Alpha Eigenvalue and Beta-Effective Calculations of High Multiplication Uranium Experiments

William Zywiec[1], Jesse Norris[1], Samuel Varghese[1], David Heinrichs[1], Daniel Siefman[1,2], Edward Lent[1], Chuck Lee[1], Anthony Nelson[1]

[1]Lawrence Livermore National Laboratory, Livermore, CA;
[2]University of California, Berkeley, Berkeley, CA

[leave space for DOI, which will be inserted by ANS]

ABSTRACT

Alpha eigenvalue (α) and beta-effective (β_{eff}) values were calculated for six different high multiplication (M > 20) uranium experiments that were performed at the Sandia Critical Experiments (SCX) facility at Sandia National Laboratories in Albuquerque, New Mexico. These experiments used U(6.90% U-235)O_2 fuel and configurations based on variations of the LEU-COMP-THERM-078 benchmark, which is taken from the International Handbook of Evaluated Criticality Safety Experimental Benchmarks. Several different methods of calculating α and β_{eff} were used, and the α calculations were compared to measured alpha eigenvalues that were fitted using variance-to-mean equations. All calculations were performed using the COG11.3 Monte Carlo radiation transport code, which is developed and maintained by the Nuclear Criticality Safety Division at Lawrence Livermore National Laboratory.

Keywords: alpha eigenvalue, beta-effective, neutron multiplicity, reactor noise, high multiplication

1. INTRODUCTION

Between June 6 and October 21, 2022, six different high multiplication (M > 20) uranium experiments were performed at the Sandia Critical Experiments (SCX) facility at Sandia National Laboratories (SNL) in Albuquerque, New Mexico. These experiments were sponsored by the US Department of Energy (DOE) Nuclear Criticality Safety Program. The experiment team was comprised of scientists from SNL, Lawrence Livermore National Laboratory (LLNL), Los Alamos National Laboratory (LANL), and the Institut de Radioprotection et de Sûreté Nucléaire (IRSN). The purpose of these experiments was to produce time-tagged neutron count data of configurations that exceeded a multiplication of 20 and to test different detector systems and methods that are used to characterize neutron multiplicity. Based on initial estimates, the multiplication of the measured configurations ranged from 6 to 1000. The configurations used in these experiments are based on variations of "Case 2" from LEU-COMP-THERM-078, which is a low-enriched uranium thermal benchmark taken from the International Handbook of Evaluated Criticality Safety Experimental Benchmarks [1].

In this paper, several different methods of calculating alpha eigenvalue (α) and beta-effective (β_{eff}) were performed using COG11.3 [2], a general-purpose Monte Carlo radiation transport code that is developed and maintained by the Nuclear Criticality Safety Division at Lawrence Livermore National Laboratory. The results of these calculations were compared to measured alpha eigenvalues that were fitted using variance-to-mean equations, originally developed by Feynman et al [3]. The form of the equations that were used to fit alpha eigenvalues are described in Albrecht [4] and Endo et al [5, 6].

2. EXPERIMENT DESCRIPTION

The subcritical benchmark experiments consisted of neutron multiplicity measurements of a water-moderated and -reflected low-enriched uranium oxide U(6.90% U-235)O_2, square-pitched, fuel lattice. The measurements described in this paper were performed by LLNL scientists. The detector configuration consisted of four helium-3 tubes that were placed in aluminum drywells surrounding the fuel lattice (Fig. 1, Fig. 2). The He-3 tubes have steel cladding and were connected directly to preamplifiers, which were connected to the Advanced List Mode Module (ALMM). The ALMM is a digitizer that was designed and built at LANL.

Fig. 1. Experimental configuration with four He-3 tubes in drywells (two fission chambers with orange cables are also pictured; fission chambers were only used to monitor neutron startup rate for safety)

