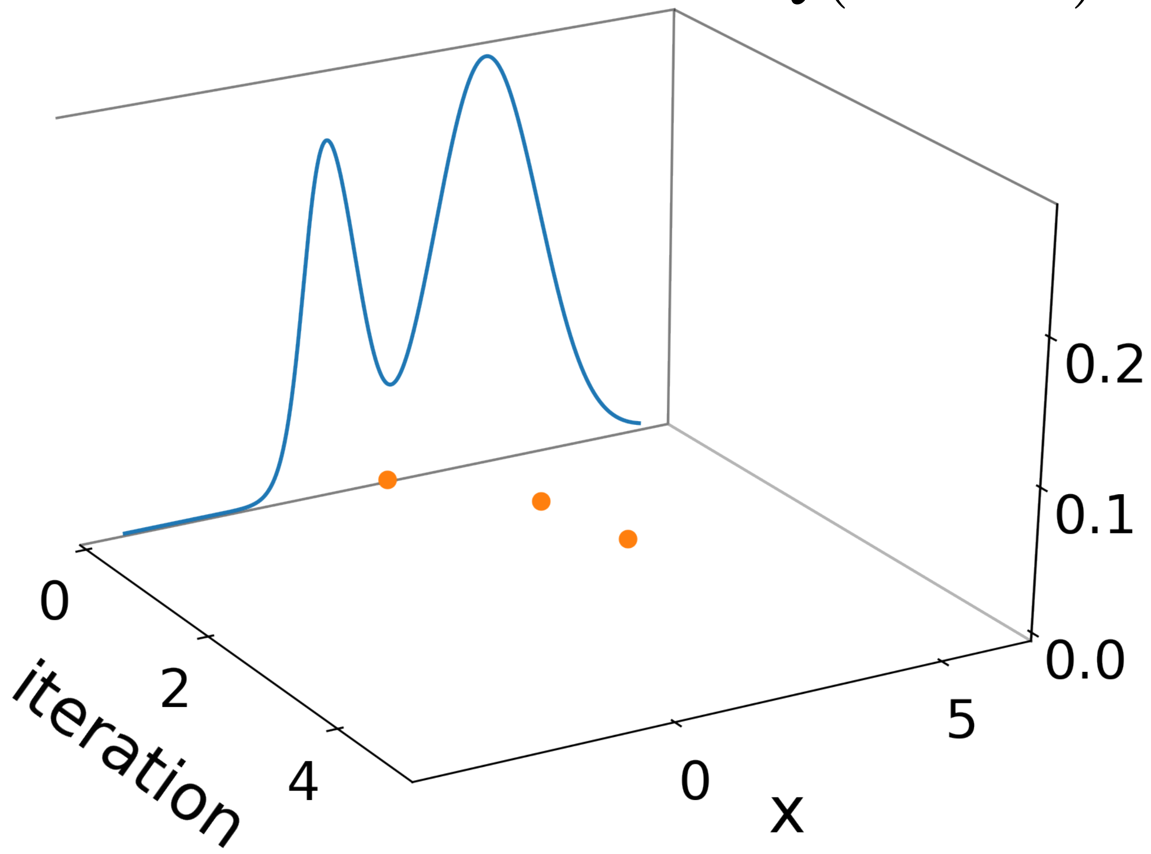
$$Q(x \to x') = \mathcal{N}(x, 1)$$




$$A(x \to x') = \min\left(1, \frac{0.28}{0.27}\right) = \min(1, 1.01)$$

# Demo

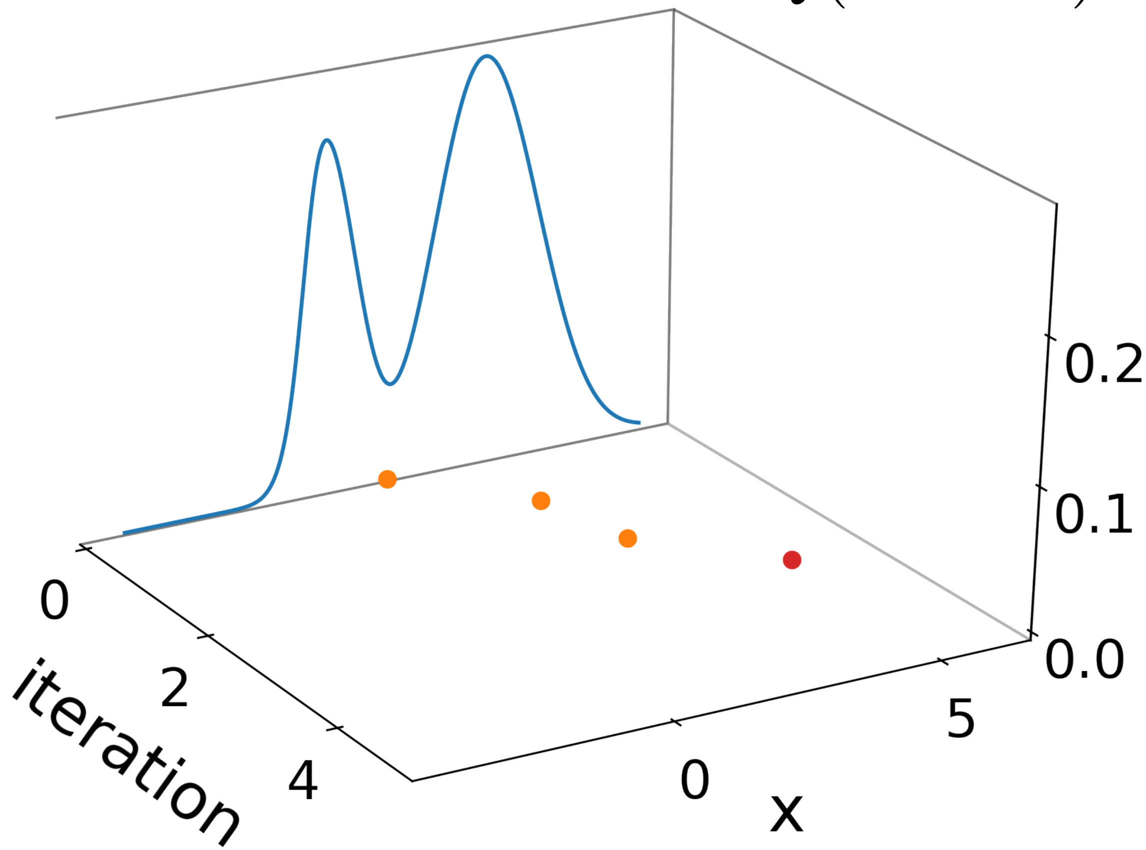
$$Q(x \to x') = \mathcal{N}(x, 1)$$




$$A(x \to x') = \min\left(1, \frac{0.28}{0.27}\right) = \min(1, 1.01)$$

# Demo

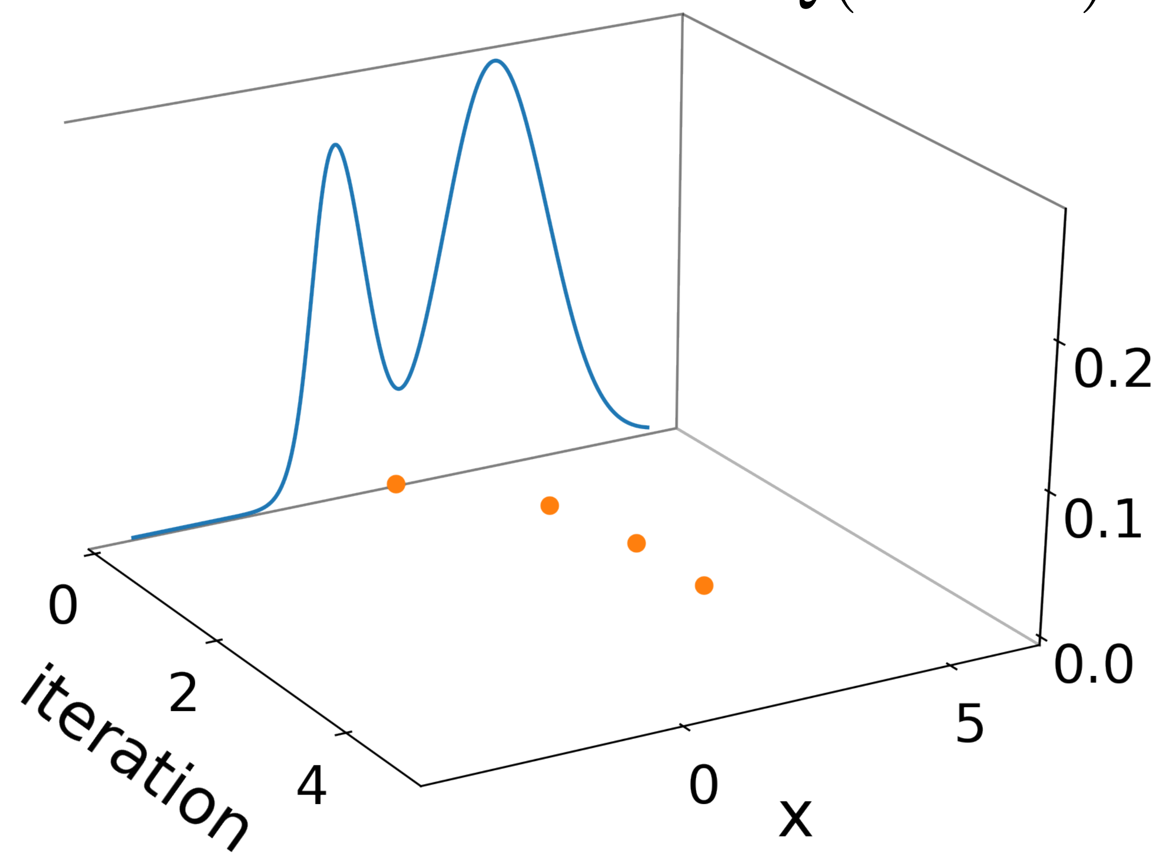
