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Title: Module 4 - Part 2 - Hand Calculation Methods

Author(s): Mitchell, Mark V.

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Module 4 – Part 2
Hand Calculation Methods

Section 1 - Array Methods

Presented by:
Mark Mitchell

Array Methods

- Limiting surface density method, NB_{N^2}
  - Calculate the critical c-t-c spacing of units of a given mass in a cubic array
  - Accounts for up to 200 mm of water reflection
- Surface density method
  - Calculate the subcritical c-t-c spacing of units of a given mass in an array that is limited in one dimension
  - Accounts for up to 155 mm of water reflection
- Density analog method
  - Calculate the subcritical c-t-c spacing of units of a given mass in a cubic array
  - Accounts for up to 200 mm of water reflection

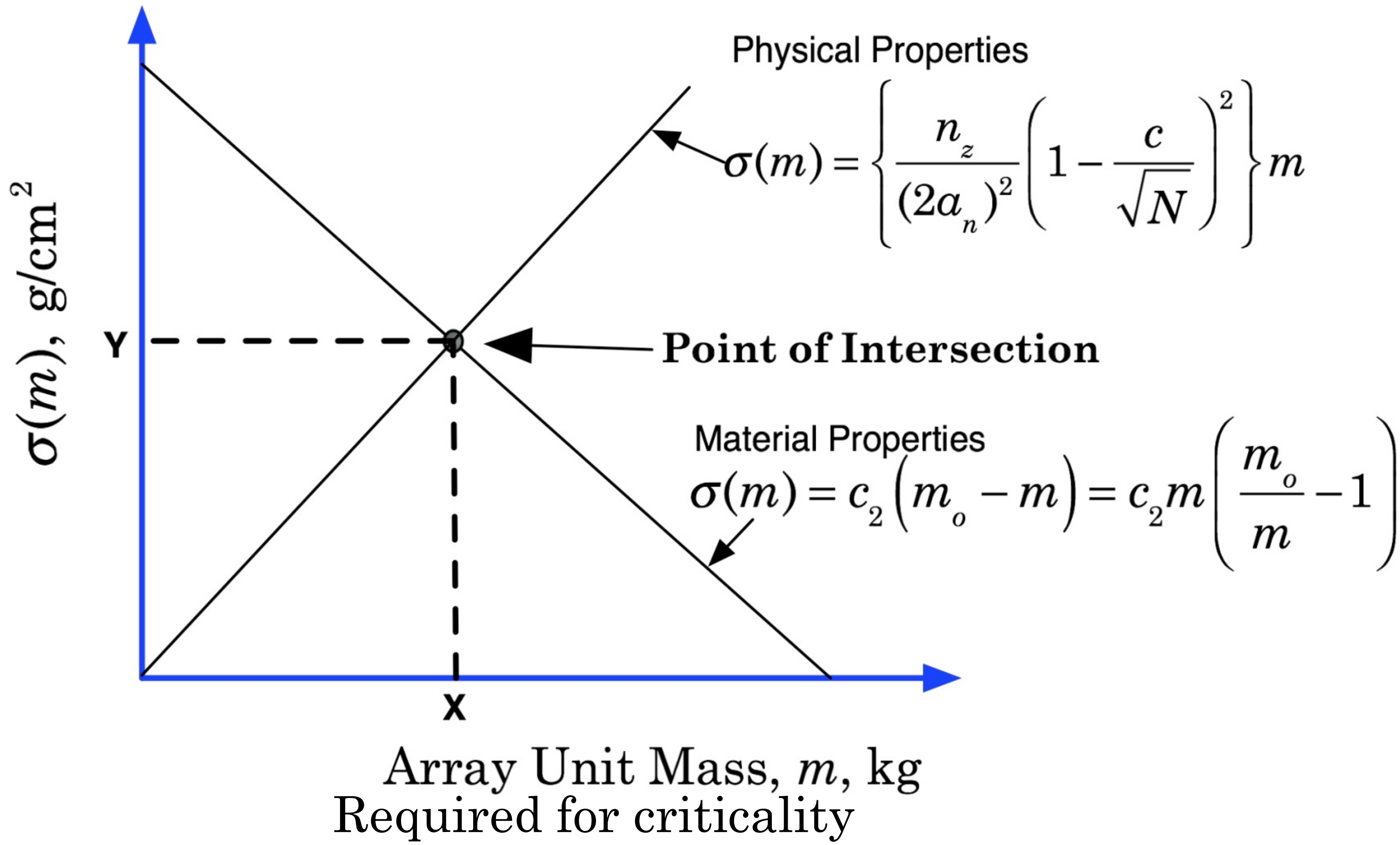
Array Methods

- Solid angle method
  - Developed as a quick, empirical means of evaluating interaction between small numbers of fissile units
    - In terms of total solid angle, Ω
  - Maximum acceptable k_{eff} of individual units a function of Ω
  - Accounts for thick water reflection