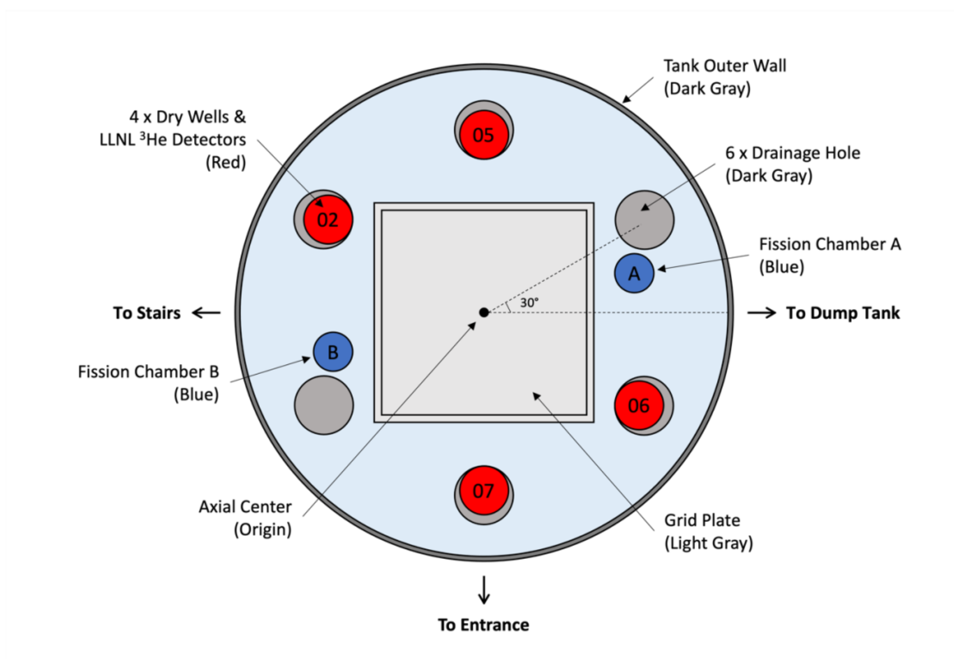

Fig. 2. Top view of experimental configuration with four He-3 tubes in drywells

The six different configurations that were measured are summarized in Table 1, with accompanying images of each fuel lattice, shown in Fig. 3. Each measurement was performed for 60 minutes. Two-hour measurements were performed using a Cf-252 source (without fuel) to estimate the efficiency of the He-3 tubes; a four-hour background measurement was also performed. Other than the detector efficiency measurement that was performed with the Cf-252 source, no sources other than the fuel were used throughout the experiments.

The He-3 tubes used in these experiments are the same as those that were used in FUND-LLNL-ALPHAN-HE3-MULT-001 [1]. A detailed description of the He-3 detection system is included in the benchmark [1]. The ALMM and He-3 detector setup, including deadtime, have not yet been fully characterized; however, deadtime is not expected to be a significant issue due to the low count rates that were observed (< 10,000 counts/second).

Table 1. Six configurations measured at SCX

Configuration	Number of Fuel Rods	Total UO2 Mass (kg)
5	948	103.06
6	1004	109.15
7	1032	112.20
8	1048	113.93
9	1056	114.80
10	1058	115.02

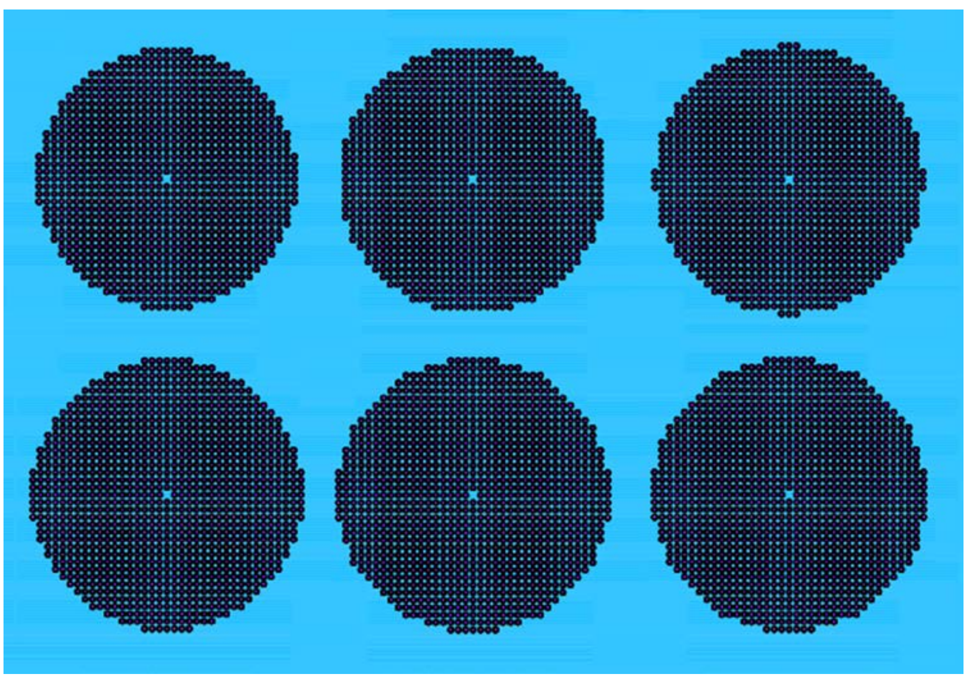

Fig. 3. Six configurations measured at SCX (top left to right: 5, 6, 7; bottom left to right: 8, 9, 10)