$$Q(x \to x') = \mathcal{N}(x, 1)$$




$$A(x \to x') = \min\left(1, \frac{0.04}{0.28}\right) = \min(1, 0.13)$$

# Demo

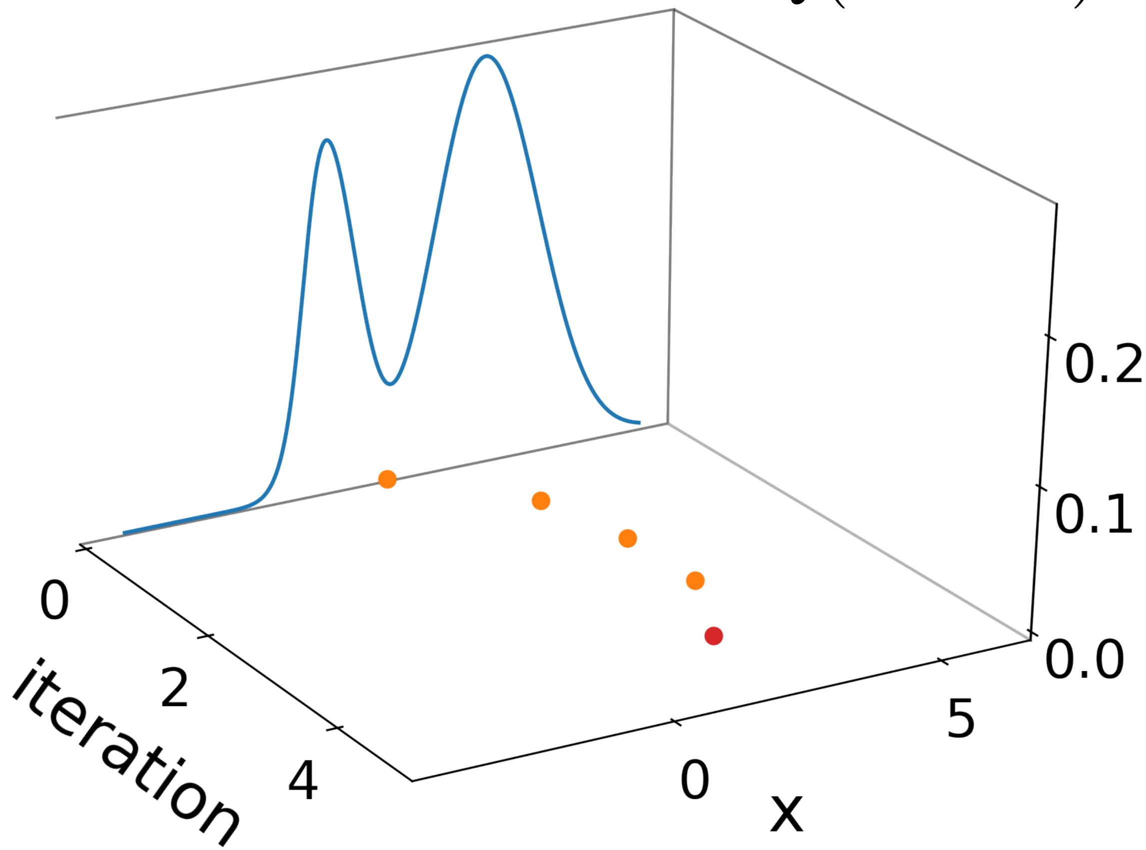
$$Q(x \to x') = \mathcal{N}(x, 1)$$




$$A(x \to x') = \min\left(1, \frac{0.04}{0.28}\right) = \min(1, 0.13)$$

# Demo

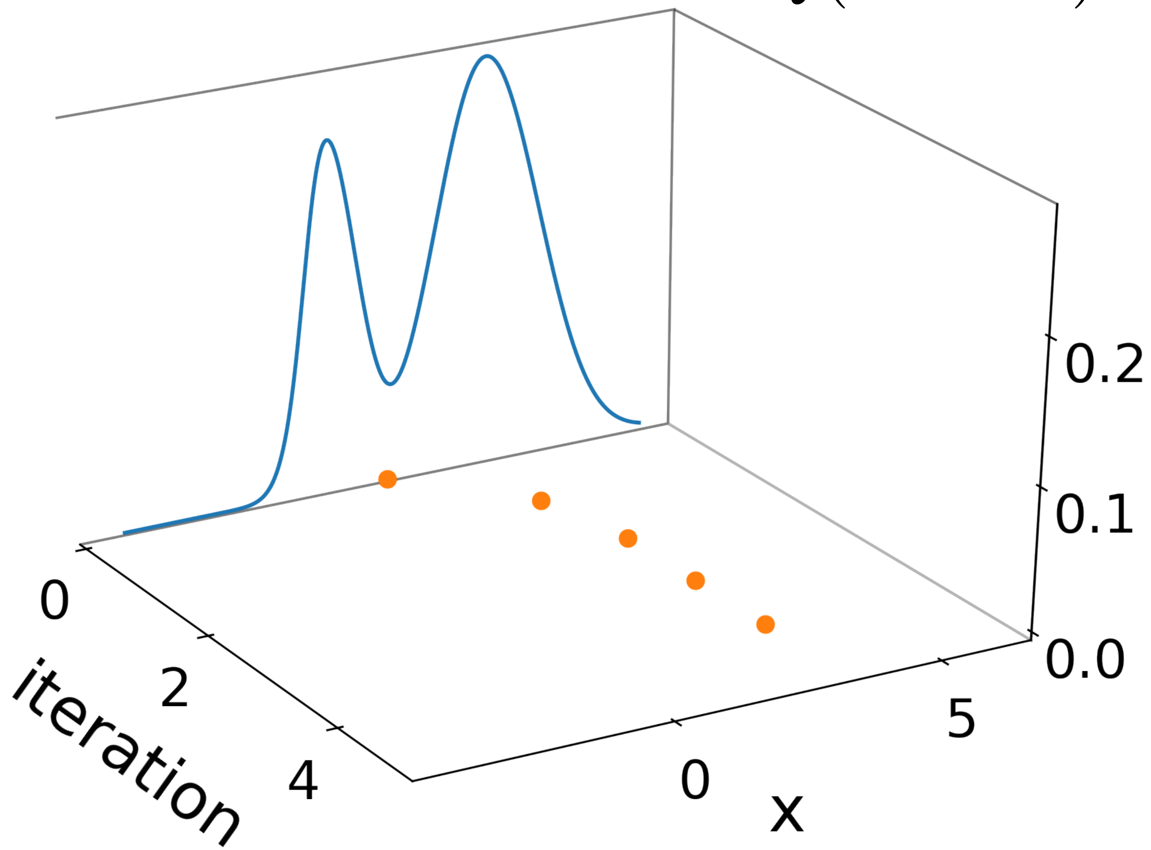
$$Q(x \to x') = \mathcal{N}(x, 1)$$




