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Title: Module 4 - Part 1 - Hand Calculation Methods

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Module 4 – Part 1
Hand Calculation Methods

Section 1 – Hand Calculations Introduction

Presented by:
Mark Mitchell

Hand Calculation Methods

Learning Objectives

- Familiarization with common hand calculation methods
  - Single Unit Methods
    - Multiplication factor vs. fraction critical
    - Core density conversions
    - Shape conversions (Buckling conversions)
  - Array Methods
    - Limiting surface density, NB_{N^2}, method
      - Surface density method
      - Density analog method
    - Solid angle method

Background

- Widely used before the existence (availability) of high-speed, large-memory, computers
- May seem obsolete today, but …
  - Useful as a starting point for more elaborate calculations
  - Can provide insight into a problem not apparent in a computer printout
  - Can provide a “sanity check”

Other Hand Calculation Methods

- Methods discussed are not exhaustive list

- Other hand calculation methods exist
  - One-group and modified one-group diffusion theory
  - Clark's Albedo Method
  - Equilateral Hyperbola Model
  - Trombay Criticality Formula
  - Etc.

Resources

- LA-3366 (Rev), *Criticality Control in Operations with Fissionable Material*
- LA-10860-MS, *Critical Dimensions of Systems Containing ^{235}U. ^{239}Pu, and ^{233}U (1986 Revision)*
- LA-14244-M, *Hand Calculation Methods for Criticality Safety – A Primer*
- NCSET Module 8, *Hand Calculations Part 1*
- TID-7016 (Revision 2), *Nuclear Safety Guide*

- Website: http://ncsc.llnl.gov/ncspMain.html

Module 4 – Part 1
Hand Calculation Methods

Section 2 – Single Unit Methods

Presented by:
Mark Mitchell

Single Unit Methods

- Multiplication factor vs. fraction critical
  - Fraction critical, *f*
  - Relationship between k_{eff} and fraction critical

- Core density conversions
  - Convert a known critical core with a density into an equivalent critical core having another density

- Shape conversions (Buckling conversions)
  - Convert a known critical shape to another critical shape consisting of the same material

Multiplication Factor Vs. Fraction Critical

- The fraction critical may be used to determine the approximate multiplication factor

- Fraction critical
  - Defined as the ratio of the mass to the critical mass of the same material conditions, m/m_c

Multiplication Factor Vs. Fraction Critical

- For instance, the critical mass of a bare sphere of α-phase ^{239}Pu metal is ~10,000 g

  - A bare sphere consisting of 4,500 g of α-phase ^{239}Pu metal has a fraction critical of:

    - $f_{bare} = 4{,}500\ g / 10{,}000\ g = 0.45$

Multiplication Factor Vs. Fraction Critical

- The critical mass of a water-reflected sphere of α-phase ^{239}Pu metal is ~5,400 g

  - A water-reflected sphere consisting of 4,500 g of α-phase ^{239}Pu metal has a fraction critical of:

    - $f_{refl.} = 4{,}500\ g / 5{,}400\ g = 0.83$

Multiplication Factor Vs. Fraction Critical

- The following figures show the k_{eff}'s as a function of the fraction critical for several spherical materials, both bare and water-reflected

- The solid line included is an empirical formula approximating the k_{eff} for each f