2.1. Measurement Summary

The total detector counts from measurements of Configurations 5 through 10 are summarized in Table 2. The Momentum software package, developed at LANL, was used to visualize He-3 tube counts during the measurements. It is important to note that the position of each detector, relative to the fuel lattice, was slightly different. These positional differences contributed to the difference in the counts for each detector. Pulse-height discrimination was performed directly by the preamplifiers, which were calibrated using a Cf-252 source, just prior to the measurements.

Table 2. Detector counts

Configuration	Total Counts	Detector 2	Detector 5	Detector 6	Detector 7
5	773,571	199,241	174,856	202.672	196,919
6	1,733,886	442,651	395,882	450,009	445,811
7	3,813,903	975,799	863,283	998,210	978,323
8	9,114,597	2,348,372	2,050,460	2,396,653	2,324,970
9	29,528,577	7,597,976	6,662,960	7,748,611	7,547,032
10	58,977,421	15,126,151	13,381,271	15,419,236	15,125,918

2.2. Fitting Alpha Eigenvalues

The ALMM generates an .lmx file for each measurement that contains a header and list-mode data, which is a series of time stamps with associated channel numbers, indicating which channel detected a neutron. The timing resolution of each channel is approximately 20 nanoseconds (ns). Individual He-3 tube dead-times are on the order of microseconds (μs). The list-mode data was postprocessed using code that was written in Python, which bins data in time gates of increasing size, calculates the first through third order central moments (Eq. 1-3), and then calculates the Y values (Eq. 4) of the binned data [5].

$$m_1 = \frac{1}{N}\sum_{i=1}^{N} \mathrm{C_i}(T) = \text{sample mean} \qquad \textbf{Eq. 1}$$

$$m_2 = \frac{1}{N-1}\sum_{i=1}^{N} (\mathrm{C_i}(T) - m_1(T))^2 = \text{second order central moment} \qquad \textbf{Eq. 2}$$

$$m_3 = \frac{1}{(N-1)(N-2)}\sum_{i=1}^{N} (\mathrm{C_i}(T) - m_1(T))^3 = \text{third order central moment} \qquad \textbf{Eq. 3}$$

$$Y(T) = \frac{m_2(T)}{m_1(T)} - 1 \qquad \textbf{Eq. 4}$$

Y(T) was fitted to Eq. 5 [5, 6] using the SciPy curve fit function. The results of this fit are plotted and shown in Fig. 4.

.

$$Y = a_1\left(1 - \frac{1-e^{(-b_1 x)}}{b_1 x}\right) + a_2 x + b_2 \qquad \textbf{Eq. 5}$$

