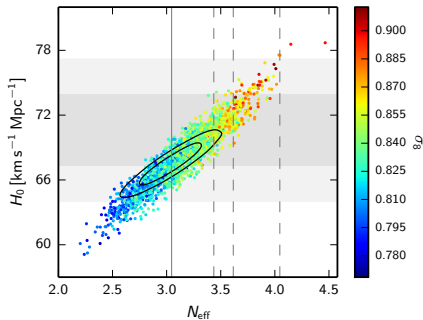


Planck (and some external data sets) constraints

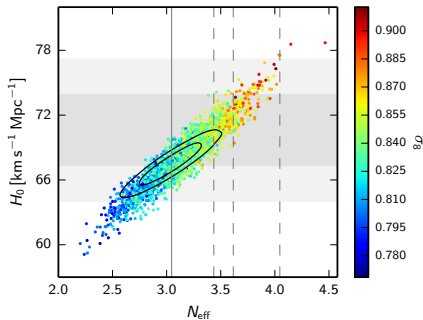
Planck (and some external data sets) constraints

TT+TE+EE+lowP+BAO

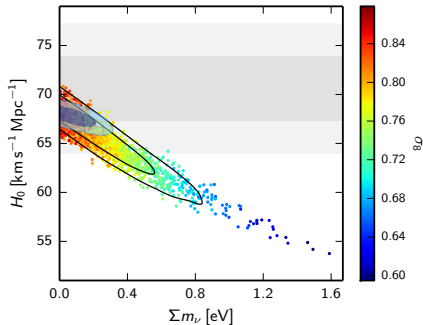


Planck (and some external data sets) constraints

TT+TE+EE+lowP+BAO

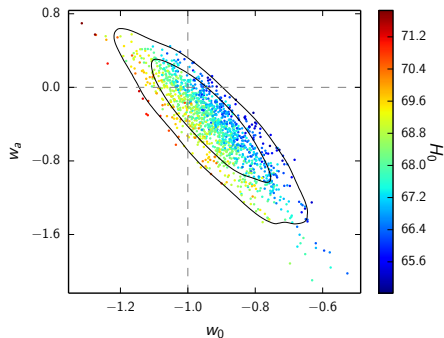


TT+lowP+lensing



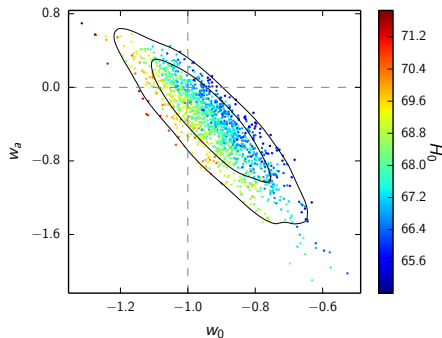
Planck (and some external data sets) constraints