Multiplication Factor Vs. Fraction Critical

- LA-14244-M

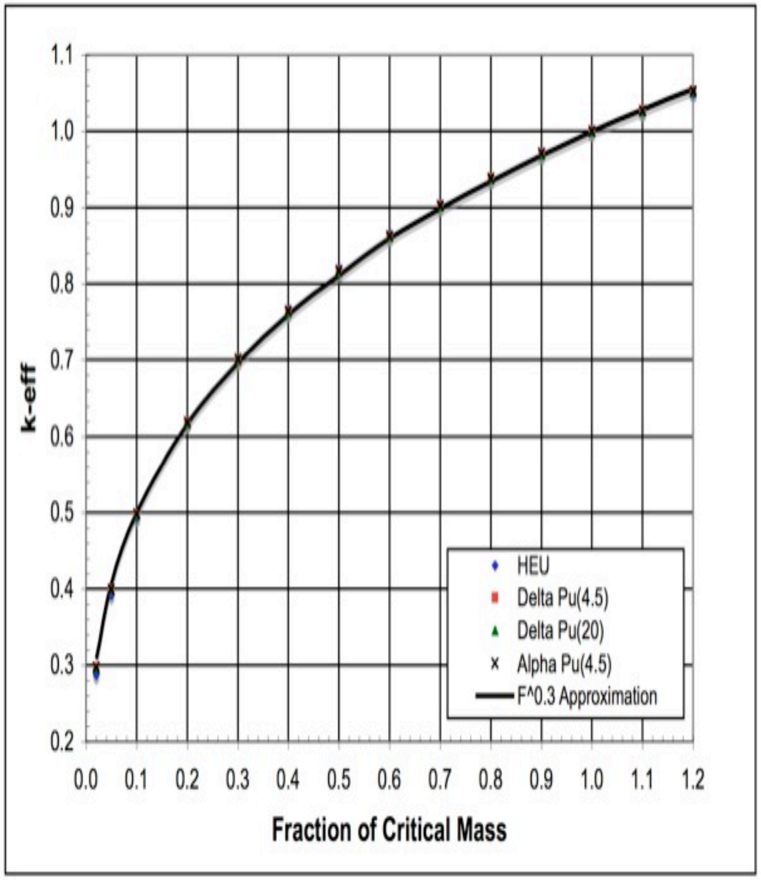


Figure 47. k_{eff} vs. Fraction of Critical Mass: Unreflected HEU and Pu Metal

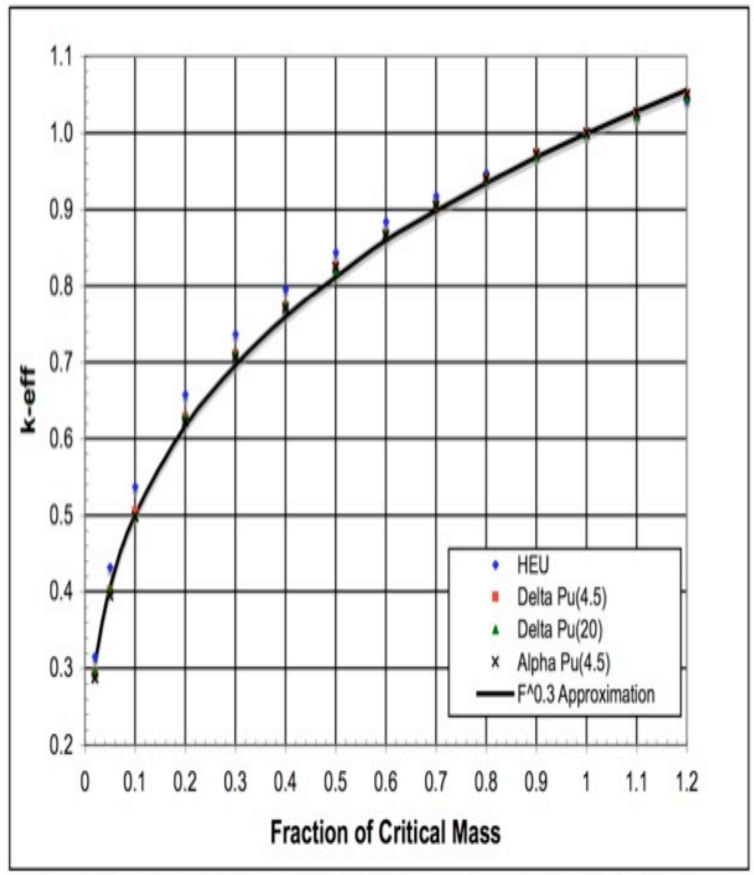


Figure 48. k_{eff} vs. Fraction of Critical Mass: Water Reflected HEU and Pu Metal

