TOWARDS HIGH-THRESHOLD DECODING OF THE GAUGE COLOR CODE

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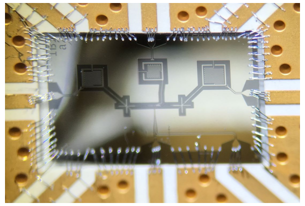
Benjamin Brown – Niels Bohr Institute, University of Copenhagen

rigetti quantum computing

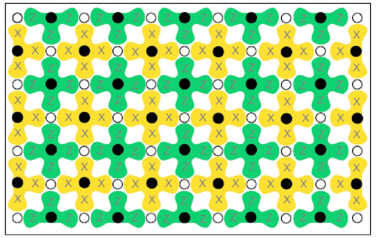
Niels Bohr Institutet

Fault-tolerant quantum computing with low overhead



Ex. Surface Code (2D):

> 3.3% Threshold (optimal phenomenological noise) [1]
> **Non-universal Encoded Gates**
> w/ Magic State distillation for T gates



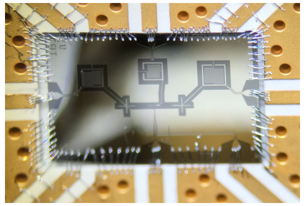
> 4,000 logical qubits for Shor's factoring algorithm

1 billion physical qubits [2]

94% are for magic state distillation

[1] Ohno, Takuya, et al. *Nuclear physics B* 697.3 (2004): 462-480.
[2] Fowler, Austin G., et al. *Physical Review A* 86.3 (2012): 032324.

Fault-tolerant quantum computing with low overhead



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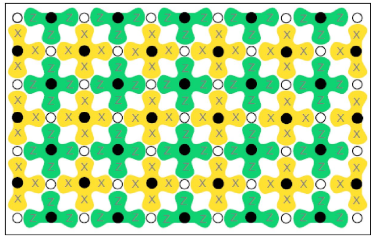

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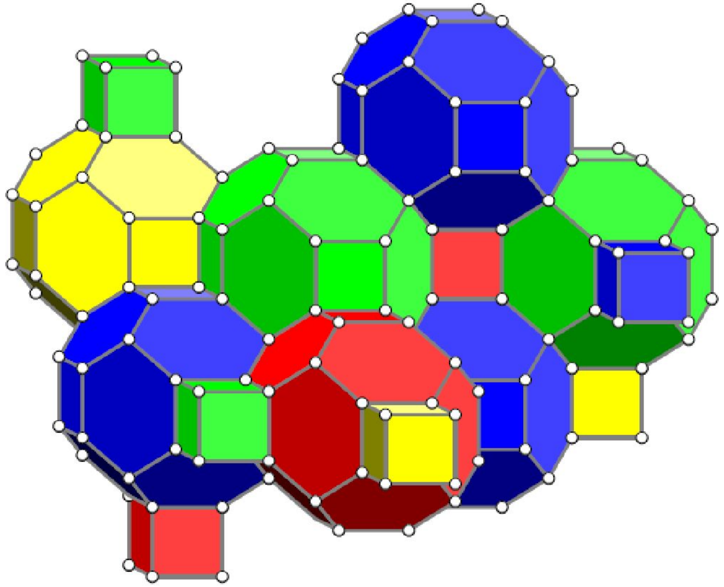
Gauge Color Code (3D):

> **Universal Encoded Gates via gauge fixing** [3]
> 0.31% Threshold (phenomenological noise) [4]
> Optimal Threshold ???



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[3] H. Bombin, New J. Phys. 17 (2015) 083002
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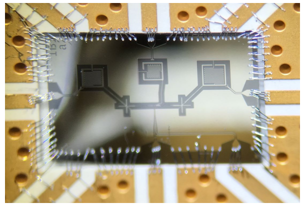
Fault-tolerant quantum computing with low overhead



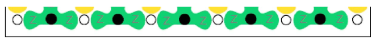
Gauge Color Code (3D):

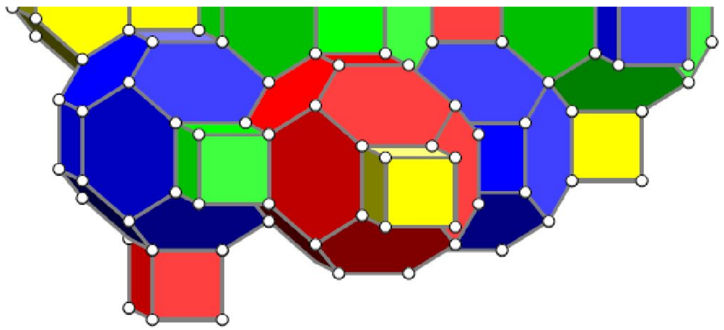
> **Universal Encoded Gates via gauge fixing** (3)
> 0.31% Threshold (phenomenological noise) (4)
> Optimal Threshold ???

Ex. Surface Code (2D):

>
>
>

Present Goal: Push the gauge color code threshold higher with:
(i) A different lattice
(ii) A higher threshold decoder
(efficient but computationally challenging)





> 4,000 logical qubits for Shor's factoring algorithm

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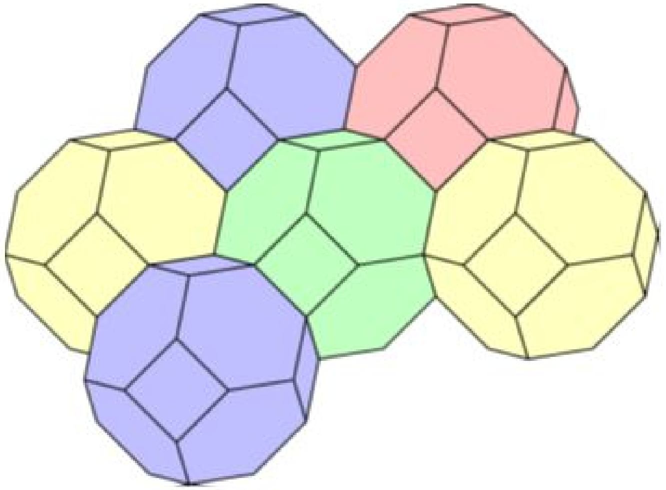
GAUGE COLOR CODES

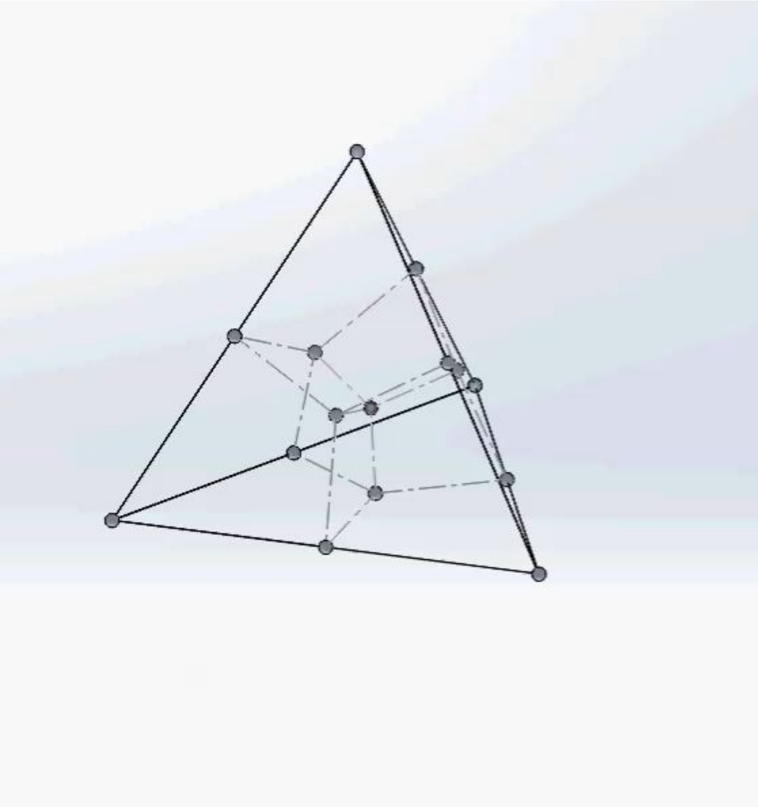
H. Bombin, New J. Phys. 17 (2015) 083002

- Four valent, four colorable lattice
- Can be implemented with only weight 4 & 6 check operators

Simplex	Represents
0-simplex (vertex)	Qubit
1-simplex (edge)	Qubit coupling
2-simplex (face)	Gauge operator
3-simplex (cell)	Stabilizer

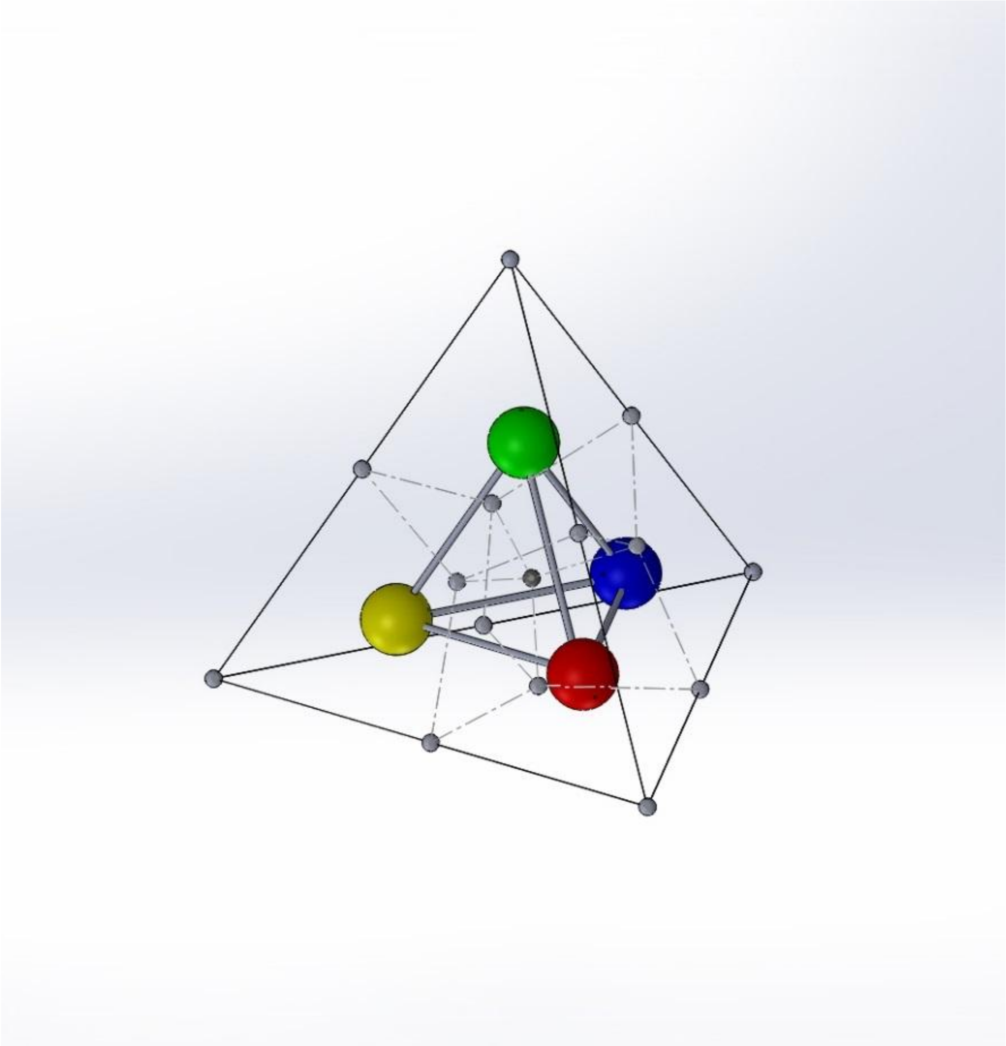


Bulk lattice



Distance 3 (primal)

ERRORS ON GAUGE COLOR CODES



Distance 3 (primal + dual)

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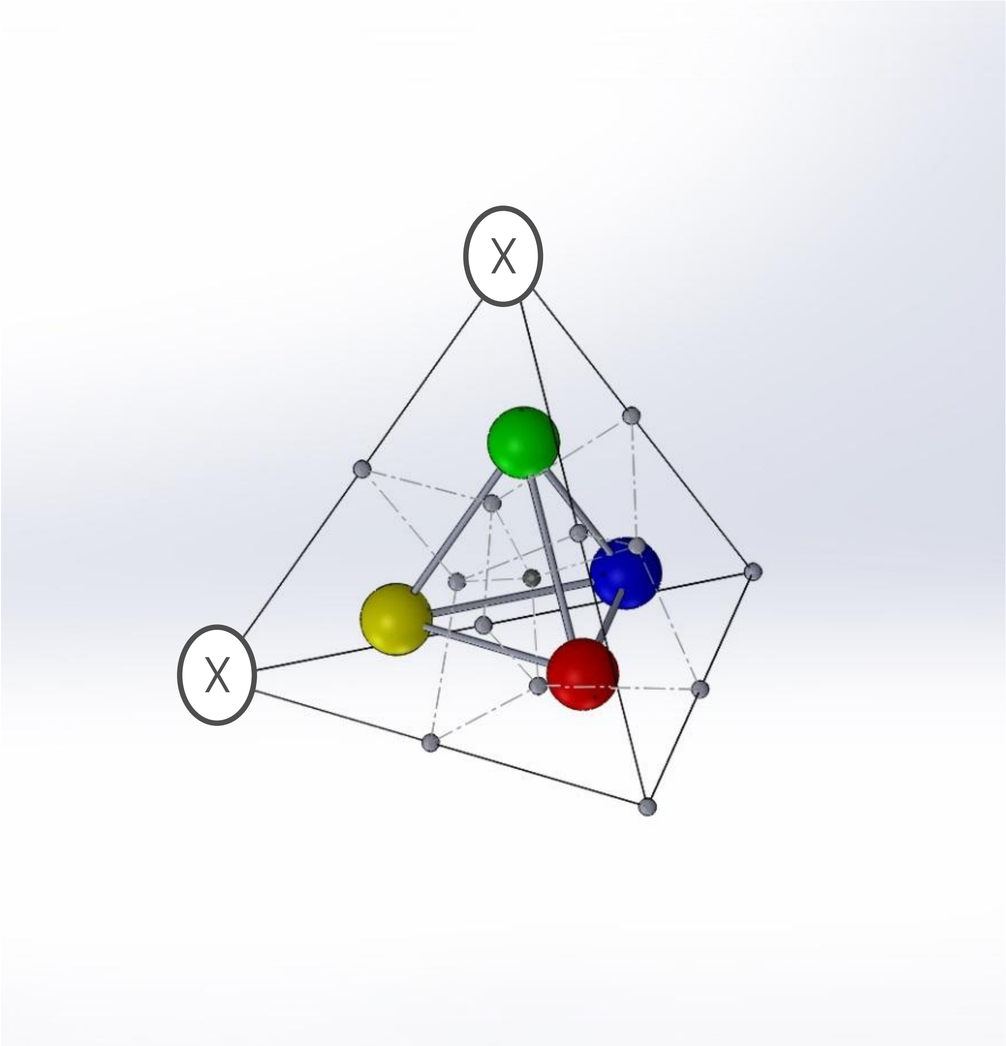