TT+lowP+BAO+JLA

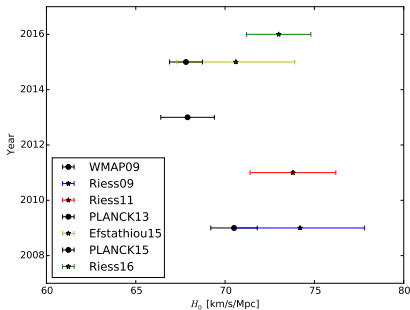


Planck (and some external data sets) constraints

TT+lowP+BAO+JLA

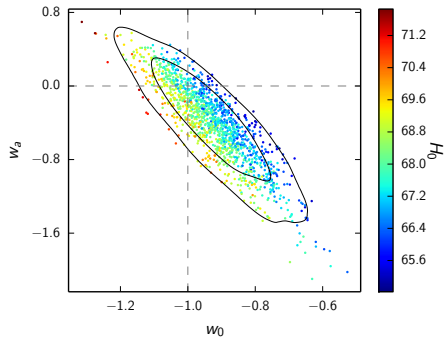


Direct vs. Indirect

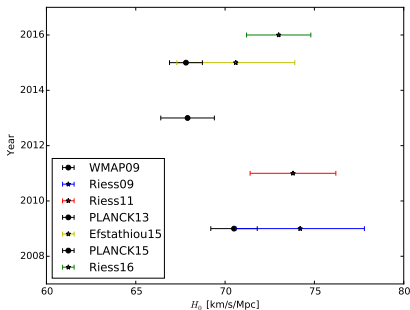


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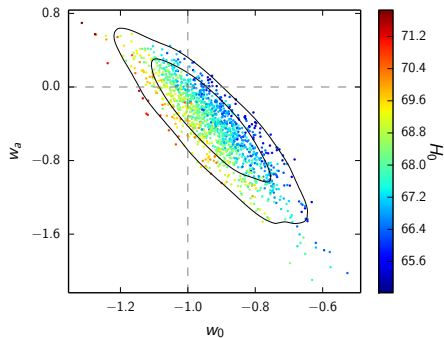
Direct vs. Indirect



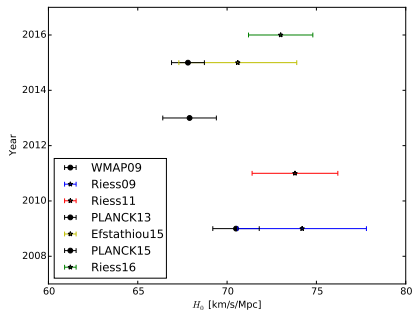
Theoretical attempts ?

Planck (and some external data sets) constraints

TT+lowP+BAO+JLA



Direct vs. Indirect



Issues in the statistical analysis ?

Determining H_0 with Bayesian hyper-parameters: preliminary results

Martin Kunz and Valeria Pettorino
(Wilmar Cardona)

September 12, 2016

Contents

What do you need to measure H_0 ?

- ▶ Distance modulus

$$\mu_0 = m_B - M_B = 5 \log_{10} \left(\frac{d_L}{1 \text{Mpc}} \right) + 25 .$$

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- ▶ Cepheid variables period-luminosity relation

$$m_{X,i,j} = \mu_{0,i} + M_X + b_X(\log P_{i,j} - 1) + Z_X \Delta \log [O/H]_{i,j},$$

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- ▶ SN Ia magnitudes

$$5 \log_{10} H_0 = m_V - \mu_0 + 5a_v + 25 .$$

Method: Bayesian hyper-parameters (HP)

- ▶ **Trust no one.** Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \rightarrow \sigma_i / \sqrt{\alpha_i}$.

Method: Bayesian hyper-parameters (HP)

- ▶ **Trust no one.** Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \rightarrow \sigma_i / \sqrt{\alpha_i}$.
- ▶ Assume Gaussian likelihood for the data D_i

$$P_G(D_i | \vec{w}) = \tilde{N}_i \frac{\exp(-\chi_i^2(\vec{w})/2)}{\sqrt{2\pi}},$$

Method: Bayesian hyper-parameters (HP)

- ▶ **Trust no one.** Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \rightarrow \sigma_i / \sqrt{\alpha_i}$.
- ▶ where

$$\chi_i^2 \equiv \frac{(x_{\text{obs},i} - x_{\text{pred},i}(\vec{w}))^2}{\sigma_i^2} \quad \text{and} \quad \tilde{N}_i = \frac{1}{\sqrt{\sigma_i^2}}.$$

Method: Bayesian hyper-parameters (HP)

- ▶ **Trust no one.** Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \rightarrow \sigma_i / \sqrt{\alpha_i}$.
- ▶ Control relative weight of data points in the likelihood with HP α_i , likelihood becomes

$$P(D_i | \vec{w}, \alpha_i) = \tilde{N}_i \alpha_i^{1/2} \frac{\exp(-\alpha_i \chi_i^2(\vec{w})/2)}{\sqrt{2\pi}}.$$

Method: Bayesian hyper-parameters (HP)