$$A(x \to x') = \min\left(1, \frac{0.20}{0.28}\right) = \min(1, 0.73)$$

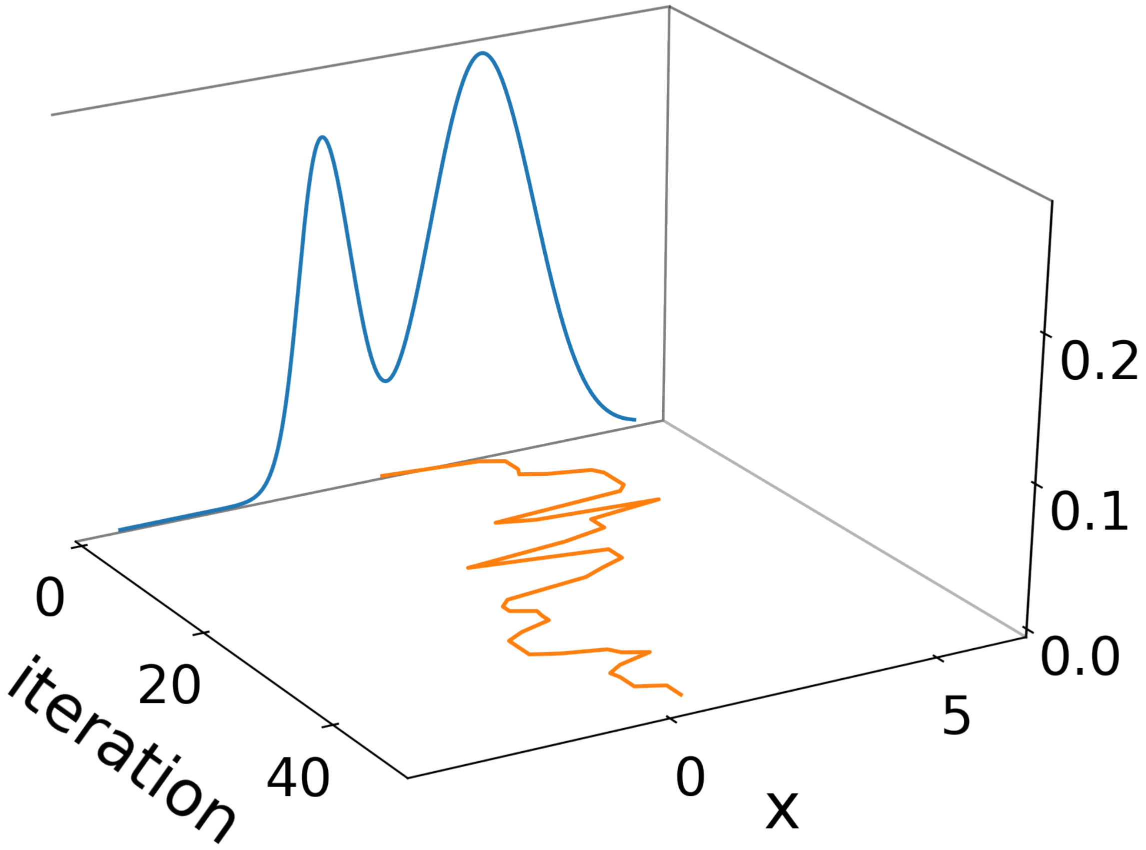
# Demo

$$Q(x \to x') = \mathcal{N}(x, 1)$$



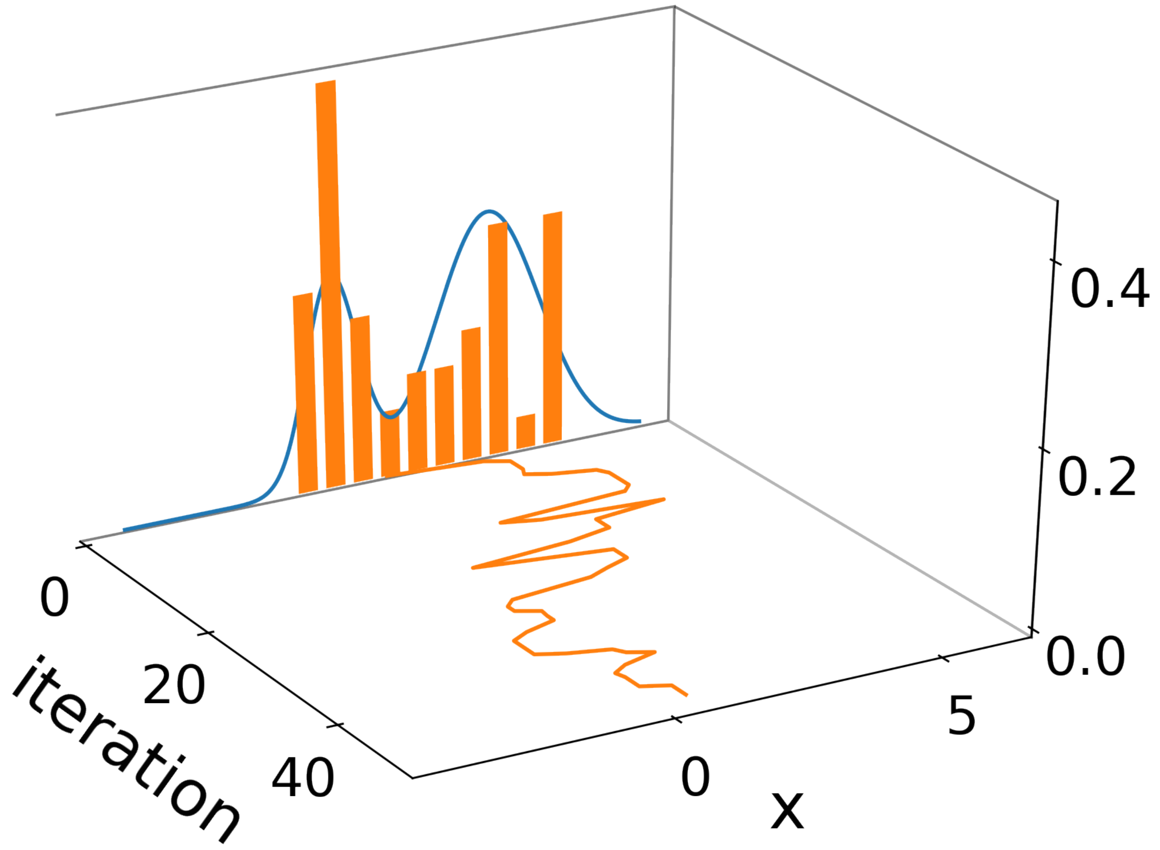

$$A(x \to x') = \min\left(1, \frac{0.20}{0.28}\right) = \min(1, 0.73)$$

# Demo



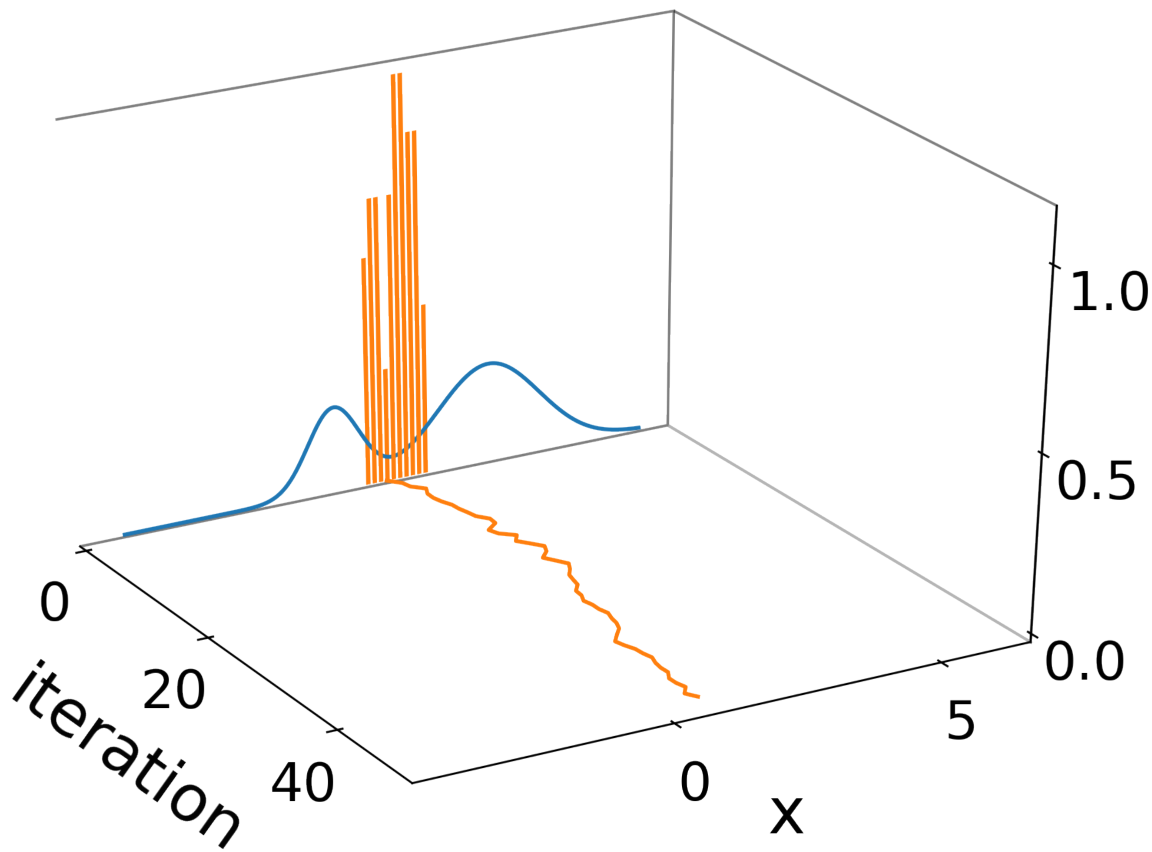

$$Q(x \to x') = \mathcal{N}(x, 1)$$