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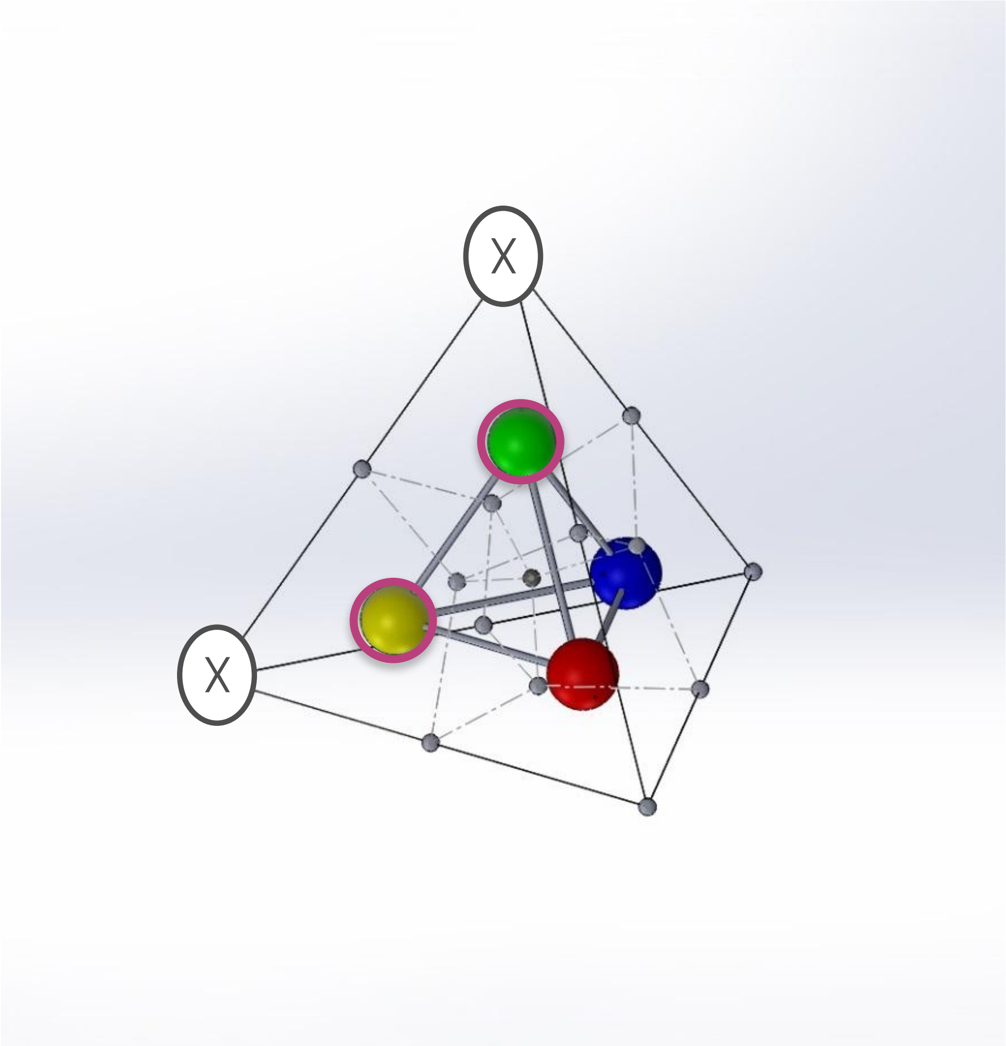

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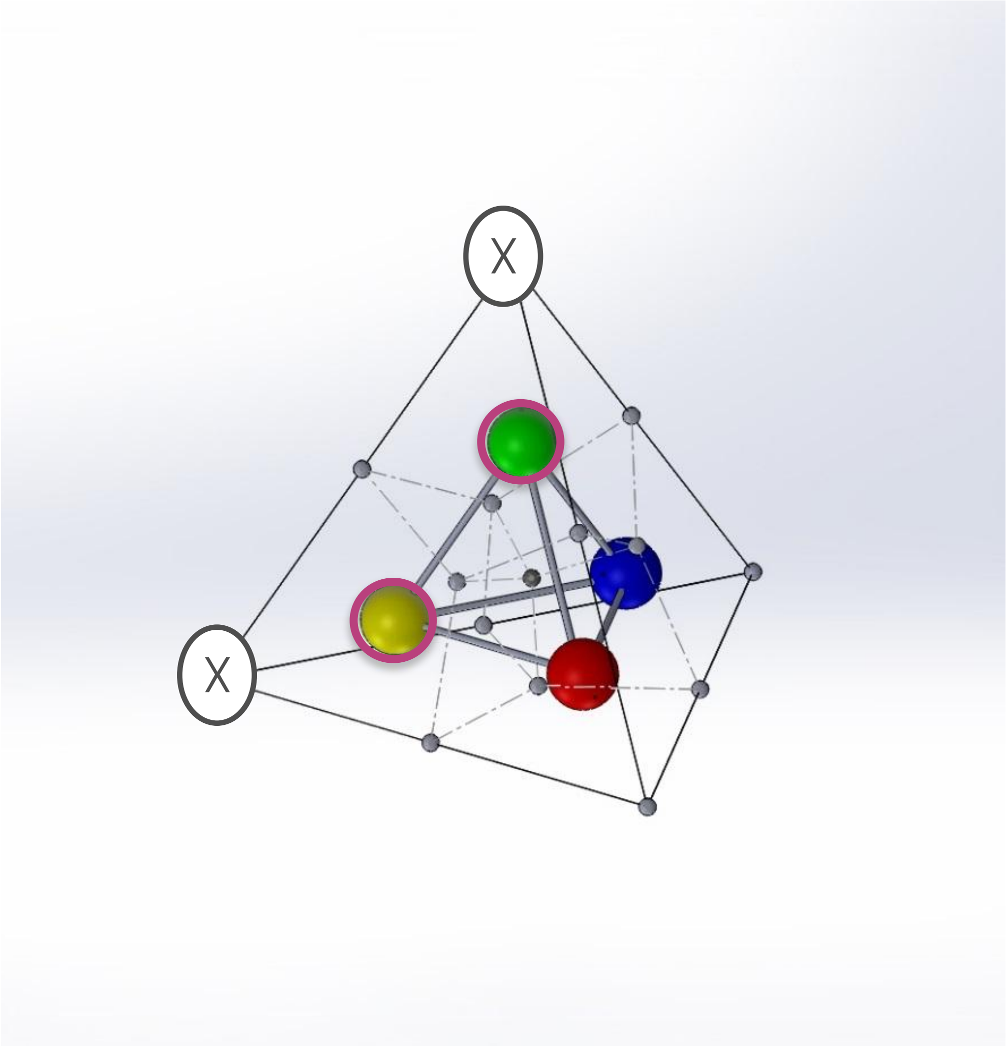

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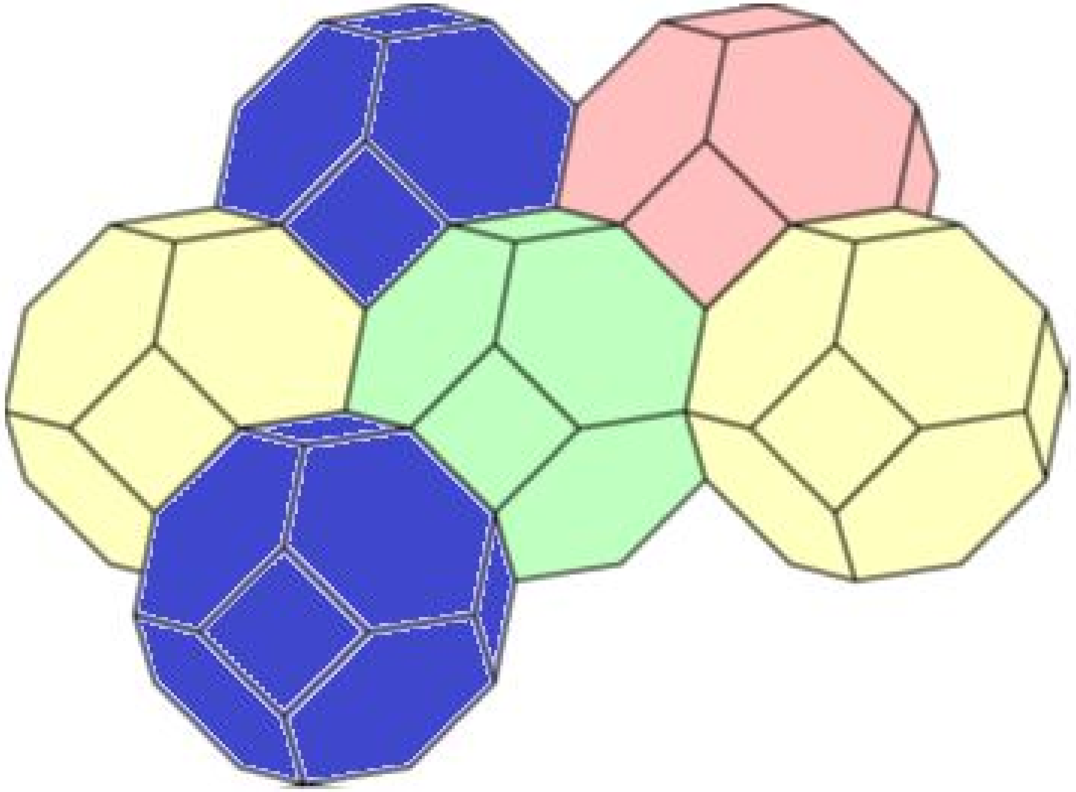

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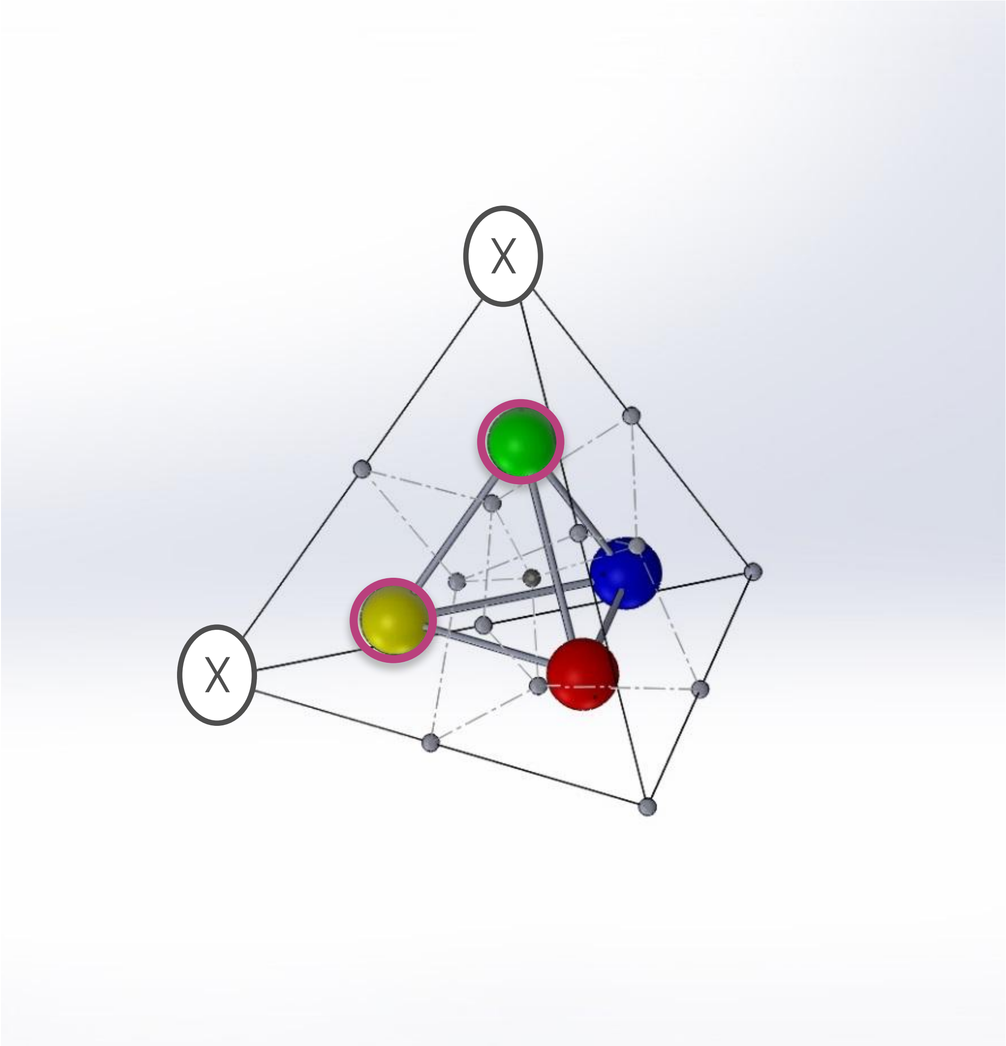