- ▶ **Trust no one.** Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \rightarrow \sigma_i / \sqrt{\alpha_i}$.
- ▶ For a given model with parameters \vec{w} and a set of N data points $\{D_i\}$

$$P(\vec{w}|\{D_i\}) = \int \dots \int P(\vec{w}, \{\alpha_i\}|\{D_i\}) d\alpha_1 \dots d\alpha_N,$$

Method: Bayesian hyper-parameters (HP)

- ▶ **Trust no one.** Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \rightarrow \sigma_i / \sqrt{\alpha_i}$.
- ▶ From Bayes' theorem

$$P(\vec{w}, \{\alpha_i\} | \{D_i\}) = \frac{P(\{D_i\} | \vec{w}, \{\alpha_i\}) P(\vec{w}, \{\alpha_i\})}{P(D_1, \dots, D_N)},$$

$$P(\vec{w}, \{\alpha_i\}) = P(\vec{w} | \{\alpha_i\}) P(\{\alpha_i\}),$$

$$P(\{D_i\} | \vec{w}, \{\alpha_i\}) = P(D_1 | \vec{w}, \alpha_1) \dots P(D_N | \vec{w}, \alpha_N),$$

$$P(\vec{w} | \{\alpha_i\}) = \text{constant}$$

$$P(\{\alpha_i\}) = P(\alpha_1) \dots P(\alpha_N)$$

Method: Bayesian hyper-parameters (HP)

- ▶ **Trust no one.** Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \rightarrow \sigma_i / \sqrt{\alpha_i}$.
- ▶ Further assuming uniform prior for HP ($P(\alpha_i) = 1$) and that errors are never smaller than their reported value ($\alpha_i \in [0, 1]$)

$$P(\vec{w}, \{D_i\}) = \frac{P(D_1|\vec{w}) \dots P(D_N|\vec{w})}{P(D_1, \dots, D_N)},$$

$$P(D_i|\vec{w}) \equiv \int_0^1 P(D_i|\vec{w}, \alpha_i) d\alpha_i.$$

Method: Bayesian hyper-parameters (HP)

- ▶ **Trust no one.** Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \rightarrow \sigma_i / \sqrt{\alpha_i}$.
- ▶ In the case of a Gaussian HP likelihood

$$P(D_i | \vec{w}) = \tilde{N}_i \left(\frac{\text{Erf} \left(\frac{\chi_i(\vec{w})}{\sqrt{2}} \right) - \sqrt{\frac{2}{\pi}} \chi_i(\vec{w}) \exp(-\chi_i^2(\vec{w})/2)}{\chi_i^3(\vec{w})} \right)$$

Method: Bayesian hyper-parameters (HP)

- ▶ **Trust no one.** Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \rightarrow \sigma_i / \sqrt{\alpha_i}$.
- ▶ Rewriting

$$\ln P(\vec{w}, \{D_i\}) = \sum_i \ln \tilde{N}_i + \ln \tilde{\chi}_i^2$$

$$\tilde{\chi}_i^2(\chi_i^2(\vec{w})) \equiv \frac{\text{Erf}\left(\frac{\chi_i(\vec{w})}{\sqrt{2}}\right) - \sqrt{\frac{2}{\pi}} \chi_i(\vec{w}) \exp(-\chi_i^2(\vec{w})/2)}{\chi_i^3(\vec{w})}$$

Method: Bayesian hyper-parameters (HP)

- ▶ **Trust no one.** Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \rightarrow \sigma_i / \sqrt{\alpha_i}$.
- ▶ Effective HP for each data point are given by

$$\begin{aligned}\alpha_i^{\text{eff}} &= 1, & \text{if } \chi_i^2 \leq 1 \\ \alpha_i^{\text{eff}} &= \frac{1}{\chi_i^2}, & \text{if } \chi_i^2 > 1.\end{aligned}$$

Applying HP

Large Magellanic Cloud (LMC)

LMC figure

Applying HP

Large Magellanic Cloud (LMC)