Multiplication Factor Vs. Fraction Critical

- LA-14244-M

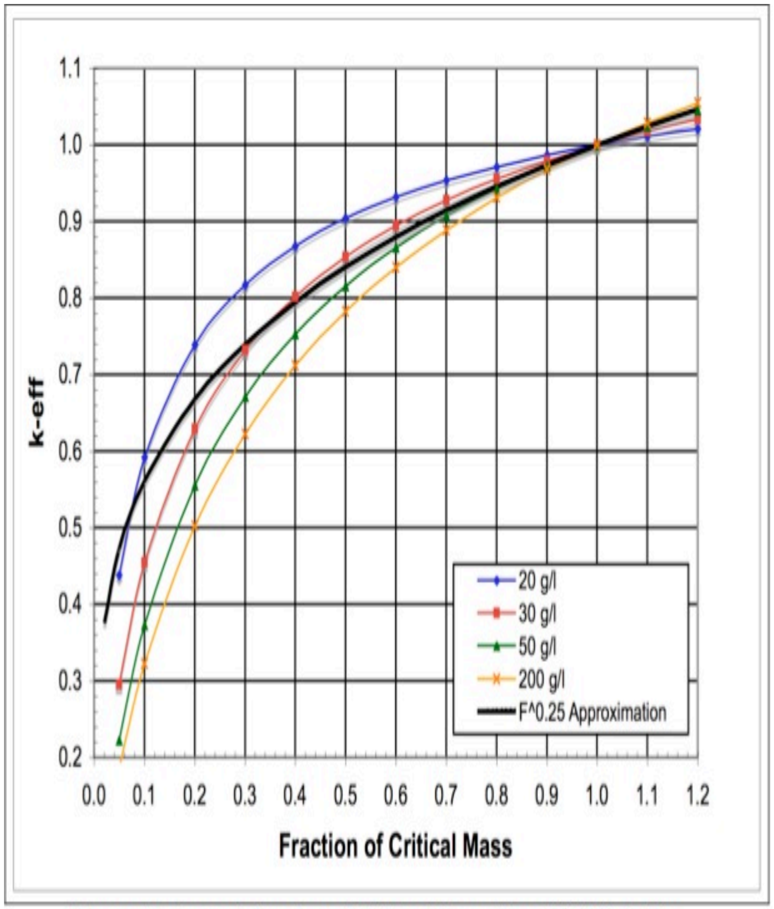


Figure 49. k_{eff} vs. Fraction of Critical Mass: Bare HEU Solution

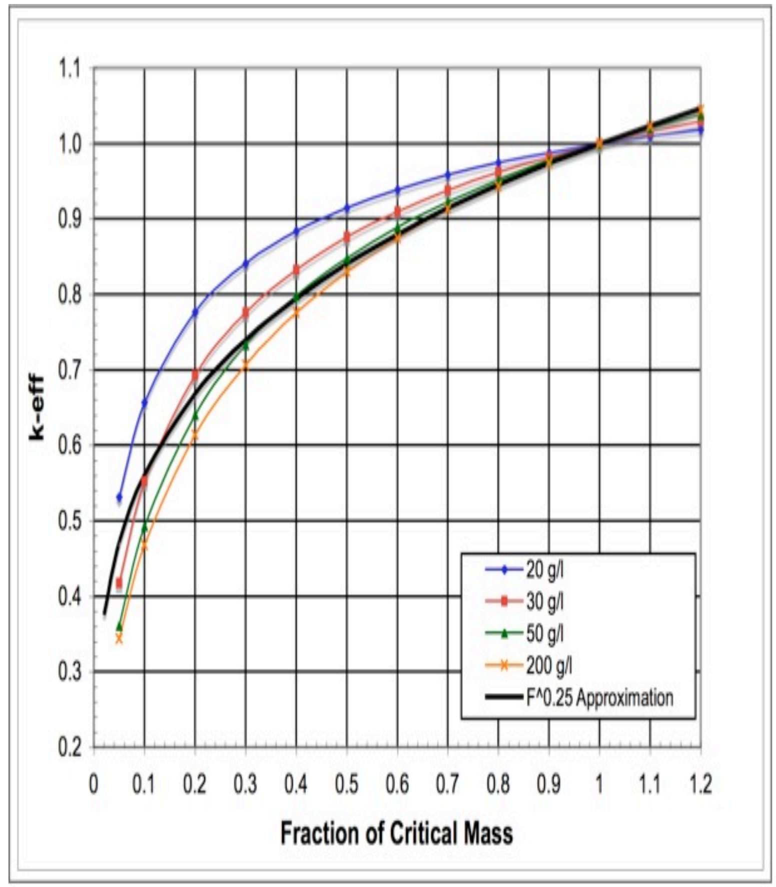


Figure 50. k_{eff} vs. Fraction of Critical Mass: Water Reflected HEU Solution

Multiplication Factor Vs. Fraction Critical

- LA-14244-M

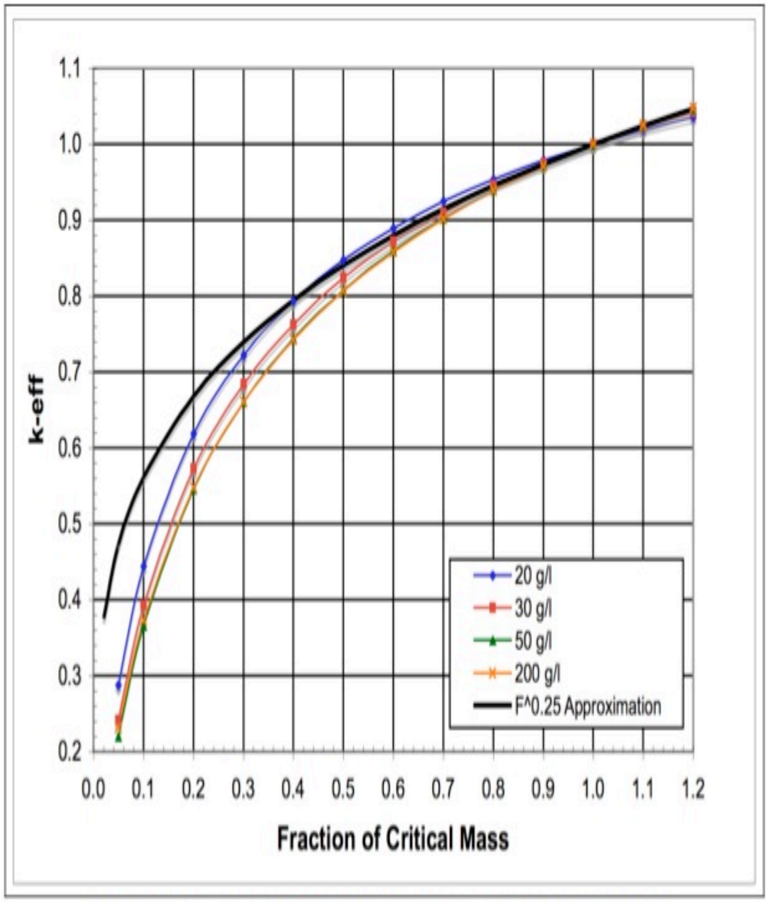


Figure 51. k_{eff} vs. Fraction of Critical Mass: Bare Pu Solution

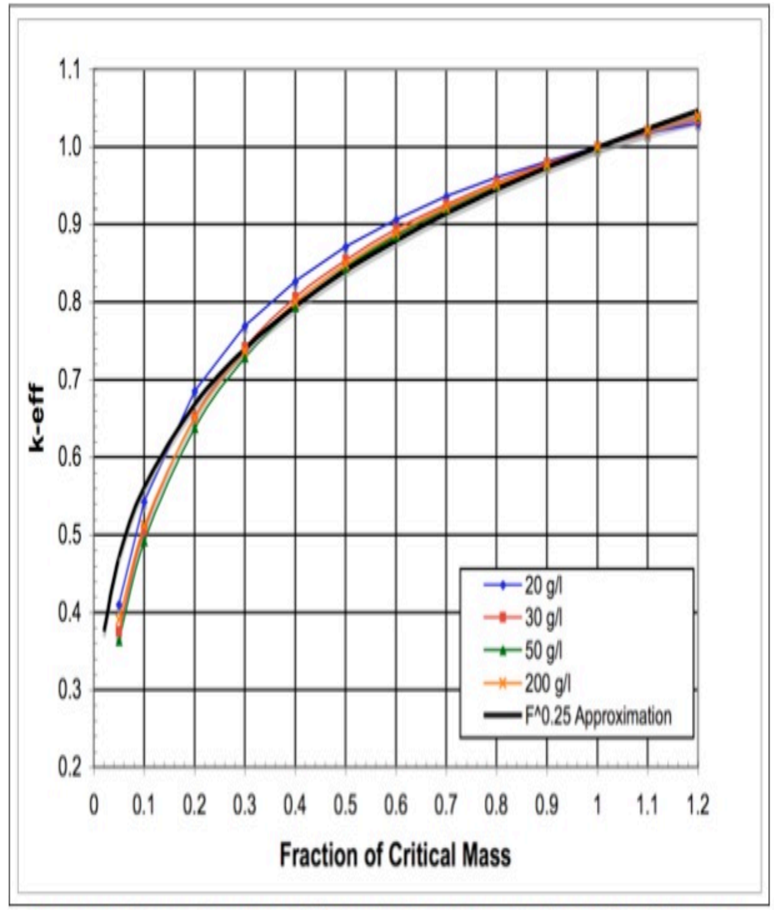


Figure 52. k_{eff} vs. Fraction of Critical Mass: Water Reflected Pu Solution

Multiplication Factor Vs. Fraction Critical

- For fast systems, both bare and reflected, a reasonable approximation of k_{eff} can be made using the following formula:

  – $k_{eff} = \left(f\right)^{\frac{1}{3}}$

- For thermal systems, a reasonable approximation of k_{eff} can be made using the following formula:

  – $k_{eff} = \left(f\right)^{\frac{1}{4}}$

Multiplication Factor Vs. Fraction Critical