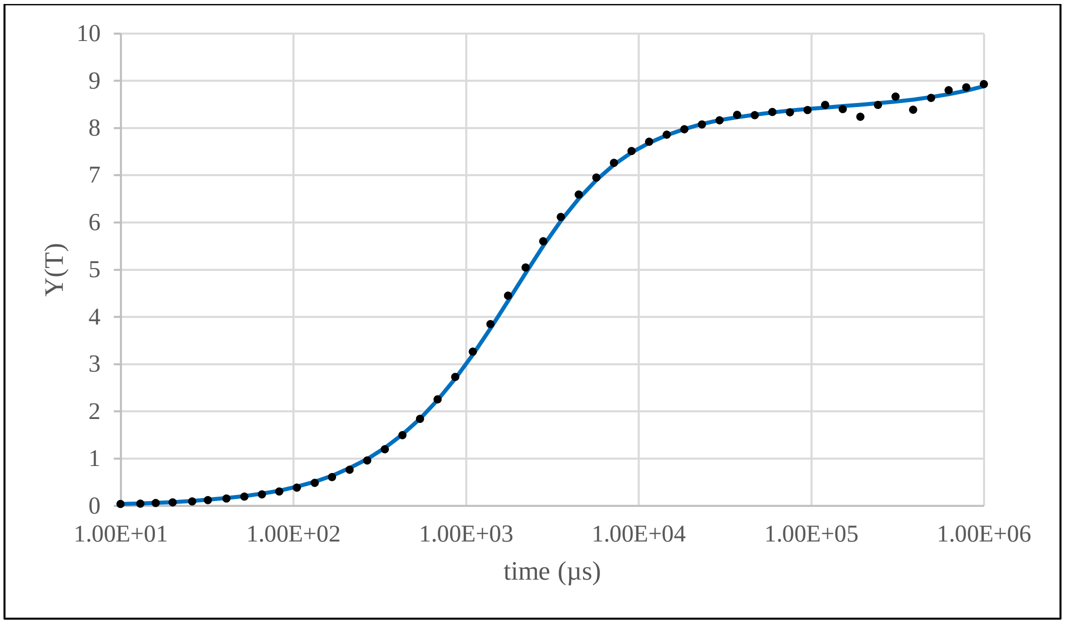

Fig. 4. Y(T) fit for Configuration 5

In Eq. 5, the leading a_1 term represents the prompt portion of Y; as x approaches infinity, the prompt term approaches an asymptotic value, known as Y_{sat}. The b_1 term in Eq. 5 represents the alpha eigenvalue (α) [4, 5]. It is important to note that Y_{sat} does not include the delayed portion of Y, which causes Y to continue to increase. The Y_{sat} and α values that correspond to the measured data are summarized in Table 3.

Table 3. Measured Y_{sat} and alpha eigenvalues

Configuration	Y_{sat}	$\alpha_{measured}$ (gen/μs)
5	8.45	-9.53e-04
6	23.1	-6.38e-04
7	48.9	-4.51e-04
8	80.8	-3.69e-04
9	112	-3.12e-04
10	120	-3.00e-04

Alpha Eigenvalue and Beta-Effective Calculations

k_{eff} calculations were performed for all six configurations using COG11.3 and cross sections based on the ENDF/B-VIII.0 nuclear data library [7]. The results of these calculations are summarized in Table 4.

Table 4. COG11.3 k_{eff} values

Configuration	k_{eff}	sd
5	0.977384	0.0003015
6	0.9887407	0.0002986
7	0.9954163	0.0003057
8	0.9981036	0.0003036
9	0.9997493	0.0003131
10	0.9996343	0.00029

Alpha eigenvalue and beta-effective calculations were performed using COG11.3 [2]. There are two different methods in COG11.3 that were used to calculate alpha eigenvalue:

Alpha eigenvalue (list-mode)

Using a settled criticality source, neutrons are tracked in *shielding mode*, in which a fission event does not terminate the particle history. Instead, secondary fission neutrons are followed, as are neutrons of all generations, until all trajectories have terminated. The alpha eigenvalue is calculated as the neutron production rate minus (-) the neutron removal rate.

Alpha eigenvalue (dynamic)

Using a settled criticality source, neutrons are tracked in *shielding mode*, in which a fission event does not terminate the particle history. Instead, secondary fission neutrons are followed, as are neutrons of all generations, until all trajectories have terminated or until alpha time cutoff is reached. The resulting event history table is analyzed, and all fission events are sorted into a set of time bins. The bin counts vs. time distribution reflects the exponential increase (or decrease) of the system, and the alpha eigenvalue is calculated by fitting an exponential to the distribution.

The comparison of measured alpha eigenvalues and COG11.3 alpha eigenvalues are summarized in Table 5 and Fig. 5.

Table 5. Measured alpha eigenvalues compared to COG11.3 alpha eigenvalues

Configuration	$\alpha_{measured}$ (gen/μs)	$\alpha_{list\text{-}mode}$ (gen/μs)	$\alpha_{dynamic}$ (gen/μs)
5	-9.53e-04	-9.22E-04	-9.57E-04
6	-6.38e-04	-6.32E-04	-6.18E-04
7	-4.51e-04	-4.35E-04	-4.47E-04
8	-3.69e-04	-3.56E-04	-3.58E-04
9	-3.12e-04	-3.09E-04	-2.91E-04
10	-3.00e-04	-3.14E-04	-2.99E-04