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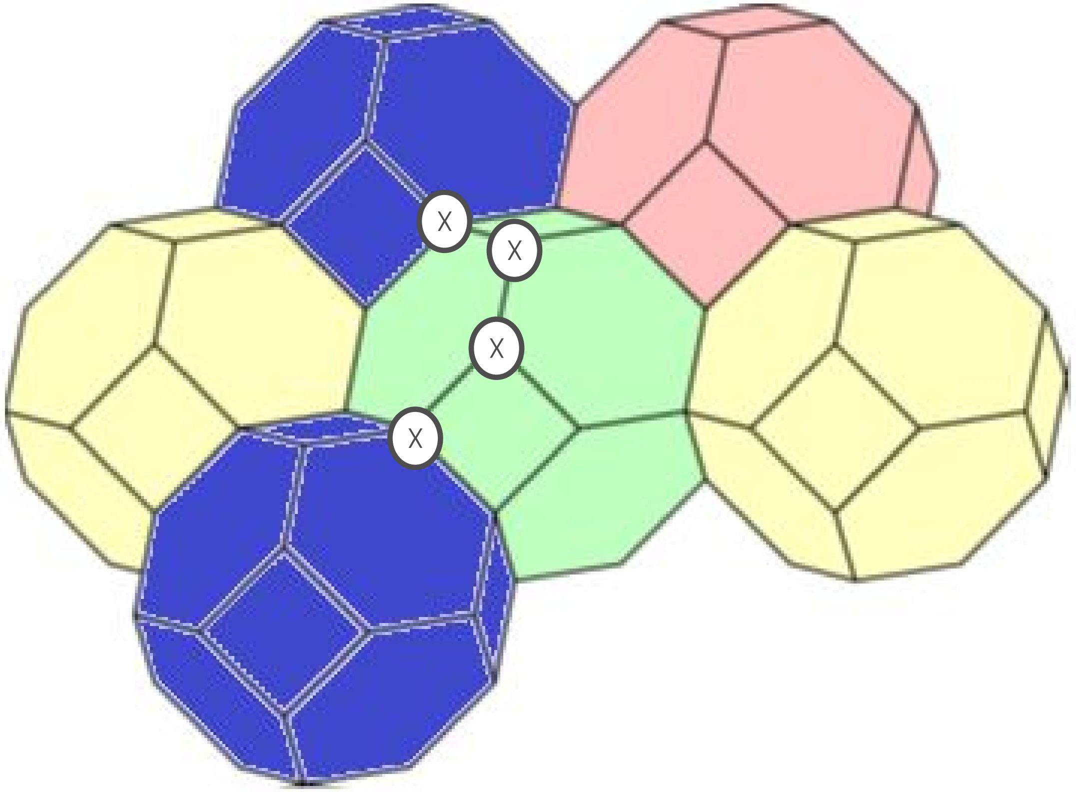

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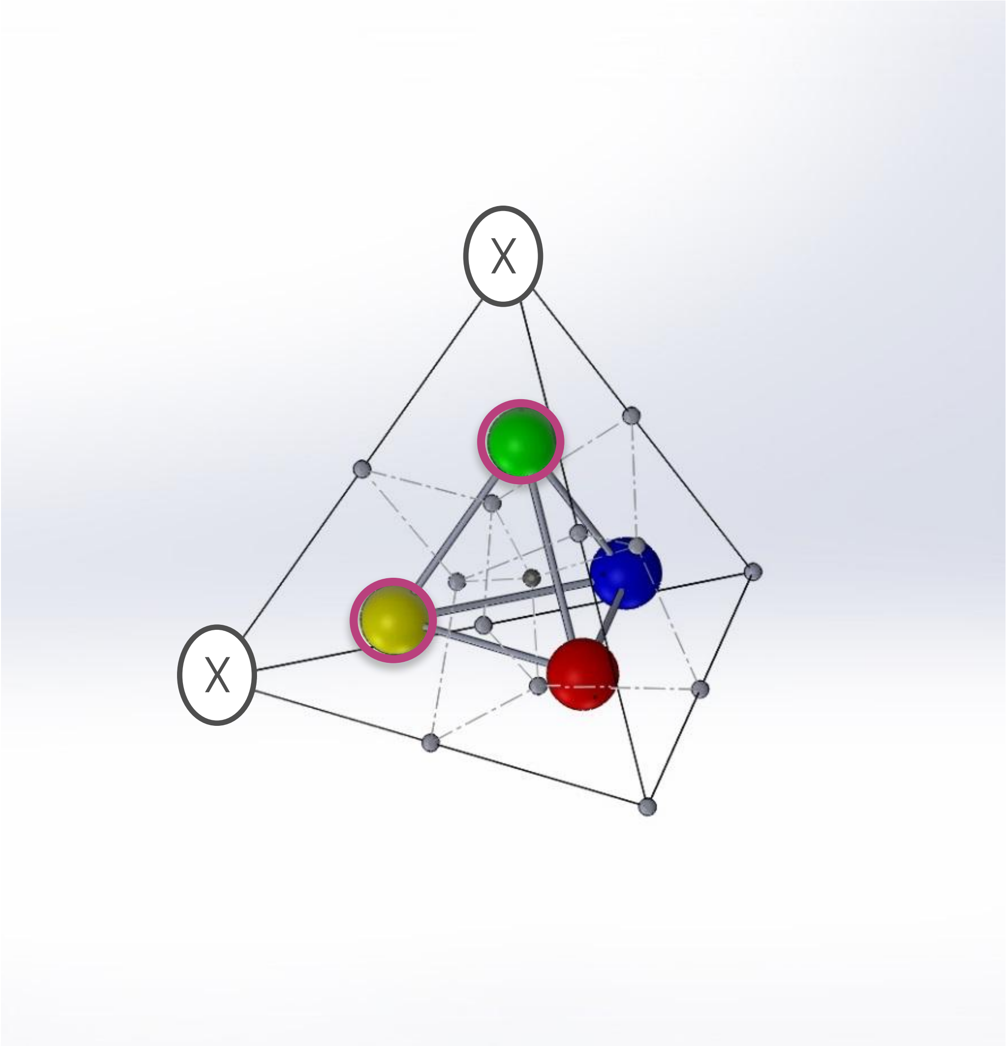

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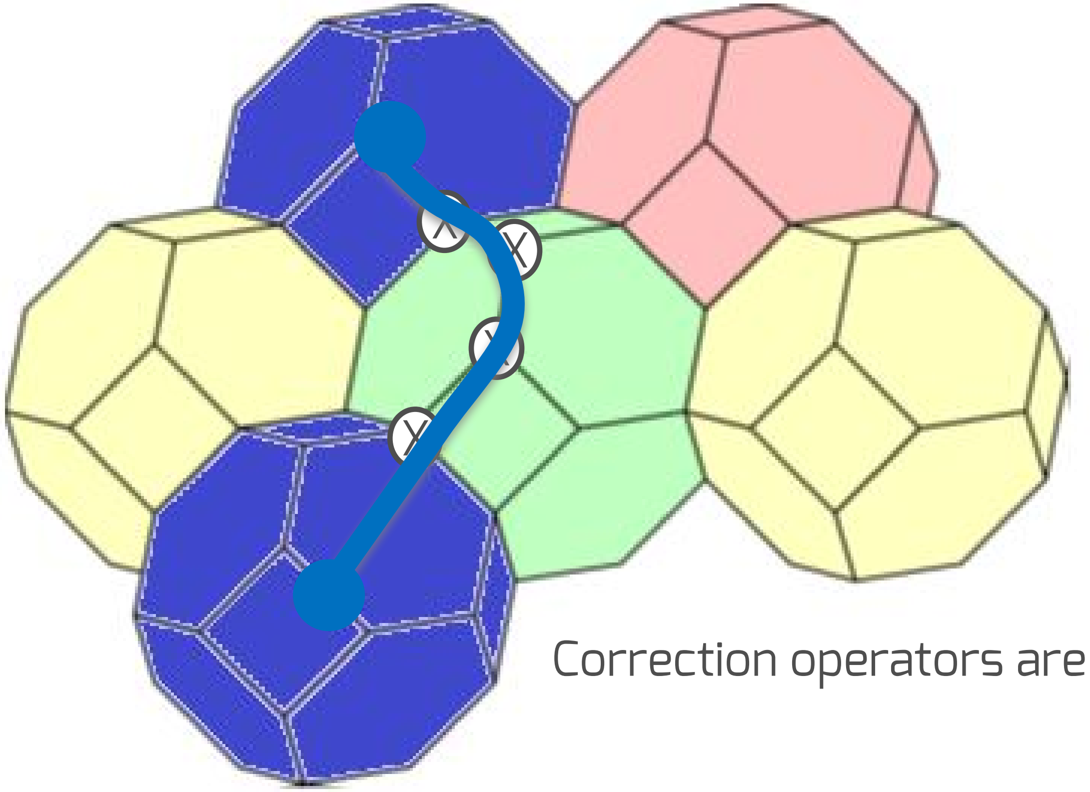


Correction operators are strings

... like in the toric code –

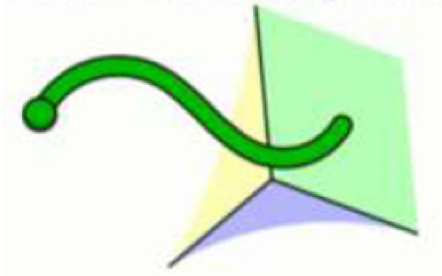
A. Kubica et al. arXiv:1503:02065 (2015)

Bulk (primal)

DECODING GAUGE COLOR CODES

Anyons are removed by either:

- Matching to the boundary of their color

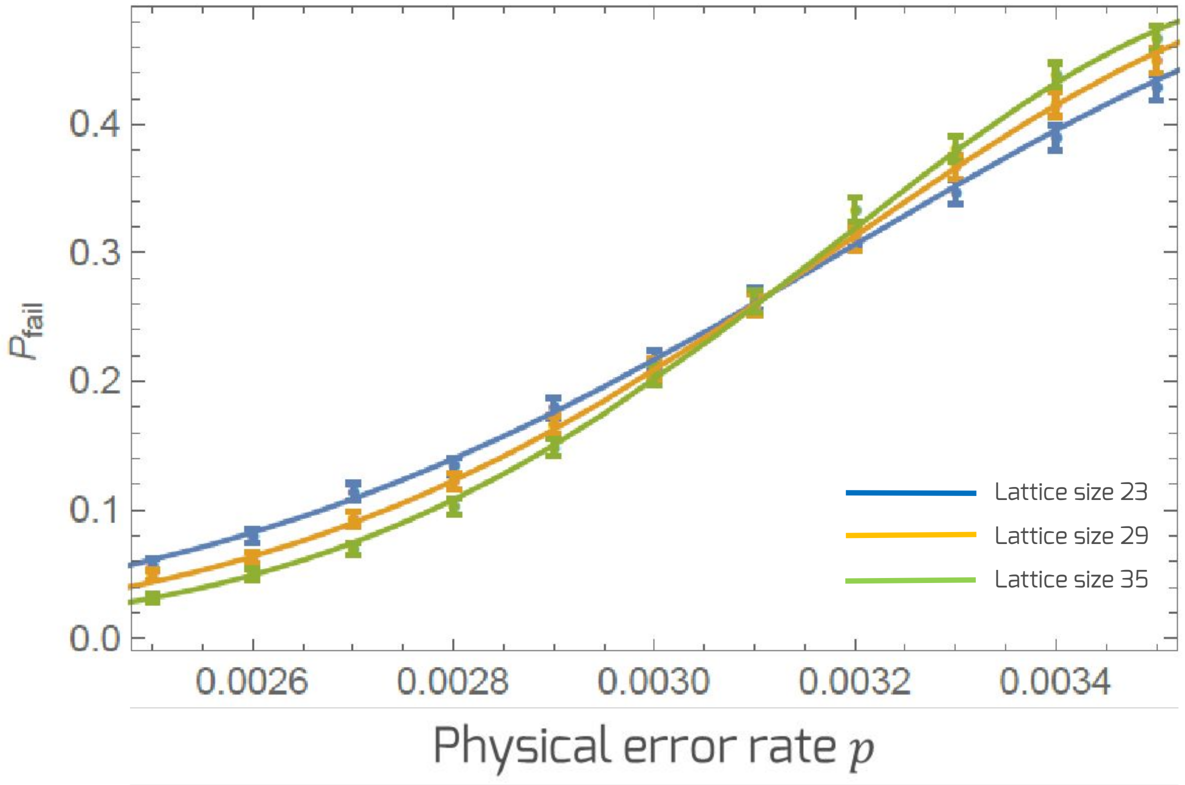


- Matching r+b+y+g to the same cell

Brown, Nickerson, Browne arXiv:1503.08217 (2015):

> 0.31% threshold for phenomenological noise (just Pauli X errors)

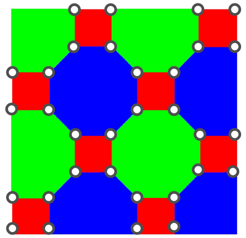
> Used an adapted clustering decoder



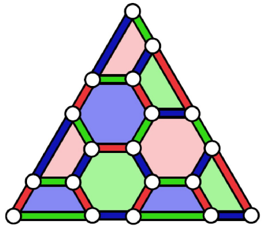
Q: How to improve on this threshold? A: (i) a different lattice & (ii) a MCMC decoder

GAUGE COLOR CODE LATTICES

2D Color Codes



4.8.8

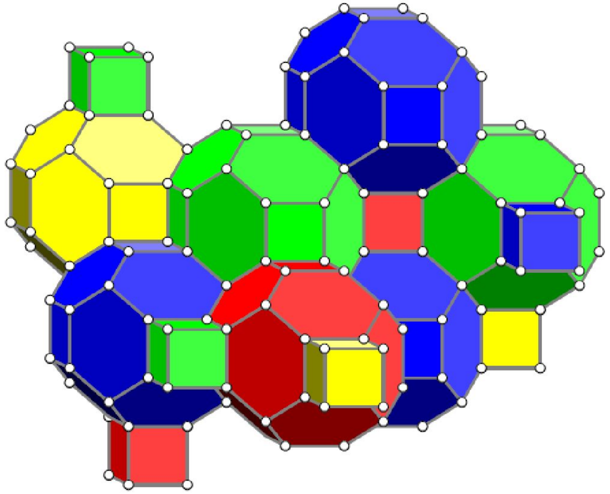


6.6.6

Landahl, et al. *arXiv preprint arXiv:1108.5738*(2011).

