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Landau–Lifshitz or Gilbert damping? That is the question

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In their seminal 1935 paper on magnetics, Landau and Lifshitz (LL) proposed a form for magnetization damping. In 1955 Gilbert proposed another form, introducing a dimensionless parameter α. We derive LL damping using the theory of irreversible thermodynamics, summarize an unbiased Langevin theory of fluctuations that yields LL damping, and argue that inhomogeneous broadening might explain the nonresonance data that led Gilbert to formulate his theory. LL versus Gilbert damping takes on special relevance in the context of bulk spin transfer torque and bulk spin pumping, where the form of damping affects the values of the "adiabatic" and "nonadiabatic" terms. We argue that the adiabatic and nonadiabatic terms are dissipative and reactive, respectively.
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I. INTRODUCTION

Landau and Lifshitz's seminal 1935 paper on magnetics[1] proposes, for the dynamics of the magnetization $\vec{M}$ of a uniform ferromagnet with gyromagnetic ratio γ in a net field $\vec{H}$,

$$\partial_t \vec{M} = -\gamma \vec{M} \times \vec{H} - \lambda \hat{M} \times (\vec{M} \times \vec{H}) \quad \text{(LL)}. \tag{1}$$

The first term is the expected precessional dynamics. With λ a quantity with the same units as γ, the second term provides a phenomenological form for the damping.

In the 1955 MMM conference proceedings, Gilbert argued that LL damping fails for large enough damping.[2] Instead, he proposed the form

$$\partial_t \vec{M} = -\gamma_G \vec{M} \times \vec{H} + \alpha \hat{M} \times \partial_t \vec{M} \quad \text{(Gilbert)}. \tag{2}$$

With $\alpha = \lambda/\gamma$ and $\gamma_G = \gamma(1+\alpha^2)$, the LL and Gilbert forms are mathematically equivalent. Good samples in ferromagnetic resonance satisfy $\alpha \ll 1$.[3,4]

Additional possible equations of motion were considered in the 1950s,[5] of which we note only one by Callen.[6] In practice, for small damping, LL and Gilbert are very nearly the same, but at issue is a question of principle.

This work first shows that irreversible thermodynamics predicts that magnetization damping takes the LL form. It then discusses Ref. 2 and the experimental basis and theoretical analysis on which Gilbert's large damping argument is based. Finally it discusses the implications of irreversible thermodynamics for the additional physics associated with spin transfer torque and with spin pumping in nonuniform ferromagnets. An unbiased Langevin theory of fluctuations leads to LL damping, with a specific expression for λ.[7]

II. IRREVERSIBLE THERMODYNAMICS AND LL DAMPING

Irreversible thermodynamics has been applied to numerous other condensed matter systems. A number of independent workers have already applied it for ferromagnets.[8–10] All obtain LL damping for low frequency, long wavelength dynamics. A recent work on damping in nonuniform ferromagnetic insulators, including a magnetism-directed introduction to irreversible thermodynamics, finds that nonuniformity introduces as many as four new damping terms, but reduces to the LL form in the uniform case.[11] Here we present a derivation restricted to the uniform case.

Irreversible thermodynamics imposes the condition that if local thermodynamics holds at the initial time, then the equations of motion (here, for ε, s, and $\vec{M}$) maintain local thermodynamics at all future times.[11]

The differential of the internal energy density ε includes an internal field $\vec{H}_{\text{int}}$ via the term $\vec{H}_{\text{int}} \cdot d\vec{M}$; the total energy density also includes the interaction term $-\vec{H} \cdot d\vec{M}$, where $\vec{H}$ includes the external field $\vec{H}_0$, lattice anisotropy from the spin-orbit interaction, and anisotropy from the dipolar interaction. For a uniform system $\vec{H}_{\text{int}}$ is along $\vec{M}$, due to a uniform exchange field, so that $\vec{M} \times \vec{H}_{\text{int}} = \vec{0}$. Using a vector generalization of Johnson and Silsbee,[12] we define $\vec{H}^* = \vec{H} - \vec{H}_{\text{int}}$. Then we take the basic thermodynamic relation to be

$$d\varepsilon = T ds - \vec{H}^* \cdot d\vec{M}. \tag{3}$$

Here the temperature T and the entropy density s both are even under time reversal. Both $\vec{M}$ and $\vec{H}^*$ are odd under time reversal. In equilibrium $\vec{H}^* = \vec{0}$, so that $\vec{H} = \vec{H}_{\text{int}}$.