- The critical mass of a bare sphere of α-phase ^{239}Pu metal is ~10,000 g
  - A bare sphere consisting of 4,500 g of α-phase ^{239}Pu metal has a fraction critical of:
    - $f_{bare} = 4{,}500\ g / 10{,}000\ g = 0.45$

- Therefore, a bare sphere consisting of 4,500 g of α-phase ^{239}Pu metal has a k_{eff} that is about:
  - $k_{eff} = (0.45)^{\frac{1}{3}} = 0.77$

Multiplication Factor Vs. Fraction Critical

- The critical mass of a water-reflected sphere of α-phase ^{239}Pu metal is ~5,400 g
  - A water-reflected sphere consisting of 4,500 g of α-phase ^{239}Pu metal has a fraction critical of:
    - $f_{refl.} = 4{,}500\ g/5{,}400\ g = 0.83$

- Therefore, a water-reflected sphere consisting of 4,500 g of α-phase ^{239}Pu metal has a k_{eff} that is about:
  - $k_{eff} = (0.83)^{\frac{1}{3}} = 0.94$

Core Density Conversions

- The core density conversion method relates a critical core with a known density to an *equivalent* critical core with *another* density

- Applies to systems with uniform densities

- Applies to both bare and reflected conditions

Core Density Conversions

- The relationship is given by:

$$m_c = m_{co}\left[\frac{\rho}{\rho_o}\right]^{-s}$$

  – Where s is determined by reflector conditions

- For a bare core, s equals 2

- For a reflected core, when both the core and reflector densities are changed by the same ratio, s also equals 2

Core Density Conversions

- For a reflected core, when the reflector density stays the same, s is relatively constant over the range:

$$0.5 \leq \rho / \rho_o \leq 1$$

  - The exponent s cannot exceed the value 2 that applies to a bare sphere

Core Density Conversions

- LA-3366

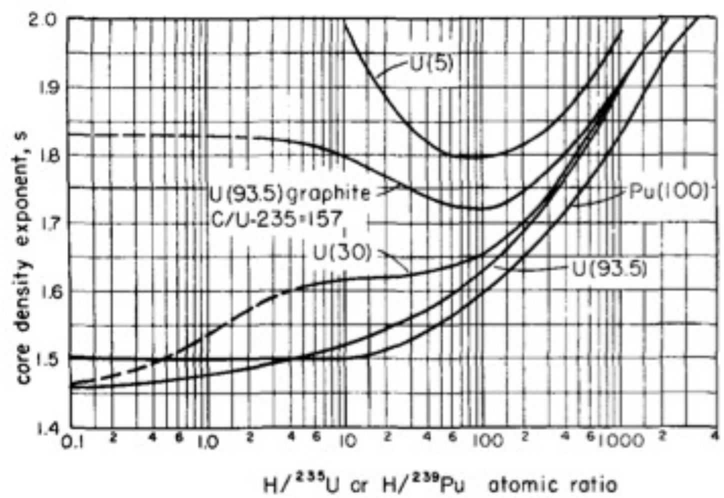


Fig. 15. Computed initial core-density exponents for water-reflected spheres of mixtures of water with ^{239}Pu, U(93.5), U(30), U(5), or U(93.5)-graphite at C/^{235}U = 157. Critical sizes are infinite where s = 2.