3D Color Codes

BNB



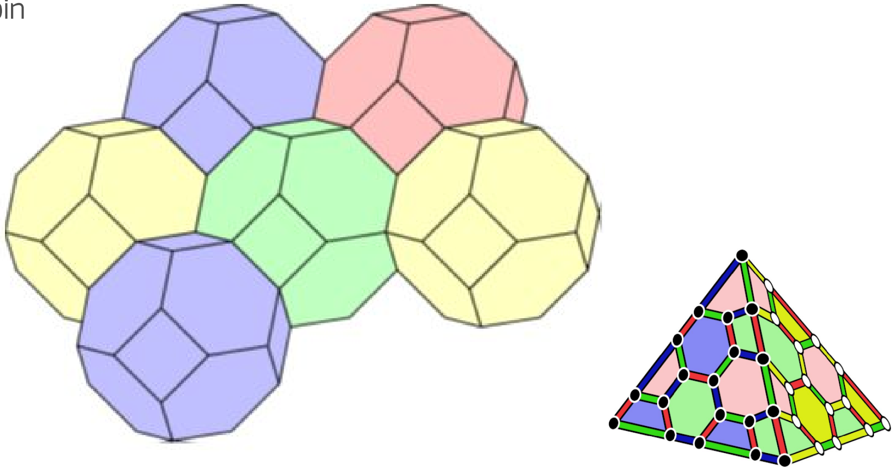
Weight 8 & 32 stabilizers

6 & 18 gauges per stabilizer

Weight 4 & 6 gauge operators

Brown, Nickerson, Browne arXiv:1503.08217 (2015)

Bombin



Weight 24 stabilizers

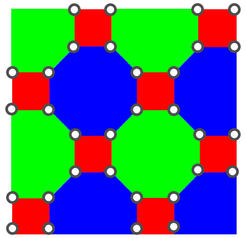
14 gauges per stabilizer

Weight 4 & 6 gauge operators

Bombin *arXiv preprint arXiv:1412.5079* (2014)

GAUGE COLOR CODE LATTICES

2D Color Codes



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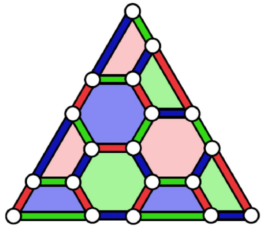

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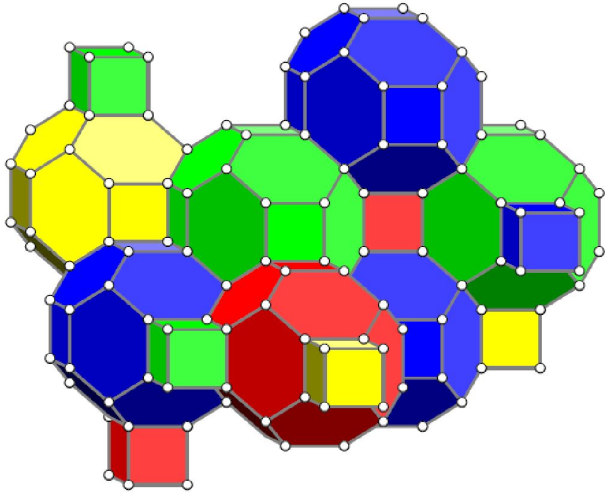

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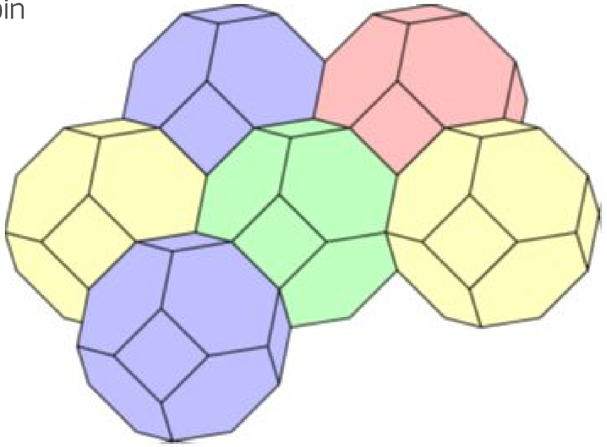

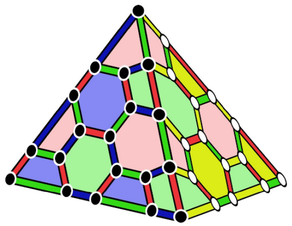

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We choose this lattice

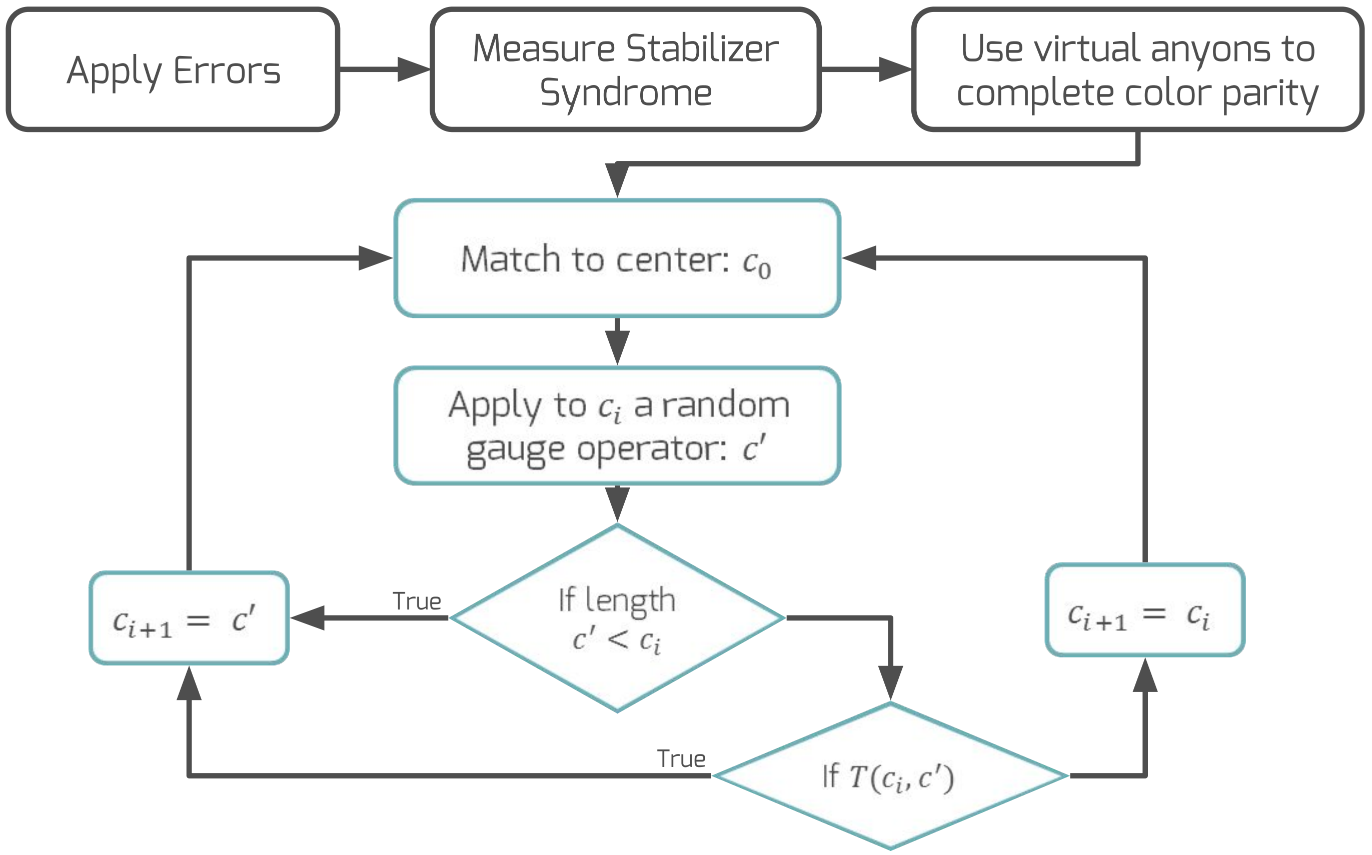
MARKOV CHAIN MONTE CARLO DECODING

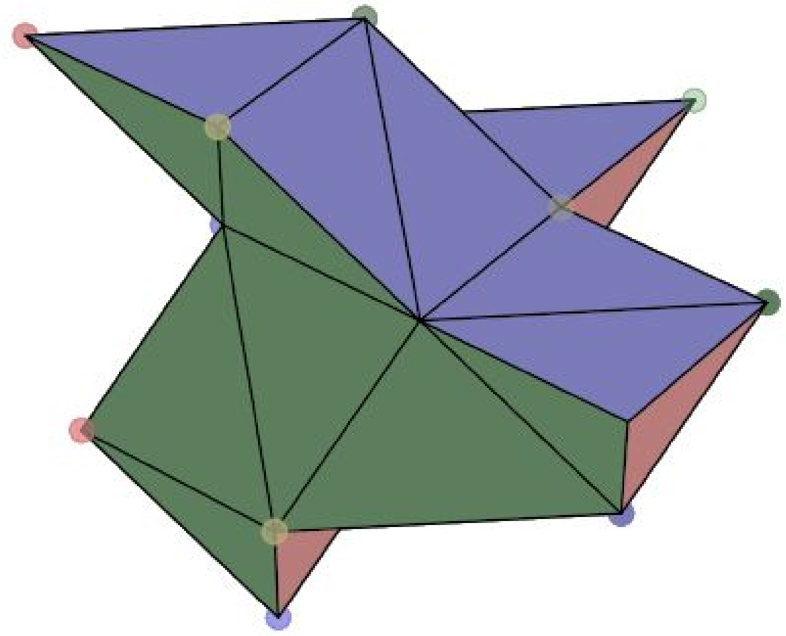
> MCMC decoders for the surface code:

- Wootton & Loss Phys. Rev. Lett, 106(16), 16053 (2012)
- Hutter et al. Phys. Rev. A 89, 022326 (2014)
- Their MCMC decoder achieves surface code threshold of 18.5%

  (upper bound is 18.9%; Masayuki. *Phys. Rev. A* 85.6 (2012): 060301.)

MARKOV CHAIN MONTE CARLO DECODING





> Run for chainlength = $\alpha \times d^6$ steps, where d is the distance of the code and α is a constant.

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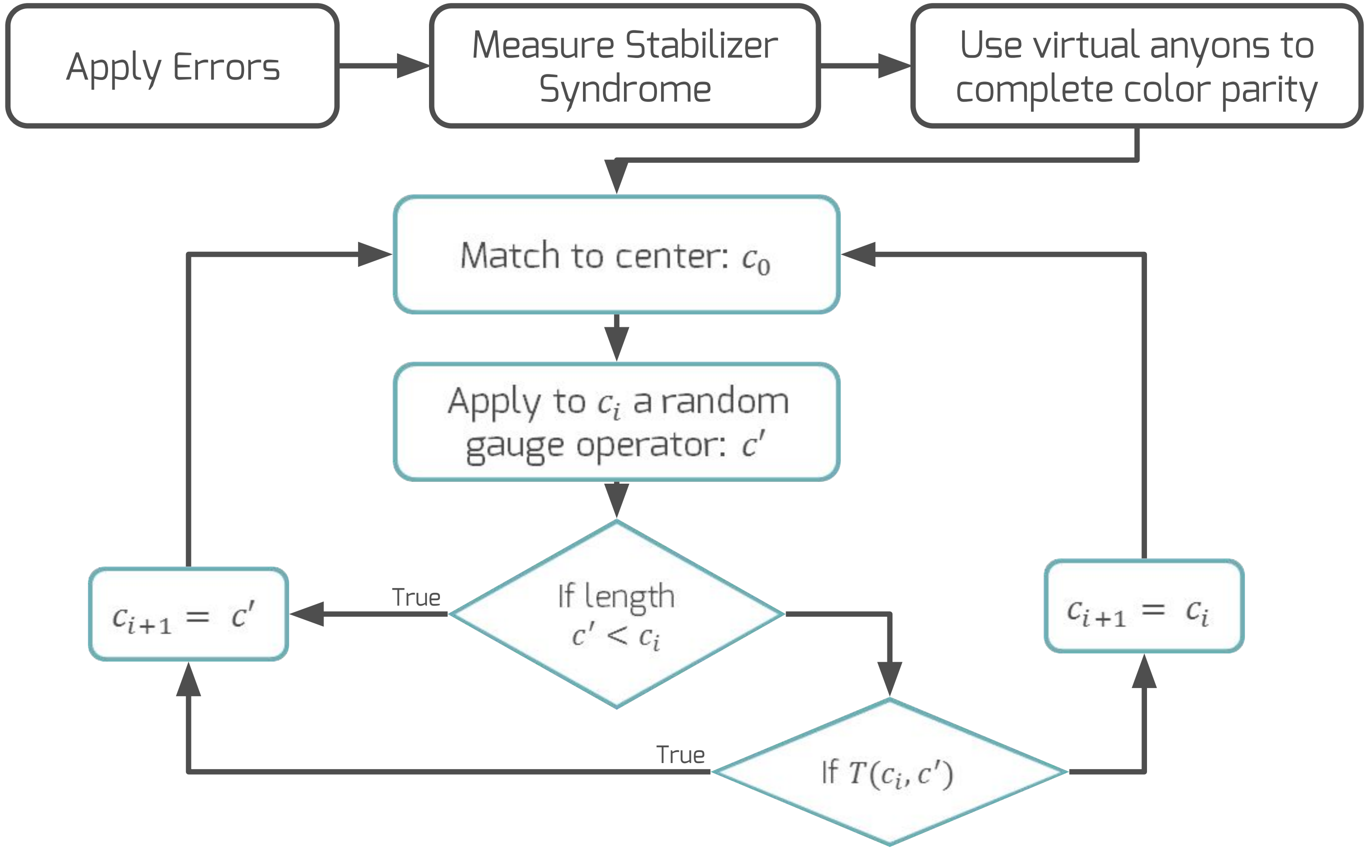