The energy density, a conserved quantity, satisfies

$$\partial_t \varepsilon + \partial_i j_i^{\varepsilon} = 0. \tag{4}$$

Here j_i^{ε} is the as-yet-unknown energy flux density. There is no energy source because energy is conserved. The intrinsic signature under time-reversal $\mathcal{T}$ of j_i^{ε} is odd. Dissipation occurs for terms in j_i^{ε} that are even under $\mathcal{T}$.

The entropy density s, a nonconserved quantity, satisfies

$$\partial_t s + \partial_i j_i^s = \frac{R}{T} \geq 0. \tag{5}$$

Here j_i^s is as-yet-unknown entropy flux density and R/T is the as-yet-unknown entropy source density, where R is the volume rate of heating. The intrinsic signatures under time reversal of j_i^s and R are odd. Dissipation occurs for terms in

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j_i^s and R that are even under $\mathcal{T}$. Heating is irreversible so in practice R contains only terms that are even under $\mathcal{T}$.

The (nonconserved) magnetization $\vec{M}$ satisfies

$$\partial_t \vec{M} = -\gamma \vec{M} \times \vec{H} + \vec{N}. \tag{6}$$

The first term is the (known) Larmor torque, with gyromagnetic ratio $\gamma > 0$ taken to include the effect of spin-orbit interactions. It is even under $\mathcal{T}$. The as-yet-unknown magnetization source (or torque density) $\vec{N}$, has even intrinsic time-reversal signature. Dissipation occurs for terms in $\vec{N}$ that are odd under $\mathcal{T}$. We take $|\vec{M}|$ to be constant so $\vec{N}$ is normal to $\vec{M}$. Hence $\vec{N}\cdot\vec{H}^* = \vec{N}\cdot\vec{H}$.

In irreversible thermodynamics each part of the unknown fluxes or sources must be proportional to the driving terms in the thermodynamic variables, called forces or affinities. Here the driving terms are $\partial_i T$, $\hat{M}\cdot\vec{H}^*$, and $\vec{M}\times\vec{H}$. In $\vec{H}^* = \vec{H} - \vec{H}_{\mathrm{int}}$ the first term is along $\vec{H}$ and the second is along $\vec{M}$, so $\vec{H}^*(-\gamma\vec{M}\times\vec{H}) = 0$, to be used shortly.

Employing Eqs. (3), (4), and (6), R in Eq. (5) becomes

$$\begin{aligned} 0 \le R &= T\partial_t s + T\partial_i j_i^s = \partial_t \varepsilon + T\partial_i j_i^s + \vec{H}^* \cdot \partial_t \vec{M} = -\partial_i j_i^\varepsilon \\ &\quad + T\partial_i j_i^s + \vec{H}^* \cdot (-\gamma \vec{M} \times \vec{H} + \vec{N}) = -\partial_i (j_i^\varepsilon - T j_i^s) \\ &\quad - j_i^s \partial_i T + \vec{N} \cdot \vec{H}. \end{aligned} \tag{7}$$

The divergence, if nonzero, could have either sign. To satisfy $R \ge 0$, we eliminate the divergence by setting

$$j_i^\varepsilon = T j_i^s. \tag{8}$$

Consistent with j_i^s being a vector in real space, the only allowed form proportional to $\partial_i T$, $\hat{M}\cdot\vec{H}^*$, and $\vec{M}\times\vec{H}$ is

$$j_i^s = -\frac{\kappa}{T}\partial_i T, \tag{9}$$

where κ is a constant called the thermal conductivity. $\partial_i T$ is even under time reversal, and thus (being opposite j_i^s's intrinsic time-reversal signature) is dissipative.