- Figure 15 shows values of s for water-reflected spheres with a density between normal and 0.8 times normal

Core Density Conversions

- The water-reflected spherical critical mass of α-phase ^{239}Pu metal (*density* = 19.86 g/cm^3) is ~5,400 g
- What is the water reflected spherical critical mass of δ-phase ^{239}Pu metal (*density* = 15.76 g/cm^3)?

$$m_c = 5{,}400\ g \left[\frac{15.76\ g \big/ cm^3}{19.86\ g \big/ cm^3} \right]^{-1.5} = 7{,}600\ g$$

Shape Conversions (Buckling Conversions)

- Converts a known critical shape to another critical shape consisting of the same material
- Originates from diffusion theory
- For a given material, shape conversion does NOT change material properties
- What changes is the neutron leakage, i.e., the shape of the neutron flux, which is related to the square of the geometric buckling, B_g^2

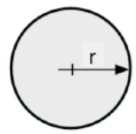
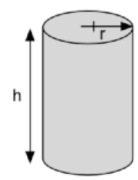
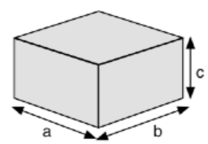
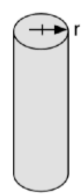
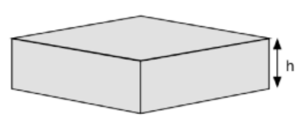
Shape Conversions (Buckling Conversions)

- To convert from one critical shape to another, we need a way to equate the neutron leakage associated with the two shapes

- Use geometric buckling equations for bare reactors

- Augmented by a “so-called” extrapolation distance, d
  - D NOT based diffusion theory
  - Look up value

Shape Conversions (Buckling Conversions)

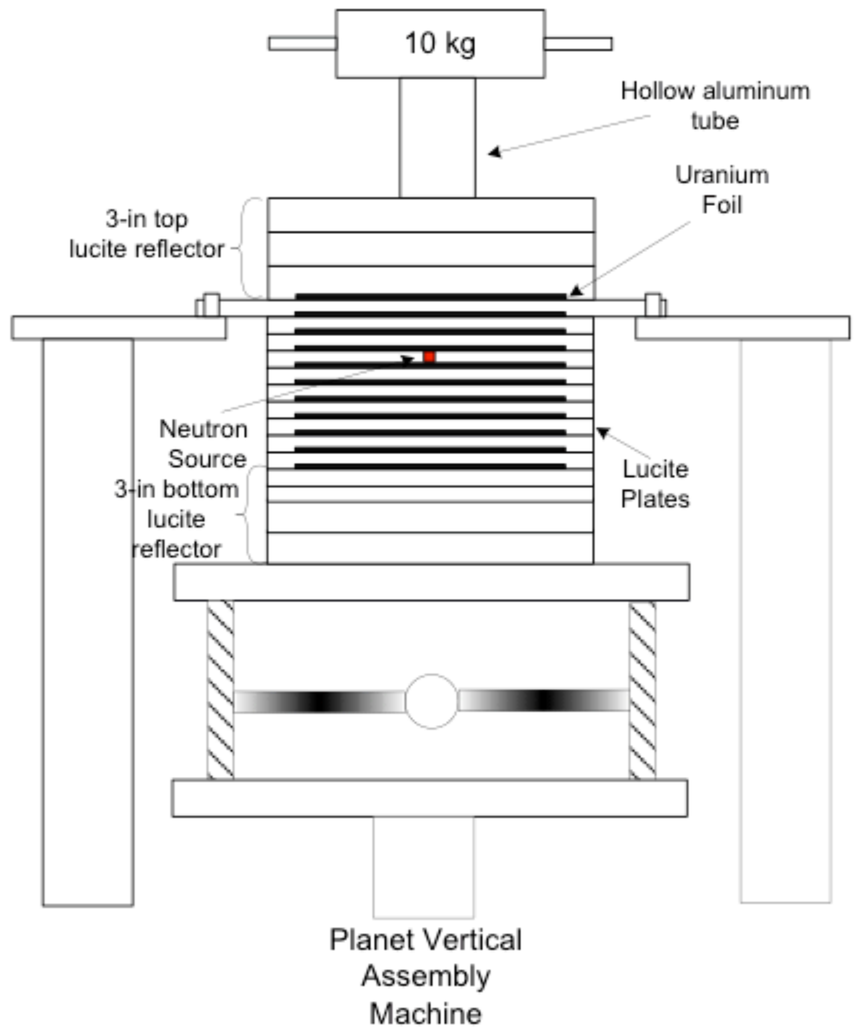
Configuration	Geometry Illustration	Geometric Buckling B_g^2 Relationship
Sphere of Radius, r		$\left(\frac{\pi}{r+d}\right)^2$
Cylinder of Radius, r, and Height, h		$\left(\frac{2.405}{r+d}\right)^2+\left(\frac{\pi}{h+2d}\right)^2$
Parallelepiped of Dimensions a, b, and c		$\left(\frac{\pi}{a+2d}\right)^2+\left(\frac{\pi}{b+2d}\right)^2+\left(\frac{\pi}{c+2d}\right)^2$
Infinite Cylinder of Radius, r		$\left(\frac{2.405}{r+d}\right)^2$
Infinite Slab of Thickness, h		$\left(\frac{\pi}{h+2d}\right)^2$