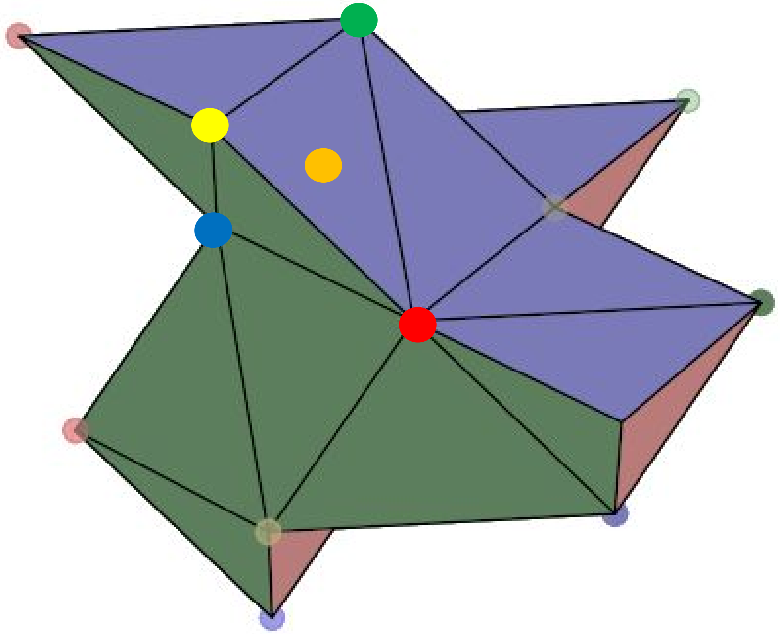

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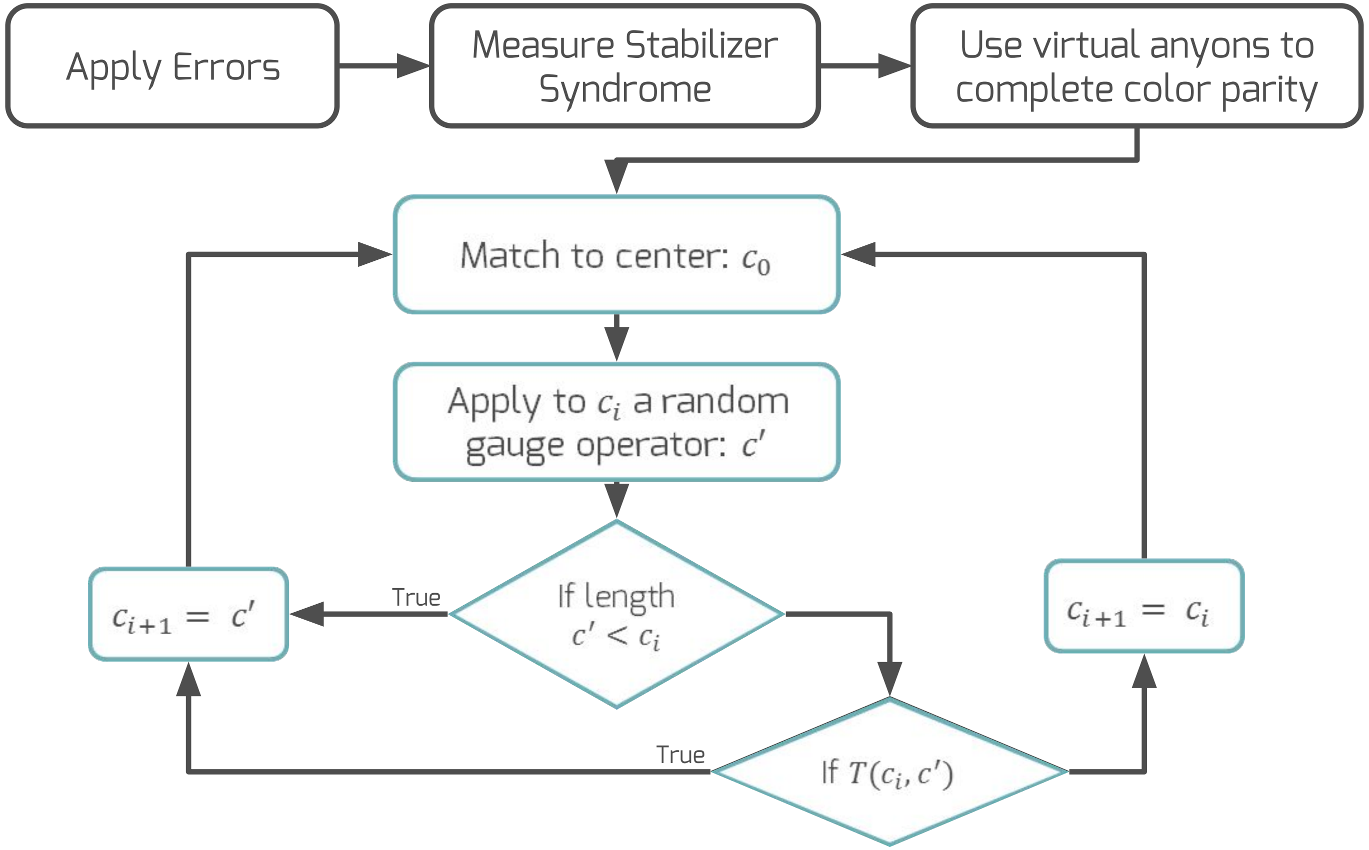

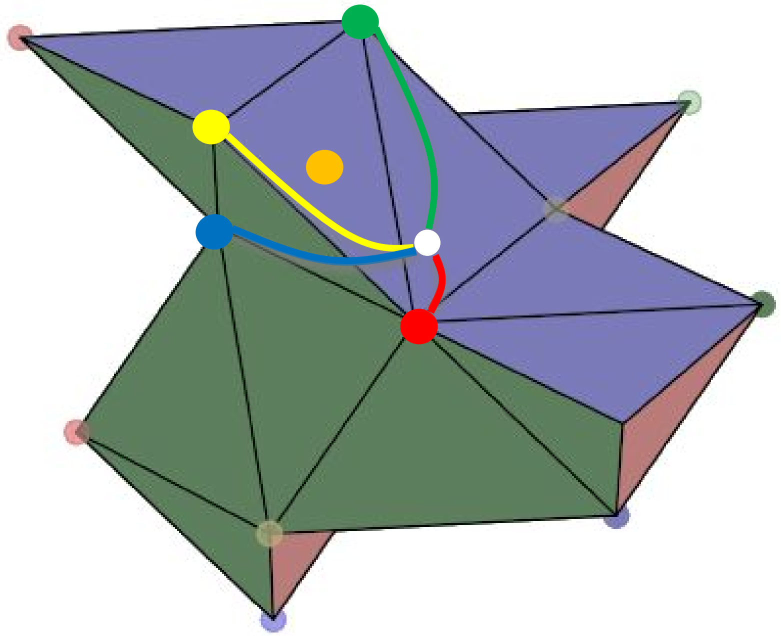

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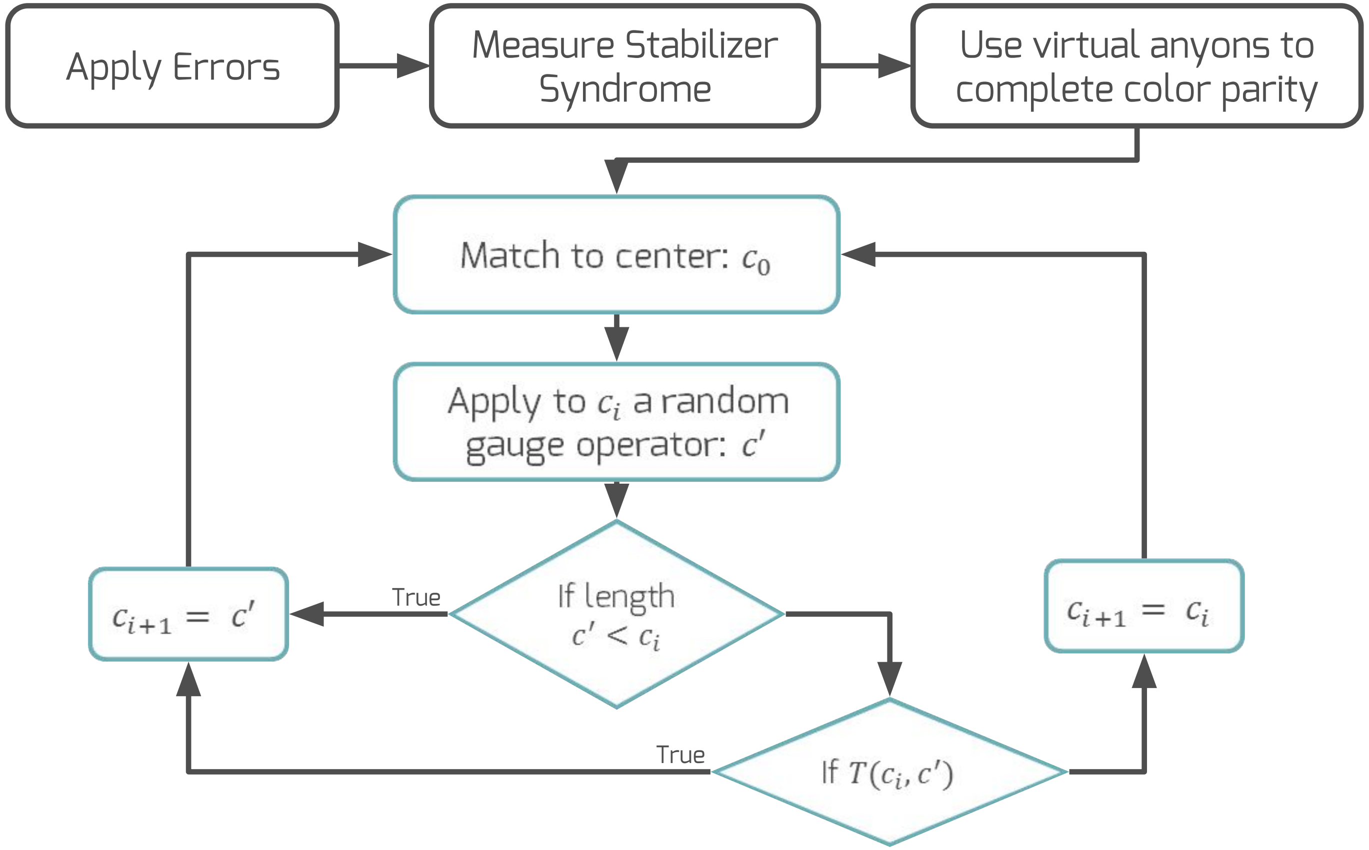

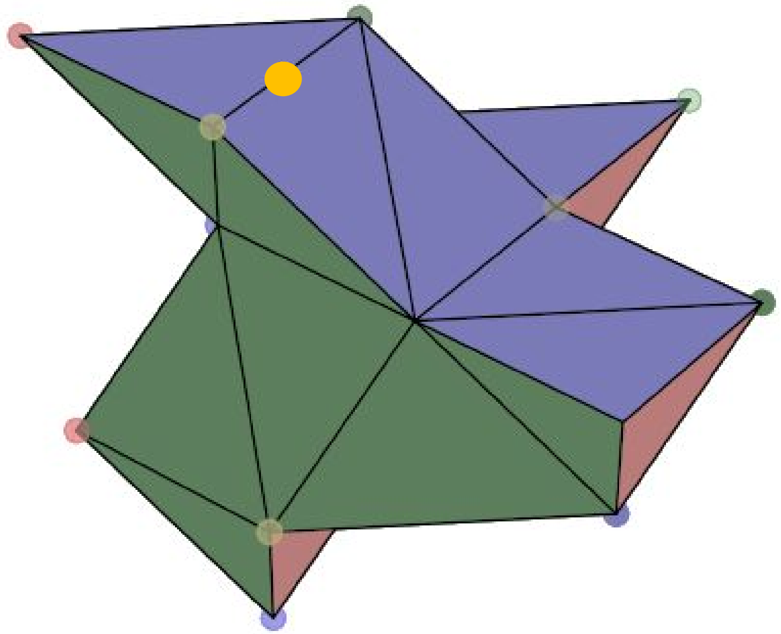

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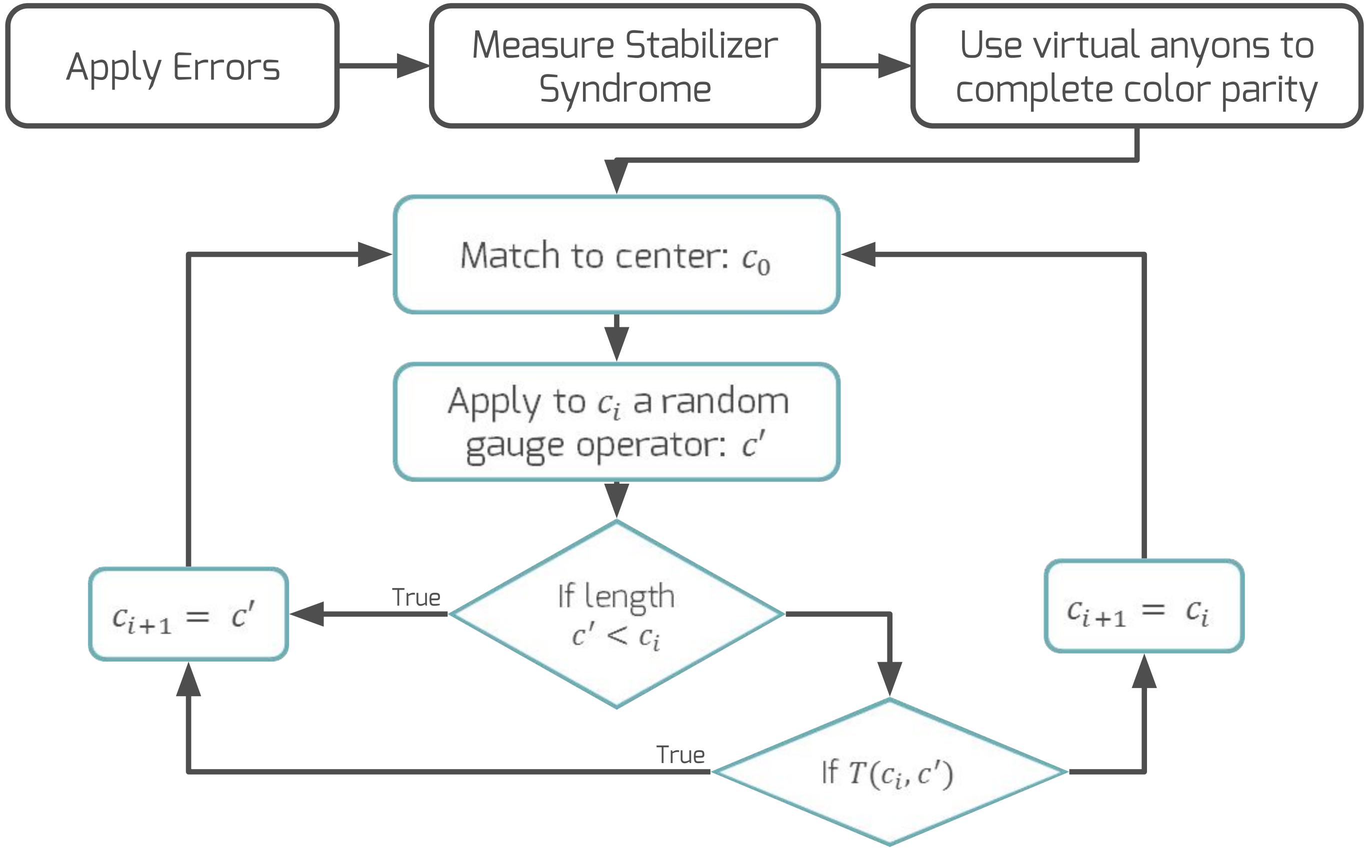