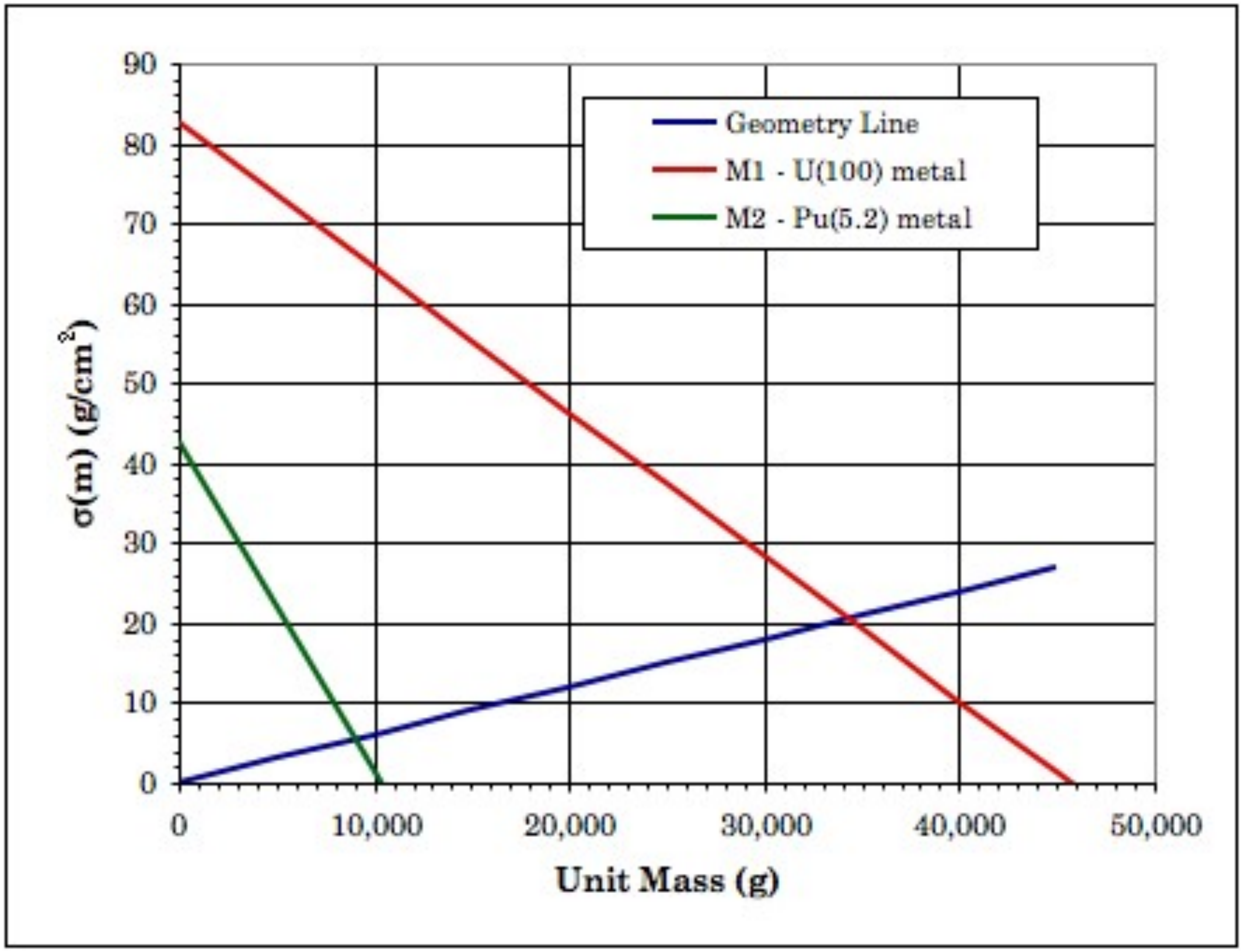
Limiting Surface Density Method

- Applicability
  - Useful for determining the critical center-to-center spacing for fissile materials stored or staged in array configurations of any shape (>64 units)
  - Useful for irregular shapes such as equipment stored on the floor
- Need to know:
  - Fissile unit stack height
  - A characteristic constant (c_2) that is a function of the material properties of the array
  - Critical mass of an unreflected sphere of the fissile material in the array
  - Mass of the fissile material in each array unit
- Method Details
  - Developed for U(93.2) spherical metal units – can be modified for all fissile materials that have Thomas data
  - Considers 20 cm of water reflection surrounding the array

Limiting Surface Density Method



Limiting Surface Density Method



Limiting Surface Density Method

$$\frac{\sigma(m)}{m} = \frac{n_z}{(2a_n)^2}\left(1 - \frac{c}{\sqrt{N}}\right)^2 = c_2\left(\frac{m_o}{m} - 1\right)$$

The array unit mass, *m*, is required for a critical array:

$$m = m_o\left[\frac{n_z}{c_2(2a_n)^2}\left(1 - \frac{c}{\sqrt{N}}\right)^2 + 1\right]^{-1}.$$

The array unit center-to-center spacing, $2a_n$, is required for a critical array:

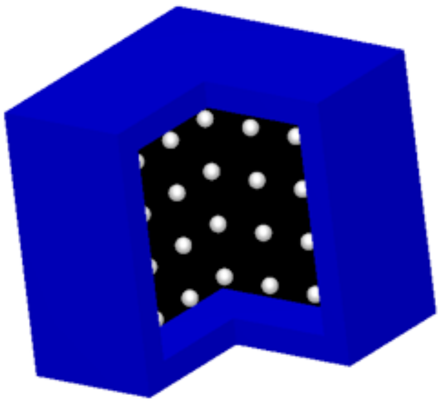
$$d = \left(2a_n\right) = \left[\frac{n_z\left(1 - \frac{c}{\sqrt{N}}\right)^2}{c_2\left(\frac{m_o}{m} - 1\right)}\right]^{\frac{1}{2}}.$$

Limiting Surface Density Method

- c_2 – a constant that depends on all of the material properties of the arrays except for the mass, *m*, and is also equal to the slope of the "material-line" (cm^{-2})
  - the Primer provides Thomas' values for this constant for various fissile systems,
- c – an empirically determined constant equal to 0.55 ± 0.18 for the range of fissile materials and arrays studied by Thomas.
  - tends toward zero in a thermal system (i.e., moderated units in the array)

Limiting Surface Density Method

Graphical LSD Example:



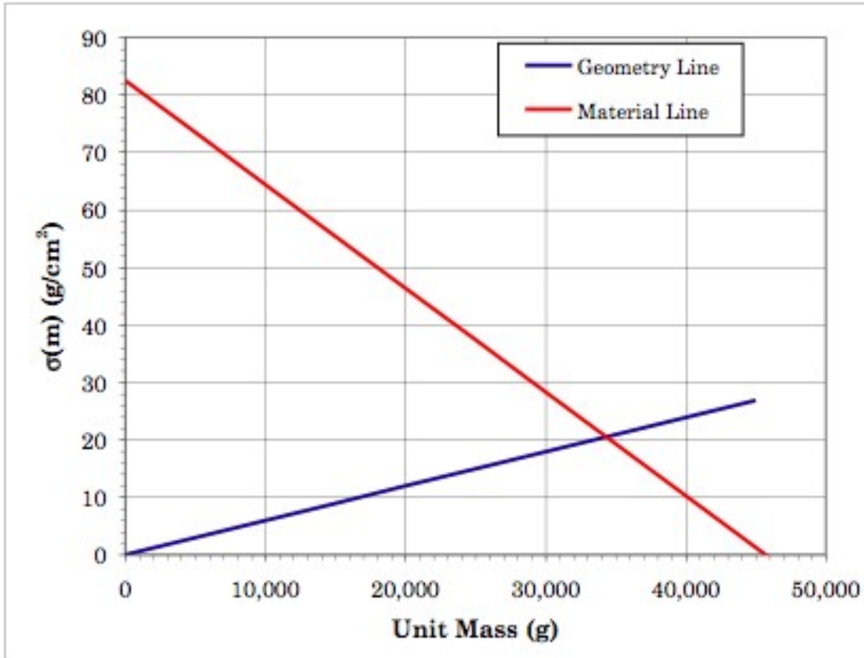
- 4x4x4 Array
- 76.2 cm (30" c-to-c spacing between units)
- U (100) metal units
- Determine unit mass for array criticality

Geometry Line: $\sigma(m) = \dfrac{n_z m}{(2a_n)^2}\left(1 - \dfrac{c}{\sqrt{N}}\right)^2 = \dfrac{4m}{(2 \times 38.1\ cm)^2}\left(1 - \dfrac{0.55}{\sqrt{64}}\right)^2$

$\sigma(m) = 5.974 \times 10^{-4} m$ or $\dfrac{\sigma(m)}{m} = 5.974 \times 10^{-4}$

Material Line: $\sigma(m) = c_2(m_o - m)$

$\sigma(m) = 1.806 \times 10^{-3}(45.68 - m)$ or $\dfrac{\sigma(m)}{m} = 1.806 \times 10^{-3}\left(\dfrac{45.68}{m} - 1\right)$



Surface Density Method

- Applicability
  - Useful for determining the subcritical center-to-center spacing for fissile materials stored or staged in finite array configurations where the size of the array is controlled in one direction
  - Useful for irregular shapes such as equipment stored on the floor
- Need to know:
  - Fissile unit stack height
  - Critical dimensions for an infinite water-reflected critical slab
  - Critical mass of an unreflected sphere of the fissile material in the array
  - Mass of the fissile material in each array unit
- Method Details
  - Infinite planar array, finite height
  - Derived from limiting surface density relationships
  - Maximum k_{eff} for an individual array unit cannot be larger than 0.9 (~73% of the critical mass)
  - Provides a center-to-center spacing result that is subcritical