Shape Conversions (Buckling Conversions)

- Calculate the bare critical height and the corresponding critical mass of a stack of the HEU foils used on the hand stack critical assembly Planet

- Foil Specifications:
  - 69 g HEU metal each
  - Uranium enriched to 93.2 w/o U-235
  - 9“ (22.9 cm) W x 9” (22.9 cm) L x 0.003” (0.00762 cm) thick
  - Assume U(93) metal has a density of 18.75 g/cm^3

Illustration of the Planet Handstack Assembly



Shape Conversions (Buckling Conversions)

- Bare sphere of U(93) metal:
  - Critical mass ~50 kg

$$\frac{50{,}000\ g}{18.75\ g/cm^3} = V_s = \frac{4}{3}\pi r_s^3$$

$$r_s = \left(\frac{(3)(50{,}000\ g)}{\left(18.75\ g/cm^3\right)(4)(\pi)} \right)^{1/3} = 8.6\ cm$$

Shape Conversions (Buckling Conversions)

- To calculate the critical mass of uranium foils for the planet assembly:

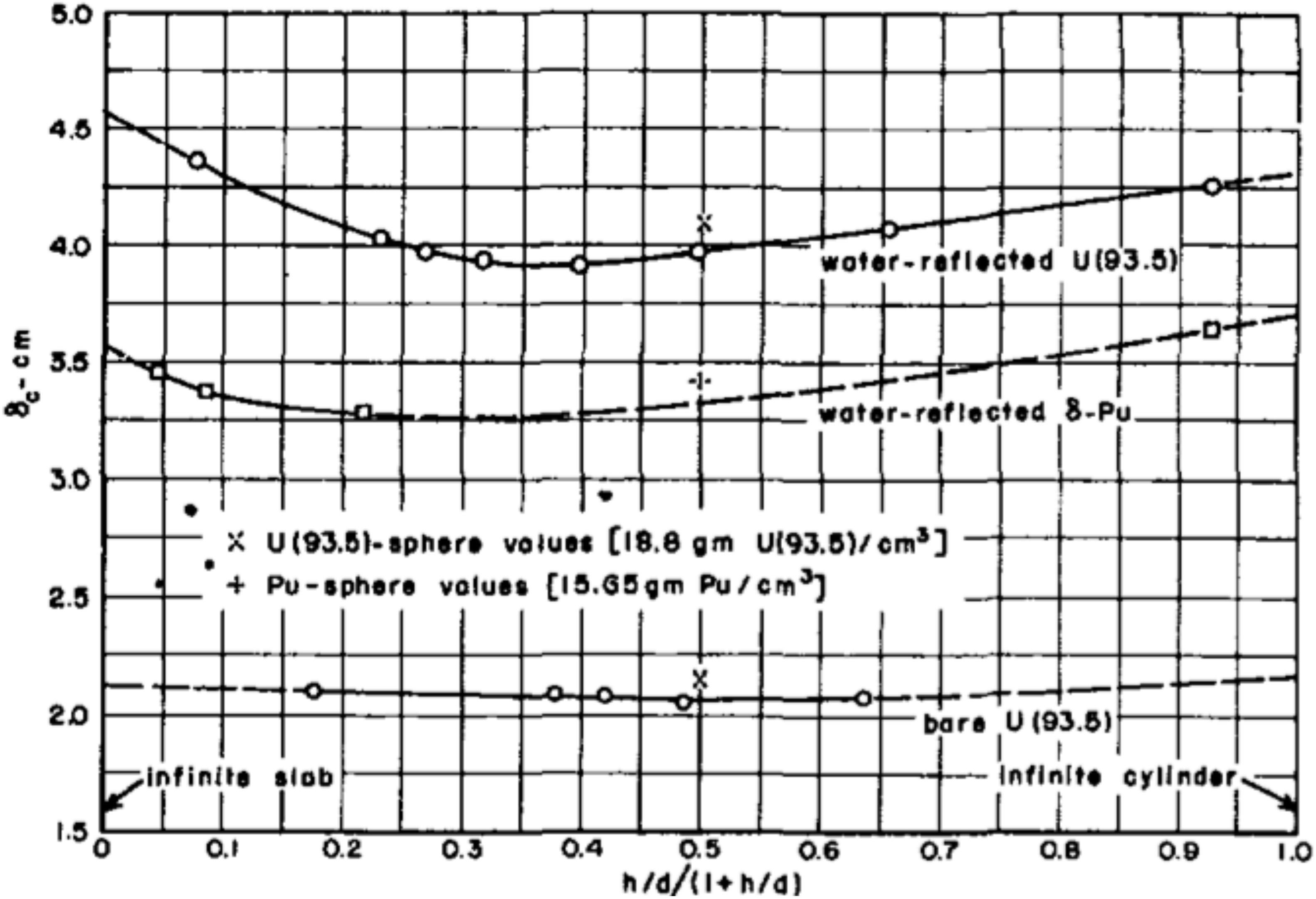
$$B_g^2(\text{sphere}) = \left(\frac{\pi}{r+d}\right)^2$$