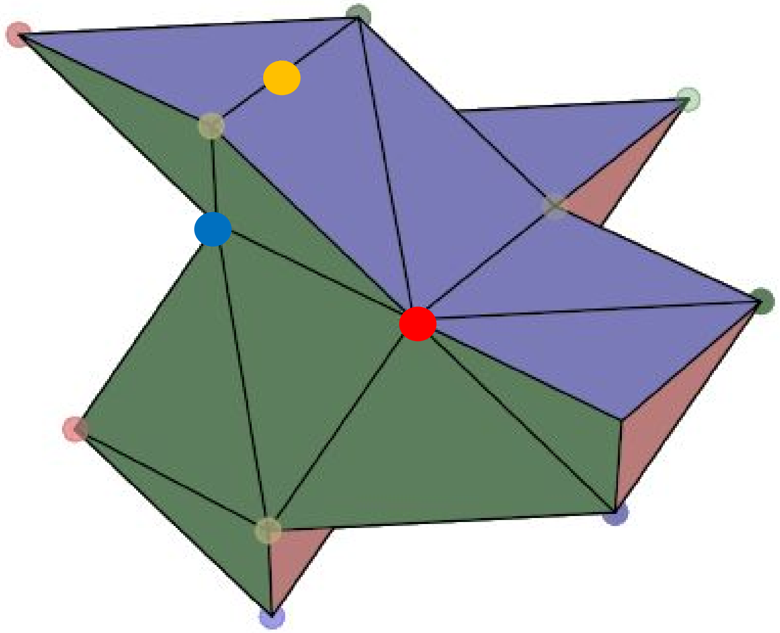

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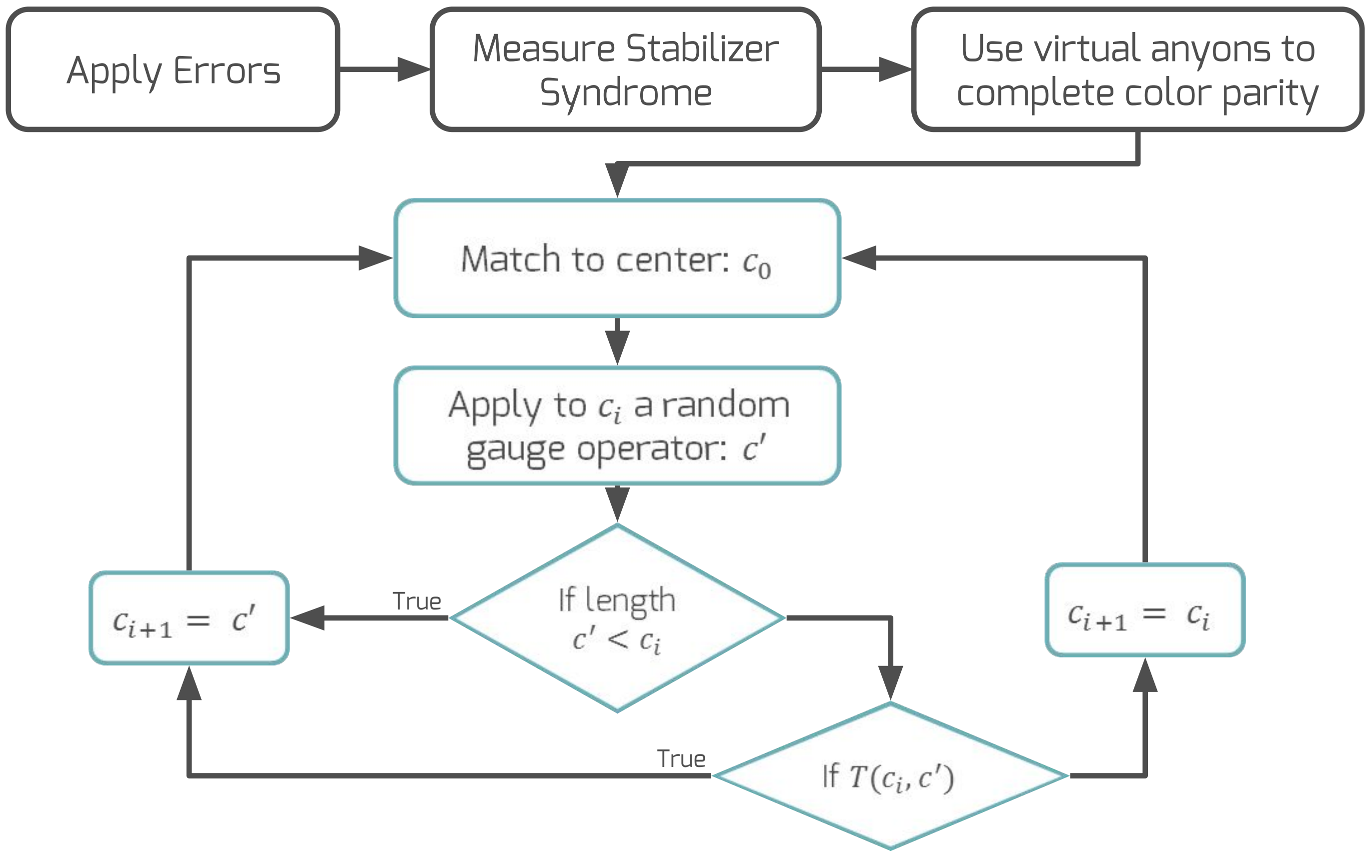

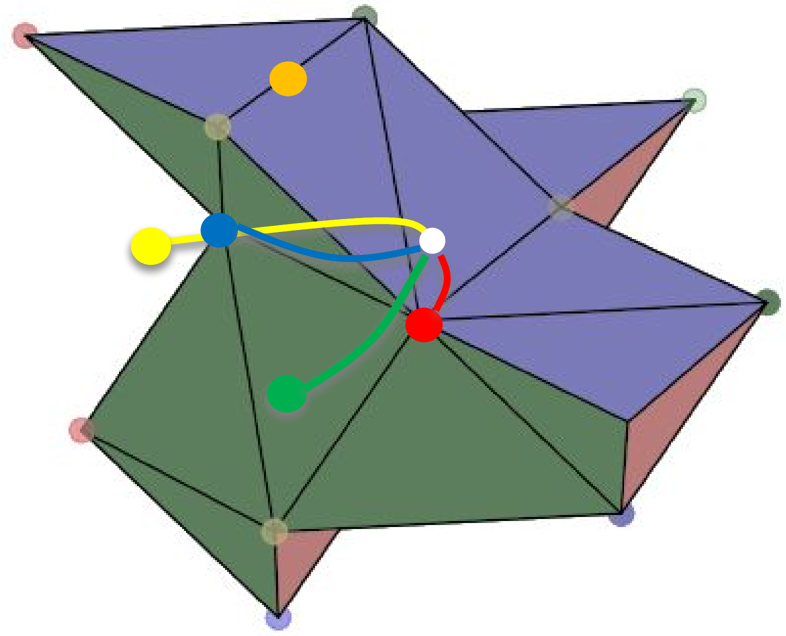

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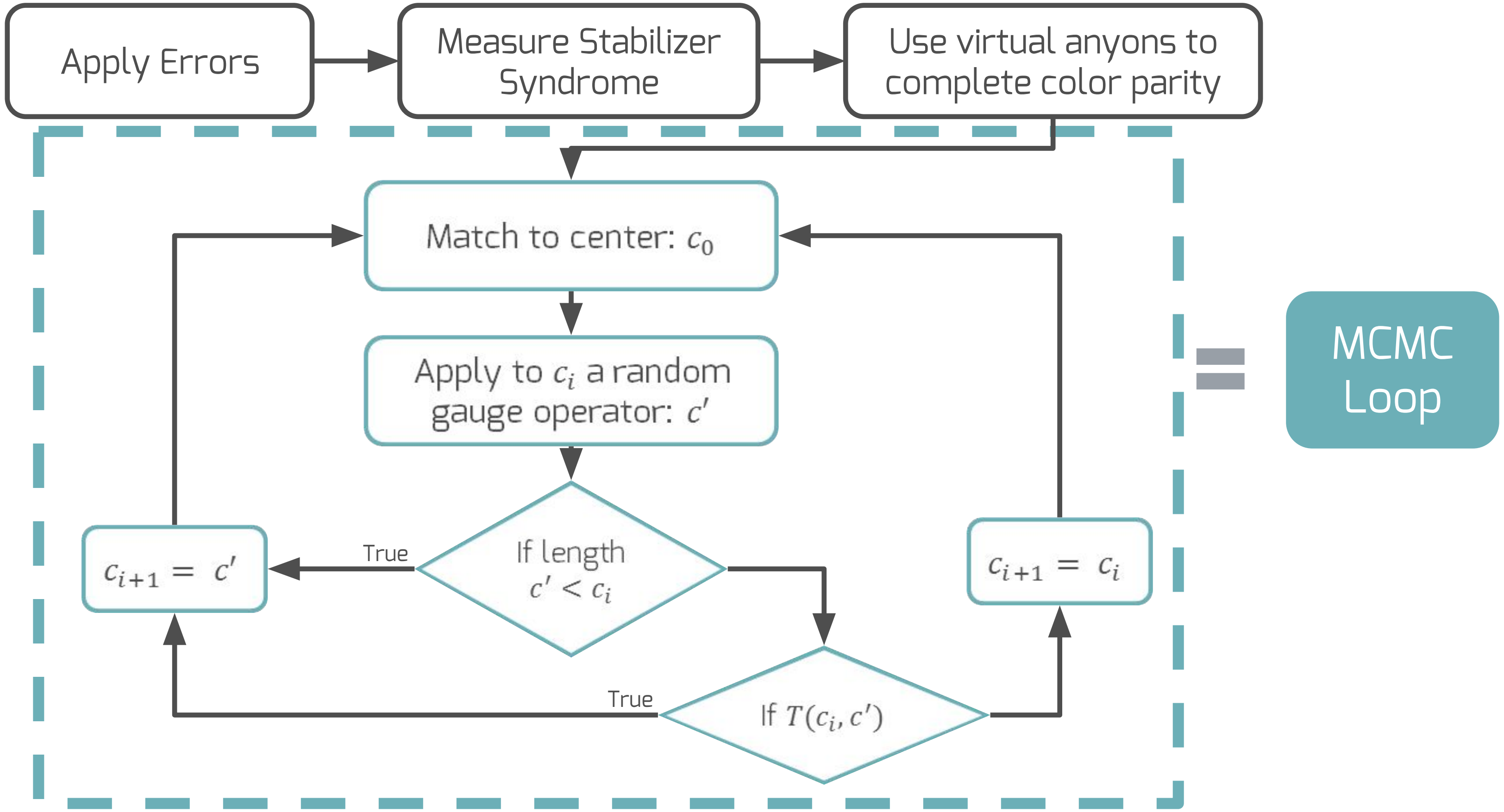

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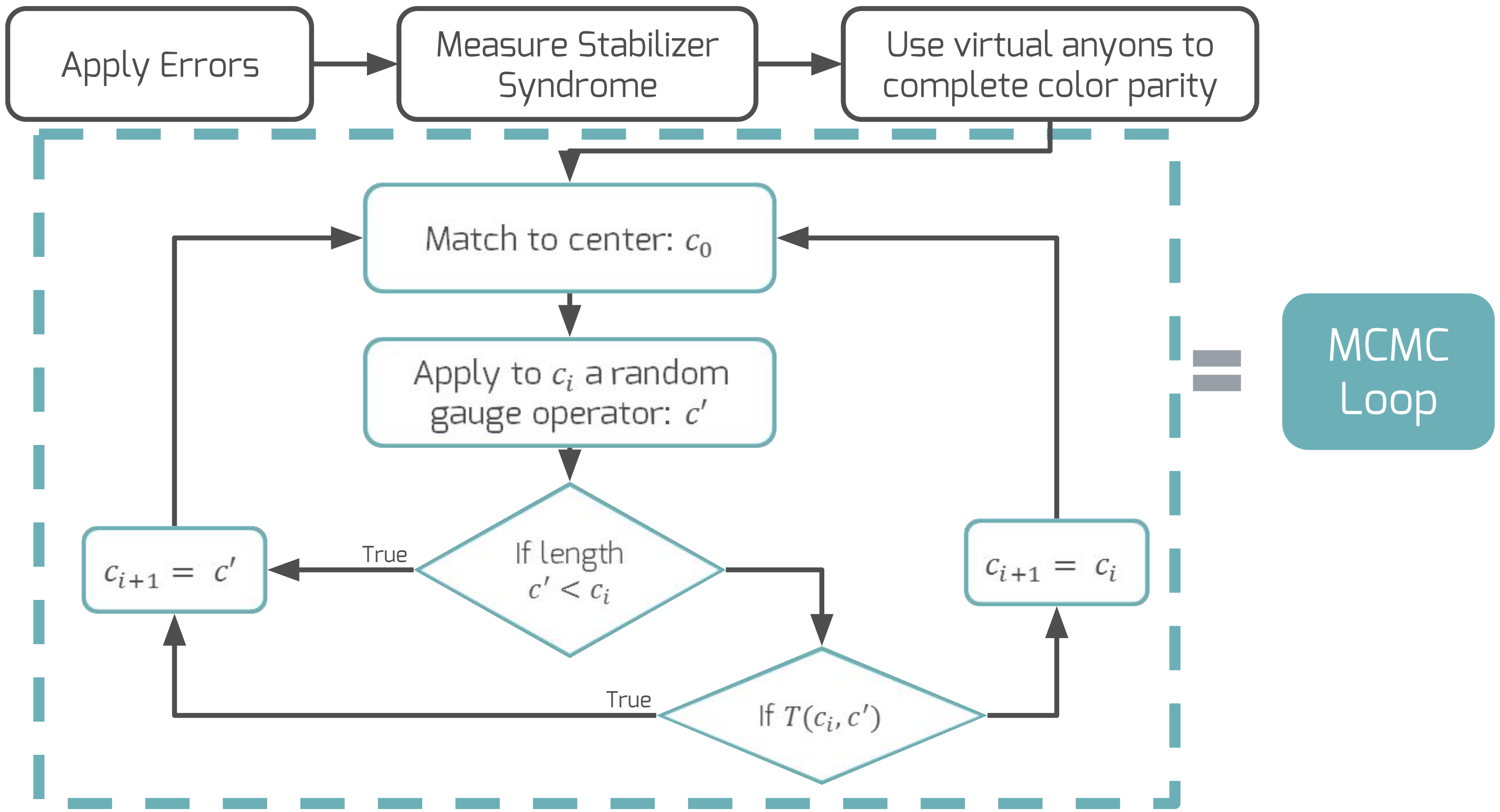