# Demo




$$Q(x \to x') = \mathcal{N}(x, 1)$$

# Demo

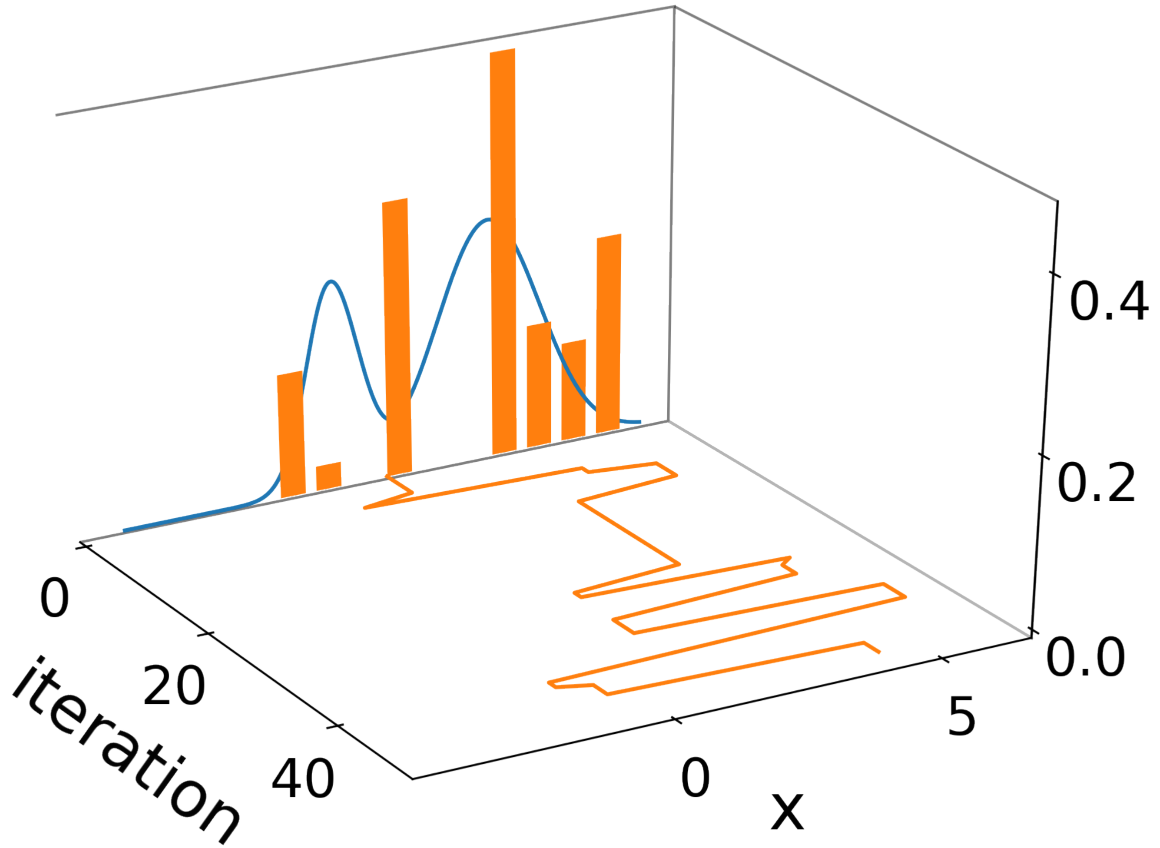



$$Q(x \to x') = \mathcal{N}(x, 0.1^2)$$

Using a gaussian with too little variance will make it too concentrated, unlike the true blue distribution.

# Demo

Text




$$Q(x \to x') = \mathcal{N}(x, 10^2)$$

With too much variance, it becomes unfocused and scattered.Can