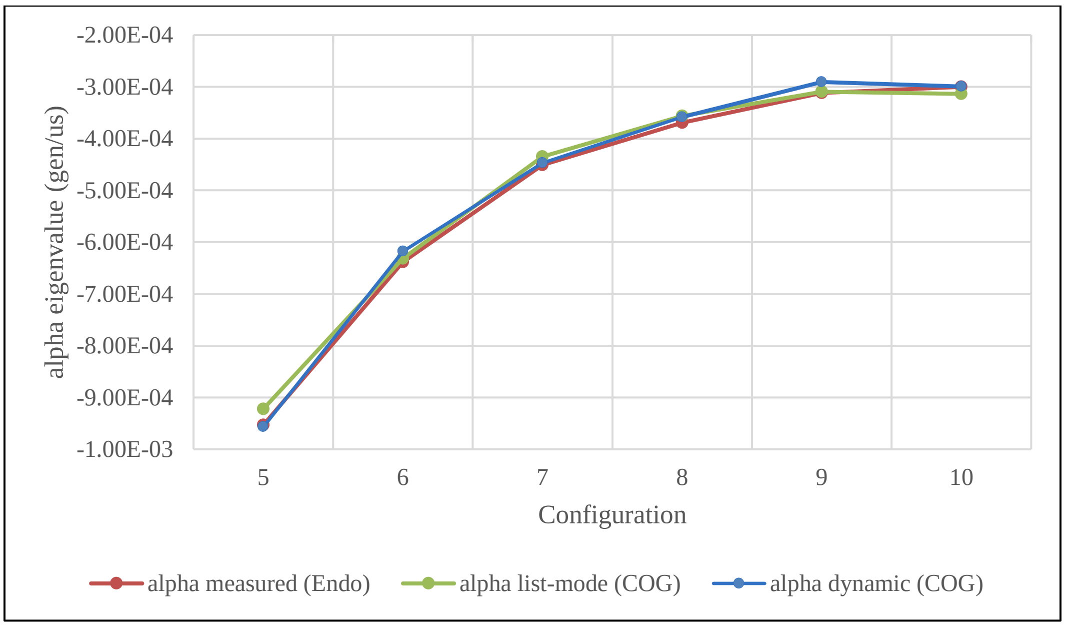

Fig. 5. Measured alpha eigenvalues compared to COG11.3 alpha eigenvalues

The results of three different beta-effective calculations are also included in the output from every k_{eff} calculation that COG performs. The three different beta-effective values were calculated as follows:

Beta-effective (fissions)

Beta-effective is calculated as the number of fission events caused by delayed neutrons, divided by all fission events.

Beta-effective (fission neutrons)

Beta-effective is calculated as the number of fission neutrons caused by delayed neutrons, divided by all fission neutrons.

Beta-effective (1 - k_{prompt} / k_{eff})

Beta-effective is calculated as 1 - k_{prompt} / k_{eff}.

The results of all beta-effective calculations are summarized in Table 6 and Fig. 6.

Table 6. COG11.3 beta-effective values

Configuration	$\beta_{eff\ fissions}$	$\beta_{eff\ fission\ neutrons}$	$\beta_{eff\ 1\ -\ kp\ /\ k}$
5	0.00808	0.00803	0.00801
6	0.00796	0.00791	0.00789
7	0.00794	0.00789	0.00787
8	0.00798	0.00793	0.00791
9	0.00794	0.00789	0.00787
10	0.00801	0.00796	0.00794