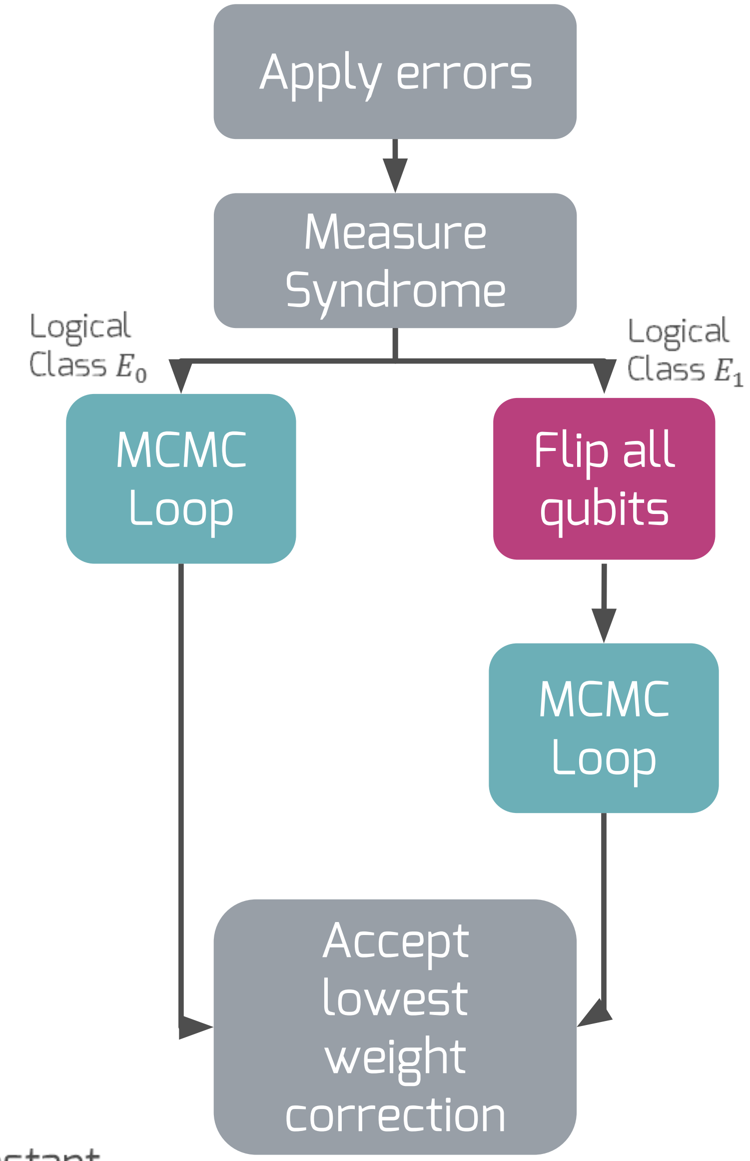

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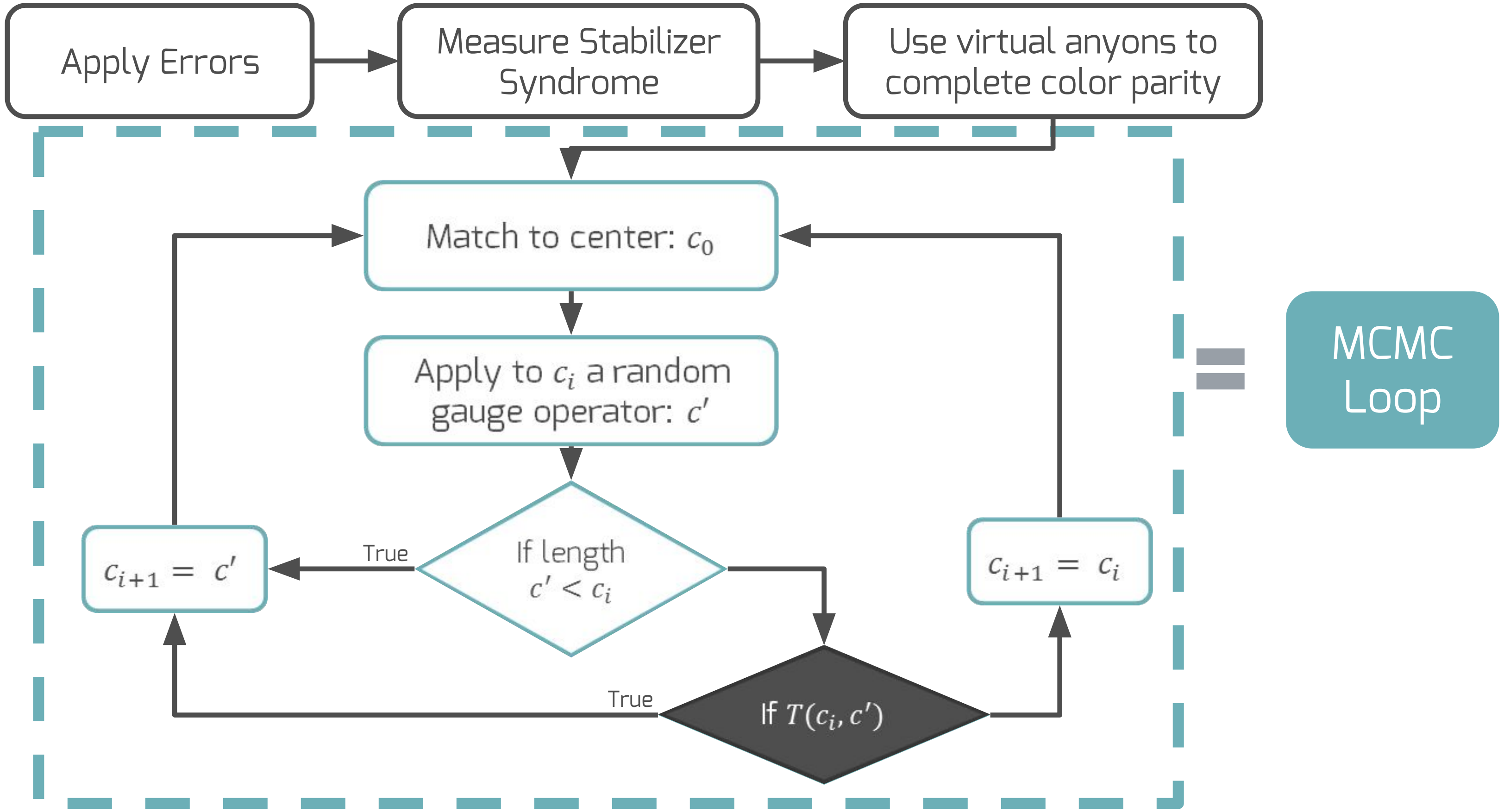

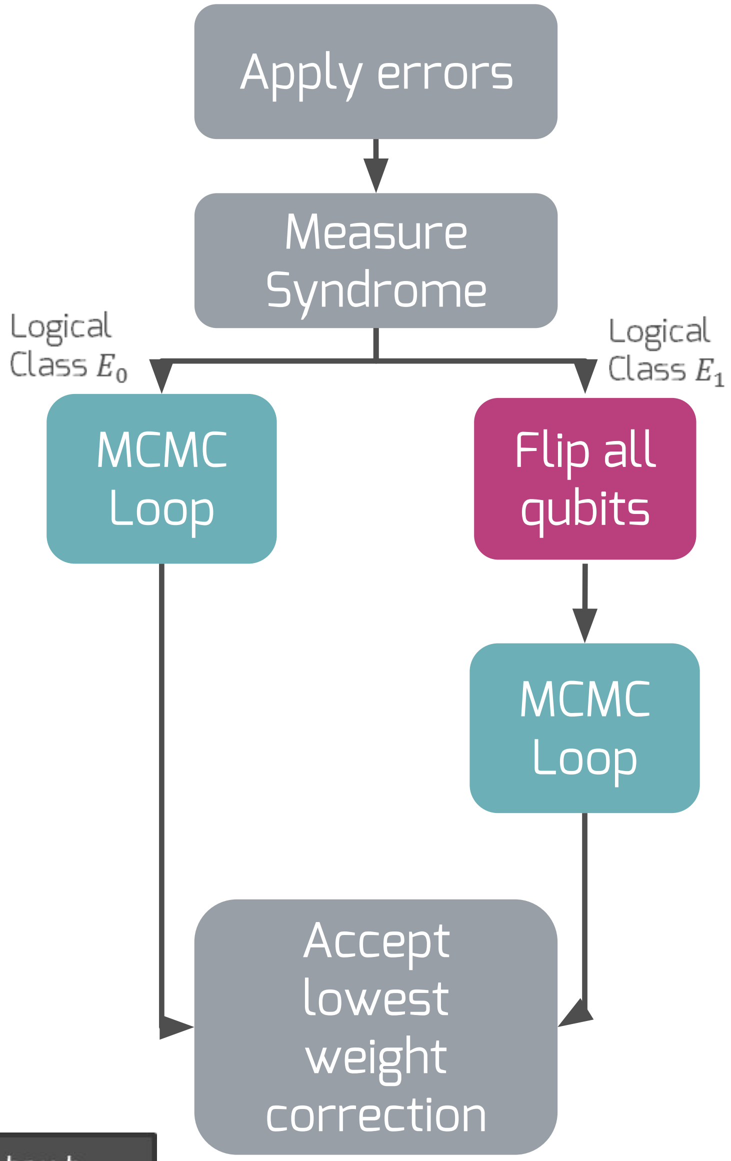

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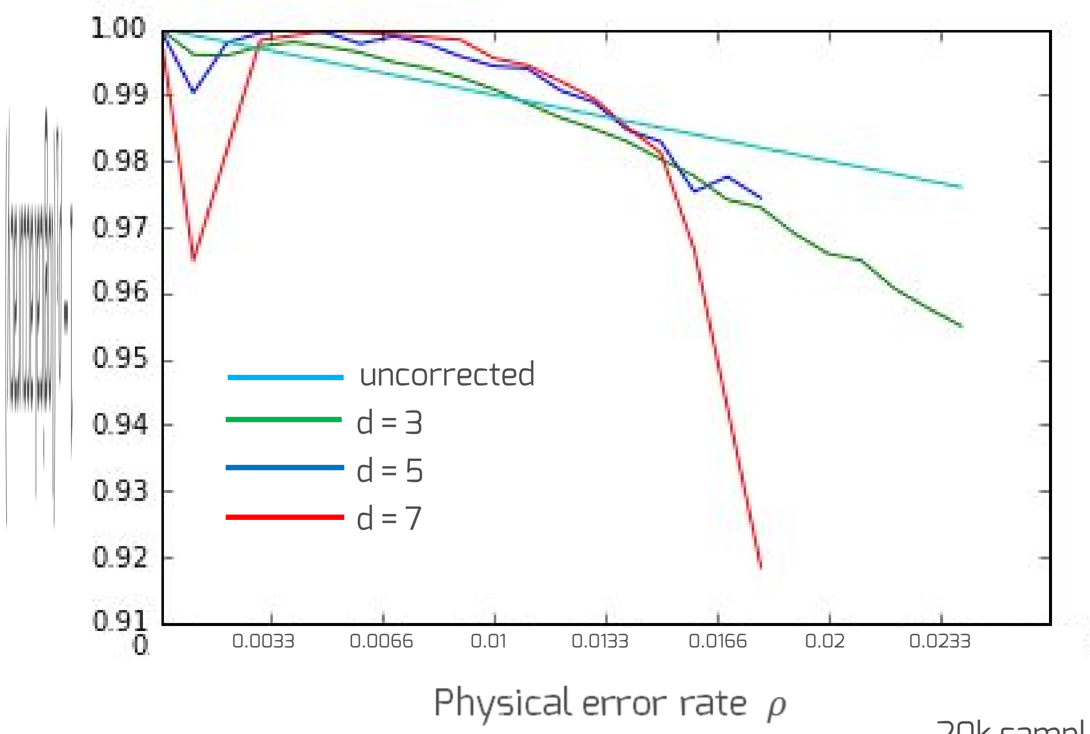
LOOKING FOR A THRESHOLD WITH MCMC DECODING

> Perfect Measurements; X errors
> Compare to 0.45% from Brown et al. arXiv:1503.08217 (2015)
> $L(c)$ = weight of correction c

$$ChainLength = 10 \times d^6$$

$$T(c_i, c') = 100\left(\frac{1-\frac{p}{3}}{p}\right)^{L(c_i)-L(c')}$$



20k samples per point (except d=11 at 2k)

