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STAT 221
Pset 4
Nov 4, 2014

question 1. 1 $p(\lambda, \theta) \propto \lambda^{-1}$

$\begin{bmatrix} N \\ \theta \end{bmatrix} = f\left(\begin{bmatrix} \lambda \\ \theta \end{bmatrix}\right)$

$p(N, \theta) = p(\lambda, \theta) * \left\| \begin{matrix} \frac{\partial \lambda}{\partial N} & \frac{\partial \theta}{\partial N} \\ \frac{\partial \lambda}{\partial \theta} & \frac{\partial \theta}{\partial \theta} \end{matrix} \right\|$

Our Jacobian is $\begin{vmatrix} \theta & -\lambda N^{-2} \\ N & 1 \end{vmatrix}$

and its determinant is $\theta + \frac{\lambda}{N}$

$p(N, \theta) \propto \frac{1}{\theta N} * (\theta + \frac{\lambda}{N})$
$= \frac{1}{\theta N} * (\theta + \frac{\theta N}{N})$
$= \frac{2}{N}$
$\propto \frac{1}{N}$

This prior favors smaller values of N.

2 It is an improper prior: $\int_0^\infty \int_0^1 \lambda^{-1} d\theta d\lambda \to \ln(\infty) - \ln(0)$. This cannot integrate to 1 with any constant factor.

3 No $p(\lambda, \theta)$ is not a non-informative prior in the sense of Jeffreys.

$p(y_i|\theta, \mu) = \frac{(\theta\mu)^{y_i}}{y_i!} e^{-\theta\mu}$

$y_i log(\theta|\mu) - log(y_i!) - \frac{\theta}{\mu}$

$\frac{\partial I}{\partial \theta} = y_i(\theta\mu)^{-1}\mu - \frac{1}{\mu} = y_i(\theta)^{-1} - \frac{1}{\mu}$

$\frac{\partial I}{\partial \mu} = y_i\mu + \theta\mu^{-2}$

$\frac{\partial^2 I}{\partial \theta^2} = -y_i\theta^{-2}$

$\frac{\partial^2 I}{\partial \mu^2} = y_i - 2\theta\mu^{-3}$

$\frac{\partial^2 I}{\partial \mu \partial \theta} = \mu^{-2}$

$\frac{\partial^2 I}{\partial \theta \partial \mu} = \mu^{-2}$

This gives us the fisher's information matrix $\begin{pmatrix} -y_i\theta^{-2} & \mu^{-2} \\ \mu^{-2} & y_i - 2\theta\mu^{-3} \end{pmatrix}$, which has determinant $(y_i\theta^{-2})^2 - 2\theta\mu^{-3}y_i - \mu^{-4}$, the square root of which is not proportional to $\frac{(\theta\mu)^{y_i}}{y_i!} e^{-\theta\mu}$.

4 See R code and MCMC design note.

5 Posterior for N

$\int_0^1 \left(\prod_{i=1}^n \binom{N}{x_i} \theta^{x_i}(1-\theta)^{N-x_i}\right) * \frac{1}{N} d\theta$
$= \int_0^1 \left(\prod_{i=1}^n \binom{N}{x_i}\right) \theta^{\Sigma x_i}(1-\theta)^{nN-\Sigma x_i} * \frac{1}{N} d\theta$
$= \frac{1}{N} \int_0^1 \theta^{\Sigma x_i}(1-\theta)^{nN-\Sigma x_i} \prod_{i=1}^n \binom{N}{x_i} d\theta$
$= \frac{1}{N} \prod_{i=1}^n \binom{N}{x_i} \int_0^1 \theta^{\Sigma x_i}(1-\theta)^{nN-\Sigma x_i} d\theta$

If we define $S = \Sigma x_i$, the integral above has the form of a
Beta($\alpha = S + 1, \beta = nN - S + 1$) pdf
Using this we get, $\frac{(nN-S)!}{(nN+1)!N)}$, which when $n = 1$, $\rightarrow \frac{x_i}{N(N+1)}$

Using our function $find.norm.log.const$ defined in $keskici\_wxiao\_ps4\_post.R$, we found that the normalizing constant for the impala dataset was $e^{15.63617}$ and the normalizing constant for the waterbuck dataset was $e^{16.83744}$.

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