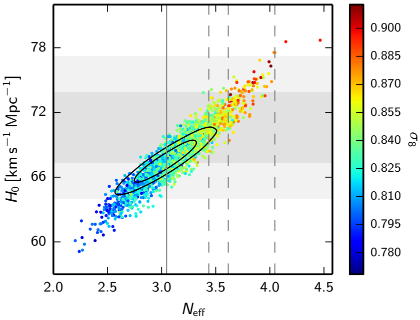
Planck (and some external data sets) constraints

Planck (and some external data sets) constraints

TT+TE+EE+lowP+BAO



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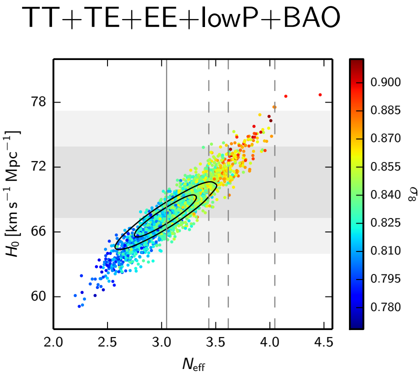

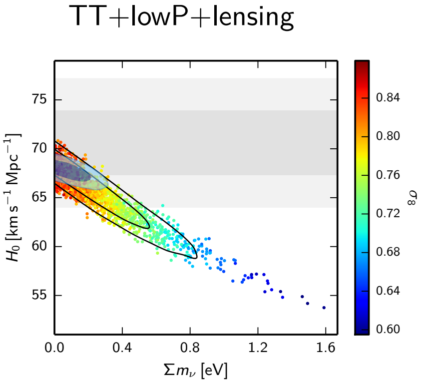

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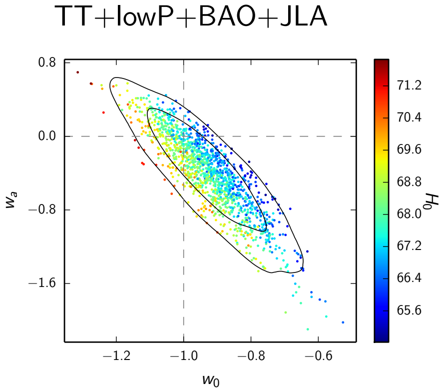
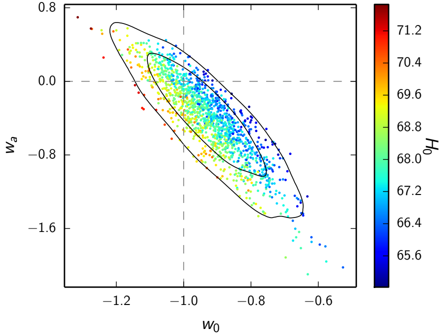
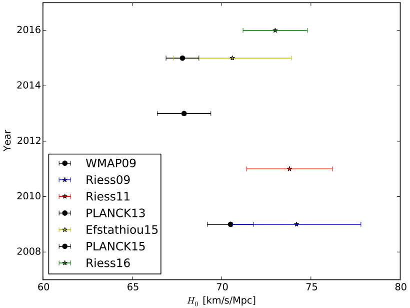