$$B_g^2(\text{cuboid}) = \left(\frac{\pi}{a+2d}\right)^2 + \left(\frac{\pi}{b+2d}\right)^2 + \left(\frac{\pi}{c+2d}\right)^2$$

$$B_g^2(\text{sphere}) = B_g^2(\text{cuboid})$$

$$\left(\frac{\pi}{r+d}\right)^2 = \left(\frac{\pi}{a+2d}\right)^2 + \left(\frac{\pi}{b+2d}\right)^2 + \left(\frac{\pi}{c+2d}\right)^2$$

Shape Conversions (Buckling Conversions)



Shape Conversions (Buckling Conversions)

Solve the buckling equation for c (finite slab thickness):

$d \approx 2.2$ cm

$$c = \left[\frac{1}{\left(r_{sph} + d\right)^2} - \frac{1}{\left(a + 2d\right)^2} - \frac{1}{\left(b + 2d\right)^2} \right]^{-1/2} - 2d$$

$$c = \left[\frac{1}{\left(8.6 \text{ cm} + 2.2 \text{ cm}\right)^2} - \frac{1}{\left(22.9 \text{ cm} + 2 \cdot 2.2 \text{ cm}\right)^2} - \frac{1}{\left(22.9 \text{ cm} + 2 \cdot 2.2 \text{ cm}\right)^2} \right]^{-1/2} - 2 \cdot 2.2 \text{ cm}$$

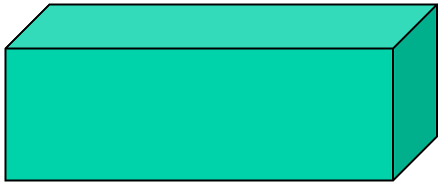
$$c = 13.03 \text{ cm - } 4.4 \text{ cm} = 8.63 \text{ cm}$$

Shape Conversions (Buckling Conversions)

Critical volume:

$$V_{critical} = 22.9 \text{ cm} \times 22.9 \text{ cm} \times 8.63 \text{ cm} = 4{,}526 \text{ cm}^3$$

$$m_{critical} = \rho \cdot V_{critical} = 18.75 \frac{g}{\text{cm}^3} \cdot 4526 \text{ cm}^3 = 84{,}857 \text{ g}$$



Why is this mass greater than the spherical critical mass (~50 kg)?

This equates to 79,087 g of U-235 in U(93.2) metal.

More than 1,229 foils are required to achieve criticality with this system.

Illustration of the Planet Handstack Assembly