Consistent with $\vec{N}$ being (a) a vector in spin space, (b) normal to $\hat{M}$, and (c) not changing the gyromagnetic ratio, the only allowed form is

$$\vec{N} = -\lambda \hat{M} \times (\vec{M} \times \vec{H}), \tag{10}$$

where λ is a constant. This is, of course, LL damping. It is odd under time reversal, and thus (being opposite $\vec{N}$'s intrinsic time-reversal signature) is dissipative.

We now can determine R. From Eqs. (7)–(10) we have

$$R = \frac{\kappa}{T}(\partial_i T)^2 + \frac{\lambda}{|\vec{M}|}|\vec{M} \times \vec{H}|^2. \tag{11}$$

Thermodynamic stability ($R \ge 0$) implies $\kappa \ge 0$ and $\lambda \ge 0$.

A recent study of magnetization damping using Langevin theory,[7] where the dominant fluctuations are characterized by thermodynamic parameters taking on nonequilibrium values, found LL damping and an expression for the LL damping constant in terms of near-equilibrium fluctuations. In contrast, the theory of Brown[13] inputs Gilbert damping to bias the fluctuations, and thus is not a Langevin theory.

III. GILBERT THEORY, KELLY'S ROTATIONAL TORQUE DATA, GILBERT'S ANALYSIS

The original literature on Gilbert theory is difficult to trace. The abstract for Gilbert's talk at a American Physical Society meeting in 1955 does not appear on the APS website,[14] although it does appear in bound copies of The Physical Review. This abstract has been of such continuing interest that it had been copied by a website available at the time of the present submission.[15] Unfortunately, the abstract is not terribly revealing.

Another early reference to Gilbert theory is an unpublished report.[16] Recently, Gilbert presented a retrospective, which was part of his Ph.D. dissertation.[17] He argues, by analogy to damping of a particle using the Rayleigh dissipation function in a modified Hamiltonian formulation of mechanics, that the damping form should go as $\hat{M}\times\partial_t\vec{M}$.[17] The most revealing article we have found is in the MMM Conference Proceedings of 1955, which presents Kelley's method and data, and Gilbert's analysis.[2]

Kelly employed a nonresonant rotating field (from crossed coils) in the plane of a Permalloy disk of thickness $h = 3.3$ μm and diameter (perhaps radius) $d = 1.3$ cm and measured the torque acting on the disk. Gilbert first employed LL damping for fixed γ and frequency-dependent λ, but the theory could not fit the data. He then introduced another form (Gilbert damping). Using fixed γ_G and frequency-dependent α,[18] he found that data for the four frequencies (in MHz) of 2.0, 1.0, 0.032, and 0.015 could be fit with values of α given by 0.3, 0.3, 3, and 9. He notes that $(\lambda/\gamma)/[1+(\lambda/\gamma)^2]$ should not exceed 0.5, and then states that using the LL form this value was exceeded for the lower two frequencies.

If γ_G is constant, then $\gamma = \gamma_G/(1+\alpha^2)$ varies. Assuming constant γ_G, the values $\alpha = 3, 9$ at the lower two frequencies imply that γ takes on values of about 0.1 and 0.01 of its high frequency value. We find it difficult to believe that dissipative processes can cause γ to decrease by such enormous factors. We think it more likely that an LL-based analysis failed because 1955 sample-preparation techniques led to large inhomogeneous broadening, which dominated at the lower frequencies. Inhomogenous broadening can be incorporated with Gilbert damping in a simple manner by taking $\lambda \to \lambda + A/f$, where A characterizes the inhomogeneous broadening and f is the frequency. Even the "high" frequency value of $\alpha = 0.3$ indicates a poor sample relative to modern ones. Such a poor sample should not be the basis for abandoning LL theory. Nevertheless, Gilbert's use of the dimensionless quantity α (proportional to the inverse of the quality factor $\mathcal{Q}$) is a valuable addition to the literature; in LL damping the form $\lambda = \gamma\alpha$ should be employed.