LMC Cepheid variables				
Fit	A	b_W	$\sigma_{\text{int}}^{\text{LMC}}$	Period cut
a	12.570 (0.035)	-3.32 (0.10)	0.06	$10 < P < 60$
b	12.562 (0.016)	-3.30 (0.05)	0.06	$P < 60$
c	12.562 (0.016)	-3.31 (0.05)	0.06	$P < 205$
d	12.555 (0.019)	-3.24 (0.06)	0.12	$P < 205$

$$\mu_{0,\text{LMC}}^{\text{M}} = 18.49 \pm 0.05, \quad M_W = -5.93 \pm 0.07$$

Applying HP

Milky Way (MW)

MW figure

$$M_W = -5.88 \pm 0.07, \quad b_W = -3.30 \pm 0.26, \quad \sigma_{\text{int}}^{\text{MW}} = 0.02,$$

Applying HP

Megamaser system NGC 4258

4258 figure

$$\mu_{0,4258} = 29.40 \pm 0.07, \quad M_W = -6.12 \pm 0.15, \quad b_W = -3.02 \pm 0.17,$$

Applying HP

Megamaser system NGC 4258

I do not see any reason to discard those data sets

The Hubble constant H_0

Data

- ▶ 8 SN Ia hosts: apparent magnitudes

The Hubble constant H_0

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- ▶ 8 SN Ia hosts: apparent magnitudes
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- ▶ Distance modulus to LMC derived from eclipsing binaries
- ▶ Distance modulus to NGC 4258 derived from geometric maser distance and from standardised candle method for type IIP SNe

The Hubble constant H_0

The likelihood

- ▶ Full likelihood

$$\ln P(\vec{w}, \{D_i\}) = \ln P^{\text{Cepheid}} + \ln P^{\text{SNe Ia}} + \ln P^{\text{Anchors}}.$$

The Hubble constant H_0

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The Hubble constant H_0

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- ▶ SNe Ia, Anchors,...

The Hubble constant H_0

Results

NGC 4258+ LMC + MW anchors				
Fit	H_0	M_W	b_W	Z_W
$M1^a$	75.0 (3.9)	-5.87 (0.18)	-3.20 (0.05) [N]	-0.005 (0.020) [S]
$M1^{af}$	74.9 (3.9)	-5.88 (0.18)	-3.20 (0.05) [N]	-0.005 (0.020) [S]
$M1^{ag}$	73.3 (2.4)	-5.88 (0.18)	-3.19 (0.05) [N]	-0.004 (0.020) [S]
$M1^{ah}$	74.6 (1.9)	-5.89 (0.17)	-3.21 (0.05) [N]	-0.005 (0.020) [S]
$M1^{aj}$	72.4 (2.2)	-5.90 (0.18)	-3.20 (0.05) [N]	-0.004 (0.020) [S]
$M1^b$	76.1 (3.1)	-5.86 (0.17)	-3.27 (0.04) [N]	-0.007 (0.019) [S]
$M1^{be}$	75.1 (3.4)	-5.89 (0.18)	-3.25 (0.07) [N]	-0.004 (0.020) [S]
$M2^a$	74.7 (4.0)	-4.68 (0.97)	-3.20 (0.05) [N]	-0.141 (0.110) [W]
$M2^b$	75.1 (3.8)	-3.84 (1.05)	-3.28 (0.04) [N]	-0.236 (0.119) [W]
$M3^a$	75.0 (3.9)	-5.92 (0.04)	-3.20 (0.05) [N]	0
$M3^b$	75.4 (3.7)	-5.92 (0.04)	-3.27 (0.04) [N]	0
$M4^a$	74.7 (4.0)	-4.35 (1.11)	-3.20 (0.05) [N]	-0.178 (0.125) [N]
$M4^b$	75.2 (3.9)	-3.18 (1.25)	-3.28 (0.04) [N]	-0.311 (0.141) [N]

The Hubble constant H_0

Results: fit $M1^a$

Triangle plot

The Hubble constant H_0

Conclusions

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The Hubble constant H_0

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- ▶ Results agree with R16: 18 SN Ia hosts, better distance modulus to NGC4258, consistent values of H_0 , no outlier rejection needed any longer (they claim), no outliers among SN Ia hosts

The Hubble constant H_0

Conclusions

H0 values