Planck (and some external data sets) constraints

TT+lowP+BAO+JLA

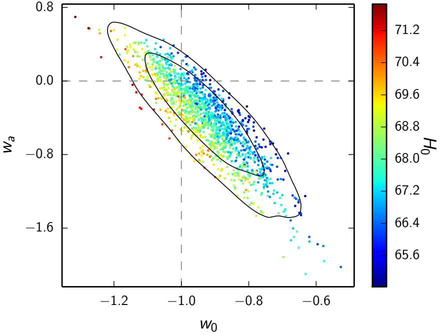


Direct vs. Indirect

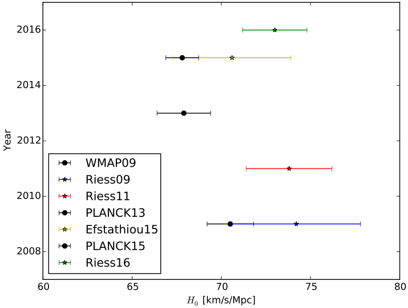


Planck (and some external data sets) constraints

TT+lowP+BAO+JLA



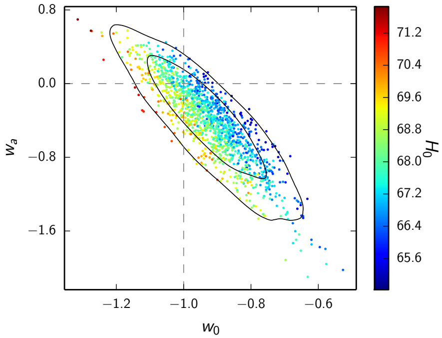
Direct vs. Indirect



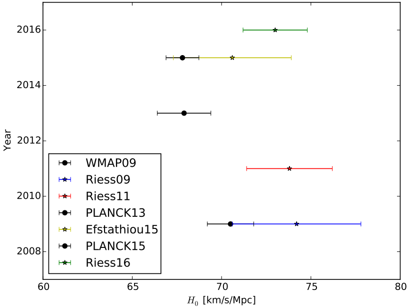
Theoretical attempts ?

Planck (and some external data sets) constraints

TT+lowP+BAO+JLA



Direct vs. Indirect



Issues in the statistical analysis ?

Determining H_0 with Bayesian hyper-parameters: preliminary results

Martin Kunz and Valeria Pettorino
(Wilmar Cardona)

April 15, 2016

Contents

What do you need to measure H_0?

- Distance modulus

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$$m_{X,i,j} = \mu_{0,i} + M_X + b_X(\log P_{i,j} - 1) + Z_X \Delta \log[O/H]_{i,j},$$

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- SN Ia magnitudes

$$5\log_{10} H_0 = m_V - \mu_0 + 5a_v + 25\,.$$

Method: Bayesian hyper-parameters (HP)

- **Trust no one**. Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \to \sigma_i/\sqrt{\alpha_i}$.

Method: Bayesian hyper-parameters (HP)

- **Trust no one**. Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \to \sigma_i/\sqrt{\alpha_i}$.
- Assume Gaussian likelihood for the data D_i

$$P_G(D_i|\vec{w}) = \tilde{N}_i \, \frac{\exp(-\chi_i^2(\vec{w})/2)}{\sqrt{2\pi}},$$

Method: Bayesian hyper-parameters (HP)

- **Trust no one**. Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \to \sigma_i/\sqrt{\alpha_i}$.
- where

$$\chi_i^2 \equiv \frac{(x_{\mathrm{obs},i} - x_{\mathrm{pred},i}(\vec{w}))^2}{\sigma_i^2} \quad \text{and} \quad \tilde{N}_i = \frac{1}{\sqrt{\sigma_i^2}}.$$

Method: Bayesian hyper-parameters (HP)

- **Trust no one**. Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \to \sigma_i/\sqrt{\alpha_i}$.
- Control relative weight of data points in the likelihood with HP α_i, likelihood becomes

$$P(D_i|\vec{w},\alpha_i) = \tilde{N}_i\,\alpha_i^{1/2}\,\frac{\exp(-\alpha_i\chi_i^2(\vec{w})/2)}{\sqrt{2\pi}}.$$

Method: Bayesian hyper-parameters (HP)

- **Trust no one**. Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \to \sigma_i/\sqrt{\alpha_i}$.
- For a given model with parameters $\vec{w}$ and a set of N data points $\{D_i\}$

$$P(\vec{w}|\{D_i\}) = \int \dots \int P(\vec{w}, \{\alpha_i\}|\{D_i\})\, d\alpha_1 \dots d\alpha_N,$$

Method: Bayesian hyper-parameters (HP)