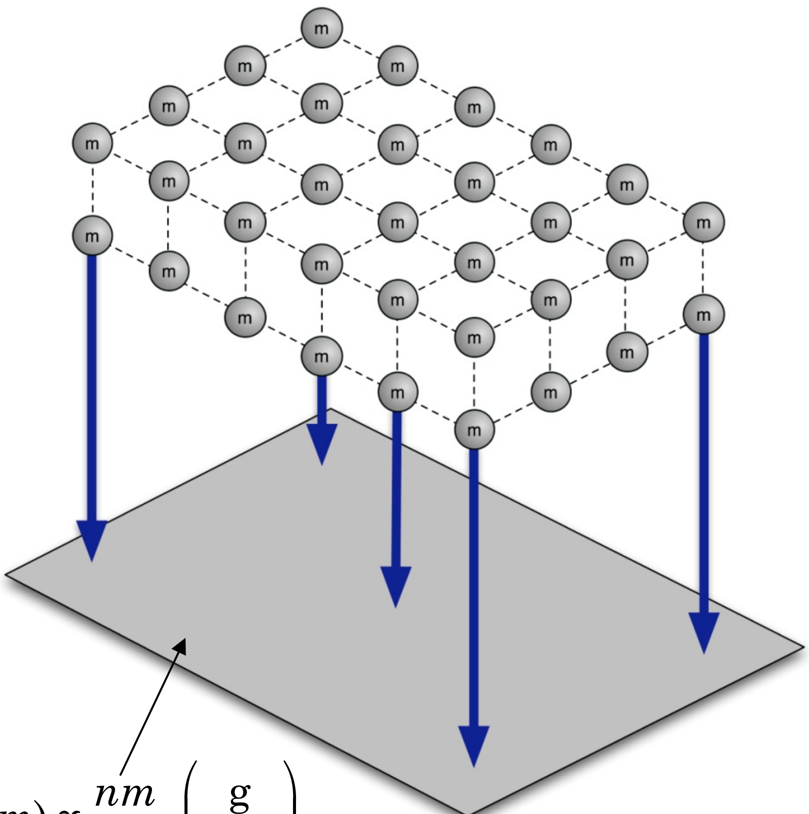
Surface Density Method

- From the limiting surface density technique:

Geometry Line: $$\sigma(m) = \frac{n_z m}{(2a_n)^2}\left(1 - \frac{c}{\sqrt{N}}\right)^2 = \frac{nm}{d^2}c_3$$

$$\sigma(m) \propto \frac{nm}{d^2}$$

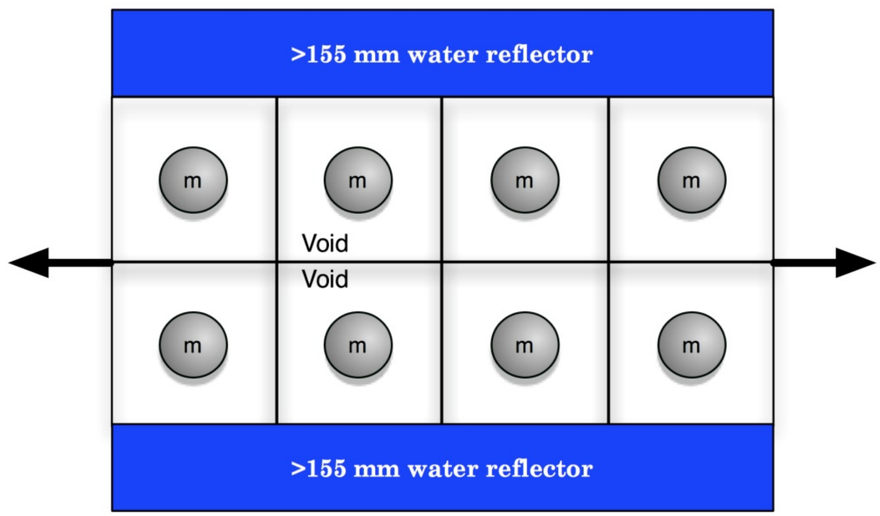
Surface Density Method



$$\sigma(m) \propto \frac{nm}{d^2}\left(\frac{\mathrm{g}}{\mathrm{cm}^2}\right)$$

Fissile 6x4x2 Array

Infinite Planar Array



Surface Density Method

- From the limiting surface density technique:

Material Line: $\sigma(m) = c_2(m_o - m) = c_2 m_o\left(1 - \frac{m}{m_o}\right) = c_4\sigma_o\left(1-f\right)$

$$\sigma(m) \propto \sigma_0\left(1-f\right)$$

- Therefore:

$$d = \left[\frac{c_3 nm}{c_4\sigma_o\left(1-f\right)}\right]^{\frac{1}{2}} = \left[\frac{nm}{\frac{c_4}{c_3}\sigma_o\left(1-f\right)}\right]^{\frac{1}{2}}$$

  – Note: d approaches infinity as *f* approaches 1

Surface Density Method

- Remember:

$$k_{eff} = (f)^{\frac{1}{3}}$$

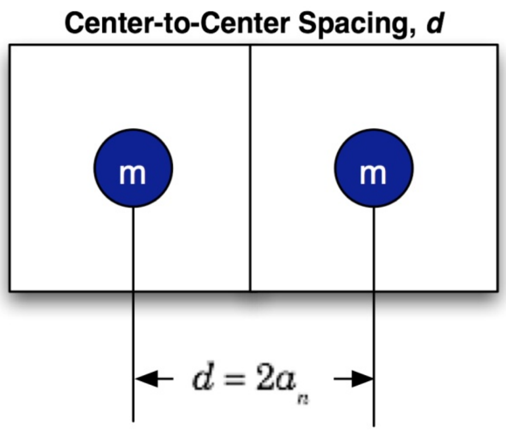
- If we want to limit k_{eff} of each array unit to a maximum of 0.90:

$$f = (k_{eff})^3 = (0.90)^3 = 0.73$$

- We must limit f to a maximum of 0.73:

$$d = \left[\frac{nm}{\frac{c_4}{c_3} \sigma_o \left(1 - \frac{f}{0.73} \right)} \right]^{\frac{1}{2}} = c_5 \left[\frac{nm}{\sigma_o \left(1 - 1.37 f \right)} \right]^{\frac{1}{2}}$$