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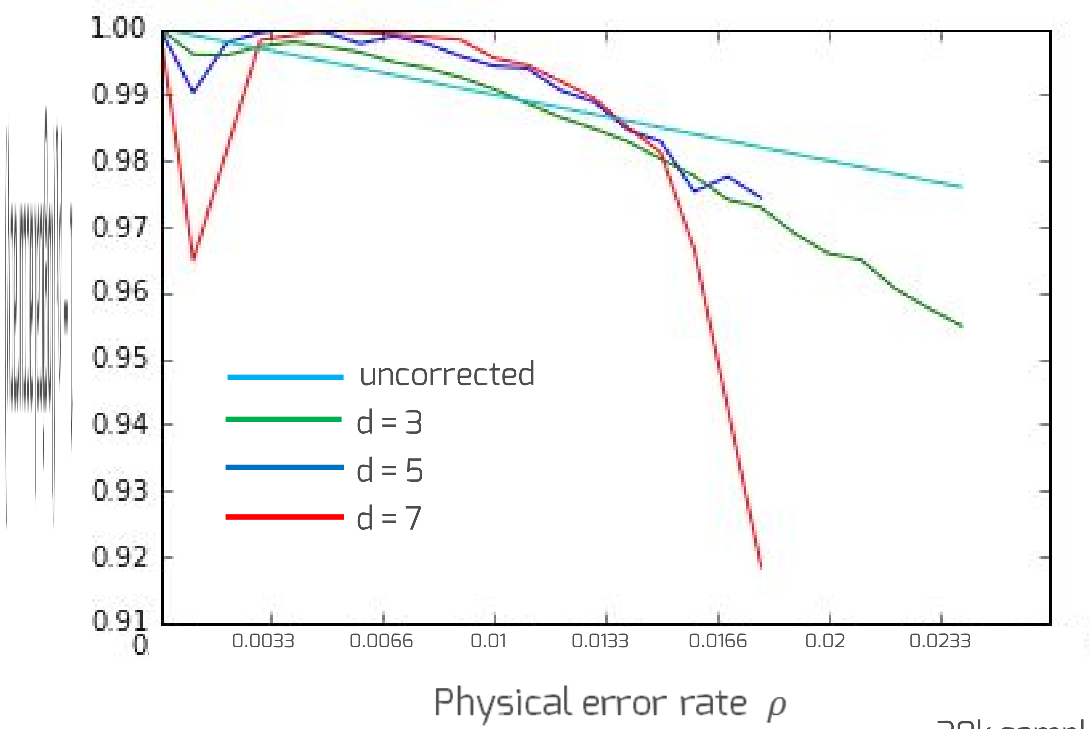
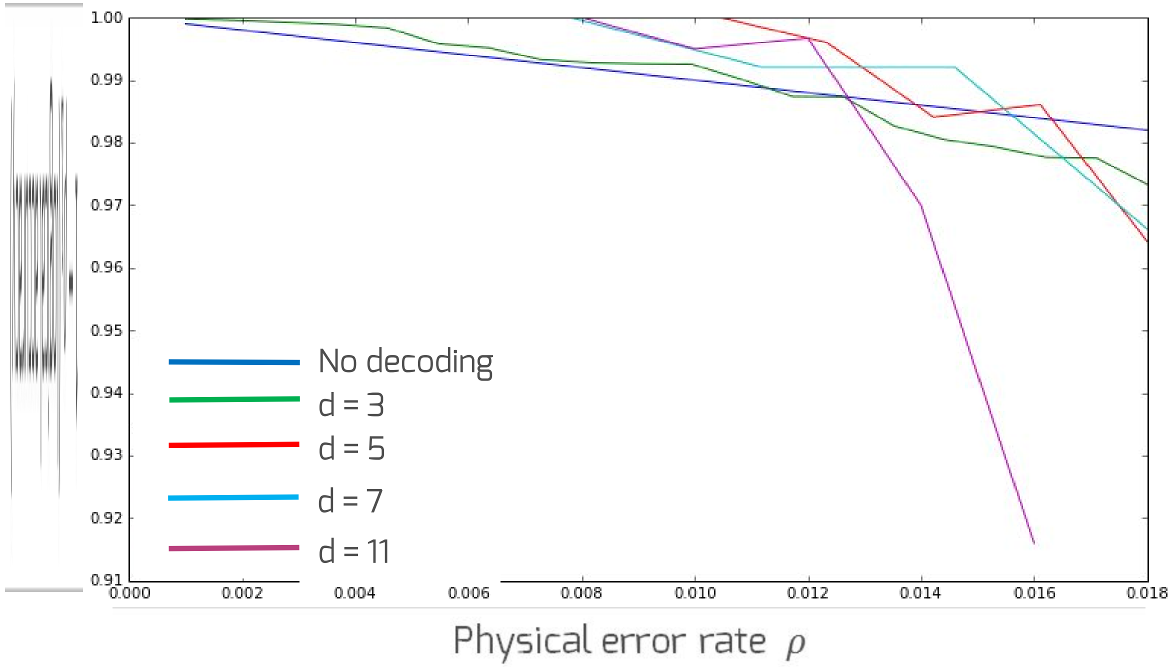
$$ChainLength = 100 \times d^6$$

$$T(c_i, c') = \frac{1}{p} \left(\frac{1 - \frac{p}{3}}{p} \right)^{L(c_i) - L(c')}$$





20k samples per point (except d=11 at 2k)

CONCLUSION

Evidence towards a GCC threshold > 1.2%*

* perfect measurements

FUTURE WORK

1) More evidence:
   - Markov chain parallelization: $O(L^4) \to O(L^2)$ in 2D case
   - Larger lattices $(\mathrm{d} \approx 41)$
   - Is this really efficient?
   - What are the optimal parameters?

2) New error models:
   - Single-shot decoding makes GCC able to easily detect measurement errors

will@rigetti.com

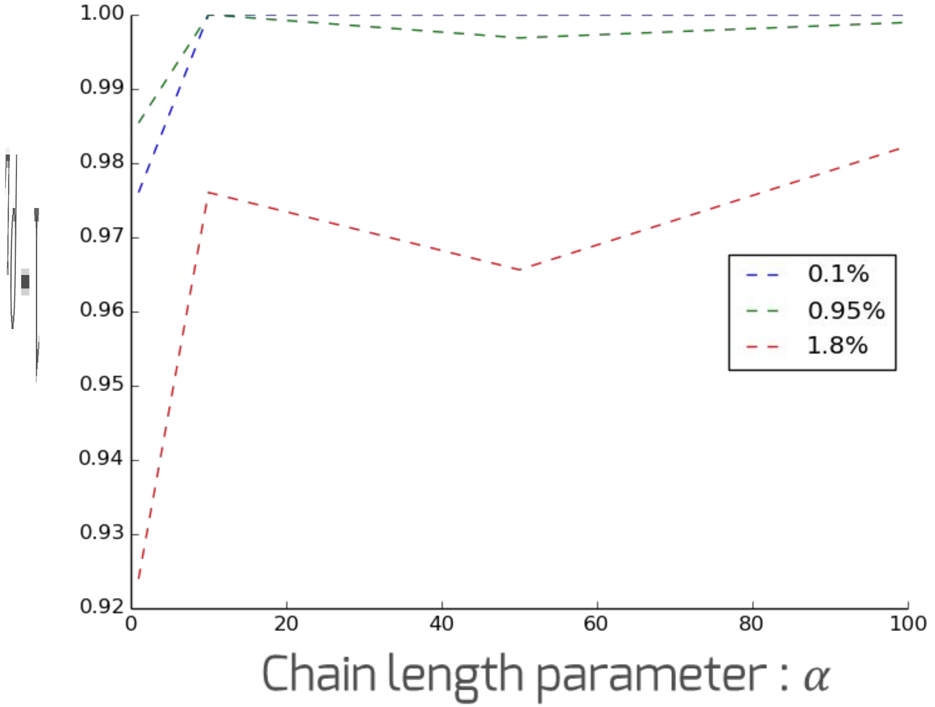
APPENDIX

LOOKING FOR A THRESHOLD WITH MCMC DECODING

$$ChainLength = \alpha \times d^6$$

$$T(c_i, c') = 100\left(\frac{1-\frac{p}{3}}{p}\right)^{L(c_i) - L(c')}$$

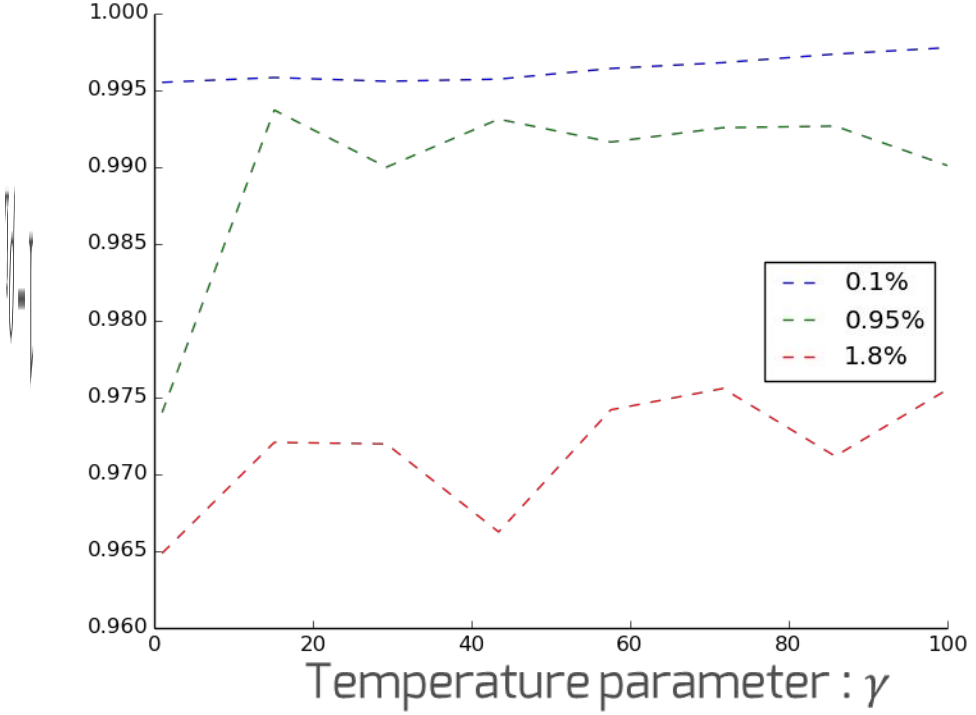
Logical accuracy vs. chainlength at different physical error rates



$$ChainLength = 30 \times d^6$$

$$T(c_i, c') = \gamma\left(\frac{1-\frac{p}{3}}{p}\right)^{L(c_i) - L(c')}$$

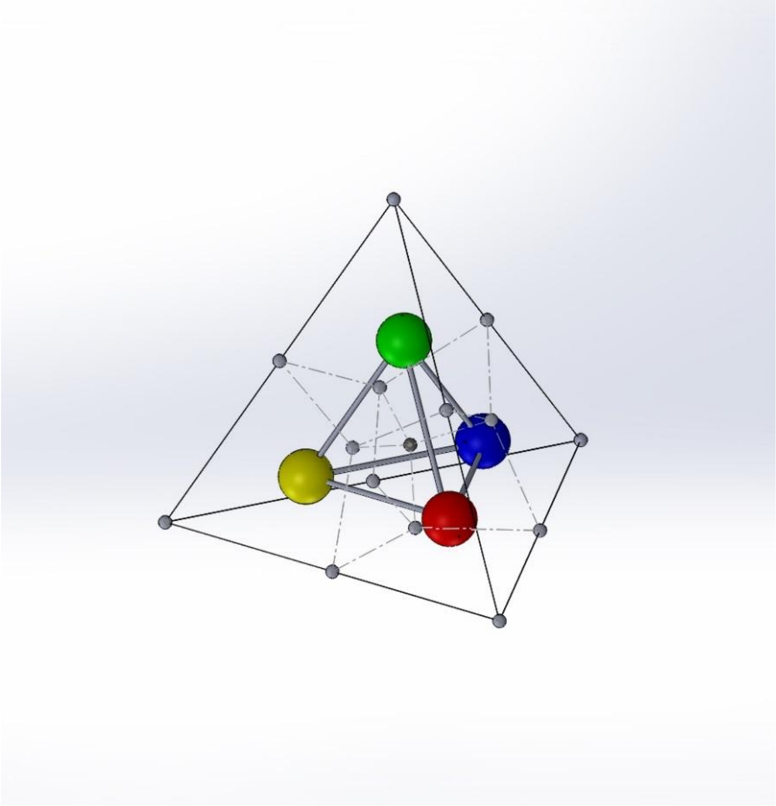
Logical accuracy vs. pseudo-temperature at different physical error rates



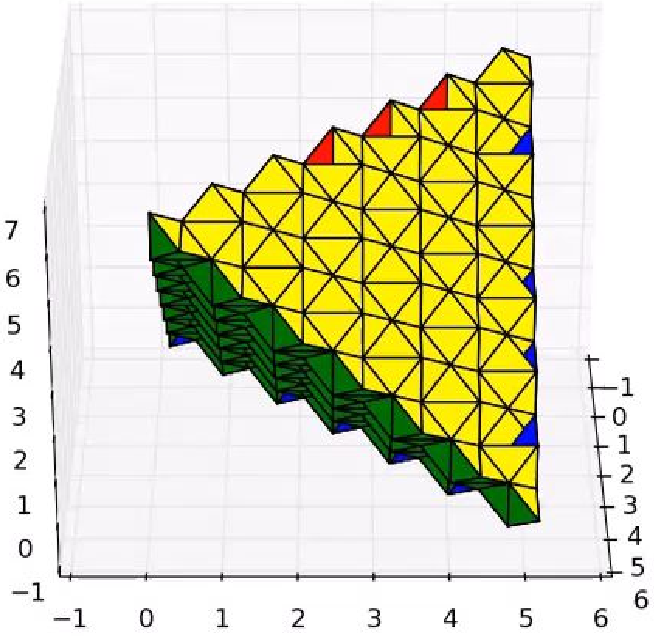
GAUGE COLOR CODES

H. Bombin, New J. Phys. 17 (2015) 083002

- A topological quantum error correcting code (3D)
- Four valent, four colorable lattice
- Admits universal transversal encoded gates via gauge fixing
- Can be implemented with only weight 4 & 6 check operators



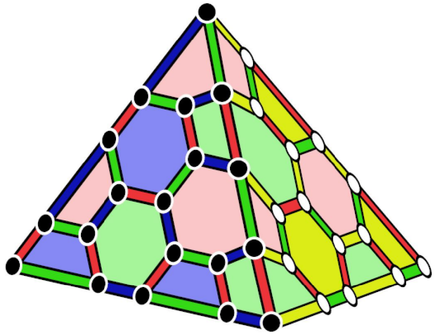
Distance 3 (primal + dual)



Distance 15 (dual):
- 671 qubits

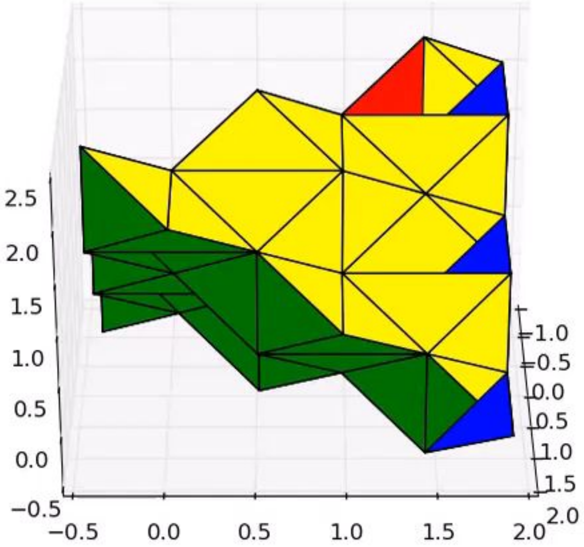
LARGER BOMBIN LATTICES

Distance 5 (primal)



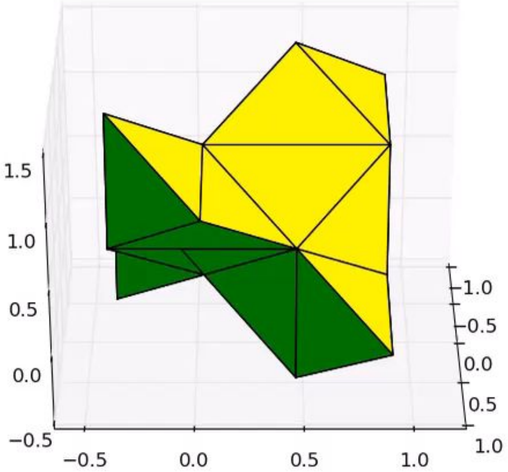
Distance 7 (dual)

175 qubits



Distance 5 (dual)

65 qubits



Distance 15 (dual)

671 qubits