- **Trust no one**. Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \to \sigma_i/\sqrt{\alpha_i}$.
- From Bayes' theorem

$$P(\vec{w}, \{\alpha_i\}|\{D_i\}) = \frac{P(\{D_i\}|\vec{w}, \{\alpha_i\})\, P(\vec{w}, \{\alpha_i\})}{P(D_1, \dots, D_N)},$$

$$P(\vec{w}, \{\alpha_i\}) = P(\vec{w}|\{\alpha_i\})\, P(\{\alpha_i\}),$$

$$P(\{D_i\}|\vec{w}, \{\alpha_i\}) = P(D_1|\vec{w}, \alpha_1) \dots P(D_N|\vec{w}, \alpha_N),$$

$$P(\vec{w}|\{\alpha_i\}) = \text{constant}$$

$$P(\{\alpha_i\}) = P(\alpha_1) \dots P(\alpha_N)$$

Method: Bayesian hyper-parameters (HP)

- **Trust no one**. Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \to \sigma_i/\sqrt{\alpha_i}$.
- Further assuming uniform prior for HP ($P(\alpha_i) = 1$) and that errors are never smaller than their reported value ($\alpha_i \in [0, 1]$)

$$P(\vec{w}, \{D_i\}) = \frac{P(D_1|\vec{w}) \dots P(D_N|\vec{w})}{P(D_1, \dots, D_N)},$$

$$P(D_i|\vec{w}) \equiv \int_0^1 P(D_i|\vec{w}, \alpha_i)\, d\alpha_i.$$

Method: Bayesian hyper-parameters (HP)

- **Trust no one**. Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \to \sigma_i/\sqrt{\alpha_i}$.
- In the case of a Gaussian HP likelihood

$$P(D_i|\vec{w}\,) = \tilde{N}_i \left(\frac{\mathrm{Erf}\left(\frac{\chi_i(\vec{w})}{\sqrt{2}}\right) - \sqrt{\frac{2}{\pi}}\chi_i(\vec{w})\exp(-\chi_i^2(\vec{w})/2)}{\chi_i^3(\vec{w})} \right)$$

Method: Bayesian hyper-parameters (HP)

- **Trust no one**. Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \to \sigma_i/\sqrt{\alpha_i}$.
- Rewriting

$$\ln P(\vec{w}, \{D_i\}) = \sum_i \ln \tilde{N}_i + \ln \tilde{\chi}_i^2$$

$$\tilde{\chi}_i^2(\chi_i^2(\vec{w})) \equiv \frac{\mathrm{Erf}\left(\frac{\chi_i(\vec{w})}{\sqrt{2}}\right) - \sqrt{\frac{2}{\pi}}\chi_i(\vec{w})\exp(-\chi_i^2(\vec{w})/2)}{\chi_i^3(\vec{w})}$$

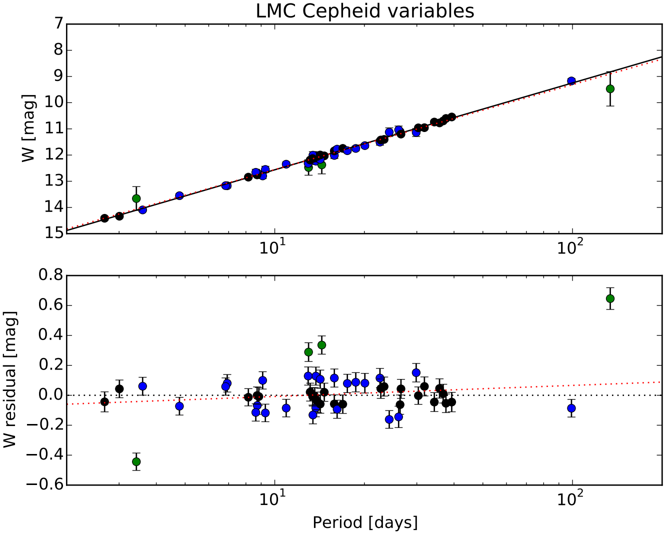
Method: Bayesian hyper-parameters (HP)

- **Trust no one**. Some error bars may have been underestimated (e.g., unrecognised systematic effects), then allow for a rescaling $\sigma_i \to \sigma_i/\sqrt{\alpha_i}$.
- Effective HP for each data point are given by

$$
\begin{aligned}
\alpha_i^{\mathrm{eff}} &= 1, \quad \text{if} \quad \chi_i^2 \leq 1 \\
\alpha_i^{\mathrm{eff}} &= \tfrac{1}{\chi_i^2}, \quad \text{if} \quad \chi_i^2 > 1.
\end{aligned}
$$