Surface Density Method



$$d = 2a_n$$

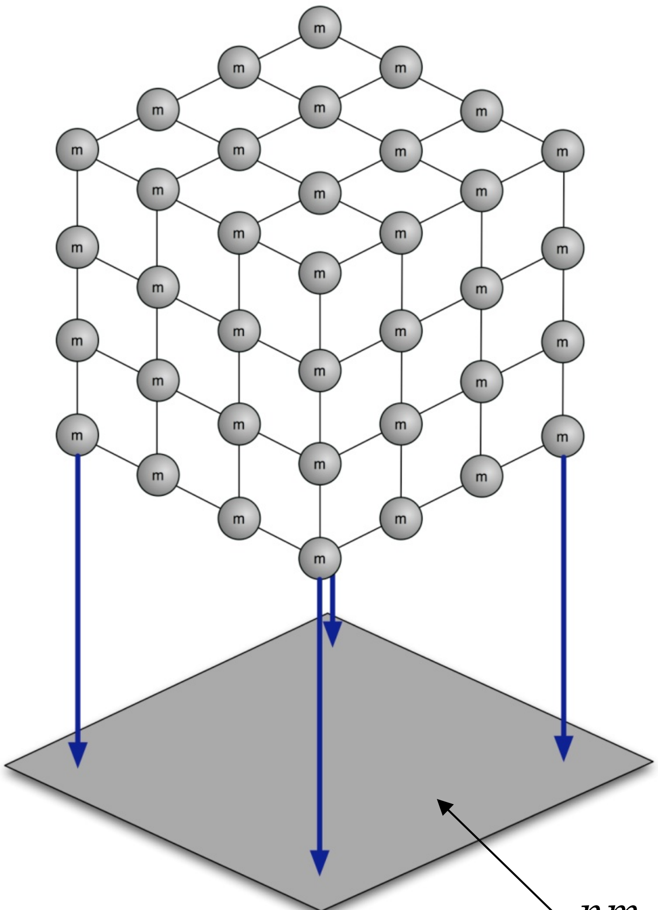
$$d = \left[\frac{nm}{0.54\sigma_0\left(1-1.37f\right)}\right]^{\frac{1}{2}} \cong 1.37\left[\frac{nm}{\sigma_0\left(1-1.37\frac{m}{m_0}\right)}\right]^{\frac{1}{2}}$$

- The coefficient 0.54 represents the product of two factors
  - Array shape
  - Array reflecting materials
    - Concrete vs. water reflected arrays
- The coefficient σ_o is the surface density of the critical water-reflected infinite slab (g/cm^2)
- Provides a subcritical result for the center-to-center spacing
- Individual units with a k_{eff}>0.9 (73% of the critical mass) results in an infinite center-to-center spacing result
- May require an administrative control on the array height

Density Analog Method

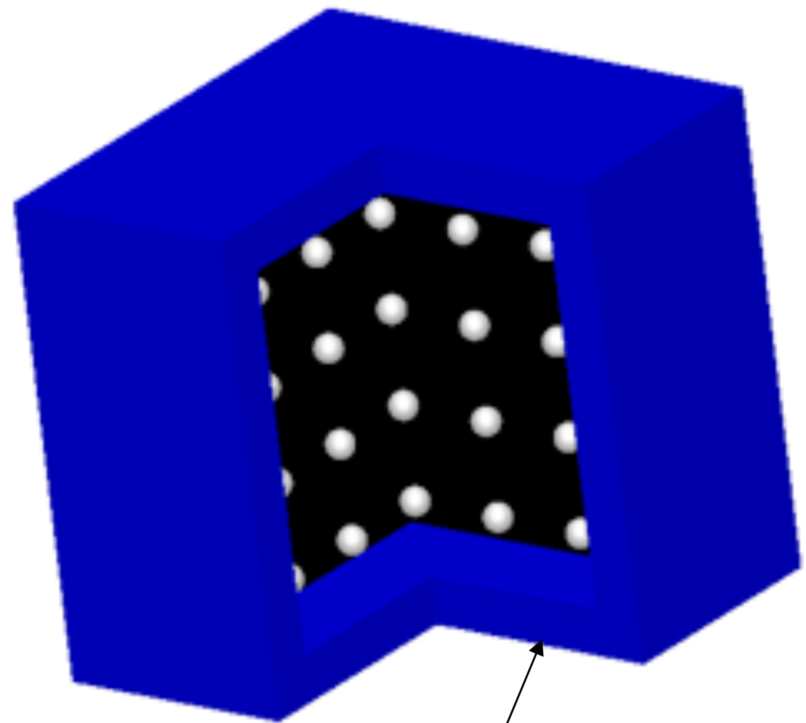
- Applicability
  - Useful for determining the subcritical center-to-center spacing for fissile materials stored or staged in array configurations of any shape
  - Useful for irregular shapes such as equipment stored on the floor
- Need to know:
  - Fissile unit stack height
  - Critical dimensions for an infinite water-reflected critical slab
  - Critical mass of an unreflected sphere of the fissile material in the array
  - Mass of the fissile material in each array unit
- Method Details
  - Cubic array configuration
  - Considers 20 cm of water reflection surrounding the cubic array

Density Analog Method

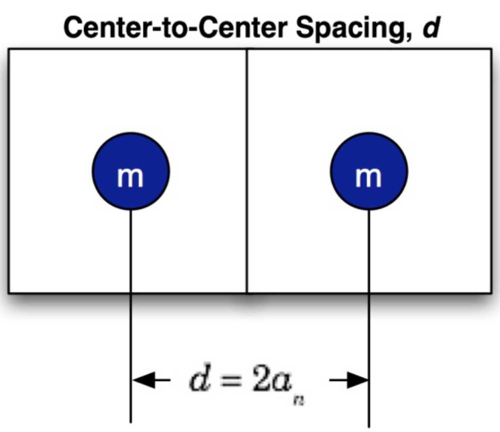


$$\sigma(m) \propto \frac{nm}{d^2} \left(\frac{\text{g}}{\text{cm}^2} \right)$$

Finite Cubic Array



Density Analog Method



$$d = \left[\frac{nm}{2.1\sigma_0\left(1-1.37f\right)}\right]^{\frac{1}{2}} \cong 0.69\left[\frac{nm}{\sigma_0\left(1-1.37\dfrac{m}{m_0}\right)}\right]^{\frac{1}{2}}$$

- The coefficient 2.1 represents experimental and calculational data for cubic arrangements of fissile materials
- σ_o is the surface density of the critical water-reflected infinite slab (g/cm^2)
- Provides a subcritical result for the center-to-center spacing
- Individual units with a k_{eff}>0.9 (73% of the critical mass) results in an infinite center-to-center spacing result
- Can be used for arrays of any shape

Array Methods Example Results/Comparison

- What is the minimum center-to-center spacing of an array of 4,500 g Pu(5) metal ingots?

Case	SD	DA	LSD	Code Result
2x2x2		26 cm	Not applicable	15 cm
10x10x10		57 cm	42 cm	41.7 cm
100x100x100		182 cm	136 cm	137 cm
Infinite Planar Array (1 unit high)	36 cm			25 cm