# Metropolis Hastings as correction scheme

Recall Gibbs sampling

Lets make it parallel

Recall that the gibbs sampling method is NOT parallel as the current step is dependent on the previous steps.

$$x_1^{k+1} \sim p(x_1 \mid x_2 = x_2^k, x_3 = x_3^k)$$

$$x_2^{k+1} \sim p(x_2 \mid x_1 = x_1^{k+1}, x_3 = x_3^k)$$

$$x_3^{k+1} \sim p(x_3 \mid x_1 = x_1^{k+1}, x_2 = x_2^{k+1})$$

# Metropolis Hastings as correction scheme

Recall Gibbs sampling

Lets make it parallel

By doing the above, we have no convergence guarantees as we proved earlier.
It is definitely parallelizable though!
However.... (see next page)

$$x_1^{k+1} \sim p(x_1 \mid x_2 = x_2^k, x_3 = x_3^k)$$

$$x_2^{k+1} \sim p(x_2 \mid x_1 = x_1^{k}, x_3 = x_3^k)$$

$$x_3^{k+1} \sim p(x_3 \mid x_1 = x_1^{k}, x_2 = x_2^{k})$$

# Metropolis Hastings as correction scheme

Recall Gibbs sampling

Can use metropolis hastings to define a critic function to reject or accept changes.

Lets make it parallel

It's wrong now, but can correct with Metropolis Hastings!

$$x_1^{k+1} \sim p(x_1 \mid x_2 = x_2^k, x_3 = x_3^k)$$

$$x_2^{k+1} \sim p(x_2 \mid x_1 = x_1^{k}, x_3 = x_3^k)$$

$$x_3^{k+1} \sim p(x_3 \mid x_1 = x_1^{k}, x_2 = x_2^{k})$$

# Summary

Rejection sampling applied to Markov Chains

**Pros:**

- You can choose among family of Markov Chains
- Works for unnormalized densities
- Easy to implement

**Cons:**

- Samples are still correlated
- Have to choose among family of Markov Chains ☺