Applying HP

Large Magellanic Cloud (LMC)



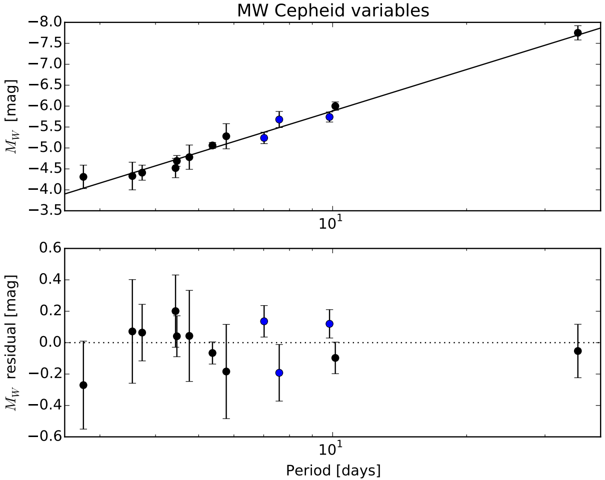
Applying HP

Large Magellanic Cloud (LMC)

LMC Cepheid variables				
Fit	A	b_W	$\sigma_{\rm int}^{\rm LMC}$	Period cut
a	12.570 (0.035)	-3.32 (0.10)	0.06	$10 < P < 60$
b	12.562 (0.016)	-3.30 (0.05)	0.06	$P < 60$
c	12.562 (0.016)	-3.31 (0.05)	0.06	$P < 205$
d	12.555 (0.019)	-3.24 (0.06)	0.12	$P < 205$

$$\mu_{0,\rm LMC}^{\rm M} = 18.49 \pm 0.05, \quad M_W = -5.93 \pm 0.07$$

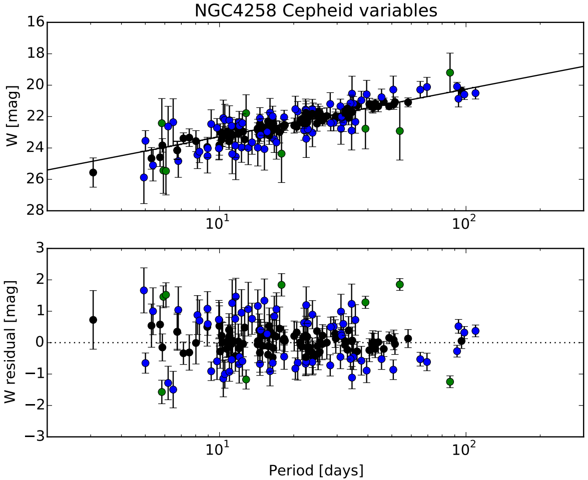
Applying HP
Milky Way (MW)



$$M_W = -5.88 \pm 0.07, \qquad b_W = -3.30 \pm 0.26, \qquad \sigma_{\rm int}^{\rm MW} = 0.02,$$

Applying HP

Megamaser system NGC 4258



$$\mu_{0,4258} = 29.40 \pm 0.07, \qquad M_W = -6.12 \pm 0.15, \quad b_W = -3.02 \pm 0.17,$$

Applying HP

Megamaser system NGC 4258

I do not see any reason to discard those data sets

The Hubble constant H_0

Data

- 8 SN Ia hosts: apparent magnitudes

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- MW Cepheid variables (parallax measurements): period, metallicity, magnitudes

The Hubble constant H_0

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- LMC Cepheid variables: period, metallicity, magnitudes

The Hubble constant H_0

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- MW Cepheid variables (parallax measurements): period, metallicity, magnitudes
- LMC Cepheid variables: period, metallicity, magnitudes
- Distance modulus to LMC derived from eclipsing binaries
- Distance modulus to NGC 4258 derived from geometric maser distance and from standardised candle method for type IIP SNe