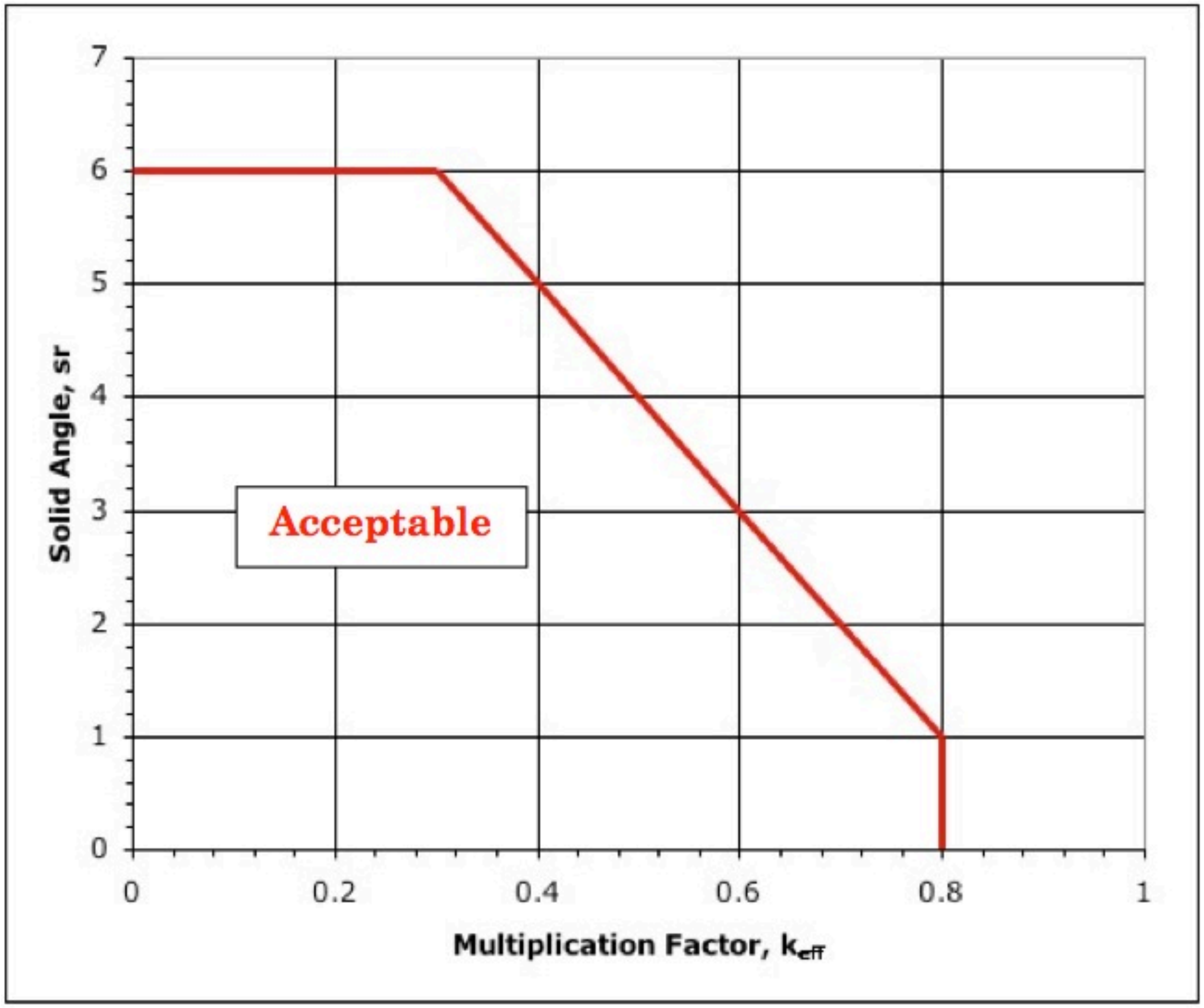
Solid Angle Method

- The basic idea behind this method is that the subcriticality of an array configuration depends on
  - The k_{eff} of a single representative unit in the array, and
  - The probability that a neutron leaking out of this unit will interact with (cause fission in) another unit in the array
- The probability that a leaking neutron will interact with another unit in the array is dependent upon total solid angle, Ω, at the most central unit, or at the most reactive unit, by all other units in the array

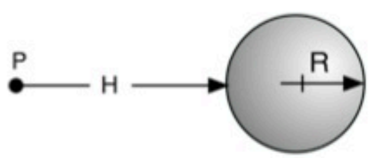
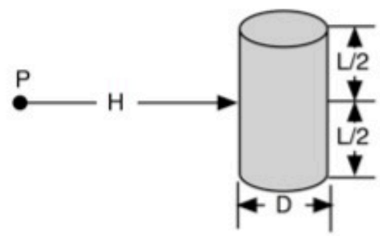
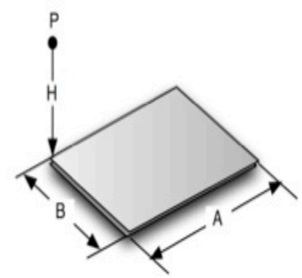
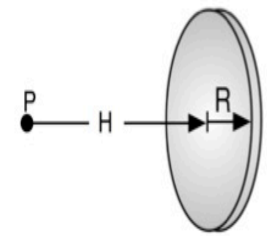
Solid Angle Method

- The following conditions must be satisfied in order to apply the method:
  - The k_{eff} of any bare unit shall not exceed 0.8
  - Each unit shall be subcritical when completely reflected by water
  - The minimum surface-to-surface separation between units shall be 0.3 m, and
  - The allowed solid angle, Ω, shall not exceed 6 sr
    - $\Omega_{allowed} = 9 - 10k_{eff}$

Solid Angle Method

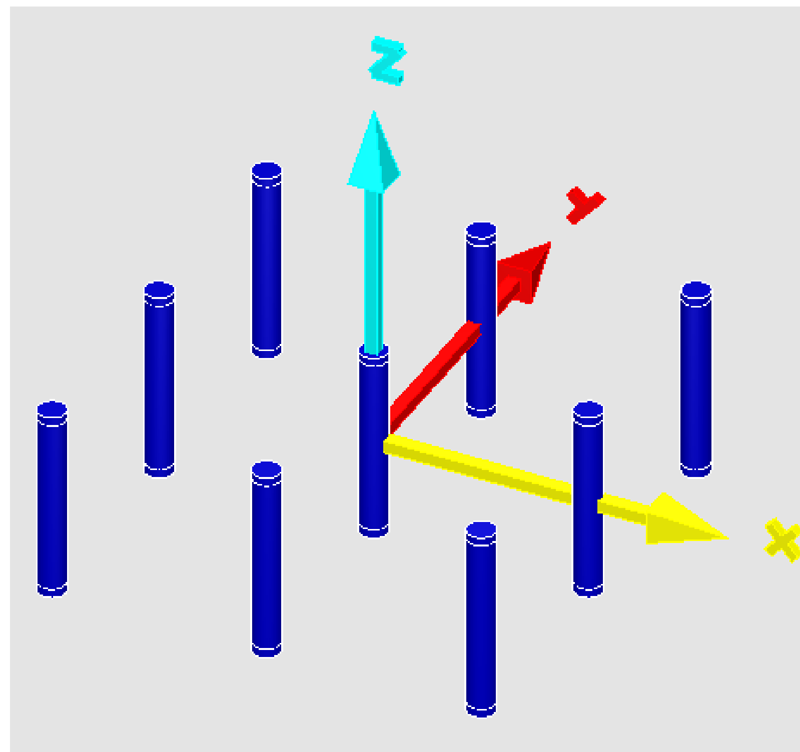


Solid Angle Method

Point-to-Sphere	Point-to-Cylinder	Point-to-Plane	Point-to-Disk
			
$\Omega = 2\pi\left(1-\frac{1}{\sqrt{1+\left(R/H\right)^2}}\right)$	$\Omega = \frac{LD}{H\sqrt{\left(L/2\right)^2+H^2}}$	$\Omega = \sin^{-1}\left(\frac{AB}{\sqrt{A^2+H^2}\sqrt{B^2+H^2}}\right)$	$\Omega = 2\pi\left(1-\frac{1}{\sqrt{1+\left(R/H\right)^2}}\right) \le \frac{\pi R^2}{H^2}$
where R = Radius of the sphere. H=Distance from the point to the surface of the sphere.	where L=Length of the cylinder D=Diameter of the cylinder H=Distance from the point to the surface of the cylinder.	where A=Length of one side of the plane B=Length of the other side of the plane H=Perpendicular distance from the point to the plane. If the point, *P*, is directly above the center of the plane (not directly over a corner as shown in the figure) with dimensions 2A x 2B, multiply Ω by 4 to obtain the solid angle.	where R=Radius of the disk H=Distance from the point P to the surface of the disk.

