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STAT 221
Pset 4
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question 1.

1. $p(\lambda, \theta) \propto \lambda^{-1}$
   $\begin{bmatrix} N \\ \theta \end{bmatrix} = f\left(\begin{bmatrix} \lambda \\ \theta \end{bmatrix}\right)$
   $p(N, \theta) = p(\lambda, \theta) * \left\| \begin{matrix} \frac{\partial\lambda}{\partial N} & \frac{\partial\theta}{\partial N} \\ \frac{\partial\lambda}{\partial\theta} & \frac{\partial\theta}{\partial\theta} \end{matrix} \right\|$
   Our Jacobian is $\begin{vmatrix} \theta & -\lambda N^{-2} \\ N & 1 \end{vmatrix}$
   and its determinant is $\theta + \frac{\lambda}{N}$

   $p(N, \theta) \propto \frac{1}{\theta N} * (\theta + \frac{\lambda}{N})$
   $= \frac{1}{\theta N} * (\theta + \frac{\theta N}{N})$
   $= \frac{2}{N}$
   $\propto \frac{1}{N}$

2. It is an improper prior: $\int_0^\infty \int_0^1 \lambda^{-1} d\theta d\lambda \to \ln(\infty) - \ln(0)$. This cannot integrate to 1 with any constant factor.

3.

4. See R code

5. Posterior for N
   $\int_0^1 \left(\prod_{i=1}^n \binom{N}{x_i} \theta^{x_i} (1-\theta)^{N-x_i}\right) * \frac{1}{N} d\theta$
   $= \int_0^1 \left(\prod_{i=1}^n \binom{N}{x_i}\right) \theta^{\Sigma x_i} (1-\theta)^{nN-\Sigma x_i} * \frac{1}{N} d\theta$
   $= \frac{1}{N} \int_0^1 \theta^{\Sigma x_i} (1-\theta)^{nN-\Sigma x_i} \prod_{i=1}^n \binom{N}{x_i} d\theta$
   $= \frac{1}{N} \prod_{i=1}^n \binom{N}{x_i} \int_0^1 \theta^{\Sigma x_i} (1-\theta)^{nN-\Sigma x_i} d\theta$
   If we define $S = \Sigma x_i$, the integral above has the form of a Beta($\alpha = S+1, \beta = nN - S + 1$) pdf
   Using this we get, $\frac{(S!)(nN-S)!}{(nN+1)!N)}$, which when $n = 1$, $\to \frac{x_i}{N(N+1)}$