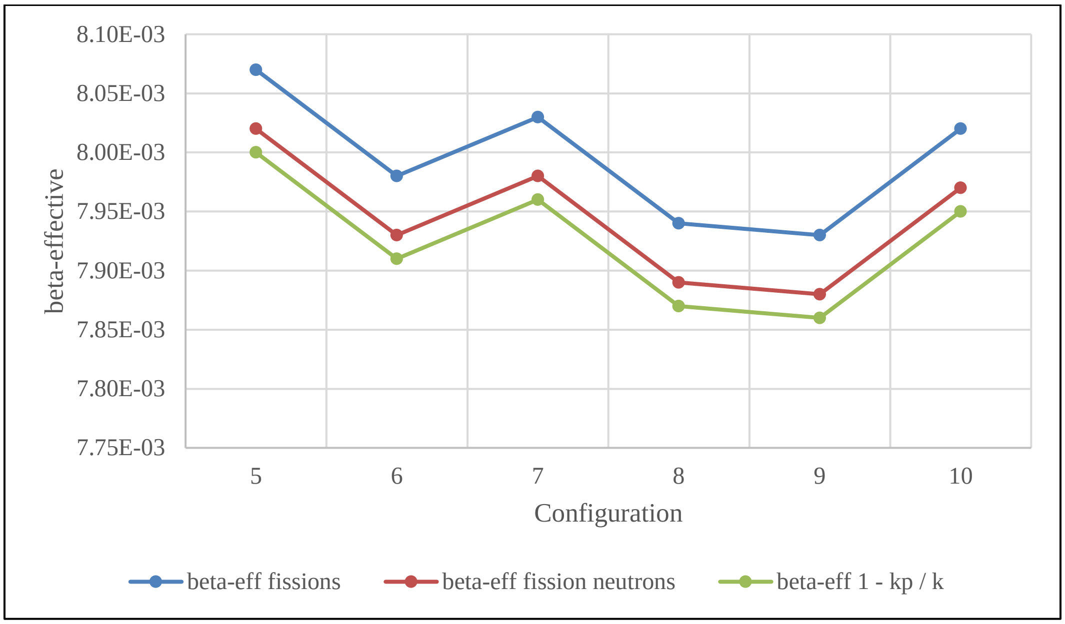

Fig. 6. COG11.3 beta-effective values

The results shown in Fig. 5 indicate that measured alpha eigenvalues and those calculated by COG11.3 show excellent agreement for all six configurations, in addition to showing a consistent and upward trend as multiplication (M) increases. The beta-effective values shown in Fig. 6 are all between 0.00785 and 0.0081, also showing excellent agreement. These beta-effective values showed general agreement with beta-effective values for other uranium systems reported in *Reactor Physics Constants* [8], as well as the NASA ZPR experiments modeled by Heinrichs et al [9].

3. CONCLUSIONS

In this paper, several different methods of calculating alpha eigenvalue and beta-effective were performed using COG11.3. The results of these calculations were compared to measured alpha eigenvalues that were fitted using variance-to-mean equations from Albrecht [4] and Endo et al [5, 6]. The results presented in this paper showed excellent agreement between measured alpha eigenvalues and those that were calculated using COG11.3. The beta-effective values calculated using COG11.3 were all between 0.00785 and 0.0081, showing general agreement with the beta-effective values of other uranium systems reported in *Reactor Physics Constants* [8], as well as the NASA ZPR experiments modeled by Heinrichs et al [9].

ACKNOWLEDGMENTS

The authors would like to thank both the US DOE Nuclear Criticality Safety Program and the experiment team at SNL, LLNL, LANL, and IRSN for their contributions. The ALMM and Momentum software package, developed at LANL, were instrumental in the success of these experiments.

The authors would especially like to thank the late Dr. Gary Harms for participating in these measurements and championing the “HiMu” experiments from the beginning. This paper and the HiMu experiments are dedicated to his memory.

REFERENCES

[1] International Handbook of Evaluated Criticality Safety Benchmark Experiments, Nuclear Energy Agency, NEA-7497, 2021.

[2] COG User’s Manual, 6th Edition, UCRL-TM-202590, Lawrence Livermore National Laboratory, 2024.

[3] Feynman, R., Hoffmann, F.D., Serber, R., “Dispersion of the neutron emission in U-235 fission”, Journal of Nuclear Energy, 1954.

[4] Albrecht, R.W., “The Measurement of Dynamic Nuclear Reactor Parameters Using the Variance of the Number of Neutrons Detected”, Nuclear Science and Engineering, 1962.

[5] Endo, T., Yamane, Y., Yamamoto, A., “Space and energy dependent theoretical formula for the third order neutron correlation technique”, Annals of Nuclear Energy 33 (2006) 521-537, 2006.

[6] Endo, T., Yamamoto, A., “Comparison of theoretical formulae and bootstrap method for statistical error estimation of Feynman-α method”, Annals of Nuclear Energy 124 (2019) 606-615, 2019.

[7] ENDF/B-VIII.0: The 8th Major Release of the Nuclear Reaction Data Library with CIELO-project Cross Sections, New Standards, and Thermal Scattering Data, Nuclear Data Sheets, Volume 148, Pages 1-142, 2018.

[8] Reactor Physics Constants, ANL-5800, Argonne National Laboratory, 1958.

[9] Heinrichs, D., Lent, E., Zywiec, W., “COG Beta-Effective Benchmarks”, LLNL-TR-843852, Lawrence Livermore National Laboratory, 2022.