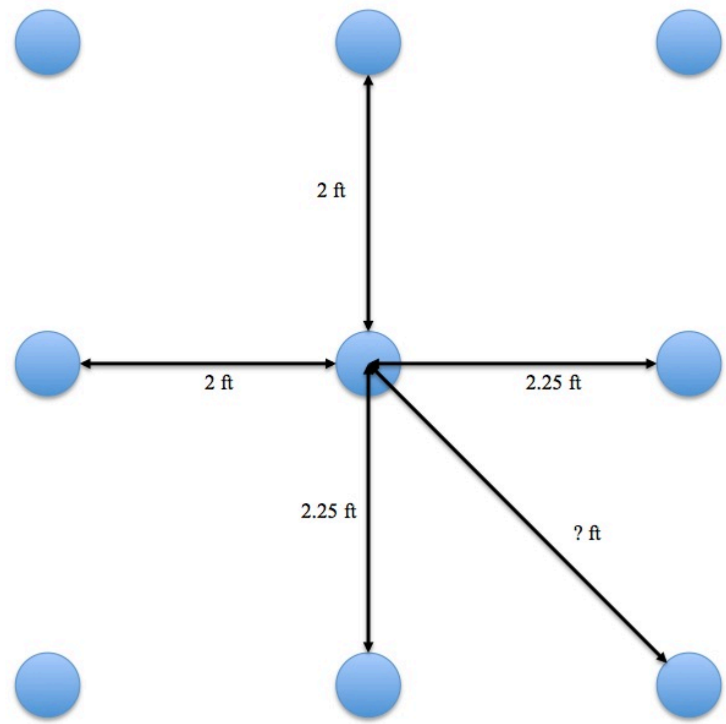
Solid Angle Method

- Problem: We want to store nine "safe"-bottles of fissile solution in a 3-by-3 array with 2-foot edge-to-edge spacing
- Question: What is the highest k_{eff} that each bottle can have as a single unit and still be subcritical in the array?
  - Each bottle is 4-feet tall
  - Each bottle diameter ½-foot



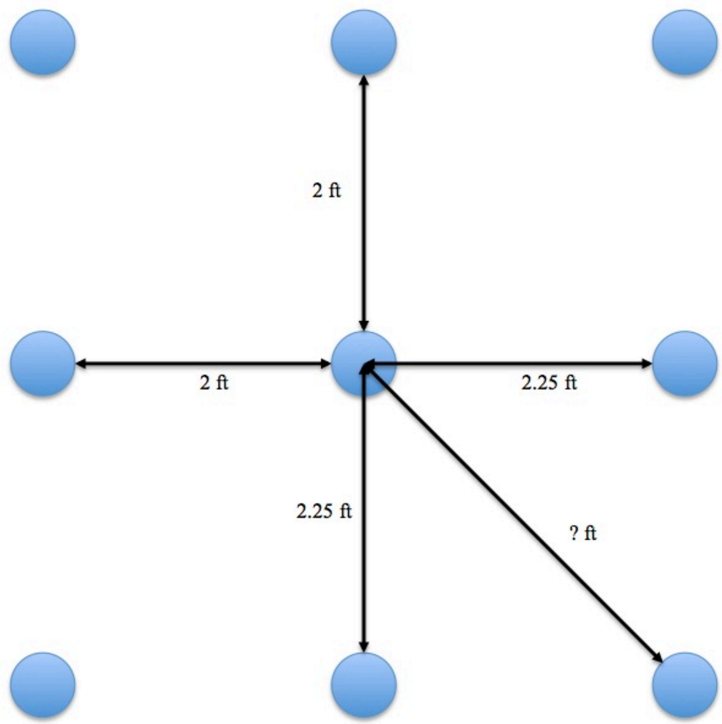
Solid Angle Method

- Question: What unit is the most reactive unit? Why?
  - $H_n = ?$
  - $H_f = ?$



Solid Angle Method

- Question: What unit is the most reactive unit? Why?
  - $$H_n = 2\,ft + \left[\frac{\frac{1}{2}\,ft}{2}\right] = 2.25\,ft$$
  - $H_f = ?$



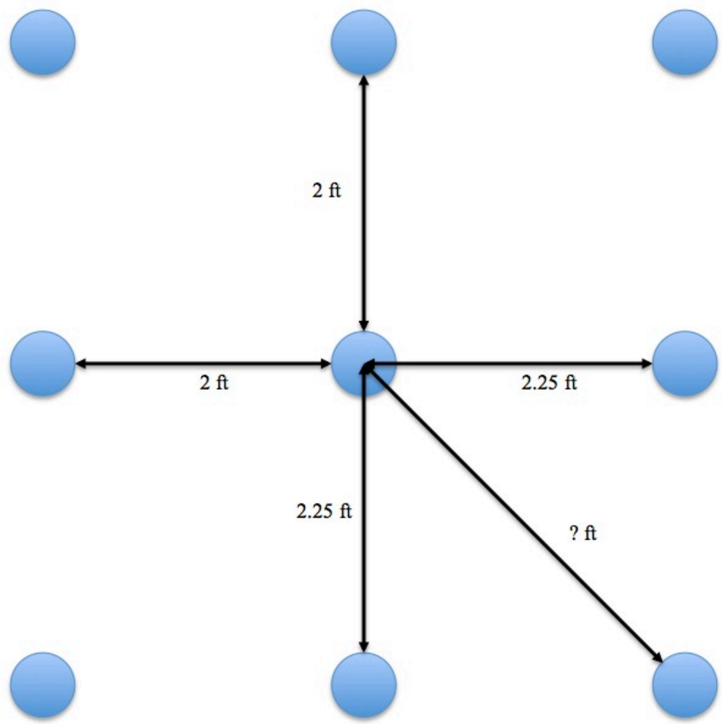
Solid Angle Method

- Question: What unit is the most reactive unit? Why?

  – $$H_n = 2\,ft + \left[\frac{\frac{1}{2}\,ft}{2}\right] = 2.25\,ft$$

  – $$\left[H_f + \left(\frac{\frac{1}{2}\,ft}{2}\right)\right]^2 = [2.5\,ft]^2 + [2.5\,ft]^2$$

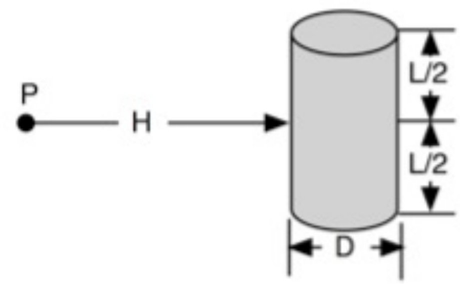
$$H_f = 3.29\,ft$$



Solid Angle Method

- $\Omega_n = ?$
- $\Omega_f = ?$
- $\Omega_{total} = ?$

Point-to-Cylinder



$$\Omega = \frac{LD}{H\sqrt{\left(L/2\right)^2 + H^2}}$$

where

L=Length of the cylinder

D=Diameter of the cylinder

H=Distance from the point to the surface of the cylinder.