Note that $\hat{M}\times\partial_t\vec{M}$ (1) is not proportional to a thermodynamic force, unlike what occurs in irreversible thermodynamics; (2) does not have a unique time-reversal signature, and thus introduces a reactive response in addition to a dissipative response; and (3) gives the equation of motion a curious self-referential character.

IV. SPIN TRANSFER TORQUE AND SPIN PUMPING—ADIABATIC AND NONADIABATIC

The distinction between LL and Gilbert damping is usually insignificant for small α. However, for spin transfer torque, this distinction matters even for small α.[7] In spin transfer torque, when the magnetization is nonuniform (e.g., at a surface or in a magnetic domain or vortex), conduction of spin-polarized conduction electrons transfers magnetization.[19,20] There are two forms of spin transfer torque, one called *adiabatic* and the other *nonadiabatic,* where adiabatic refers to slow spatial variations.

Including the spin transfer torque term for current density j along x, the LL equation reads[7]

$$\partial_t\vec{M} = -\gamma\vec{M}\times\vec{H} - \lambda\hat{M}\times(\vec{M}\times\vec{H}) \quad (\mathrm{LL}) - v[\partial_x\vec{M} - \beta\hat{M}\times\partial_x\vec{M}]. \tag{12}$$

Here v is proportional to j and the fractional magnetization P and β is a constant. For a two-band model, the form is even more complex than Eq. (12).[21] Here v has units of velocity and is proportional to current, which for ordinary conducting magnets should be thought of as proportional to a gradient in the electrochemical potential, which is even under $\mathcal{T}$. Therefore for ordinary conducting magnets the term in $v\partial_x\vec{M}$, called adiabatic, has the same time-reversal properties as the LL damping term, and leads to damping. The term in $v\beta\hat{M}\times\partial_x\vec{M}$, called nonadiabatic, has the same time-reversal properties as the Larmor term and is reversible. In the irreversible thermodynamics, a term equivalent to v appears in the dissipation rate, but not a term equivalent to $v\beta$.[21]

Using vector identities and $\alpha\equiv\lambda/\gamma$, Eq. (12) can be rewritten as the corresponding Gilbert equation,

$$\partial_t\vec{M} = -\gamma(1+\alpha^2)\vec{M}\times\vec{H} + \alpha\hat{M}\times\partial_t\vec{M} \quad (\mathrm{Gilbert}) - v(1-\beta\alpha)\partial_x\vec{M} + v(\beta+\alpha)\hat{M}\times\partial_x\vec{M}. \tag{13}$$

The choice of LL or Gilbert damping clearly leads to significant differences in the assessing the two types of spin transfer torque. Microscopic theory and data analysis should indicate which of LL or Gilbert is employed.

A number of recent works consider spin pumping (of the current) for a system with nonuniform magnetization, three of them[22–24] using spin-Berry phase arguments[25] and one using the methods of irreversible thermodynamics.[21] Spin-pumping is closely related to spin transfer torque.[21] In the spin-Berry phase-based works[22–24] current and spin current are driven by phase gradients, and thus are odd under time reversal, as for a magnetic superconductor, where they are nondissipative. It follows that for a magnetic superconductor, the adiabatic spin pumping and adiabatic spin transfer torque terms are nondissipative, whereas the nonadiabatic spin pumping and nonadiabatic spin transfer torque terms are dissipative. In this case, just opposite to what one has for an ordinary conducting magnet, a term proportional to $v\beta$ would appear in the dissipation rate, but not a term proportional to v alone.

V. CONCLUSIONS

The present work argues for LL rather than Gilbert damping, as follows from many independent studies using the methods of irreversible thermodynamics, a near-equilibrium Langevin theory for magnetization damping, and an examination of the original arguments of Ref. 2. In particular, the data from Ref. 2 was obtained by a nonresonant method that to our knowledge has not been employed since then, and the neglect of inhomogeneous damping may not have been valid. Recent work by Smith[26] favors Gilbert damping; we have not determined its relationship to that of irreversible thermodynamics.

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