- Full likelihood

$$\ln P(\vec{w}, \{D_i\}) = \ln P^{\mathrm{Cepheid}} + \ln P^{\mathrm{SNe\,Ia}} + \ln P^{\mathrm{Anchors}}.$$

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- For instance

$$\ln P^{\mathrm{Cepheid}} = \sum_{ij} \ln \tilde{\chi}^2(\chi_{ij}^{2,\mathrm{Cepheid}}) + \ln \tilde{N}_{ij}^{\mathrm{Cepheid}},$$

- Full likelihood

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- For instance

$$\ln P^{\text{Cepheid}} = \sum_{ij} \ln \tilde{\chi}^2(\chi_{ij}^{2,\text{Cepheid}}) + \ln \tilde{N}_{ij}^{\text{Cepheid}},$$

- SNe Ia, Anchors,...

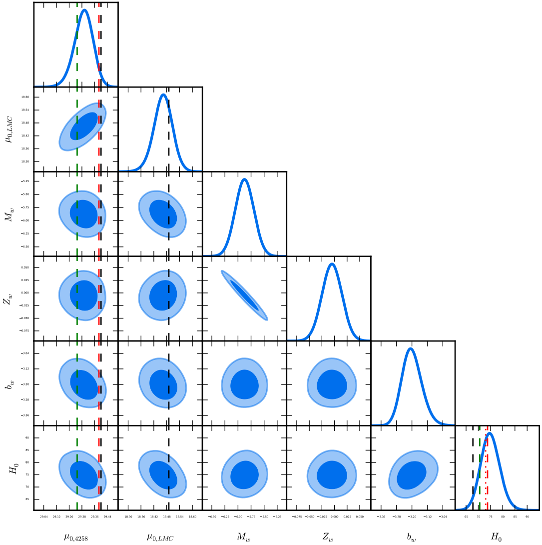
The Hubble constant H_0

Results

	NGC 4258+ LMC + MW anchors			
Fit	H_0	M_W	b_W	Z_W
$M1^{a}$	75.0 (3.9)	−5.87 (0.18)	−3.20 (0.05) [N]	−0.005 (0.020) [S]
$M1^{af}$	74.9 (3.9)	−5.88 (0.18)	−3.20 (0.05) [N]	−0.005 (0.020) [S]
$M1^{ag}$	73.3 (2.4)	−5.88 (0.18)	−3.19 (0.05) [N]	−0.004 (0.020) [S]
$M1^{ah}$	74.6 (1.9)	−5.89 (0.17)	−3.21 (0.05) [N]	−0.005 (0.020) [S]
$M1^{aj}$	72.4 (2.2)	−5.90 (0.18)	−3.20 (0.05) [N]	−0.004 (0.020) [S]
$M1^{b}$	76.1 (3.1)	−5.86 (0.17)	−3.27 (0.04) [N]	−0.007 (0.019) [S]
$M1^{be}$	75.1 (3.4)	−5.89 (0.18)	−3.25 (0.07) [N]	−0.004 (0.020) [S]
$M2^{a}$	74.7 (4.0)	−4.68 (0.97)	−3.20 (0.05) [N]	−0.141 (0.110) [W
$M2^{b}$	75.1 (3.8)	−3.84 (1.05)	−3.28 (0.04) [N]	−0.236 (0.119) [W
$M3^{a}$	75.0 (3.9)	−5.92 (0.04)	−3.20 (0.05) [N]	0
$M3^{b}$	75.4 (3.7)	−5.92 (0.04)	−3.27 (0.04) [N]	0
$M4^{a}$	74.7 (4.0)	−4.35 (1.11)	−3.20 (0.05) [N]	−0.178 (0.125) [N]
$M4^{b}$	75.2 (3.9)	−3.18 (1.25)	−3.28 (0.04) [N]	−0.311 (0.141) [N]

The Hubble constant H_0

Results: fit $M1^a$



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The Hubble constant H_0

Conclusions

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The Hubble constant H_0

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- Results agree with R16: 18 SN Ia hosts, better distance modulus to NGC4258, consistent values of H_0, no outlier rejection needed any longer (they claim), no outliers among SN Ia hosts

The Hubble constant H_0

Conclusions