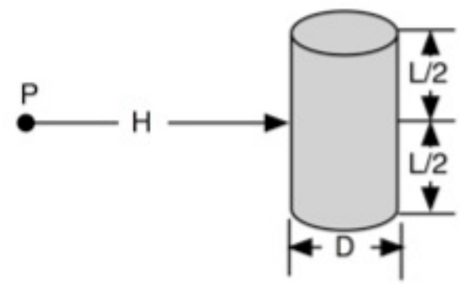
Solid Angle Method

- $\Omega_n = \dfrac{(4\ ft)\left(\frac{1}{2}\ ft\right)}{2.25\ ft\left[\left(\frac{4\ ft}{2}\right)^2 + (2.25\ ft)^2\right]^{\frac{1}{2}}} = 0.30\ SR$

- $\Omega_f = ?$

- $\Omega_{total} = ?$

Point-to-Cylinder



$$\Omega = \frac{LD}{H\sqrt{\left(L/2\right)^2 + H^2}}$$

where

L=Length of the cylinder

D=Diameter of the cylinder

H=Distance from the point to the surface of the cylinder.

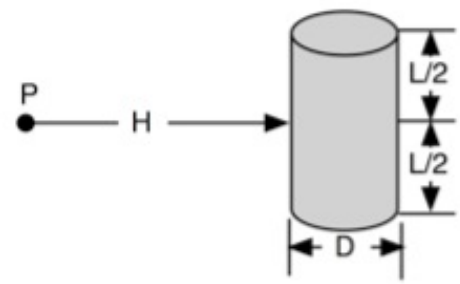
Solid Angle Method

– $$\Omega_n = \frac{(4\ ft)\left(\frac{1}{2}\ ft\right)}{2.25\ ft\left[\left(\frac{4\ ft}{2}\right)^2 + (2.25\ ft)^2\right]^{\frac{1}{2}}} = 0.30\ SR$$

– $$\Omega_f = \frac{(4\ ft)\left(\frac{1}{2}\ ft\right)}{3.29\ ft\left[\left(\frac{4\ ft}{2}\right)^2 + (3.29\ ft)^2\right]^{\frac{1}{2}}} = 0.16\ SR$$

– $\Omega_{total} = ?$

Point-to-Cylinder



$$\Omega = \frac{LD}{H\sqrt{\left(L/2\right)^2 + H^2}}$$

where

L=Length of the cylinder

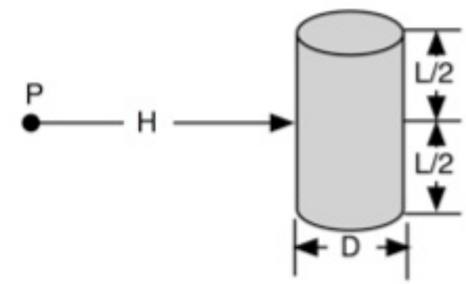
D=Diameter of the cylinder

H=Distance from the point to the surface of the cylinder.

Solid Angle Method

- $\Omega_n = \dfrac{(4\ ft)\left(\frac{1}{2}\ ft\right)}{2.25\ ft\left[\left(\frac{4\ ft}{2}\right)^2 + (2.25\ ft)^2\right]^{\frac{1}{2}}} = 0.30\ SR$
- $\Omega_f = \dfrac{(4\ ft)\left(\frac{1}{2}\ ft\right)}{3.29\ ft\left[\left(\frac{4\ ft}{2}\right)^2 + (3.29\ ft)^2\right]^{\frac{1}{2}}} = 0.16\ SR$
- $\Omega_{total} = 4\Omega_n + 4\Omega_f = 1.84\ SR$

Point-to-Cylinder



$$\Omega = \frac{LD}{H\sqrt{(L/2)^2 + H^2}}$$

where

L=Length of the cylinder

D=Diameter of the cylinder

H=Distance from the point to the surface of the cylinder.

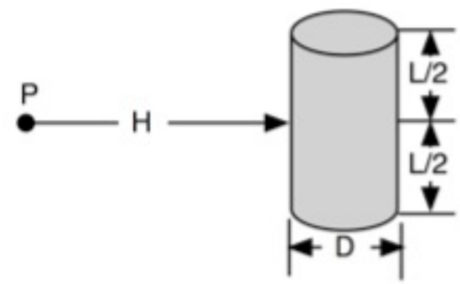
Solid Angle Method

– $$\Omega_n = \frac{(4\ ft)\left(\frac{1}{2}\ ft\right)}{2.25\ ft\left[\left(\frac{4\ ft}{2}\right)^2 + (2.25\ ft)^2\right]^{\frac{1}{2}}} = 0.30\ SR$$

– $$\Omega_f = \frac{(4\ ft)\left(\frac{1}{2}\ ft\right)}{3.29\ ft\left[\left(\frac{4\ ft}{2}\right)^2 + (3.29\ ft)^2\right]^{\frac{1}{2}}} = 0.16\ SR$$

$$\Omega_{total} = 4\Omega_n + 4\Omega_f = 1.84\ SR$$

Point-to-Cylinder



$$\Omega = \frac{LD}{H\sqrt{\left(L/2\right)^2 + H^2}}$$

where

L=Length of the cylinder

D=Diameter of the cylinder

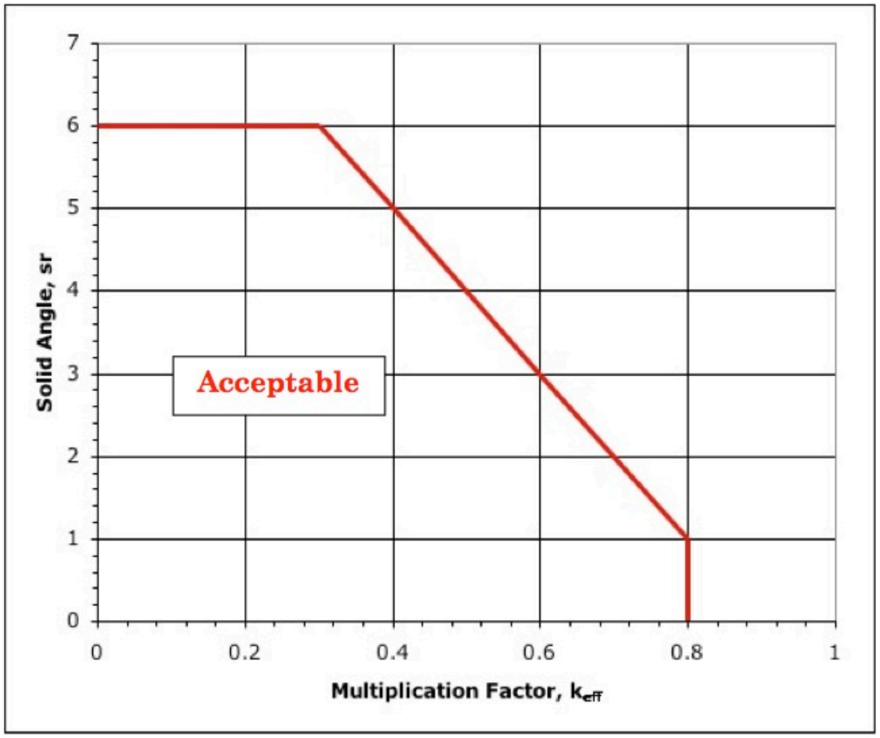
H=Distance from the point to the surface of the cylinder.

- Question: What is the highest allowed k_{eff}?

Solid Angle Method

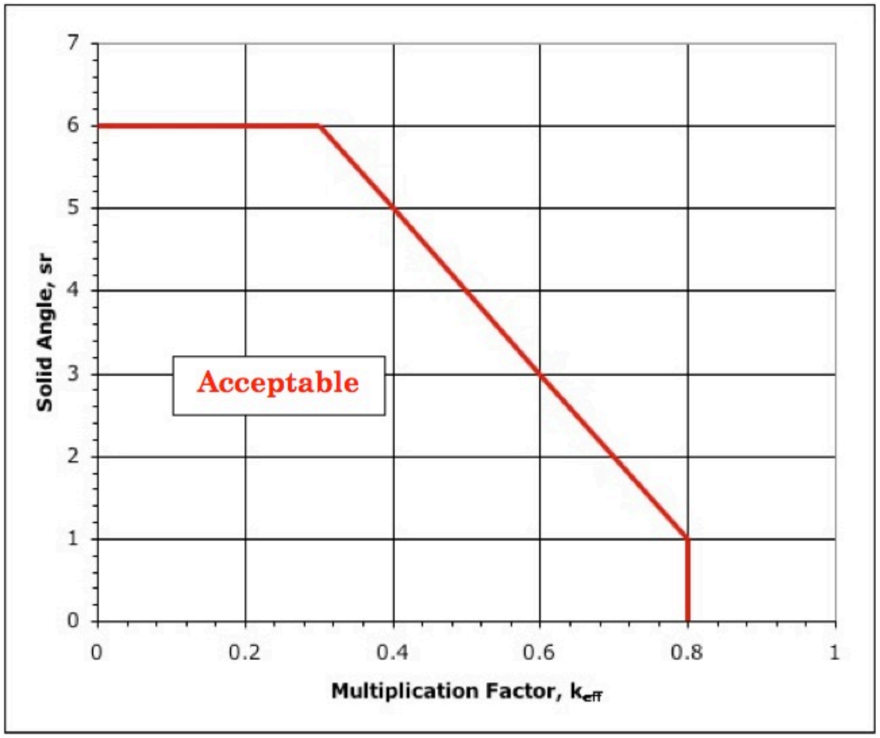
- $\Omega_{total} = 1.84\ SR$

- $k_{eff} = ?$



Solid Angle Method

- $\Omega_{total} = 1.84\ SR$
- $1.84 = 9 - 10k_{eff}$
- $k_{eff} = ?$

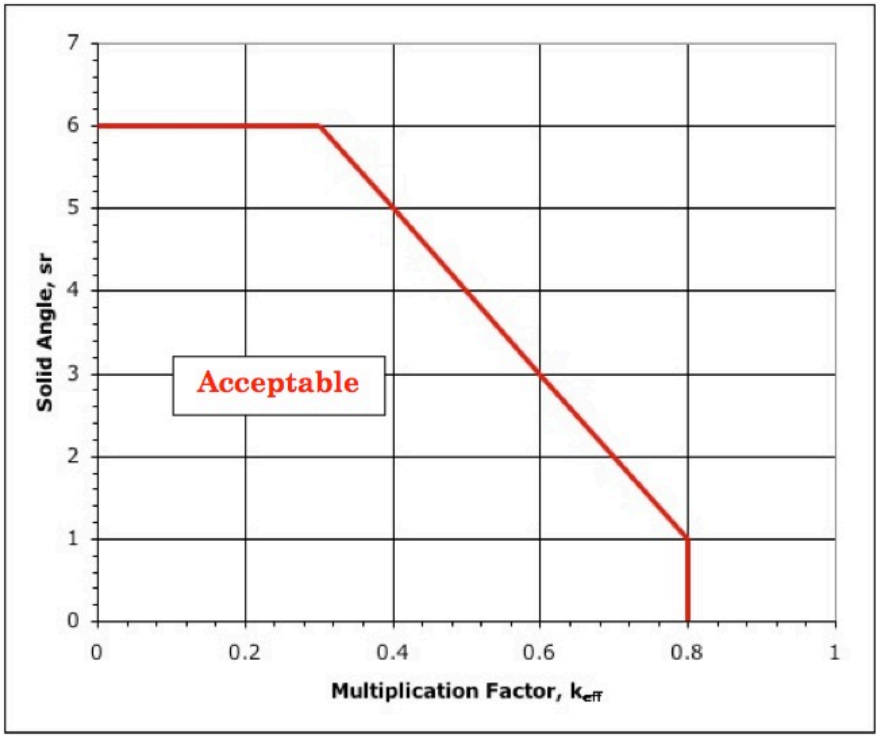


Solid Angle Method

- $\Omega_{total} = 1.84\ SR$

- $1.84 = 9 - 10k_{eff}$

- $k_{eff} = \left[\dfrac{9 - 1.84}{10}\right] = 0.72$



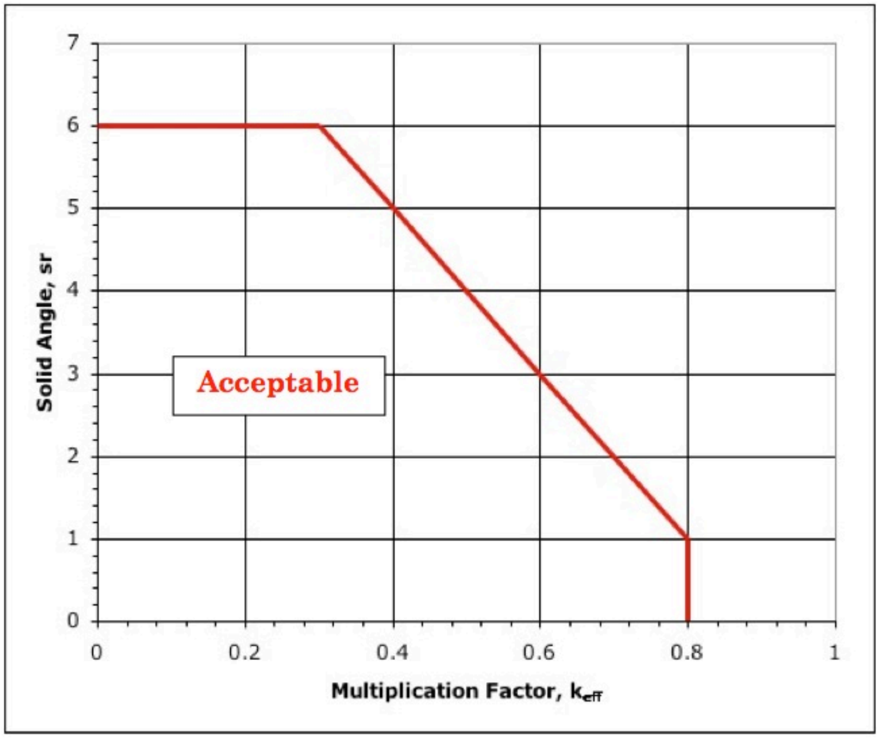
Solid Angle Method

– $\Omega_{total} = 1.84\,SR$

– $1.84 = 9 - 10k_{eff}$

– $k_{eff} = \left[\dfrac{9 - 1.84}{10}\right] = 0.72$

– What else is required?



Solid Angle Method

- $\Omega_{total} = 1.84\ SR$

- $1.84 = 9 - 10k_{eff}$

- $k_{eff} = \left[\dfrac{9 - 1.84}{10}\right] = 0.72$

- What happens if we move the bottles closer together?